Università degli Studi di Napoli
Federico II

Tesi di Dottorato in
*Matematica per l'Analisi Economica
e la Finanza*

XV Ciclo

Informational Cascades in Capital Markets with Trading Frictions

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Napoli, Novembre 2004

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Introduction

The main goal of this thesis is to investigate imitative behavior arising from informational externalities in capital markets where information is asymmetrically distributed among market participants.

Herding has always attracted the attention of financial economists because it can help to explain many interesting regularities observed in capital markets that are not easily addressed in theories based on the efficient markets hypothesis. For example, the empirical evidence of episodes of increasing prices and subsequent collapse, like the Dutch Tulip Mania in the seventeenth century, the stock market crashes of 1929 and 1987, and more recently, the international financial crisis in southeast Asia in 1997-8, and the overpricing of US technology stocks in the late 1990s, would be consistent with a short-run mispricing caused by rational herd.[1] Yet, the high volatility of prices that frequently financial data exhibit even in the absence of justifying news, could be the consequence of the information aggregation failure due to herding behaviors.[2]

Originally, imitative behavior was not considered rational: herding was associated with people blindly following the decision of others.[3] In the last decades, however, the literature on social learning has reconciled herding with rational behavior in many economic environments. Informational cascades models belong to this strand of literature on "rational herding".

The standard models of informational cascades apply to environments in which prices are taken to be exogenously given, and therefore they can hardly be applied to asset markets, where prices adjust continuously to reflect the changing information revealed by

[1] See Avery and Zemsky [1].
[2] See Lee [26].
[3] For references about irrational herding see Devenow and Welch [14].

the orders and trades effected by market participants. However, some recent theoretical models have analyzed mechanisms that may generate informational cascades also in the context of financial markets.

This thesis aims at contributing to this research area by analyzing the existence and the empirical implications of informational cascades arising from transaction costs, in financial markets with asymmetric information, sequential trading and competitive price mechanism. The main contributions of this work are the followings. In an asset market with exogenous transaction costs, the competitive price mechanism does not prevent the occurrence of informational cascades. These may develop not only when beliefs converge to a specific asset value, but also when in the market there is complete uncertainty about the asset's fundamental value. With proportional transaction costs, cascades are asymmetric in that they are more likely to develop in a context of high prices. Both bid-ask spreads and trading volume tend to shrink before an informational cascade, and attain their lowest level during the cascade.

The outline of the thesis is as follows.

The first chapter, titled "Informational cascades in financial economics: A review", surveys and appraises the recent theoretical research on informational cascades and herding behavior in capital markets.

This survey is divided in two parts. The first part presents a basic model of informational cascades, similar to that proposed by Bikhchandani, Hirshleifer and Welch [3], and describes the main extensions to the standard cascading models proposed by the recent literature on social learning (Chamley and Gale [8]; Gale [17]; Smit and Sorenson [29]; Chamley [6]). Moreover, it highlights the difference between the concepts of informational cascade and herd behavior, and introduces the notion of partial informational cascade.

The second part reviews the main studies on cascades in sequential asset markets with asymmetrically informed agents. The most interesting results of this strand of literature are the following. In the standard setting proposed by Glosten and Milgrom [19], price adjustment prevents the occurrence of an informational cascade in equilibrium. Nevertheless, multidimensional uncertainty and, more generally, non-monotonic signals open the possibility of herd behavior that may lead to a significant, short-run mispricing of assets (Avery and Zemsky, [1]). Moreover, informational cascades may develop in the case of different risk aversion among traders and market makers (Decamps and Lovo [13]; Cipriani and Guarino [10]), when traders care about their reputation for ability (Dasgupta and Prat [12]), or if informed market participants must pay a brokerage fee to trade (Lee [26]).

The second chapter, titled "Learning, cascades and fixed transaction costs" studies the equilibrium in an asset market *à* la Glosten-Milgrom [19] with costly market-making. Trades are assumed to entail a fixed transaction cost, and the focus of the analysis is on how this cost affects social learning. The starting point is the analysis of market crashes and informational avalanches by Lee [26], who shows that in a sequential asset market with fixed brokerage costs social learning fails, and partial or total informational cascades occur.

The shortcoming of Lee's analysis is that its results depend on the assumption of non-rational market maker. In this chapter, instead, this assumption is removed: sequential trades for a single risky asset are channeled through risk neutral market makers who set competitive bid and ask prices. Some traders have superior information about the asset's fundamental value and trade to exploit their informational advantage, while the remaining traders are uninformed and trade for exogenous reasons. A key assumption is that the competitive market makers have to pay an exogenous fixed cost to execute each order placed with them.

The fixed transaction cost is shown to lead to an informational cascade, despite the competitive price mechanism. Moreover, since all informed traders act alike, during an informational cascade the adverse selection component of the bid-ask spread disappears and the spread shrinks. Moreover, high transaction costs may generate an informational cascade even in a situation where market participants are completely uncertain about the true value of the asset. This contrasts with results in the literature on informational cascades in stock markets which highlight that cascades develop when there is a convergence of beliefs in the market.

The third chapter, titled "Informational cascades with proportional trading costs", analyzes the robustness of the main results in the previous chapter, with respect to different kinds of transaction costs. To accomplish this task, it presents two models that are similar in spirit to that presented in the previous chapter, but assuming that transaction costs are a function of the size of the order rather than being fixed. In one of the two models, they are assumed to be proportional to the asset price. The other model allows for different-sized orders, and transaction costs are taken to be increasing in the quantity of the asset traded. In both cases, transaction costs turn out to reduce the informational content of orders and may generate cascades.

With proportional transaction costs, if the asset's fundamental value in the bad state of nature is sufficiently low, an informational cascade may develop only when the prices are very high. This has the interesting implication that cascades will be asymmetric: they will be likely in bull markets and rare in depressed ones. By the same token, informational cascades are more prone to result in crashes than in frenzies.

In the second model, traders can choose between two order sizes: a large or a small one. Since informed traders always prefer to trade larger quantities (other things being equal), allowing for variable trade size induces an adverse selection problem. It turns out that, for any public belief about the true asset value, an informational cascade develops if the transaction cost exceeds the informational advantage of informed traders: in this case, since the latter will refrain from using their superior information in their trading strategy, no private information is impounded in market prices. As in the analysis of the second chapter, trading volume gradually is shown to decrease before a cascade occurs, and to reach its lowest value as the cascade develops.

If instead transaction costs are below this critical threshold, a cascade cannot develop, because at least some of the private information possessed by informed traders will be reflected in market prices. More specifically, two situations can arise, depending on the magnitude of the transaction cost. For intermediate values of the transaction cost, a pooling equilibrium exists, where informed traders place both large and small orders. For very low values of the transaction cost and a sufficiently large difference between the size of large and small orders, a semi-separating equilibrium exists, in which informed traders place only large orders and thereby reveal even more of their private signal to other market participants.

Chapter 1

Informational cascades in financial economics: A review

1.1 Introduction

A crucial goal of financial markets is to aggregate information about fundamentals which is widely dispersed among the market participants.

The recent literature on herding suggests that the information aggregation sometimes may fail because agents prefer to imitate others rather than act following their own information.[1]

The idea that individuals are influenced by others in investment decisions and financial transactions has had always great allure. For Keynes [21] financial markets were very similar to a "beauty context" in which the judges picked who they thought other judges would pick rather than who they considered to be the most beautiful. Similarly, in stock markets, investors - rather than focusing on the fundamental value - are often interested in the asset assessment of others traders.

Payoff externalities and informational externalities are the main sources of rational herding.[2] In most theoretical models both externalities are simultaneously present. Herding patterns based on reputational concerns in a principal-agent setting are an example.

Herding due to payoffs externality can arise when the payoffs depend directly on the behavior of other market participants. Such externalities cause herding of analysts or fund managers in models of reputational herding[3] or herd behavior of depositors in bank runs.[4]

Informational based herding arises when an agent gains useful information from observing actions of previous agents, to the point where he optimally acts as prior agents even when his own private information advises him to take a different action. This herding may lead to an informational cascade[5], that is an extreme example of failure of social learning in which agents' decisions do not covey any information to successors. Hence, the occurrence of a cascade leads to a complete information blockage.

[1] See Hirshleifer and Teoh [20] for a survey on the theoretical and empirical literature on herd behavior and cascades in capital markets.

[2] Brunnermeier [4].

[3] For example, see Scharfstein and Stein [28].

[4] For example, see Diamond and Dybving [15].

[5] Hirshleifer and Welch [3].

The main purpose of this chapter is to provide a critical review of cascading models applied to capital markets.[6]

The term "informational cascade" was introduced by Bikhchandani, Hirshleifer and Welch in their seminarial paper *A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades* [3], to denote a situation in which, in a sequential trade framework, every agent, based on the observation of previous agents, takes a decision that is independent of his private information.[7] The idea they illustrate by their simple model is the following. When individuals receive limited private information and make publicly observed actions, every individual tries to infer his predecessors' information from their choices. If early actions show a clear pattern, the information collected from the history of actions may dominate the private information of later agents. In this case, later agents may optimally choose to imitate the action of previous ones regardless of their private information. Hence, once an informational cascade starts, actions do not convey private information, the social learning fails and the cascade goes on forever, possibly with incorrect decisions. Bikhchandani, Hirshleifer and Welch [3] show that this type of behavioral convergence is: idiosyncratic, because the behavior of many followers depends on the first few individuals; path dependent, because the order of moves and information arrival affects the outcomes; fragile, because the arrival of a little new information may breakdown easily a cascade.

The standard cascade model applies in fixed-price contexts. Thus, it cannot be directly applied to asset markets, where prices change to take into account the information revealed by trades. Avery and Zemsky [1] prove that, in the standard Glosten-Milgrom [19] setting, price adjustments prevent informational cascades. Besides, they find that with multidimensional uncertainty, individuals may optimally act in opposition to their private information. Intuitively, this happens because price is a single-dimensional instrument and it only assures that the market learns about one dimension of un-

[6] The task of surveying the existing literature on informational cascades exhaustively is somehow ambitious and this chapter is not aimed at this, rather intending to introduce the part of literature most related to the present research.

[7] Banerjee [2] proposed at the same time another paper on imitative behaviors due to informational externalities. But the results of his model are more idiosyncratic and one cannot analyze their robustness.

certainty at a time. Hence, in a market with multiple dimensions of uncertainty, a short run mispricing may occur because market makers and informed traders interpret differently the history of trades. However, in the Avery and Zemsky model, the flow of information to the market never stops because the market continues to learn about at least one of the dimensions of uncertainty. Therefore, herding does not impede prices to converge to the fundamental value, that is, in the long run, the market is informational efficient.

In the standard Glosten-Milgrom [19] setting agents are homogeneous in preferences. Decamps and Lovo [13] show that differences in risk aversion between market makers and informed traders can generate informational cascades if the minimum and maximum size of trade per period are fixed. More specifically, they analyze the case of risk averse traders and risk neutral market makers. In their model, an informational cascade occurs because as prices become more informative, the traders' informational advantage vanishes and orders only reflect the inventory imbalance of traders. Cipriani and Guarino [10] reach similar results by considering heterogeneous traders in a multiple security setting. Moreover, they find that informational cascades can spill over from one asset to the other pushing prices far from fundamentals value, even in the long run.

Dasgupta and Prat, [12] adjust the standard sequential sequential trading model to allow for reputational concern. They show that if traders care about their reputation for ability, the market is informational inefficient and traders behave in a conformist manner.

Lee [26] argues that in a capital market with fixed transaction costs and sequential trading, the aggregation mechanism of dispersed information may fail and partial or total informational cascades occur. Moreover, during a partial cascade, the market accumulates a large amount of hidden information. If it is inconsistent with the current public belief, a small triggering event can reverse the cascade and induce agents to reveal their private information. Lee [26] terms "informational avalanche" the sudden release of hidden information due to the triggering event.

The remainder of the chapter is structured as follows. Section 1.2 presents the basic model for cascades and illustrates some interesting extensions. Section 1.2.1 also clarifies the difference between information cascade and herd behavior, and introduces the concept of partial informational cascade. Section 1.3 provides a review of the main works on cascading in sequential capital markets *à* la Glosten and Milgrom [19]. Section 1.4 concludes the chapter.

1.2 The basic model for informational cascades

In this section we describe the simplest model used by Bikhchandani, Hirshleifer, and Welch [3] to introduce the concept of informational cascades. In what follows we will use modifications of this model to show how the idea of herding due to informational cascades has been applied to capital markets.

The market is for an investment project whose liquidation value is low or high, depending on the realization of the bad or the good state of nature. Thus, the project can be viewed as a claim on a random variable $\widetilde{V} \in \{\underline{V}, \overline{V}\}$, with $\overline{V} > \underline{V} \geq 0$. Without loss of generality, $\overline{V} = 1$ and $\underline{V} = 0$. The initial prior probability of the high value is π_1. We assume that it is non-degenerate, that is, $\pi_1 \in (0, 1)$.

A sequence of risk neutral agents, indexed by t, face the choice of whether invest or not in the project. Each agent privately receives a conditionally independent imperfect signal θ on the true project value. The set of private signals is $\Theta = \{\underline{\theta}, \overline{\theta}\}$. The probability to receive the signal $\overline{\theta}$ is $p \in (0.5, 1)$ if the true project value is $\overline{V}$, and $(1-p)$ otherwise. Symmetrically, the signal $\underline{\theta}$ is observed with probability p if the true project value is $\underline{V}$, and $(1-p)$ otherwise. We say that agents receiving $\overline{\theta}$ are endowed with a good signal whereas agents receiving $\underline{\theta}$ are endowed with a bad signal.

Agents make their choice in an exogenous order. With the index t we denote the agent who takes his action in period t (and only in this period).

The information of agent t is his private signal θ_t and the publicly observable history h_t of the actions up until time t. The public belief about the liquidation value of the project, at time t, is the probability of the high value conditional on the prior history h_t, that is:

$$\pi_t = P(\widetilde{V} = 1 \mid h_t).$$

By bayes' rule, the expected liquidation value of the project for the agent t is:

$$\overline{\pi} = \frac{\pi p}{\pi p + (1-\pi)(1-p)} > \pi,$$

if he observes a good signal, and:

$$\underline{\pi} = \frac{\pi(1-p)}{\pi(1-p) + (1-\pi)p} < \pi,$$

if he observes a bad signal. The choice of the agent depends on whether his private belief is greater than the cost to invest.

The cost of the investment c is fixed and, without loss of generality, we assume that it is equal to 0.5. It is easy to notice that if the public belief is above p, then $\underline{\pi}$ is greater than the cost c; both agents endowed with a good signal and agents endowed with a bad signal find worthwhile to invest. Otherwise, if it is lower than $(1-p)$, then $\overline{\pi}$ is lower than c, and no agent chooses to invest. So, in any period t, if the public belief π_t belongs to $[(1-p), p]$, the agent t chooses to invest only if his signal is good; if π_t is above p, the agent t invests regardless of his signal; if π_t is smaller than $(1-p)$, he does not invest regardless of his signal. As a consequence, the choice of agent t does not convey information on his private signal both when π_t is greater than p and when it is lower than $(1-p)$. This blockage of the information is called informational cascade.

In order to shows that in a such benchmark an informational cascade starts in finite time with probability 1, we assume, for simplicity, $\pi_1 = 0.5$. Therefore, the private belief of an agent endowed with a good signal is $\overline{\pi} = p$, and the private belief of an agent endowed with a bad signal is $\underline{\pi} = (1-p)$.

Since $p > 0.5$, the agent first arriving in the market follows his private information: if he observes $\overline{\theta}$, he invests; if he observes $\underline{\theta}$, he rejects the project. The second agent observes the decision of the first one and infers his signal. Suppose that he infers $\theta_1 = \overline{\theta}$. If his signal is $\theta_2 = \overline{\theta}$, he invests. If, instead, his signal is $\theta_2 = \underline{\theta}$, his expected liquidation value of the project is 0.5; therefore, he is indifferent between investing and rejecting. Assume, as a tie-breaking convention, that an agent indifferent between to invest and to reject invests and rejects with equal probability.[8] If the second agent

[8] This tie-breaking convention is the same as Bikhchandani, Hirshleifer, and Whelch [3].

invests, the third agent infers that the first agent saw $\overline{\theta}$ and the second one is more likely to have seen $\overline{\theta}$ than $\underline{\theta}$. Clearly, if his signal is $\theta_3 = \overline{\theta}$ he invests. But, even if his signal is $\theta_3 = \underline{\theta}$ he invests, because his private belief on the project value exceeds the cost. Thus, the third agent always invests regardless of his private information. This implies that his decision is uninformative to others. The fourth agent faces exactly the same position as the previous one. Since all signals are drawn independently from the same distribution, he too invests independently of his own information. And so on all later agents arriving in the market. We can conclude that a cascade with investment starts in period 3. Similarly, if both first and second agents choose to reject the projet, a cascade with no investment begins in period 3. If π_1 is greater than c, and the first agent optimally chooses to invest, a cascade with investment develops at second agent; if π_1 is lower than c, and the first agent rejects the project, a cascade with no investment develops at second agent.

We can conclude that a necessary condition to have no cascade is that actions alternate consecutively between investing and rejecting. The probability that actions alternate consecutively for t periods, is decreasing in t, and converges to 0.

Proposition 1.1 *(Chamley [7]) When agents have a binary signal, an informational cascade occurs after some finite date, almost surely. The probability that the cascade has not started by date t converges to* 0 *like β^t for some β^t with* $0 < \beta < 1$.

Our simple model points out that, when the decisions are sequential with subsequent actors observing only predecessors' decisions (and not their signals), and the action space is discrete (e.g., an adopt/reject decision), the choice of few may influence the behavior of a large number of agents. Since only a small part of private information becomes public, the probability that a wrong informational cascade starts is positive. The fundamental reason the outcome with observable actions is different from the observable-signals

Koessler and Ziegelmeyer [22] show that, in the model of Bikhchandani, Hirshleifer, and Whelch, by relaxing their tie-breaking convention, there exist other equilibria in which informational cascades are not necessarily observed. More precisely, they consider a new tie-breaking rule: the *non-confident* tie breaking rule. Under this rule, an indifferent agent simply imitates the action of his predecessor.

benchmark is that if the signals received by predecessors are observable (instead of actions taken), later decision-makers would have almost perfect information about the true value of the project and would tend to take the correct action.

An important feature of informational cascades is that they are fragile with respect to small shocks, where a small shock refers to a change in the public belief due to a public signal less informative than the private signal of a single agent. Cascades aggregate the information of only a few early agents' actions. The shock thus needs only to offset the information conveyed by the action of the least agent before the start of the cascade.

1.2.1 Informational cascades and herd behavior

In the literature, the terms informational cascade and herd behavior are often used interchangeably, but the two concepts are not equivalent.[9] An informational cascade is said to occur when all agents ignore their private information when choosing an action; whereas a herd take place when all agents act alike after some period.

In a herd, all agents choose the same action, but some of them may have acted differently if the realization of their private signal had been different. During an informational cascade, agents optimally follow their predecessors disregarding their private signal since the public belief is so strong to outweigh any private signal. Thus, a cascade implies a herd but the converse is not true.[10]

The distinction between cascades and herd is significant because in a cascade the learning process stops since agents behavior became purely imitative and hence in uninformative. In contrasts, in a herd, agents become more and more likely to imitate but their actions still may provide information. Clearly, if the herd is sustained, the amount of social learning is very small because the probability that the herd is broken must be vanishingly small.

Herd always occurs with probability 1, while cascades take place only in very specific models. In the framework described to illustrate the concept of informational cascade, we assumed a binary signal

[9] Smith and Sorenson [29] first empathize the difference between cascades and herd.
[10] See Chamley [7] for a more formal explanation.

space. Bikhchandani, Hirshleifer and Welch [3] present a model for cascades with a finite set of conditionally independent and identically distributed private signals. Smith and Sorensen [29] show that the hypothesis of bounded distribution of private signals is crucial for the existence of informational cascades. The intuition is easy: in a model with identical preferences, binary actions space and two states, a complete information blockage can takes place only if the public belief from the history of actions dominates any private signal. This condition cannot be satisfied if the distribution of private signals is unbounded. If this is the case, the public belief eventually converges to the truth.

However, this result does not reduce the relevance of cascading models to explain many economic and financial phenomena. Although cascades do not arise with unbounded distributions of private signals, under many plausible distributions the learning process is so much slow[11] that the convergence to the truth does not matter and the behavior of agents exhibits the main properties of cascading models. Chamley [6] illustrates with a numerical example how a model with unbounded private signals can generate long regimes where agents herd and sudden changes of the public belief due to contrarian actions. Therefore, informational cascades, although may not exist, are a good stylized description of the properties of social learning in contexts where the actions space is discrete.

1.2.2 Informational cascades with continuum action space

In the model illustrated in section 1.2, which is similar to that presented in Bikhchandani, Hirshleifer, and Welch [3], informational cascades arise because of the discrete action space.[12]

Lee [25] shows that the hypothesis of discrete action space is essential for the occurrence of cascades in the Bikhchandani, Hirshleifer, and Welch [3] setting. Discreteness reduces the social learning because of two effects. First, it prevents the actions from fully revealing the private information of agents. Actions are the main tool

[11] Chamley [6] demonstrates that when the actions space of agents is discrete and the signals distribution is unbounded, the rate of convergence to the truth is exponentially slower than the rate when private signals are observable.

[12] To simplify, we have assumed two actions, but the results could be generalized to a finite set of actions.

for the communication of information between agents. A finite set of actions is a strong limitation on the capability of social communication. Second, the discreteness prevents agents from fully using their private information. As a consequence, the likelihood of a wrong cascade decreases as the action space grows.

In Banerjee [2] informational cascades (termed herd in the paper) develop despite the continuum action space. More precisely, an informational cascade occurs because the one-dimensional action space, even if continuous, cannot reflect two-dimensional uncertainty.

In the Banerjee model, risk neutral agents sequentially choose an asset on a continuum. Each agent observes the decisions of all those ahead of him. Only one asset has certain positive payoff. All the others have zero payoff. With positive probability an agent privately gets an imperfect signal about the asset with positive payoff. If the signal is false, it is uniformly distributed on the continuum. This implies that the probability that two agents get the same wrong signal is equal to zero. Banerjee assumes that when agents are indifferent between two or more actions, they follow their private signal, if they are informed; they choose the asset number 0, otherwise. This implies that, if the first agent does not receive a signal, he chooses the asset number 0.

The assumptions on the distribution of wrong signals and on the tie-breaking rule are crucial in the Banerjee model. The first agent buying an asset different from "0" is following his private signal, for the assumption on the tie-breaking rule. If the following agent makes that same choice, two scenarios are possible. Either he is uninformed, or he observes the same private signal as the previous agent, and this signal is true for the assumption on the distribution of wrong signals. If the third agent gets a signal, in the first scenario he is indifferent between following the predecessors' decision and his own signal; in the second scenario he strictly prefers following his predecessors. If he does not get a signal he always prefers following his predecessors. Since the agents do not observe the signals of those ahead of them, the third agent will follow his predecessors and ignore his own signal. Following agents face exactly the same situation as the third agent and they will make the same choice. If the first agent receives a wrong signal and the second agent does not receive signal, all agents will choose the wrong action. In this case herding leads to an informational cascade.

1.2.3 Endogenous sequencing

A crucial assumption for the occurrence of an informational cascade is that agents choose sequentially, with subsequent agents observing actions, and not information, of their predecessors. The basic informational cascades model makes the hypothesis of exogenous sequencing of decisions. In both models described in the previous sections we suppose that agents enter the market to make their choice in a pre-specified order. Yet, cascades are robust to the relaxation of this assumption.

In a dynamic game with asymmetric information, waiting allows agents to take advantage of the information revealed by others. Hence, in a setting where delay is costless and agents can choose the timing of their decision, everybody would prefer to decide last. Since someone has to decide first, individual compete for the best place in the decision-making queue.

Strategic delay due to informational externalities, has been analyzed by Chamley and Gale [8]. They study a strategic model of investment, with discrete time, in which a random number of agents receive a real option with fixed exercise price but a risky payoff. To have an investment opportunity is private information, and the value of the underlying asset is increasing in the number of real options. Agents who exercise the option early reveal that they had and investment opportunity. This positive externality allows the successors to update their belief about the true value of the underlying asset. The cost of waiting is modelled by assuming that agents discount the future using a common discount factor $\delta \in (0, 1)$.

Chamley and Gale [8] show that if beliefs about the number of real options are sufficiently pessimistic, no agent will exercise the option. Since no information is revealed in the absence of investment, the situation will not change in the next period. Therefore, if no one invests in one period, the investment stops forever. The probability of informational collapse depends on the speed with which agents can react to the information inferred from their predecessors: it increases as the period length reduces.

1.2.4 Heterogeneous preferences

Smith and Sorensen [29] investigate the social learning in a exogenous sequencing model with heterogeneous preferences of agents. They find that with multiple rational preferences types, not all ordinally alike, an incomplete learning informational pooling outcome exists even when an informational cascade is impossible.[13] The intuition is that if agents'preferences differ, the successor does not know whether the action of a predecessor is due to a different private signal of to a different preference ordering. As a consequence, it may be impossible to draw any clear inference from the history of actions. Smith and Sorensen [29] denote these outcome confounded learning.

1.2.5 Partial informational cascades

Most recent works on social learning focus on the completeness of information revelation and the rate at which information is revealed. In particular, cascading models emphasize the complete failure of learning process due to informational externalities. Gale [17] remarks that, a setting where information arrives too slowly to be helpful for most individuals decisions is essentially the same from the point of view of both welfare and predicting behavior as one where there is a complete information blockage. An economy is in a partial information cascade when all agents with mildly bad or good news make an identical choice regardless of their private information. Therefore, during a partial informational cascade, the learning process goes on, but very slowly.

Gale [17] proposes a model with a continuous signal space and a binary action space. N agents choose sequentially whether to invest or not. Each agent privately observes a signal θ_n uniformly distributed on $[-1, 1]$ correlated with a payoff relevant variable. If an agent invests, he makes a return $\widetilde{V} = \sum_{n=1}^{N} \theta_n$,; if he rejects he obtains a zero payoff. The first agent invests if $E[\widetilde{V} \mid \theta_1] = \theta_1$ is greater than 0. The second agent invests if $E_2[\widetilde{V}] = \theta_2 + E[\theta_1 \mid action_1] > 0$. If the first agent has invested, $E_2[\widetilde{V}] = \theta_2 + E[\theta_1 \mid \theta_1 > 0] = \theta_2 + \frac{1}{2}$. When the second agent sees the first agent invest,

[13] This is the case when beliefs are unbounded.

he will invest as long as $\theta_2 > \frac{1}{2}$. When both the first and the second agent have invested, the third agent will invest as long as $\theta_3 > \frac{3}{4}$, and so on. So, all subsequent agents may imitate the action of the first agent, even though they would not make that choice if they had to choose by considering only their private information. This is the sense in which agents ignore their private information during a partial information cascade. Clearly, the longer the partial cascade, the harder is for a single agent not to make an identical choice, even if his private information strongly indicates an opposite action. However, although herding behavior may occur, a full informational cascade can never develop.

1.3 Informational cascades in financial markets

Cascades can help to explain many empirical phenomena in financial economics. Welch [30] in his cascade model explains the decisions of IPO (initial public offering) investors to ask for share allocations.[14] He shows that if sufficiently many investors underwrite early to receive shares, all later investors rationally and optimally ignore their own private information and imitate earlier investors. Corb [11] models intra-bank panics and Chen [9] models inter-bank panics using the concept of informational cascade.

The basic cascades model applies in fixed-price situations. In an asset market, prices are not fixed; they every time move to incorporate all new publicly available information. The instantaneous prices adjustment should prevent informational cascades. As Brunnermeier [4] remarks, in these frameworks, the predecessors' actions not only cause an *informational externality* as in Bikhchandani, Hirshleifer, and Welch [3], but also a *payoff externality*. In fact, changes in prices alter the payoffs structures for all successors. To show this result, we need modify the simple framework formerly considered.[15]

The investment opportunity is represented by a financial asset whose value can be low ($= 0$) or high ($= 1$), depending on the true state of nature. The investors are traders who exchange the asset with a competitive risk neutral market maker responsible for quoting prices. Transactions occur sequentially in a discrete time, with

[14] Offering prices are fixed by regulation in the U.S.

[15] The model we present is a special case of Glosten and Milgrom [19] model.

one trader allowed to transact in each period t. With probability $(1-\mu)$, the trader who comes in the market is a profit maximizing informed agent who is endowed with a private signal $\theta \in \{\underline{\theta},\, \overline{\theta}\}$ correlated with the fundamental value; with probability μ, he is a noise trader who transacts for exogenous reasons.[16] The type of a trader is private information to that trader. However, the probability that the trader is informed is common knowledge, as well as the history if transactions H_t. The choice of a trader arriving in the market is whether to buy or sell one unit of the asset, or to refrain from trading. We suppose that noise traders buy, sell, or do nothing with equal probability $\frac{1}{3}$. As in the previous section, we denote π the public belief at the beginning of an arbitrary period.

Since some traders have a superior information, then the market maker takes losses, on average, to those traders. To remain solvent, the market maker has to offset these losses by making gains from noise traders. Consequently, he sets a bid and an ask price respectively below and above the expected asset value in the common knowledge. Since the market maker acts competitively, he posts a bid price B and an ask price A such that his expected profit is equal to zero on both sides of the market; that is:

$$B = E[\widetilde{V} \mid a\, sell,\, H] = \pi \frac{P(a\, sell \mid \widetilde{V} = 1,\, H)}{P(a\, sell,\, H)} < \pi, \tag{1.1}$$

and

$$A = E[\widetilde{V} \mid a\, buy,\, H] = \pi \frac{P(a\, buy \mid \widetilde{V} = 1,\, H)}{P(a\, buy,\, H)} > \pi. \tag{1.2}$$

The probability of a sell order is the same as the probability of meeting a trader with a bad signal or a selling noise trader. Consequently, the bid price is greater than $\underline{\pi}$, whatever the public belief is. Likewise, the ask price is always below $\overline{\pi}$ since a buy order may come from either a trader observing a good signal or a buying noise trader.

It is easy to show that this price mechanism prevents the occurrence of a (full) informational cascade. Indeed, in a cascade the

[16] The presence of noise traders is needed to guarantee that trading occurs. Indeed, without the presence of traders who trade for reasons other than speculation, the no-trade theorem of Milgrom and Stokey [27] applies and the market breaks down.

decisions of traders are uninformative about the true state of nature. This means that the expected asset value given a sell order or given a buy order is, in both cases, equal to the unconditional expectation. From the zero expected profit condition, it follows that both the bid and the ask prices would be equal to the public belief π during a cascade. In this case, however, traders receiving a bad signal would sell the asset (since $\underline{\pi} < \pi$) and traders with a good signal would buy the asset (since $\overline{\pi} > \pi$). Hence, trades would convey some information. This contradicts the hypothesis that a cascade is begun.

Our simple model showed that with binary signals, herd behavior never arises when prices adjust to reflect available information. Avery and Zemsky [1] generalize this property to the case where private signals are monotonic.

1.3.1 Herd behavior with multidimensional uncertainty

In the previous section we showed that in sequential trading models à la Glosten and Milgrom [19], in which the market maker modifies the price on the basis of the order flow, it is never the case that traders neglect their information and imitate previous traders' decisions. In those models, herd does not occur because prices fully reflect available information. However, as Avery and Zemsky [1] remark, the price is a single-dimensional instrument and it only allows to learn about a single dimension of uncertainty at one time. Therefor, when there are multiple sources of uncertainty, herd could arise even in the Glosten-Milgrom framework.

Avery and Zemsky [1] find that multidimensional uncertainty can lead traders and market maker to a different interpretation of the history and it may produce herd behavior in the short-run. In any case, multidimensional uncertainty does not prevent beliefs to converge to the true asset value in the long-run.

The definition of herding they adopt differ from the standard one in the literature. More precisely, they say that an informed investor engages in buy (sell) herding behavior if initially he strictly prefers not to buy (resp. not to sell) and after observing a positive history of trades (resp. negative), he strictly prefers to buy (resp. to sell). They also introduce the notion of contrarian behavior: they say that an informed investor engages in buy (sell) contrarian behavior if initially he strictly prefers not to buy (resp. not to sell) and after observing a negative history of trades (resp. negative), he strictly prefers to buy (resp. to sell).

In order to obtain herding behavior, they add to the value uncertainty, another dimension of uncertainty: the *event uncertainty*.[17] In the standard trade framework used by Glosten and Milgrom [19], an information event is assumed to have occurred. Avery and Zemsky, following Easley and O' Hara [16], consider a market in which an information event occurs with positive probability $\alpha < 1$.

More precisely, they assume that $\widetilde{V} \in \{0,\, \frac{1}{2},\, 1\}$. If no information arrives, the asset value remains $\widetilde{V} = \frac{1}{2}$. Otherwise, $\widetilde{V} \in \{0,\, 1\}$ and some traders receive a private signal on the true asset value, whose precision is $p \in (\frac{1}{2},\, 1)$. Besides, all traders know whether an information event has occurred, while the market maker does not; this is the second dimension of uncertainty.

After the information event, in the Easley and O' Hara [16] model informed traders know for certain the true asset value. In contrast, in the Avery and Zemsky model, signals are imperfect.[18] As the market maker does not know whether an information event has occurred, he learns from the trading sequence less than informed traders and adjusts prices slowly.[19] Slow price adjustment reduces the payoff externality. As a consequence, informed traders might rationally ignore their own private information about value uncertainty in order to trade following the trend in past trades.[20] During the herding, new information still reaches the market because the market maker collects information about the occurrence of an information event[21]; hence a cascade never starts. Moreover, since when herding takes place prices move slowly, this information structure cannot lead to a price bubble and crash.

Avery and Zemsky also consider a setting with two types of signals. Some traders observe a signal with low precision ($p_L < 1$),

[17] Easley and O' Hara, [16].

[18] The information structure they consider makes signals nonmonotonic.

[19] The model of Bikhchandani, Hirshleifer, and Welch [3] can be viewed as an extreme case where prices are fixed.

[20] The authors prove that when an information event is very rare, the probability of herding goes to 1 and this herd behavior is misdirected with a strictly positive probability.

[21] Herd behavior can occur only after an information event.

whereas the other know for certain the true asset value ($p_H = 1$). The fraction of traders receiving a perfect signal can be high (the market is said well-informed) or low (the market is said poorly-informed); no market participant knows this fraction for sure. The probability β of a well-informed market is common knowledge. The authors denote this information structure *composition uncertainty*. They show that when imperfect signals are very noisy, composition uncertainty can generate, in the short-run, contrarian behavior of less informed traders.

Finally, Avery and Zemsky investigate the combination of event uncertainty and composition uncertainty, by making the hypothesis that some information events have a high fraction of perfectly informed traders, while others have only a few. They show, by means of a simulation, that with a such information structure traders may herd for an extended length of time and a sudden price change may occur.

To illustrate this result, we consider a market with an extreme information event (low α) and high probability of well-informed market (conditional on the occurrence of the information event). Besides, by following Avery and Zemsky, we assume that in a poorly informed market there are not perfectly informed traders, and that the precision of imperfect signals is very low.

We suppose that a sequence of buy orders take place. If an information event has occurred, they can come from either informed traders or noise traders. Since β is very high, poorly informed traders believe that buy orders are generated by perfectly informed traders with high probability. Hence, their expectation about the true asset value increases significantly. The market maker does not know if an information event has occurred. Since α is low, the asset price does not change very much.

After a few of periods, informed traders with low precision signals choose to buy regardless of their private information. In a first phase, the market maker believes to be trading with noise traders, but, at some point, he realizes that all these purchases are very unlikely in the absence of an information event. Given the long sequence of previous buy orders, the price shoots op to near 1 since a well-informed market is more likely. At this point, if the market is poorly informed, all informed traders stop to buy the asset because the price is too high, and the trading volume reduces significantly.

After a while, the market maker realizes that the low volume is incompatible with a well-informed market. Consequently, the asset price suddenly falls to near $\frac{1}{2}$. Avery and Zemsky refer to this sequence of events as a bubble.

The price path described by Avery and Zemsky is really spectacular. Nevertheless, as stressed in Chamley [9], the empirical relevance of their example is not convincing. Indeed, a price bubble develops only if a unlikely state of nature occurs, and a specific sequence of traders arrive to the market.

The impact of multidimensional uncertainty on the bayesian learning process in an asset market is also analyzed by Gervais [18]. As in Avery and Zemsky [1], in Gervais [18] there is uncertainty about traders' information precision. Before transactions start, if an information event occurs any insider receives a signal on a certain aspect of the liquidation asset value[22], whose precision (equal for all signals) can be high or low. The market maker does not observe the quality of signals. He can only seek to infer that quality from the history of trades. In contrast to Avery and Zemsky [1], Gervais [18] finds information blockage due to the bid-ask spread. Indeed, the competitive bid-ask spread decreases as trades go on. When it reduces below a certain threshold, low precision informed traders make the same choice as high precision informed traders. As a consequence, trades do not convey any information about precision and the market is in a cascade. This result, even if fascinating, is not robust. The occurrence of a cascade strongly depends on many assumptions very restrictive of the model.

1.3.2 Risk aversion and informational efficiency

In the standard Glosten and Milgrom [19] setting, both informed traders and market maker are assumed risk neutral. Decamps and Lovo [13] relax this hypothesis, by assuming risk averse informed traders. They find that differences in the degree of risk aversion between market makers and traders can generate history dependent

[22] In Avery and Zemsky [1], signals refer to the liquidation value of the asset. Here, the future payoff of the risky asset is equal to the sum of N independent incremental payoffs, that is: $\widetilde{V} = \sum_{n=1}^{N} \widetilde{\xi}_n$. Each private signal refers to a specific incremental payoff.

behaviors and long run mispricing, in asset markets where the maximum and the minimum size of trade per period are fixed.[23]

As Avery and Zemsky [1], they consider a sequential trade model similar to Glosten and Milgron [19]. The peculiarity of their model is that traders are risk averse[24] and can choose to trade any amount nq, with q fixed and $n \in \mathbb{N}$, of the asset, up to a maximum quantity $Q = Nq$. The information structure is similar to that described in section 1.3, with the only difference that there are not noise traders.[25]

The risk aversion hypothesis has relevant consequences on the trading motivations of informed traders. In the basic framework presented in section 1.3, risk neutral informed traders only transact to exploit their informational advantage. By assuming aversion to the risk, Decamps and Lovo add to the trading motivations of informed traders the inventory component, which reflects the traders preference for low-risk-portfolio.

Decamps and Lovo demonstrate that as the public belief gets concentrated in the extreme tails of asset value distribution, the information component of traders decisions reduces and, since the action space is discrete, transactions only reflect the traders inventory unbalance. Thus, depending on the distribution of their portfolio composition, traders may engage in herding or contrarian behavior. At this point, trades stop providing information on the true asset value and an informational cascade starts. Besides, as a finite history of trade is, in general, compatible with any realization of liquidation asset value, the informational cascade may be incorrect.

These results are in contrast to the findings of Avery and Zemsky [1]. In their model, the competitive price mechanism eliminates the possibility of informational cascades, and the price ultimately converges to the fundamental value. Decamps and Lovo show that if there is a difference in the degree of risk aversion between market makers and traders, the market may not be strong-form efficient in the long run.

The findings of Decamps and Lovo are strongly dependent on the exogenous distribution of the traders portfolio composition. In

[23] This is what happens, for instance, in the Nasdaq SOES market.

[24] Whereas the market maker is risk neutral.

[25] Differences in risk aversion prevent the market to break down. Thus, the presence of noise traders is irrelevant.

my opinion, risk averse informed traders who have not risky asset in their initial portfolio, would always choose to buy the asset with positive probability[26] if he observes a good signal, and to sell the asset (with positive probability) in the opposite case. Hence, in the absence of exogenous initial portfolio exposure to risk, the learning process should never stop and the price, eventually, converge to the fundamental value.

Analogous results have been founded by Cipriani and Guarino [10], which extend the Glosten and Milgrom [19] model to a multiple security setting. They consider heterogenous informed traders. More precisely, they suppose that some traders have an extra utility and other suffer a disutility from holding an asset. When there is a convergence of belief, the informational advantage of informed traders is small. Then, the gains and the losses from holding an asset may exceed the gains due to the private information. In that case, traders may optimally choose to buy the asset even if they observe a bad signal, or choose to sell the asset even if they have a good signal. When the public belief on the asset value becomes such that all traders optimally make their choice regardless of their private signals, an informational cascades develops. Furthermore, Cipriani and Guarino show that informational cascades can lead to contagion across markets.

1.3.3 Informational cascades due to reputational concerns

Scharfstein and Stein [28] present a principal-agent model where reputational concerns of fund managers can generate herd behavior. Managers care about reputation because there is uncertainty about their ability to forecast the true investment liquidation value. Since the managers payoff depends only on the beliefs of the principal about their ability, they may optimally choose to ignore their private information and mimic decisions of previous managers.

As in the model of Bikhchandani, Hirshleifer and Welch [3], Scharfstein and Stein [28] assume that the investment is available to all agents at the same price. Dasgupta and Prat [12] develop a sequential trading model *à* la Glosten and Milgrom [19] which cap-

[26] In order to guarantee the existence of equilibrium bid and ask prices, Decamps and Lovo allow traders to use any mixed strategy when they are indifferent between trading or not.

tures reputational concerns of informed traders in a setting where prices are endogenously determined.

In their paper informed traders are found managers who trade on behalf of other investors. They can be of two types: smart or dump. The precision of their signal depends on their unknown type. The payoff that found managers receive depends both on the trading profits and on the reputation that they earn with their clients.

Dasgupta and Prat [12] introduce an incentive to imitate others by assuming that the reputational payoff reduces when the manager takes an opposite position with respect to his predecessor.

As the uncertainty over the fundamental value is resolved and price becomes sufficiently precise, trading profits reduce and found managers ignore their own information and behave in a conformist way because of reputational concerns. Thus, financial markets whit reputationally sensitive traders are informationally inefficient.

1.3.4 Bubbles, crashes, and informational avalanches

An interesting feature of many market crashes is that, at the time of the crash, no major event changing the state of nature happens. Moreover, the empirical evidence shows that, before a market crash, prices grow steadily and, after the crash, they remain low for a substantial length of time before rising again[27]. Lee [26] tries to explain market crashes exhibit a such price path, by the failure of information aggregation due to the transaction costs in the capital markets.

The model described by Lee [26] differs from the previous ones in many aspects. A sequence of risk averse traders exchange a risky asset whit a risk neutral market maker, who is responsible for quoting the price, over $T+1$ trading rounds. At the end of each trading period, there is a positive probability β that the liquidation asset value becomes common knowledge and the trading activity stops.[28]

In contrast to the previous models, traders can decide when to trade, and are allowed to transact several times. In each trading round a new trader reaches the market and makes his choice.

[27] See Lee [26].

[28] This assumption prevents traders to place an infinite trading order for a certain profit in any period.

Traders arrived in the previous trading rounds remain active. The first time that a trader wants to trade he incurs a fixed transaction cost $c > 0$ to open an account with a broker.

Each trader privately observes a signal $\theta_i \in \Theta$ correlated with the liquidation value of the risky asset. Signals differ in their precision and satisfy the monotone likelihood property. Strategic behaviors are ruled out by assuming that the traders behave as price takers.

Prior to any trading round the market maker sets the price at which all orders will be executed. The market maker does not take into account all the relevant information: by setting the price, he ignores the information provided by orders in the current trading round. The equilibrium price is indeed equal to the expected asset value conditional on the history of previous trades. The model of Lee thus neglects a step that is crucial in all models of trade between rational agents[29]. The market maker loses money on average since he is trading with traders better informed than him and does not offset these losses by making gain with noise traders. As Brunnermeier [4] emphasizes, the assumption of non-full rationality for the market maker is necessary to induce informed traders to trade. Otherwise, the no-trade speculation theorem of Milgrom and Stokey [27] would apply in a setting without noise traders.

Before to proceed to the equilibrium analysis, we need some definitions.

A *no-trade informational cascade* with respect to the subset $\widehat{\Theta}$ of Θ (having more than 2 elements) is said to be developed if, given the public belief π on the liquidation asset value, all agents with signal in $\widehat{\Theta}$ prefer to refrain from trading.

An *informational avalanche* is said to be occurred in trading round t if all the agents arrived in previous trading rounds without trading, simultaneously opt for trade.

The definition of informational cascade is different from that of Bikhchandani, Hirshleifer, and Welch [3] in that it allows a partial informational cascade because $\widehat{\Theta}$ can be a proper subset of Θ. During a partial informational cascade, all the traders endowed with private signal $\theta \in \widehat{\Theta}$ make an identical choice. Hence, their signals are not distinguishable and the market accumulates a lot of hidden

[29] Chamley [7].

information not incorporated in the asset price. A trader with an extreme signal (that is, a signal $\theta \in \widehat{\Theta}^c$) might shatter the partial cascade by taking an action which contradicts the state of nature implied by the cascade. If a such action is observed in the market, an informational avalanche occurs and all the hidden information accumulated during the partial informational cascade is revealed.

In order to find the security market equilibrium, Lee demonstrates that the first non-zero trade fully reveals the private signal of trader. This implies that after the first trading order, the informational advantage of traders disappears. Because of risk aversion, in the absence of fixed costs, a trader whose private belief is the same as the public belief, strictly prefers to get perfect insurance.[30] Hence an informed trader trades at the most twice, first to buy or sell risky asset based on his private signal and second to unload the risky asset at the fair price, after the price reflects his private signal.

Clearly, a trader makes the first non-zero trading order in period t if, and only if, the expected payoff from that strategy is greater than the payoff from permanently no trading.[31] Indeed, it may be the case that the expected capital gains are so small that it is not worthwhile for a trader to pay the fixed transaction cost.

Lee shows that, due to the transaction cost, new traders with a middle informational advantage do not trade and a partial informational cascade develops as the public beliefs get concentrated either on the bad or on the good state of nature. More precisely, he proves that for every signal θ there exist unique public beliefs $\pi^u(\theta)$ and $\pi^l(\theta)$ (with $\pi^u(\theta) \geq \pi^l(\theta)$) such that, given a public belief π lower than $\pi^l(\theta)$ or greater than $\pi^u(\theta)$, traders with signal θ optimally choose to refrain from trading.

To explain the meaning of this result we analyze the simple example proposed by Lee. We consider an economy with only three private signals: H, L, and R.[32], such that $\pi^u(R) > \pi^u(H) > \pi^u(L) > \pi^l(L) > \pi^l(H) > \pi^l(R)$,. R is rarely observed under both states.

[30] In the absence of informational advantage, the only trading motivation of traders is the inventory component.

[31] By assumption, traders behave as price takers. As a consequence, they do not compare the profit from the optimal non-zero trade against the future trading profit from a price path which does not reflect their own private signal.

[32] H=high liquidation value; L=low liquidation value; R=rare extreme low liquidation value signal.

The initial prior on the asset liquidation value is such that all traders prefer to trade following their private information.

We suppose that the true asset value is 0 and consider the following sequence of arrivals.

In the first K periods traders observing the signal H reach the market and place buy orders. The market maker infers their private information and the asset price rises. This phase is called by Lee the "boom". If K is big enough (so that the public belief becomes greater than $\pi^u(H)$), a partial informational cascade including signals H and L develops, strongly indicating the high asset value.

In the following phase, denoted "euphoria", a sequence of traders with mildly bad and good news (signals L and H) arrive at the market without trading. A large amount of hidden information is accumulated because traders with low-precision signals are not distinguishable. If this phase is very long a total informational cascade develops. On the contrary, if a trader with rare very informative bad signal arrives at the market before public belief grows bigger than $\pi^u(R)$, he places a sell order. This period is the "trigger".

Subsequent to trigger, traders with signal L who arrived in the market during the euphoria phase place selling orders, since their mild skepticism is strengthened by the trigger. An informational avalanche develops, resulting in the phase of "panic".

Therefore, during the phase of panic a dramatic price movement may happen even in the absence of correspondingly dramatic news. The variability of the price at the time of informational avalanche exceeds what would be possible if only the trading order from the new agent is taken into account. If the R signal arrives early in the euphoria phase, the price change is not too dramatic since the traders with moderate bad signal who are active in the market are not much. But, if there are many traders observing a private information inconsistent with the present price, the price fall is very big. Lee demonstrates that the volatility of price increases whit the length of the partial informational cascade preceding the informational avalanche.

Moreover, although we cannot entirely rule out the possibility of the price being incorrect after the informational avalanche, when the informational avalanche occurs the price is likely to be closer to the correct one because it incorporates the hidden information accumulated during the partial cascade.

In the model of Lee a market crash is described as a procedure which corrects the public belief inconsistent with the distribution of private information in the market. In that, the model is very similar to Caplin and Leahy [5], where a market crash is viewed as a mechanism by which markets aggregate previously received, but hidden, private information, and only a small piece of additional information is required to price collapse.[33]

However, the model of Lee, as that of Caplin and Leahy [5], does not present a specific bias for a crash. By changing the signal distribution, the same mechanism described previously to illustrate a crash can generate a boom.

The main criticisms of the model are related to the price mechanism. The market maker, by setting the asset price, does not take into account the information content of current orders. In any period, the asset price is equal to the public belief given the history of previous trades. Moreover, the market maker cannot adjust the price within the trading round even when his belief moves because of trading orders. Those assumptions are not realistic in capital markets. But they are essential for the occurrence of an informational avalanche. The large trading volume which characterizes avalanches would probably not occur if the market maker were to adjust the asset price after each trading order.

1.4 Concluding remarks

In this chapter we have reviewed the recent theoretical researches on cascading in capital markets with sequential trading mechanism.

In the basic models of informational cascades, the price for taking an action is fixed ex ante and remains so. This assumption is inappropriate for market microstructure models, since in asset markets prices adjust to incorporate any new relevant information.

[33] In Caplin and Leahy [5], private signals of informed traders are revealed to other market participants by alterations to their routine behavior. Fixed costs incurred by traders when they change their behavior, prevent the gradual release of private information.

In sequential models *à* la Glosten and Milgrom [19], in the absence of market frictions, an informational cascade does not arise, and prices are strong-form efficient in the long run. Moreover, as long as the private signals of informed traders are monotonic, the competitive price mechanism prevents any herd behavior.

However, if there are multiple sources of uncertainty, herd behavior due to informational externalities can arise because traders and market maker interpret differently the history if trades. In Avery and Zemsky [1], imitative behaviors lead to an interesting short run mispricing and, with a more complex informational structure, can generate bubbles and crashes. In contrast, in Gervais [18], the uncertainty about the precision of private information can lead to a complete blockage of the learning process.

Decamps and Lovo [13] show that, by relaxing the hypothesis of risk neutral informed traders, history dependent behaviors and long run mispricing can arise, because the inventory component of trading motivations may outweigh the information one. A similar result is illustrated in Cipriani and Guarino [10].

Dasgupta and Prat [12] argue that the presence of reputational concerns induce found managers to ignore their private information and to act in a conformist way. This prevents prices to converge to the true asset value.

Finally, Lee [26] finds that total or partial informational cascades with no trade may develop if traders have to pay a one-time fixed fee to transact. Moreover, he introduces the concept of informational avalanche to explain a market crash as a procedure which corrects a public belief inconsistent with the distribution of private information.

A central purpose of almost all models on herd behavior and informational cascades in financial markets is to explain the empirical evidence of bubbles and market crashes, not fully addressed in standard asset pricing models. But, all models discussed so far have not a particular bias for a crash. In principle, in all those models, herd behavior can generate frenzies as often as crashes. This is a general criticism of almost all models given the empirical evidence that frenzies do not happen as often as crashes.

All the models described predict that herd behaviors and/or informational cascades can occur as the public belief gets concentrated in the extreme tails of the asset value distribution. May rational traders herd when the market is completely uncertain about the fundamental value?

Finally, in my view, the effect of informational cascades on the trading volume is not really clear. It would be interesting to investigate whether, as the informational content of transactions reduces, the size of orders decreases or not.

Chapter 2

Learning, cascades and fixed transaction costs

2.1 Introduction

Standard market microstructure models predict that when the information is dispersed among many agents and trades occur sequentially, the market gradually learns fundamentals and, eventually, prices converge to fundamental value.[1] In other words, in the long run, asset markets are viewed as being informational efficient.[2] However, recent theoretical literature has suggested new models of asset market behavior which conflict with the efficient market hypothesis.[3] In particular, the literature on informational cascades shows that sequential actions can produce herd behavior by rational agents and lead to a complete or partial information blockage.

In the previous chapter we discussed main issues achieved in the existing literature about informational cascades in securities markets with sequential trading mechanism. Recent research has emphasized several plausible market frictions which may generate informational cascades in financial economics although prices are endogenously determined.

In this chapter we investigate the effect of fixed processing order costs on the social learning in a capital market whit sequential trades and asymmetric information.

Market microstructure is mainly about the effect of trading frictions on price formation, so it is natural to ask whether such frictions are more conducive to informational cascades. Lee [26] finds that in an asset market with sequential trades,if traders bear a fixed brokerage cost to transact, the aggregation of the information may fail and a partial or total informational cascade develops. The main criticism of the model of Lee [26] is that the market maker ignores the implication of current trades for the information. At each trading round, he is assumed to set the price equal to the expected asset value conditional on the history of past trades. This assumption,

[1] See Glosten and Milgrom [19], Kyle [24].

[2] Informational efficiency refers to how much information is revealed by the price process. This is important in economies where information is dispersed among many individual. Prices are informational efficient if they fully and correctly reflect the relevant information. If prices do not correctly and fully reflect public information, then there would be a profitable trading opportunity for individuals. In general, this is ruled out in models with rational utility maximizing agents.

[3] See Kortian [23] for a survey of recent theoretical literature on asset price formation.

implausible in asset markets, is crucial for the main properties of the model.

In this work, we relax this hypothesis and analyze a Glosten - Milgrom type model with fixed transaction costs.[4] We prove that when the bid and ask prices are set by competitive market makers who bear a fixed cost per transaction, the market is not informational efficient. In other words, the price mechanism cannot prevent the occurrence of an informational cascade. In tune with the finding of Lee [26], we show that during the informational cascade, no informed traders will trade. This implies that in a cascade, the adverse selection component of bid-ask spread disappears and the spread decreases.

We also find a positive correlation between bid-ask spreads and trading volume. This is because traders endowed with private signals with moderate informational content place high weight on the previous price history in their investment decisions. Therefore, they will prefer to refrain from trading because of transaction costs. Namely, before the cascade occurs, the trading volume will gradually decrease and it will reach the minimum as the cascade develops.

Moreover, we show that if transaction costs are high enough, an informational cascade may occur even when the market is completely uncertain about the fundamental value. This is in contrast with results in the previous literature on informational cascades in stock markets which highlights that cascades develop when there is a convergence of beliefs in the market.

The outline of this chapter is as follows. The basic model with fixed transaction costs is presented in section 2.2. In section 2.3 we define and derive the market equilibrium. In 2.4 we prove the occurrence of informational cascades, while in section 2.5 we analyze the equilibrium during a cascade. Finally section 2.6 concludes. The appendix at the end of the chapter contains proofs of propositions, corollaries and lemmas.

[4] Glosten and Milgrom ([19]) introduce fixed transaction costs in their model to show that if the market maker pay a fixed processing cost then transaction prices changes exhibit negative serial correlation.

2.2 The environment

We consider a sequential trade model similar to Glosten and Milgrom [19]. The market is for a single risky asset whose value $\widetilde{V}$ depends on the state of nature. If the true state of nature is good, the asset value is $\overline{V}$; if it is bad, the value of the asset is $\underline{V}$, with $\overline{V} > \underline{V} \geq 0$. We assume that the initial prior probability $\pi_0 = P(\overline{V})$ of the high value is non-degenerate, that is, $\pi_0 \in (0, 1)$. The asset is exchanged among traders and market makers who are responsible for quoting prices. Trades take place in a sequential fashion, with one trader allowed to transact at any point in the time. Before a trader arrives, market makers simultaneously announce the bid and ask prices at which they are willing to buy and sell one unit of the asset. We assume that market makers are risk neutral and act competitively. In addition, we suppose that they incur a fixed order-processing cost c. The trader arriving in the market observes the prices and has the option to sell or buy one unit of the asset at the most attractive bid and ask prices, or to refrain from trading. The trader leaves the market after he had the opportunity to trade. He may trade further, but only after returning to the pool of traders and being selected again to trade.

A fraction μ of traders are uninformed liquidity traders while $1-\mu$ are informed. Liquidity traders trade for reasons exogenous to the model. To simplify the analysis, we assume that they choose to sell, buy, or refrain from trading with equal probability.[5] Informed traders are risk neutral, price taking agents[6] that privately observe a signal θ correlated with the asset value. They trade to maximize their expected profit. We denote $\Theta = \{\theta_1,\ \theta_2, \ldots,\ \theta_N\}$ the set of private signals, and assume that the private signals are conditionally independent and satisfy the monotone likelihood property:

$$0 < \lambda_1 < \lambda_2 < \cdots < 1 < \cdots < \lambda_{N-1} < \lambda_N < \infty$$

[5] In Glosten and Milgrom [19], liquidity traders trade as long as the reservation value dictated by their liquidity needs exceed the ask (bid) price for the buy (sell) order. As a consequence, if there are too many informed traders, then the market maker may have to set a spread so large as to preclude any trading at all. In order to avoid market collapse, we assume that the probability that liquidity traders buy and sell in each period is sufficiently high and is stationary.

[6] This assumption simplify the analysis by ruling out strategic behavior of informed traders. It is reasonable given that the trader's chance to trade again is zero since there are infinitely many informed traders in the pool.

where $\lambda_n = \frac{P(\theta_n|\widetilde{V}=\underline{V})}{P(\theta_n|\widetilde{V}=\overline{V})}$.

The signal θ_n is denoted "good" if the probability that a trader observes θ_n when the true asset value is $\overline{V}$, exceeds the probability of θ_n conditional on $\underline{V}$; that is, $\lambda_n < 1$. It is denoted "bad" in the opposite case. For simplicity, we assume that good and bad signals are symmetric, that is $\lambda_1 = \frac{1}{\lambda_N}$, $\lambda_2 = \frac{1}{\lambda_{N-1}}$, and so on.

Both market makers and traders are Bayesians who know the structure of the market. We denote by π_t the probability of $\overline{V}$ conditional on the publicly observable history of trades up until time t. Thus, the public belief about the true asset value at t is:

$$E_t[\widetilde{V}] = \pi_t \cdot \overline{V} + (1 - \pi_t) \cdot \underline{V} = \underline{V} + \pi_t \cdot (\overline{V} - \underline{V})$$

2.3 The equilibrium

Prior to each trading round, market makers set their prices. Bertrand competition and risk neutrality lead, in equilibrium, market makers to earn zero expected profit for every possible trade. After prices are set, a trader is randomly selected to trade. If he is informed, he chooses his optimal strategy given the prices. By assuming that market makers act competitively and that informed traders are price takers, we rule out all strategic behavior. Therefore, any trading round can be viewed as a four-stage game.

At stage 1, nature chooses the true asset value. Agents do not observe the realization of $\widetilde{V}$: they know only the probability π of $\widetilde{V} = \overline{V}$, which is common knowledge.

At stage 2, nature selects a trader to transact. With probability μ the trader will be uninformed and with probability $1 - \mu$ he will be informed. Informed traders differ from one another in the private signal $\theta \in \Theta$ they observe. There are N types of signals and then N types of informed traders. The signal distribution depends on the true asset value. The conditional distributions $P(\theta|\,\widetilde{V})$ are common knowledge. Agents do not observe the type of the selected trader.

At stage 3, market makers announce the price B_t at which they are willing to buy the asset and the price A_t at which they are willing to sell the asset. Perfect competition restricts the market makers to set these prices so as they earn zero expected profit.

At stage 4, the selected trader observes the prices and plays his strategy. He can choose to place a sell order (SO) at the highest bid price, to place a buy order (BO) at the lowest ask price, to refrain from trading (RT). We denote by $\mathcal{A} \equiv \{SO,\ BO,\ NT\ \}$ the traders' action space. Uninformed traders trade for exogenous reasons and submit sell and buy orders in the ex-ante specified probabilistic way. Informed traders choose the strategy that maximizes their expected profit given the price schedule. We denote $\sigma \equiv \{\sigma_\theta\}_{\theta\in\Theta}$ the informed traders' strategies, where σ_θ is the mixed strategy of the informed if he observes θ. Clearly:

$$\sigma_\theta \equiv (\sigma_{\theta,SO},\ \sigma_{\theta,BO},\ \sigma_{\theta,RT})$$

where $\sigma_{\theta,i}$ is the probability of i, with $i \in \mathcal{A}$, if the informed observes θ, $\Sigma_{i\in\mathcal{A}}\ \sigma_{\theta,i} = 1$, and $\sigma_{\theta,i} \geq 0\ \ \forall\, i \in \mathcal{A}$.

The expected profit of a market maker is equal to $E_t[\widetilde{V}|SO\ at\ B] - B - c$, if he buys at B, and it is equal to $A - E_t[\widetilde{V}|BO\ at\ A] - c$, if he sells at A. The expected profit of a trader endowed with signal θ, when the price schedule is $P = \{B,\ A\}$ and he plays the strategy $\sigma \in \Delta(\mathcal{A})$, is $E_t[\Pi_\theta(\sigma|P)] = \sigma_{\theta,SO}(B - E_t[\widetilde{V}|\theta]) + \sigma_{\theta,BO}(E_t[\widetilde{V}|\theta] - A)$.

Next, we define the equilibrium in the asset market as a sequentially rational Nash equilibrium.

Definition 2.1 *At each trading round* t*, the equilibrium in the asset market consists of a trading strategy correspondence* $\sigma^*_\theta(P|t) : R^2_+ \longrightarrow \Delta(\mathcal{A})$ *for each informed trader* $\theta \in \Theta$*, and a price schedule* $P^*_t = \{B^*_t,\ A^*_t\}$ *such that:*

1. $\sigma^*_\theta(P|t) = \arg\max_{\sigma\in\Delta(\mathcal{A})} E_t[\Pi_\theta(\sigma|P)], \quad \forall P \in R^2_+,\ \ \forall\theta \in \Theta$
2. $B^*_t \in E_t[\widetilde{V}|SO\ at\ B^*_t,\, \sigma^*(P^*_t|t)] - c$
3. $A^*_t \in E_t[\widetilde{V}|BO\ at\ A^*_t,\, \sigma^*(P^*_t|t)] + c$
4. $B^*_t \leq A^*_t$

where $\sigma^*(P^*_t|t) = \{\sigma^*_\theta(P|t)\}_{\theta\in\Theta}$.

The first equilibrium condition means that informed traders maximize their expected profit given the price schedule, trade by trade. The second and third conditions imply that market makers determine the price schedule such that they anticipate zero expected profit from each trade. It is clear that, if there are no fixed transaction costs, the equilibrium bid and ask prices are equal to the conditional expectation of $\widetilde{V}$ given a sell or a buy order respectively. The last condition says that at the equilibrium the price at which market makers buy the asset must be lower than (or at most equal to) the price at which they sell the asset.

We can now provide an useful characterization of the equilibrium in term of informed traders' optimal strategy, and then prove the existence and the uniqueness of zero profit equilibrium prices. We will see that equilibrium bid and ask prices straddle the public belief about the true asset value.

The last condition of definition 2.1 states that the equilibrium bid price does not exceed the equilibrium ask price. Proposition 2.1 describes the optimal strategies of informed traders when they face a price schedule that satisfies this condition.

Proposition 2.1 *For any $P = \{B, A\}$ such that $B \leq A$, the optimal strategy of the traders endowed with the private signal $\theta \in \Theta$ is $\sigma^*_\theta(P|t)$ equal to:*

$$\left\{ \begin{array}{lll} \sigma^*_{\theta,SO}(P|t) = 1 & & if\ E_t[\widetilde{V}|\theta] < B \\ \sigma^*_{\theta,SO}(P|t) = \alpha & \sigma^*_{\theta,RT}(P|t) = 1 - \alpha & if\ E_t[\widetilde{V}|\theta] = B \\ \sigma^*_{\theta,RT}(P|t) = 1 & & if\ B < E_t[\widetilde{V}|\theta] < A \\ \sigma^*_{\theta,BO}(P|t) = \beta & \sigma^*_{\theta,RT}(P|t) = 1 - \beta & if\ E_t[\widetilde{V}|\theta] = A \\ \sigma^*_{\theta,BO}(P|t) = 1 & & if\ E_t[\widetilde{V}|\theta] > A \end{array} \right. .$$

where α and β are any real number belonging to the interval $[0, 1]$.

Proof: See Appendix.

Informed traders are profit maximizing agents. Hence, for any price schedule $P = \{B, A\}$ such that $B \leq A$, traders observing the private signal $\theta \in \Theta$ optimally prefer to sell the asset if their conditional expectation of its value is below the bid price; to place a buy order if their conditional expectation is above the ask price; to refrain from trading if the bid and ask prices straddle their valuation. Finally, if their valuation is equal to the bid or to the ask prices, they are indifferent between all mixed strategies defined on the simplex $\Delta(SO,\, RT)$ in the first case or on the simplex $\Delta(BO,\, RT)$ in the second case.

The next lemma establishes a link between the optimal trading strategies of traders endowed with different signals.

Lemma 2.1 *If $\sigma^*_{\theta_n,\, SO}(P|t) \neq 0$, then $\sigma^*_{\theta_{n+i},\, SO}(P|t) = 1$ for any $i \in \{1, 2, ..., N-n\}$. If $\sigma^*_{\theta_n,\, BO}(P|t) \neq 0$, then $\sigma^*_{\theta_{n-j},\, BO}(P|t) = 1$ for any $j \in \{1, 2, ..., n-1\}$.*

Proof: See Appendix.

Informed traders maximize the expected profit given their information set. The information set includes both the whole history of trades and the trader's private signal. For the maximum likelihood ratio property of signals, the expected asset value of traders observing the signal θ_i exceeds the expectation of traders observing θ_j for any $j < i$. Hence, if it is profitable for traders endowed with signal θ_n to sell the asset when the price schedule is P, then all traders observing signals θ_{n+i}, with $i \in \{1, 2, ..., N-n\}$, optimally prefer to sell the asset as well. Similarly, if the traders endowed with the signal θ_n prefer to buy the asset, then all traders endowed with signals θ_{n-j}, with $j \in \{1, 2, ..., n-1\}$, prefer to do the same.

Let define the informational content of a trading order as the likelihood ratio of that order. More precisely, the informational content λ_t^{SO} of a sell order arriving at t is :

$$\lambda_t^{SO} \equiv \frac{P_t(SO \text{ } at \text{ } B|\underline{V})}{P_t(SO \text{ } at \text{ } B|\overline{V})}$$

and the informational content λ_t^{BO} of a buy order arriving at t is :

$$\lambda_t^{BO} \equiv \frac{P_t(BO \text{ } at \text{ } A|\underline{V})}{P_t(BO \text{ } at \text{ } A|\overline{V})}.$$

It is straightforward to notice that an order indicates $\overline{V}$ if the probability of its occurrence is greater in the good state of nature, that is, the likelihood ratio is lower than 1. It indicates the low asset value if the likelihood ratio is greater than 1. Finally, a trading order is uninformative about the true asset value if the probability of observing it does not depend on the true asset value, that is, likelihood ratio is exactly equal to 1. Clearly, the more the likelihood ratio differs from 1, the more informative the order is about the true asset value.

An implication of Lemma 2.1 is that a sell order generally indicates $\underline{V}$, while a buy order generally indicates $\overline{V}$. To see that, let define the "marginal selling trader" at t, given the price schedule $P = \{B, A\}$ with $B \leq A$, as the trader endowed with signal $\theta_{n_t^b(B)}$ such that:

$$E_t[\widetilde{V}|\theta_{n_t^b(B)}] \leq B \quad \text{and} \quad E_t[\widetilde{V}|\theta_{(n_t^b(B)-1)}] > B$$

and the "marginal buying trader" at t as the trader endowed with signal $\theta_{n_t^a(A)}$ such that:

$$E_t[\widetilde{V}|\theta_{n_t^a(A)}] \geq A \quad \text{and} \quad (E_t[\widetilde{V}|\theta_{n_t^a(A)+1)}] < A.$$

Given the traders' strategy described above, the likelihood ratio of a sell order arriving at t, when the price schedule is $P = \{B, A\}$, is equal to:

$$\lambda_t^{SO}(B) = \frac{\frac{\mu}{3} + (1-\mu)\sum_{i=n_t^b(B)}^{N} P(\theta_i|\underline{V})}{\frac{\mu}{3} + (1-\mu)\sum_{i=n_t^b(B)}^{N} P(\theta_i|\overline{V})}$$

and the likelihood ratio of a buy order is equal to:

$$\lambda_t^{BO}(A) = \frac{\frac{\mu}{3} + (1-\mu)\sum_{i=1}^{n_t^a(A)} P(\theta_i|\underline{V})}{\frac{\mu}{3} + (1-\mu)\sum_{i=1}^{n_t^a(A)} P(\theta_i|\overline{V})}$$

where $\frac{\mu}{3}$ is the probability that the trading order comes from a liquidity trader, and $\theta_{n_t^b(B)}$ and $\theta_{n_t^a(A)}$ are respectively the signals of marginal selling and buying traders.[7]

[7] For simplicity, here we suppose that the marginal traders choose to place an order also if they are indifferent between trade and to refrain from trading.

Since $\sum_{i=n}^{N} P_t(\theta_i|\underline{V}) \geq \sum_{i=n}^{N} P_t(\theta_i|\overline{V})$ for all $n = 1,\ 2, \ldots, N$, the probability of a sell order is greater in the bad state of nature. Similarly, since $\sum_{i=1}^{n} P_t(\theta_i|\underline{V}) \leq \sum_{i=1}^{n} P_t(\theta_i|\overline{V})$ for all $n = 1,\ 2, \ldots, N$, the probability of a buy order is greater in the good state of nature. This shows that a sell order generally indicates $\underline{V}$, and a buy order generally indicates $\overline{V}$.

We now analyze the equilibrium behavior of market makers. From Lemma 2.1, it follows that if equilibrium bid and ask prices exist, they straddle the unconditional expected asset value, which is the price that would prevail in the absence of adverse selection and fixed transaction costs.

Proposition 2.2 *If a price schedule $P_t^* = \{B_t^*,\ A_t^*\}$ satisfying conditions 2 and 3 of Definition 2.1 exists, the equilibrium bid and ask prices are such that:*

$$B_t^* \leq E_t[\widetilde{V}] \leq A_t^*.$$

Proof: See Appendix.

Market makers know that orders may come from either a liquidity trader or an informed trader, but they cannot tell them apart. When they are trading with informed traders, they lose on average. Hence, they have to balance the losses to the informed traders with the gains from the liquidity traders. Moreover, market makers pay a fixed cost c to process each trading order. To recover the losses due to both the adverse selection and the order processing cost, market makers set a spread between the price at which they are willing to sell the asset and the price at which they are willing to buy the asset. Namely, if the probability that an informed trader places a trading order is greater than 0 and/or the fixed transaction cost is strictly positive, then $B_t^* \leq E_t[\widetilde{V}] \leq A_t^*$.

The next proposition states the existence and the uniqueness of equilibrium bid and ask prices.

Proposition 2.3 *In each period t there exists a unique price schedule $P_t^* = \{B_t^*,\ A_t^*\}$ that satisfies conditions 2 and 3 of Definition 2.1.*

Proof: See Appendix.

2.4 Informational cascades with costly market-making

In this section we study the occurrence of informational cascades in the asset market just described.

If all informed traders make the same choice regardless of their private signal, no new information reaches the market. Hence, during an informational cascade, the market equilibrium is such that: $\sigma^*_\theta(P^*_t|t) = \sigma^*(P^*_t|t)$ for all $\theta \in \Theta$, and $B^*_t = E_t[\widetilde{V}] - c$ and $A^*_t = E_t[\widetilde{V}] + c$ since $\lambda^{SO}_t(B^*_t) = \lambda^{BO}_t(A^*_t) = 1$.

It is straightforward to notice that in equilibrium traders endowed with a bad signal never place a buy order, and traders endowed with a good signal never place a sell order. Indeed, the asset valuation of traders with a bad signal is always lower than the unconditional expected asset value, while that of traders observing a good signal always exceeds it. Proposition 2.2 dictates that the unconditional expected asset value is the upper bound of the equilibrium bid price and the lower bound of the equilibrium ask price. Therefore, traders endowed with a bad signal will never find worthwhile to buy the asset, and traders endowed with a good signal will never find worthwhile to sell the asset. This implies that informational cascades characterized by all informed traders placing orders in the same direction will never occur in the market equilibrium. Nevertheless, if fixed transaction costs are positive, the equilibrium price schedule may be such that all informed traders choose to refrain from trading, since transaction costs induce higher bid ask spreads. On the other hand, the expectation of market makers and that of informed traders converge as the number of transactions increases.[8] Hence, there will be a moment when the informational advantage of the informed is so small relative to the fixed transaction cost that it is no longer profitable for any informed trader to place an order.

A no-trade informational cascade starts if $E_t[\widetilde{V}|\theta] > B^*_t$ and $E_t[\widetilde{V}|\theta] < A^*_t$ for any $\theta \in \Theta$. Clearly, if an informational cascade oc-

[8] See Glosten and Milgrom [19].

curs, orders have no informational content and hence $B_t^* = E_t[\widetilde{V}] - c$ and $A_t^* = E_t[\widetilde{V}] + c$.

Proposition 2.4 $B_t^* = E_t[\widetilde{V}] - c$ *if, and only if:*

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c$$

and $A_t^* = E_t[\widetilde{V}] + c$ *if, and only if:*

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c.$$

Proof: See Appendix.

Proposition 2.4 establishes that when (and only when) the informational advantage of traders with the most informative bad signal θ_N is lower than the fixed transaction cost, the equilibrium bid price is equal to the public belief about the true asset value minus the transaction cost c. Symmetrically, when (and only when) the informational advantage of traders endowed with the most informative good signal θ_1 is lower than the fixed transaction cost, the equilibrium ask price is equal to the public belief about the true asset value plus c.

The valuation of traders endowed with bad signals less informative than θ_N, exceeds that of traders observing θ_N. This means that:

$$E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c \Longrightarrow E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta] \leq c$$

for all bad signals $\theta \in \Theta$. Likewise, the valuation of traders endowed with good signals less informative than θ_1, is lower than that of traders who observe it. This means that:

$$E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \leq c \Longrightarrow E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}] \leq c$$

for all good signals $\theta \in \Theta$. Hence, if the informational advantage of all informed traders is lower than the transaction cost, no trader observing a private signal will place a trading order. Since all informed traders act alike, no new information reaches the market and an informational cascade starts.

It is straightforward that until the uncertainty is resolved, the informational advantage of informed traders is strictly positive. As a consequence, in the absence of fixed transaction costs, an informational cascade would never occurs. On the other hand, if for any possible history of trades the fixed transaction cost is greater than the advantage of informed traders, then orders will never be information-based.

To determine the minimum level of c such that no informed trader prefers to trade, define the functions $h_\theta(\pi)$ as:

$$h_\theta(\pi) \equiv |E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}]| = \frac{\pi - \pi^2}{\pi + (1-\pi)\lambda_\theta} \cdot |1 - \lambda_\theta| \cdot (\overline{V} - \underline{V}).$$

$h_\theta(\pi)$ gives the informational advantage of traders observing θ for any $\pi \in [0,\ 1]$. Since signals are assumed to be symmetric, it is easy to see that:

$$\max h_{\theta_N}(\pi) = \max h_{\theta_1}(\pi) = \frac{|1 - \sqrt{\lambda_1}|}{1 + \sqrt{\lambda_1}} \cdot (\overline{V} - \underline{V}) \equiv \overline{c}.$$

We know that the informational advantage of traders endowed with the signals θ_N and θ_1 always exceeds that of traders observing a different signal. Hence, if $c > \overline{c}$, all informed traders will prefer to refrain from trading whatever the history of trades is.

The next proposition states that if transaction costs are below $\overline{c}$ then an informational cascade occurs as the public belief approaches the extreme values of the distribution of the true asset value.

Proposition 2.5 *If* $c \in (0,\ \overline{c}\ )$ *there exist unique* $\pi^l_{\theta_1}$ *and* $\pi^u_{\theta_N}$*, with* $\pi^l_{\theta_1} < \pi^u_{\theta_N}$*, such that when* $\pi \in [0,\ \pi^l_{\theta_1}) \cup (\pi^u_{\theta_N},\ 1]$ *all informed traders refrain from trading in equilibrium.*

Proof: See Appendix.

Proposition 2.5 establishes that, if the transaction cost is small enough, there exist a lower bound $\pi^l_{\theta_1}$ and an upper bound $\pi^u_{\theta_N}$ of public beliefs such that if the unconditional expected asset value is lower than $E[\widetilde{V}|\pi^l_{\theta_1}]$ or greater than $E[\widetilde{V}|\ \pi^u_{\theta_N}]$, then no informed trader prefers to trade.

The last result is a consequence of the beliefs convergence. Suppose that at $t = 0$ the unconditional expected asset value $E_0[\widetilde{V}]$ is

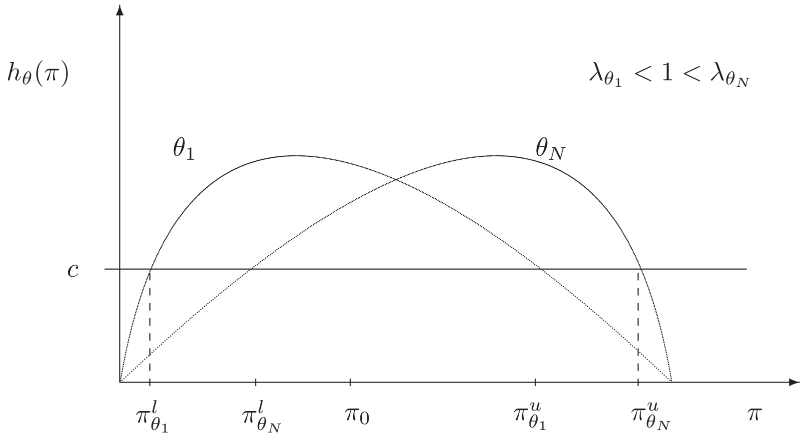

Figure 2.1: *Informational cascades in a market with positive fixed transaction costs.*

such that at least traders endowed with signals θ_1 and θ_N prefer to trade (see figure 2.1). Suppose also that a sequence of buy orders arrive. Because of the new information that reaches the market, the public belief about the true asset value moves toward $\overline{V}$. Hence the bid and ask prices increase as well as the asset assessment of informed traders. If the public belief rises too much, the informational advantage of traders endowed with signal θ_1 becomes so small that it is not worthwhile for them to buy the asset. Moreover, if $E_0[\widetilde{V}]$ is low enough, at the beginning of the arrival process the expected profit from selling of traders endowed with signal θ_N increases because the bid price rises. Nevertheless, if the buy orders are very numerous, the expected profit from selling begins to fall off until it becomes negative. This occurs because in the traders' asset assessment the relative weight of the information inferred from the history of trades grows with respect to the traders' private information. If enough buy orders are placed, so that the unconditional expected asset value grows bigger than $E[\widetilde{V}|\pi^u_{\theta_N}]$, no informed trader places a sell order although the bid price is very high. At that point, an informational cascade occurs. A similar argument can be used to show the occurrence of an informational cascade when the public belief tends to $\underline{V}$.

An informational cascade develops because the asset assessment of each informed trader depends not only on his private signal but also on the history of actions taken by previous traders. If in the market there are perfectly informed traders, an informational cascade never occurs. Indeed, if signals θ_1 and θ_N are perfectly informative (that is, $P(\theta_1|\underline{V}) = P(\theta_N|\overline{V}) = 0$) then when the public belief tends to $\underline{V}$, traders endowed with signal θ_1 will always buy the asset, and when the public belief tends to $\overline{V}$, traders endowed with signal θ_N will always sell the asset.

The next proposition states that if transaction costs are high enough, an informational cascade may start even when the market is completely uncertain about the true asset value, that is π close to $\frac{1}{2}$. To see this, notice that if the most informative good and bad signals are symmetric then $h_{\theta_N}(\frac{1}{2}) = h_{\theta_1}(\frac{1}{2}) = \frac{|1-\lambda_1|}{2(1+\lambda_1)} \cdot (\overline{V} - \underline{V}) = \underline{c}$. That is, if the market is completely uncertain about the true asset value, then the informational advantage of traders endowed with signal θ_1 is equal to that of traders observing signal θ_N. Proposition 2.6 establishes that if the fixed transaction cost exceeds this bound, there exists a neighborhood of $\frac{\overline{V}+\underline{V}}{2}$ such that an informational cascade occurs also if the unconditional expected asset value, $E_t[\widetilde{V}]$, is within this neighborhood.

Proposition 2.6 *If* $\underline{c} < c < \overline{c}$ *then there exist* $\pi^u_{\theta_1} \in (\pi^l_{\theta_1}, \frac{1}{2})$ *and* $\pi^l_{\theta_N} \in (\frac{1}{2}, \pi^u_{\theta_N})$ *such that an informational cascade occurs when* $\pi \in (\pi^u_{\theta_1}, \pi^l_{\theta_N})$.

Proof: See Appendix.

To understand intuitively why cascades can occur under the conditions described in Proposition 2.6, consider the following argument. The informational advantage of traders endowed with a good signal exceeds that of traders observing an equally informative bad signal when $\pi < \frac{1}{2}$, and it is lower in the opposite case. This implies that if the transaction cost is high, no trader with a bad signal chooses to place a sell order when $E_t[\widetilde{V}]$ is below $\frac{\overline{V}-\underline{V}}{2}$, and no trader observing a good signal chooses to buy the asset when $E_t[\widetilde{V}]$ exceeds

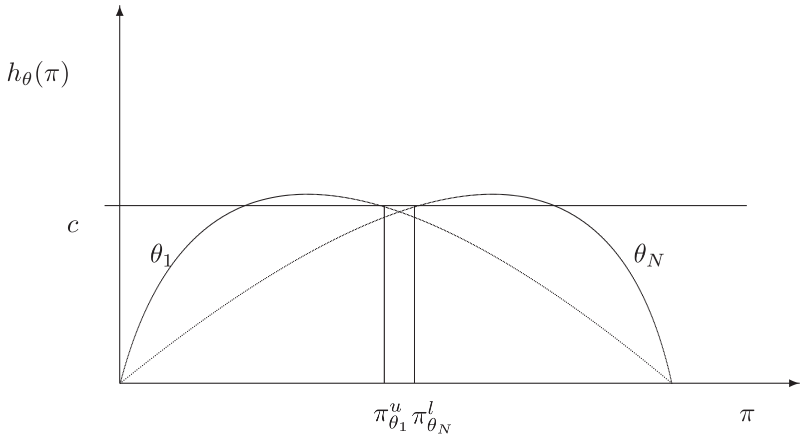

Figure 2.2: *Informational cascades in a market with high fixed transaction costs.*

$\frac{\overline{V}-\underline{V}}{2}$ (see figure 2.2). As a consequence, when the bid price is low, sell orders are uninformative about the true asset value and when the ask price is high, buy orders are uninformative. Moreover, if $E_t[\widetilde{V}]$ is into the interval $(E[\widetilde{V}|\pi^u_{\theta_1}],\ E[\widetilde{V}|\pi^l_{\theta_N}])$ no informed trader will place an order and an informational cascade occurs. Therefore, if transaction costs are large enough, an informational cascade may develop even when the public belief is not concentrated on $\overline{V}$ or $\underline{V}$. This result contrasts with the typical finding of the literature that informational cascades tend to occur when there is convergence of beliefs.

2.5 Informational content of orders and fixed transaction costs

In this section we analyze the effect of fixed transaction costs on the informational content of trading orders in equilibrium. To this aim, we first prove that in the absence of order processing costs, the competitive price mechanism leads to equilibrium bid and ask prices that maximize the informational content of orders. Then we show that positive transaction costs reduce the information that the market can infer from both buy and sell orders.

The informational content of a trading order depends on the set of informed traders who prefer to place the order. From Lemma 2.1 it follows that, given the price schedule $P = \{B, A\}$, the set of informed traders who sell the asset is $\Theta_t^b(B) = \{\theta_n \in \Theta : n \geq n_t^b(B)\}$ and the set of informed traders who buy the asset is $\Theta_t^a(A) = \{\theta_n \in \Theta : n \leq n_t^a(A)\}$, with $n_t^b(B)$ and $n_t^a(A)$ being the signals respectively of the marginal selling and buying traders.

Lemma 2.2 *There exist a bad signal θ_{n^b} and a good signal θ_{n^a} such that, for all trading histories:*

- *the informational content of a sell order is maximum if $n_t^b(B) = n^b$*
- *the informational content of a buy order is maximum if $n_t^a(A) = n^a$.*

Proof: See Appendix.

Lemma 2.2 establishes that there exist a set of selling traders, $\Theta_{n^b} = \{\theta_n \in \Theta : n \geq n^b\}$, and a set of buying traders, $\Theta_{n^a} = \{\theta_n \in \Theta : n \leq n^a\}$, which maximize the informational content respectively of sell and buy orders, for any given history of trades. Then the price schedule $P = \{B, A\}$ maximize the information of orders at t if $\Theta_t^b(B) = \Theta_{n^b}$ and $\Theta_t^a(A) = \Theta_{n^a}$.

The next proposition states that in the absence of fixed transaction costs, perfect competition among market makers leads to equilibrium bid and ask prices such that: $\Theta_t^b(B_t^*) = \Theta_{n^b}$ and $\Theta_t^a(A_t^*) = \Theta_{n^a}$ for all trading histories.

Proposition 2.7 *If $c = 0$, the equilibrium bid and ask prices, B_t^* and A_t^*, are such that $\Theta_t^b(B_t^*) = \Theta_{n^b}$ and $\Theta_t^a(A_t^*) = \Theta_{n^a}$ $\forall$ t.*

Proof: See Appendix.

To gain intuition about this result, denote $E_t[\widetilde{V}|\Theta_{n^a}]$ the expected asset value conditional on a buy order, when the set of informed buying traders is Θ_{n^a}. Clearly: $E_t[\widetilde{V}|BO\ at\ A] \leq E_t[\widetilde{V}|\Theta_{n^a}] \leq E_t[\widetilde{V}|\theta_{n^a}]$ for all ask price A. In the absence of order processing costs, the expected profit of market makers is equal to the difference between the ask price and the conditional expectation. It is easy to see that if traders observing θ_{n^a} refrain from trading, that is the ask price exceeds $E_t[\widetilde{V}|\theta_{n^a}]$, the expected profit from selling of market makers is positive. On the other hand, if traders observing good signals less informative than θ_{n^a} buy the asset, that is A is below $E_t[\widetilde{V}|\theta_{(n^a+i)}]$, the information that market makers can infer from a buy order is more precise than the signal $\theta_{(n^a+i)}$. This implies that $E_t[\widetilde{V}|BO\ at\ A]$ exceeds the valuation of marginal traders and hence the expected profit from selling of market makers is negative. As a consequence, the equilibrium ask price belongs to the interval $(E_t[\widetilde{V}|\theta_{(n^a+1)}], E_t[\widetilde{V}|\theta_{n^a}]]$, and then $n^a_t(A^*_t) = n^a$.

The results of Proposition 2.7 suggest that in an asset market with competitive price mechanism, if the adverse selection is the only source of the bid-ask spread, the informational content of both sell and buy orders is always maximum in equilibrium.

When market makers pay a fixed cost to process orders, the equilibrium bid price is lower than the conditional expectation of the asset value, given a sell order, and the equilibrium ask price is greater than the conditional expectation of the asset value, given a buy order. As a consequence, fixed transaction costs reduce the informational content of both sell and buy orders, for any given history of trades.

An implication of Proposition 2.7 is that, in the absence of transaction costs, the set of informed traders whose asset assessment is lower than the equilibrium bid price, and the set of informed traders whose asset assessment is greater than the equilibrium ask price do not depend on the trading history and are constant over time. As a result, the occurrence probability of a trading order, conditional on the true asset value, is the same at each t. If we consider the expected trading volume as the occurrence probability of a trading order, Proposition 2.7 suggests that in equilibrium, the expected trading volume is constant over time and does not depend on the public belief about the true asset value.

This result does not apply to a market characterized by fixed order processing costs. We know that when an informational cascade occurs, no trading order comes from an informed trader. This means that, during an informational cascade, the expected trading volume is equal to the probability that a liquidity trader is selected to trade and places an order. Moreover, before an informational cascade starts, the probability of an order gradually decreases because traders endowed with less informative signals prefer to refrain from trading. To show this, suppose that at $t = 0$ traders endowed with signal θ_n prefer to buy the asset, that is:

$$E_0[\widetilde{V}|\theta_n] - A_0^* > 0.$$

Suppose then, that a sequence of buy orders arises. The informational advantage of traders endowed with signal θ_n decreases as the public belief about the true asset value approaches $\overline{V}$. In particular, it becomes negative before the public belief grows larger than $E[\widetilde{V}|\pi^u_{\theta_1}]$ because the valuation of traders observing θ_n is lower than the valuation of traders endowed with θ_1. Because the ask price exceeds the public belief, traders endowed with signal θ_n prefer to refrain from trading before an informational cascade starts. Therefore, the probability of a trading order is not constant over time. We conclude that, in an asset market with fixed transaction costs, the trading volume decreases as the public belief approaches $\overline{V}$ or $\underline{V}$, and it is positively correlated with the bid-ask spread.

2.6 Conclusions

We have presented a model of the effect of fixed transaction costs on the informational efficiency of prices in an asset market characterized by asymmetric information and sequential trading mechanism.

Standard microstructure models predict that prices ultimately converge to the true asset value. However, most of the models that analyze the price mechanism in asset markets with asymmetrically informed agents, do not take into account transaction costs. Lee [26] who does consider fixed brokerage costs in a setting with asymmetric information and sequential trading, does not allow for optimizing behavior by market makers. In contrast, in this paper market makers are assumed to behave optimally and competitively in setting prices. The results show that the competitive price mechanism does not prevent the occurrence of informational cascades. Moreover, large fixed transaction costs may lead to an informational cascade not only when the market attaches a large probability to the high or to the low asset value and then prices are very high or very low, as in Lee [26], but even when there is complete uncertainty about the asset's fundamental value.

Appendix

Proof of Proposition 2.1

The proposition is an immediate consequence of definition 2.1. □

Proof of Lemma 2.1

We prove the result for the ask side of the market. The proof for the bid side is similar. Suppose that $\sigma^*_{\theta_n, BO}(P|t) \neq 0$. This means that, when the price schedule is P and the probability of $\overline{V}$ is π_t, the expected profit from buying of traders observing θ_n is not negative and greater or at most equal to the expected profit from selling. For the maximum likelihood ratio property of private signals, $E_t[\widetilde{V}|\theta_n] < E_t[\widetilde{V}|\theta_{n-j}]$. Then, the expected profit from buying of traders observing θ_{n-j} is always greater than the one of traders endowed with θ_n. This proves the thesis. □

Proof of Proposition 2.2

Bertrand competition among market makers leads to equilibrium bid and ask prices respectively equal to the expected asset value conditional to a sell order minus the transaction cost and the expected asset value conditional to a buy order plus the transaction cost. When the price schedule is $P = \{B, A\}$, the expected asset value given a sell order is equal to:

$$E_t[\widetilde{V}|SO\ at\ B,\, \sigma^*(P|t)] = \underline{V} + (\overline{V} - \underline{V})\frac{\pi_t}{\pi_t + (1-\pi_t)\lambda_t^{SO}(B)}$$

and the expected asset value given a buy order is equal to:

$$E_t[\widetilde{V}|BO\ at\ A,\, \sigma^*(P|t)] = \underline{V} + (\overline{V} - \underline{V})\frac{\pi_t}{\pi_t + (1-\pi_t)\lambda_t^{BO}(A)}.$$

Since $\lambda_t^{SO}(B) \geq 1$ and $\lambda_t^{BO}(A) \leq 1$ for all π and P, the expected asset value conditional to a sell order is always lower or equal to the unconditional expectation, while the expected asset value conditional to a buy order is always greater or equal to it. □

Lemma 2.3 *The marginal selling trader affects positively the informational content of a sell order if, and only if:*

- $\lambda_{\theta_{n_t^b(B)}} > \lambda_t^{SO}(B)$.

The marginal buying trader affects positively the informational content of a buy order if, and only if:

- $\lambda_{\theta_{n_t^a(A)}} < \lambda_t^{BO}(A)$.

Proof

We prove the result for the ask side. The result for the bid side follows from symmetry.

Let $f^a(n) = \frac{\frac{\mu}{3}+(1-\mu)\sum_{i=1}^{n} P(\theta_i|\underline{V})}{\frac{\mu}{3}+(1-\mu)\sum_{i=1}^{n} P(\theta_i|\overline{V})}$. In order to prove the lemma for the ask side, we have to show that:

$$f^a(n) \geq f^a(n-1) \Longleftrightarrow \lambda_n \geq f^a(n).$$

By using algebraic calculus, it is easy to show that:

1. $f^a(n) \geq f^a(n-1) \Longleftrightarrow \lambda_n \geq f^a(n-1)$
2. $\lambda_n \geq f^a(n-1) \Longleftrightarrow \lambda_n \geq f^a(n)$.

By combining these results, we obtain:

$$f^a(n) \geq f^a(n-1) \Longleftrightarrow \lambda_n \geq f^a(n).$$

Since $f^a(n_t^a(A)) = \lambda_t^{BO}(A)$, the lemma for the ask side is proved. □

Proof of Proposition 2.3

We prove the proposition for the ask price. The proof for the bid price can be obtained with a symmetric argument.

For the monotone likelihood property of signals, the expected asset value conditional on a buy order is always greater or equal to the unconditional expected asset value. This implies that, if the ask price is lower than $E[\widetilde{V}|H] + c$, the market maker's expected profit is negative. Moreover, $E[\widetilde{V}|H,\ BO\ at\ A]$ is upper bounded by $\overline{V}$. Hence, for the zero expected profit condition, the equilibrium ask price cannot be greater than $\overline{V}+c$. Let define the correspondence $F_t^a : [E_t[\widetilde{V}]+c,\ \overline{V}+c] \rightsquigarrow [E_t[\widetilde{V}]+c,\ \overline{V}+c]$ as:

$$F_t^a(A) \equiv E_t[\widetilde{V}|BO\ at\ A]+c$$

$F_t^a(A)$ is an upper semicontinuous convex valued correspondence that maps the set $[E_t[\widetilde{V}]+c,\ \overline{V}+c]$ in itself. For the Kakutani's fixed point theorem, $F_t^a(A)$ has a fixed point A_t^*, that is, an equilibrium ask price always exists.

The uniqueness is proved by using the results of Lemma 2.3, that states that the marginal buying (selling) trader affects positively the informational content of a buy (sell) order, that is the likelihood ratio of the buy (sell) order increases (decreases) when the marginal buying (selling) trader refrains from trading, if the signal that he observes is more accurate than the information that the market maker infers from a buy (sell) order.

Suppose, by way of obtaining a contradiction, that there exist two equilibria ask prices: A_1 and A_2, with $A_1 < A_2$. Denote $n_t^a(A_1)$ and $n_t^a(A_2)$ respectively the marginal buying trader given A_1 and the marginal buying trader given A_2. Clearly, since we suppose $A_1 < A_2$, it has to be $\lambda_{n_t^a(A_1)} > \lambda_{n_t^a(A_2)}$. First, we notice that at the equilibrium, the asset assessment of the marginal buying trader is greater or at most equal to the market maker's conditional expected asset value. Hence $\lambda_{n_t^a(A_1)} \leq \lambda_t^{BO}(A_1)$. This implies that $\lambda_t^{BO}(A_1) < \lambda_t^{BO}(A_2)$ because, when the ask price grows to A_2, the traders endowed with the signal $\theta_{n_t^a(A_1)}$ prefer to refrain from trading, and then the likelihood ratio of a buy order increases by lemma 2.3. As a consequence, if the ask price is equal to A_2, the market maker's expected profit is strictly positive. Hence, for the zero expected profit condition, A_2 cannot be an equilibrium ask price. $\square$

Proof of Proposition 2.4

We prove the proposition for the ask price; the proof for the bid price is symmetric. First we assume that $A_t^* = E_t[\widetilde{V}]+c$ and we prove that this implies $E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N] \leq c$. Suppose by way of obtaining a contradiction that $E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] > c$. Since at least the traders observing θ_1 prefer to buy the asset when the ask price is $E_t[\widetilde{V}] + c$, the expected asset value conditional to a buy order at $E_t[\widetilde{V}] + c$ is greater than the unconditional expected asset value. This implies that the market maker's expected profit is negative, which contradict the fact that $E_t[\widetilde{V}] + c$ is the equilibrium ask price.

On the other hand, if $E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}] \le c$ then no informed trader prefers to buy the asset. Hence $\inf\{E_t[\widetilde{V}|BO\ at\ E_t[\widetilde{V}] + c,\ \sigma^*(P_t^*|t)] + c\} = E_t[\widetilde{V}] + c = A_t^*$. $\square$

Proof of Proposition 2.5

Consider the function h_θ defined at page 47. Notice that:

- $h_\theta(0) = h_\theta(1) = 0$
- $\max h_\theta(\pi) = \frac{|1-\sqrt{\lambda}|}{1+\sqrt{\lambda}} \cdot (\overline{V} - \underline{V})$
- $h_\theta^{''}(\pi) = -\frac{2\lambda}{(\pi+(1-\pi)\lambda)^3} \cdot |1-\lambda_i| \cdot (\overline{V} - \underline{V}) < 0\ \forall \pi \in [0,\ 1]$.

Since by assumption $0 < c \le \frac{|1-\sqrt{\lambda}|}{1+\sqrt{\lambda}} \cdot (\overline{V} - \underline{V})$, from the strictly concavity of $h_\theta(\pi)$ it follows that there exist unique π_θ^l and π_θ^u, with $\pi_\theta^l < \pi_\theta^u$, such that $h_\theta(\pi_\theta^l) = h_\theta(\pi_\theta^u) = c$ and $h_\theta(\pi) > c$ if, and only if, $\pi \in (\pi_\theta^l,\ \pi_\theta^u)$. Thus, when $\pi \in (0,\ \pi_\theta^l) \cup (\pi_\theta^u,\ 1)$, traders endowed with signal θ prefer to refrain from trading. Moreover

$$\arg\max h_{\theta_1}(\pi) = \frac{\sqrt{\lambda_1}}{1+\sqrt{\lambda_1}} < \frac{1}{2} < \arg\max h_{\theta_N}(\pi) = \frac{\sqrt{\lambda_N}}{1+\sqrt{\lambda_N}}$$

and

$$h_{\theta_1}(\pi) \ge h_{\theta_N}(\pi) \quad \forall \pi \le \frac{1}{2}$$

As $\pi_{\theta_1}^l < \arg\max h_{\theta 1}(\pi)$ and $\pi_{\theta_N}^u > \arg\max h_{\theta_N}(\pi)$, then $\pi_{\theta_1}^l < \pi_{\theta_N}^l$ and $\pi_{\theta_N}^u > \pi_{\theta_1}^u$. Hence, when either $\pi < \pi_{\theta_1}^l$ or $\pi > \pi_{\theta_N}^u$, all informed traders prefer to refrain from trading. $\square$

Proof of Proposition 2.6

From the proof of Proposition 2.5, we know that there exist $\pi_{\theta_1}^u > \pi_{\theta_1}^l$ and $\pi_{\theta_N}^l < \pi_{\theta_N}^u$ such that $h_{\theta_1}(\pi_{\theta_1}^u) = h_{\theta_N}(\pi_{\theta_N}^l) = c$. Since

$h'_{\theta_1}(\frac{1}{2}) < 0$ and $h'_{\theta_N}(\frac{1}{2}) > 0$, and since $\frac{|1-\lambda_1|}{2(1+\lambda_1)} \cdot (\overline{V} - \underline{V}) < c < \overline{c}$, then $\pi^u_{\theta_1} < \pi^l_{\theta_N}$ and both $c > h_{\theta_1}(\pi)$ and $c > h_{\theta_N}(\pi)$ for any $\pi \in [\pi^u_{\theta_1},\ \pi^l_{\theta_N}]$. $\square$

Proof of Lemma 2.2

We prove the lemma for the ask side. The proof for the bid side is analogous.

A buy order indicates the good state of nature. Hence, the informational content of a buy order is maximum when $f^a(n)$ (defined in the proof of lemma 2.3) reaches its minimum value. If all informed traders prefer to buy the asset, then $f^a(N) = 1 < \lambda_N$. By lemma 2.3, it follows that the marginal buying trader affects negatively the informational content of the buy order; that is: $f^a(N-1) < f^a(N)$. If no informed traders prefer to buy the asset, then $f^a(0) = 1 > \lambda_1$, and the marginal buying trader affects positively the informational content of the buy order; that is: $f^a(1) > f^a(0)$. For the maximum likelihood property of signals, it follows that there exists an unique signal θ_{n^a} such that:

$$f^a(n^a+1) > f^a(n^a) \geq \lambda_{n^a} \ \ and \ \ f^a(n^a) \leq f^a(n^a-1).$$

If the equilibrium ask price A^*_t is such that $n^a_t(A^*_t) = n^a$, then the equilibrium informational content of a buy order is maximum. $\square$

Proof of Proposition 2.7

We prove the proposition for the ask side of the market. The proof for the bid side can be obtained by using symmetric arguments.

Since $c = 0$, the competitive ask price has to be equal to the expected asset value conditional to a buy order, that is:

$$A^*_t \in E_t[\widetilde{V}|BO\ at\ A^*_t,\, \sigma^*(P^*_t|t)].$$

Lemma 2.2 states that there exists a good signal θ_{n^a} such that $f^a(n^a) \leq f^a(n)$ for every $n \in \{1, 2, ..., N\}$, and $\lambda_{n^a} \leq f^a(n^a)$. Moreover, by combining lemmas 2.3 and 2.2 it easy to see that $\lambda_{(n^a+1)} > f^a(n^a+1)$. As a consequence, for any history of trades:

$$E_t[\widetilde{V}|\theta_{n^a+1}] < \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t)f^a(n^a+1)} \cdot (\overline{V} - \underline{V}) \tag{2.1}$$

and

$$E_t[\widetilde{V}|\theta_{n^a}] \geq \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t)f^a(n^a)} \cdot (\overline{V} - \underline{V}). \tag{2.2}$$

2.1 and 2.2 imply that there exists an ask price A_t^* that belongs to $(E_t[\widetilde{V}|\theta_{n^a+1}],\ E_t[\widetilde{V}|\theta_{n^a}]]$, that satisfies the zero market maker's expected profit condition. By Proposition 2.3, we know that there exists an unique ask price that satisfies the zero market makers' expected profit condition. Thus, in equilibrium, $n_t^a(A_t^*) = n^a$. $\square$

Chapter 3

Informational cascades with proportional trading costs

3.1 Introduction

In the last chapter we have modified the standard sequential trading model of Glosten and Milgrom [19] to allow for fixed transaction costs. We have shown that fixed costs reduce the informational content of orders and lead to informational cascades. Besides, despite of the existing literature which predicts informational cascades when beliefs converge to a specific asset value, we have shown that if transaction costs are very high, an informational cascade may develop when in the market there is complete uncertainty about the asset's fundamental value.

The aim of this chapter is twofold: to verify the robustness of results presented in the previous chapter with respect to different kinds of exogenous transaction costs and, more importantly, to better investigate the empirical implications of cascades in financial markets. To this purpose, we develop two sequential trading models where transaction costs are proportional rather than fixed. More precisely, in one model we suppose that market makers bear a cost per transaction which is proportional to the asset price. In the other model, traders are allowed to transact different trade sizes, and transaction costs are an increasing function of the quantity of the asset traded. We find that in both cases transaction costs negatively affect the informational content of orders and yield informational cascades.

The first model is similar to that presented in the previous chapter, with the only difference that the transaction cost payed by market makers is proportional to prices.

We find that in this setting, if the true asset value in the bad state of nature is sufficiently low, then the probability of an informational cascade approaches zero when the prices are low. This implies that with transaction costs depending on bid and ask prices cascades will tend to be asymmetric. They will seldom emerge in depressed markets, while they are more likely to develop in bull markets. As a consequence, they are more likely to result in market crashes than in price jumps.

The second model differs from the previous one mainly along two dimensions. First, traders are allowed to transact different trade sizes. This ability to transact orders for large or small quantities introduces a strategic element into the trading game. Informed traders choose their optimal order size given the price schedule. If risk neutral informed traders wish to trade, they prefer to trade larger amount to any given price. Consequently, the market maker sets a larger spread for larger trades in every equilibrium with information based trading. A second difference in this model lies in the nature of transaction costs which are assumed to increase with the order size. Such transaction costs give informed traders an incentive to purchase small, rather than large quantity.

We show that, for any public belief about the true asset value, three types of equilibria can arise depending on the magnitude of the transaction cost. In the separating equilibrium, informed traders place only large orders. Hence, small orders are uninformative and the spread for small quantities only reflects the exogenous transaction cost. This outcome occurs if the transaction cost is very low and the difference between the size of large and small orders is sufficiently large. For intermediate values of the transaction cost, a pooling equilibrium prevails in the market. In this equilibrium, informed trade both the large and the small amounts. The large trade spread reduces with respect to the separating equilibrium, whereas the spread for small quantities increases because of information costs. Finally, if the transaction exceeds the informational advantage of traders observing a private signal on the true asset value, all informed prefer to refrain from trading. Orders are not information based and an informational cascade develops.

The informational content of orders reaches its maximum level in the separating equilibrium. It reduces when a pooling equilibrium prevails in the market and tends to zero in the no-trading equilibrium, as an informational cascade develops. As in the analysis of the second chapter, trading volume gradually is shown to decrease before a cascade occurs, and to reach its lowest value as the cascade develops.

The remainder of this chapter is structured as follows. In section 3.2 we extend the model presented in the previous chapter to the case of transaction costs proportional to the asset price. In sections 3.3 and 3.4 we analyze the case of transaction costs increasing with the trade size. In section 3.5 we conclude. All the proofs are in the appendix at the end of the chapter.

3.2 The case of transaction costs proportional to the asset price

In this section, we discuss the occurrence of informational cascades when the market makers pay a proportional transaction cost to execute each trading order. The only difference relative to the setting of the previous sections is in the expected profit of market makers. Because of proportional transaction costs, the expected profit of a market maker is equal to $E_t[\widetilde{V}|SO\ at\ B] - (1+c)B$ if he buys at B, and it is equal to $(1-c)A - E_t[\widetilde{V}|BO\ at\ A]$ if he sells at A.

Perfect competition leads market makers to earn zero expected profit on any side of the market. Hence, the equilibrium price schedule, $P_t^* = \{B_t^*,\, A_t^*\}$, is such that:

$$B_t^* \in \frac{E_t[\widetilde{V}|SO\ at\ B_t^*,\, \sigma^*(P_t^*|t)]}{1+c} \tag{3.1}$$

$$A_t^* \in \frac{E_t[\widetilde{V}|BO\ at\ A_t^*,\, \sigma^*(P_t^*|t)]}{1-c}. \tag{3.2}$$

Since equilibrium bid and ask prices straddle the unconditional expectation of the asset value, in equilibrium traders with a bad signal will never buy the asset, and traders with a good signal will never sell the asset. Then, as before, informational cascades characterized by all informed traders placing orders in the same direction will never occur in equilibrium. But, the equilibrium price schedule may be such that all informed traders refrain from trading. If this occurs, orders have no informational content and hence $B_t^* = \frac{E_t[\widetilde{V}]}{1+c}$ and $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$.

Proportional transaction costs do not affect the optimal correspondence strategies of informed traders. Hence, no informed trader wishes to sell the asset when the bid price is lower than the valuation of traders with the most informative bad signal, that is $E_t[\widetilde{V}|\theta_N] > B_t^*$. Symmetrically, no informed trader wishes to buy the asset when the ask price exceeds the valuation of traders with the most informative good signal, that is $E_t[\widetilde{V}|\theta_1] < A_t^*$.

Proposition 3.1 $B_t^* = \frac{E_t[\widetilde{V}]}{1+c}$ *if, and only if:*

$$\frac{E_t[\widetilde{V}] - E_t[\widetilde{V}|\theta_N]}{E_t[\widetilde{V}|\theta_N]} \leq c$$

and $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$ *if, and only if:*

$$\frac{E_t[\widetilde{V}|\theta_1] - E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c.$$

Proof: See Appendix.

Proposition 3.1 states that if (and only if) the relative informational advantage of traders observing the most informative bad signal θ_N is lower than the c, the equilibrium bid price is equal to the public belief about the true asset value times $\frac{1}{1+c}$. Symmetrically, if (and only if) the relative informational advantage of traders observing the most informative good signal θ_1 is lower than the transaction cost, the equilibrium ask price is equal to the public belief about the true asset value times $\frac{1}{1-c}$. Clearly, an informational cascade develops when the proportional transaction cost c exceeds the relative informational advantages of both traders with θ_N and traders with θ_1.

In the previous model, where transaction costs are fixed, the condition for the occurrence of informational cascades concerns the absolute informational advantage of traders observing private signals. Here, the amount of the costs payed by the market maker depends on the price at which he buys or sells the asset. Higher bid and ask prices produce higher transaction costs. So, their impact on equilibrium prices is larger when the probability that the market attaches to $\overline{V}$ is high. As a consequence, when the transaction costs are proportional to prices, the condition for informational cascades depends on the relative informational advantage of informed traders rather than their absolute one.

If in the bad state of nature the true asset value is zero, the impact of the transaction costs on the equilibrium bid and ask prices vanishes as π converges to 0. Hence, it is worthwhile to consider separately the two cases of $\underline{V} > 0$ and $\underline{V} = 0$.

First we consider the case of $\underline{V} > 0$. If for any possible history of trades the proportional transaction cost is greater than the relative informational advantages of both traders endowed with θ_N and traders endowed with θ_1, then orders will never be information-based.

To determine the minimum level of c such that no informed trader prefers to trade, define the functions $g_\theta(\pi)$ as:

$$g_\theta(\pi) \equiv \frac{|E_t[\widetilde{V}|\theta] - E_t[\widetilde{V}]|}{E_t[\widetilde{V}|\theta]} = \frac{\pi - \pi^2}{\pi\overline{V} + (1-\pi)\lambda_\theta \underline{V}} \cdot |1 - \lambda_\theta| \cdot (\overline{V} - \underline{V}).$$

$g_\theta(\pi)$ gives the relative informational advantage of traders observing θ for any $\pi \in [0,\ 1]$. It is easy to see that:

$$\max g_{\theta_1}(\pi) < \max g_{\theta_N}(\pi) = \frac{\overline{V} - \underline{V}}{(\sqrt{\overline{V}} + \sqrt{\lambda_N \underline{V}})^2} \cdot (\lambda_N - 1) \equiv \overline{c}_p.$$

The relative informational advantage of traders endowed with the signals θ_N and θ_1 always exceeds that of traders observing a different signal. Hence, if $c > \overline{c}_p$, all informed traders will prefer to refrain from trading whatever the history of trades is.

The next proposition states that, if $\underline{V}$ is strictly positive and the proportional transaction cost is below $\overline{c}_p$, an informational cascade develops both when the public belief about the true asset value tends to $\overline{V}$, and when it approaches $\underline{V}$. Besides, if the transaction costs are large enough and if the difference between $\overline{V}$ and $\underline{V}$ is not too large, an informational cascade may occur also when the probability that the market attaches to $\underline{V}$ and $\overline{V}$ are close.

Proposition 3.2 *If $c \in (0, \overline{c}_p)$ and if $\underline{V}$ is strictly positive, there exist unique $\underline{\pi}$ and $\overline{\pi}$, with $\underline{\pi} < \overline{\pi}$, such that when $\pi \in [0,\ \underline{\pi}) \cup (\overline{\pi},\ 1]$ all informed traders refrain from trading in equilibrium. Besides, if $c \in (\underline{c}_p,\ \overline{c}_p)$, with $\underline{c}_p = \frac{\overline{V} - \underline{V}}{\overline{V} + \underline{V}} \cdot \frac{1-\lambda_1}{1+\lambda_1}$, and $\underline{V}$ is greater than $\lambda_1 \overline{V}$, there exist $\underline{\underline{\pi}} \in (\underline{\pi},\ \frac{\underline{V}}{\overline{V}+\underline{V}})$ and $\overline{\overline{\pi}} \in (\frac{\underline{V}}{\overline{V}+\underline{V}},\ \overline{\pi})$ such that an informational cascade occurs also when $\pi \in (\underline{\underline{\pi}},\ \overline{\overline{\pi}})$.*

Proof: See Appendix.

Too large transaction costs inhibit informed traders from trading whatever is the public belief about the asset value. If transaction costs are not too large, informed traders find worthwhile to trade until their relative informational advantage exceeds c. When the low asset value is greater than zero, the relative informational advantage of traders observing a private signal decreases both when π approaches 0 and when π tends to 1, and it is zero when π is exactly equal to 0 or 1. As a result, an informational cascade occurs both when the public belief about the asset value is close to $\underline{V}$, and when it is near to $\overline{V}$.

The relative informational advantage of traders endowed with a bad signal exceeds that of traders observing an equally informative good signal for all $\pi > \frac{\underline{V}}{\overline{V}+\underline{V}}$, and it is lower than it in the opposite case.

If $\underline{V}$ is not too small respect to $\overline{V}$, that is $\underline{V} > \lambda_1 \overline{V}$, the behavior of the relative informational advantages of traders is similar to the absolute ones (see figure 2.1). Hence, if c exceeds the relative informational advantages of traders with θ_1 and θ_N when $\pi = \frac{\underline{V}}{\overline{V}+\underline{V}}$, an informational cascade can occur even when the probability that the market attaches to $\overline{V}$ is close to $\frac{\underline{V}}{\overline{V}+\underline{V}}$. Clearly, if the difference between $\overline{V}$ and $\underline{V}$ is small, $\frac{\underline{V}}{\overline{V}+\underline{V}}$ is near to $\frac{1}{2}$. We can conclude that if $\overline{V}$ is not too small respect to $\underline{V}$, the results about the occurrence of an informational cascade due to proportional transaction costs, are similar to those obtained for a market characterized by fixed transaction costs.

If $\underline{V}$ is low, the relative informational advantage of traders observing θ_1 exceeds that of traders observing θ_n only if the probability that the market attaches to $\underline{V}$ is very high (see figure 3.1). As a consequence, an informational cascade can occur only when π approaches 0 or 1. Moreover, if $\underline{V}$ is close to zero, the amount of the executing cost payed by the market maker is very low when π is near to zero. Then, the probability of an informational cascade with low prices, given that true asset value is $\overline{V}$, approaches zero. In particular, it is exactly equal to zero in the extreme case of $\underline{V} = 0$. This result is stated in the next proposition.

Proposition 3.3 *If $c \in (0, \overline{c}_p)$ and if $\overline{V} = 0$, there exists unique $\overline{\pi}$*

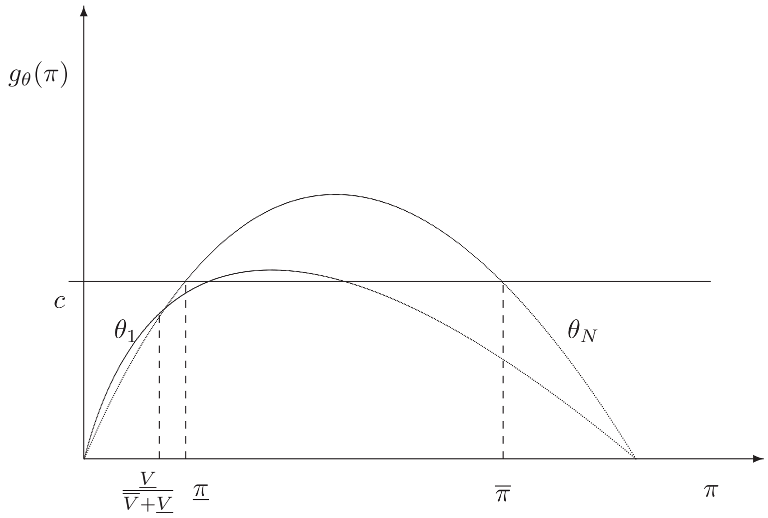

Figure 3.1: *Informational cascades in a market with proportional transaction costs and* $\underline{V} < \lambda_1 \overline{V}$.

such that if, and only if, $\pi > \overline{\pi}$, *all informed traders refrain from trading in equilibrium.*

Proof: See Appendix.

Intuitively, transaction costs payed by market makers reduce as π tends to zero, and they reach the minimum value, $c\underline{V}$, when $\pi = 0$. Clearly, if $\underline{V}$ is equal to zero, proportional transaction costs vanish when $\pi = 0$. As a consequence, an informational cascade will never occur when the equilibrium prices are low.

This result has the interesting implication that cascades will tend to be asymmetric. They will almost never emerge in depressed markets, while they are more likely to be present in bull markets. As a consequence, they are more likely to result in market crashes than in price jumps.

3.3 The case of transaction costs proportional to the trade size: the framework

We consider an extension of model presented in section 2.2 to account for different trade sizes. In the previous, we discussed sequential models where sell and buy orders of traders are restricted to a fixed size of one unit. In this section, we analyze a market where traders are allowed to place large and small orders.[1]

The market is for a risky asset which is exchanged among a sequence of risk neutral traders and competitive market makers who are responsible for quoting prices. The value $\widetilde{V}$ of the asset can be low ($\widetilde{V} = \underline{V}$) or high ($\widetilde{V} = \overline{V}$). The nature probability of $\widetilde{V} = \overline{V}$ is $\pi_0 \in (0,\, 1)$.

There are two types of traders: informed traders (fraction $1 - \mu$) and liquidity traders (fraction μ). Each informed trader is endowed with a private signal θ correlated with the true asset value. The set of private signals is $\Theta = \{\underline{\theta}, \overline{\theta}\}$; the signal $\underline{\theta}$ indicates $\underline{V}$ and the signal $\overline{\theta}$ indicates $\overline{V}$. We assume that signals are symmetric and let be $p = Pr(\underline{\theta}|\underline{V}) = Pr(\overline{\theta}|\overline{V}) > \frac{1}{2}$.

Trades occur sequentially, with one trader allowed to transact at any point in the time. A trader whose turn is to transact may either buy a small or a large quantity, or sell a small or a large quantity, or refrain from trading. We denote Q_S and Q_L, with $Q_S < Q_L$, the size, respectively, of small and large orders. To simplify the analysis, we assume that liquidity traders choose to submit large or small sell and buy orders with equal probability $\frac{\gamma}{4}$; clearly $(1-\gamma)$ is the probability that they refrain from trading. Before a trader arrives, the market maker sets competitive prices. We suppose that any transaction costs to the market maker $c \cdot Q$ euros, where $Q \in \{Q_S,\, Q_L\}$.

3.4 The case of transaction costs proportional to the trade size: the equilibrium

At the beginning of any trading round t, the market maker sets bid and ask prices at which he is willing to trade each asset quantity.

[1] The approach taken in this section involves a sequential trade model similar to that of Easley and O'Hara [16].

We denote P_t the price schedule at time t. Clearly:

$$P_t = \{B_{L,t},\ B_{S,t},\ A_{S,t},\ A_{L,t}\}$$

where $B_{L,t}$ denotes the bid price for the large quantity, $B_{S,t}$ the bid price for the small quantity, $A_{S,t}$ the ask price for the small quantity, and $A_{L,t}$ the ask price for the large quantity.

After prices are set, a trader, randomly selected to trade, observes the price schedule and plays his strategy.[2] The traders' action space is $\mathcal{A} \equiv \{SL,\ SS,\ BS,\ BL,\ NT\}$, where SL = *to Sell the Large quantity*, SS = *to Sell the Small quantity*, BS = *to Buy the Small quantity*, BL = *to Buy the Large quantity*, and RT = *to Refrain from Trading.*

If the trader selected is a liquidity trader, he acts in the ex-ante specified probabilistic way. If the trader selected is an informed trader, he chooses the strategy which maximizes his expected profit given the price schedule. We denote $\sigma \equiv \{\sigma_{\underline{\theta}},\ \sigma_{\overline{\theta}}\}$ the informed traders' strategies, where σ_θ is the mixed strategy of the informed if he observes θ. Clearly:

$$\sigma_\theta \equiv (\sigma_{\theta,SL},\ \sigma_{\theta,SS},\ \sigma_{\theta,BS},\ \sigma_{\theta,BL},\ \sigma_{\theta,RT})$$

where $\sigma_{\theta,i}$ is the probability of i, with $i \in \mathcal{A}$, if the informed observes θ, $\Sigma_{i\in\mathcal{A}}\, \sigma_{\theta,i} = 1$, and $\sigma_{\theta,i} \geq 0\ \ \forall\, i \in \mathcal{A}$.

The expected asset value for a trader observing signal θ is equal to:

$$E_t[\widetilde{V} \mid \theta] = \underline{V} + \frac{\pi_t}{\pi_t + (1-\pi_t)\frac{Pr(\theta|\underline{V})}{Pr(\theta|\overline{V})}} \cdot (\overline{V} - \underline{V}),$$

which exceeds $E_t[\widetilde{V}]$ if $\theta = \overline{\theta}$, and which is lower than $E_t[\widetilde{V}]$ if $\theta = \underline{\theta}$. Given the price schedule P and the strategy $\sigma = (\sigma_{\underline{\theta}},\ \sigma_{\overline{\theta}})$, the expected profit of a trader observing θ is:

$$E_t[\Pi_\theta(\sigma,\ P)] = \sigma_{\theta,SL}(B_L - E_t[\widetilde{V} \mid \theta])Q_L + \sigma_{\theta,SS}(B_S - E_t[\widetilde{V} \mid \theta])Q_S +$$

$$+\sigma_{\theta,BS}(E_t[\widetilde{V} \mid \theta] - A_S)Q_S + \sigma_{\theta,BL}(E_t[\widetilde{V} \mid \theta] - A_L)Q_L.$$

The equilibrium strategy of informed traders will be the correspondence $\sigma^*(P|t) = (\sigma^*_{\underline{\theta}}(P|t),\ \sigma^*_{\overline{\theta}}(P|t))$ which associates to any price

[2] By assuming that the market maker acts competitively and that the informed traders are price takers, we exclude all strategic behaviors, and any trading round can be viewed as an independent game.

schedule P, the strategies maximizing expected profits for any type $\theta \in \Theta$; that is:

$$\sigma_\theta^*(P|t) = \arg \max_{\sigma \in \Delta(\mathcal{A})} E_t[\Pi_\theta(\sigma|P)], \quad \forall P \in R_+^4, \ \forall \theta \in \Theta \tag{3.3}$$

The market maker anticipates the optimal strategies correspondence and announces his price schedule. Bertrand competition restricts the market maker to earn zero expected profit from each trade. Hence, the equilibrium price schedule $P_t^* = \{B_{L,t}^*,\ B_{S,t}^*,\ A_{S,t}^*,\ A_{L,t}^*\}$ will be such that:

$$\begin{array}{l} B_{i,t}^* = E_t[\widetilde{V} \mid SOQ_i \ at \ B_{i,t}^*, \sigma^*(P_t^*|t)] - c \quad \forall\, i \in \{S,\ L\} \\ A_{i,t}^* = E_t[\widetilde{V} \mid BOQ_i \ at \ A_{i,t}^*, \sigma^*(P_t^*|t)] + c \quad \forall\, i \in \{S,\ L\}, \end{array} \tag{3.4}$$

where SOQ_i = *Sell Order for quantity* Q_i and BOQ_i = *Buy Order for quantity* Q_i.

Following Easly and O'Hara [16], we are interested in an equilibrium price schedule P_t^* in which bid prices $B_{S,t}^*$ and $B_{L,t}^*$ and ask prices $A_{S,t}^*$ and $A_{L,t}^*$ straddle the unconditional expected asset value $E_t[\widetilde{V}]$. Given a such equilibrium, traders observing a good signal will never prefer to sell the asset, and traders with a bad signal will never prefer to buy the asset. However, depending on the parameters of the model, different outcomes may prevail. If informed traders prefer to trade only the large quantity, they are separated from small liquidity traders. We call this a separating equilibrium. If informed traders submit either small or large orders with positive probability, a pooling equilibrium occurs. If they prefer to refrain from trading, a no trading equilibrium occurs.

It is useful to stress that the equilibrium on one side of the market may differ from the equilibrium on the other side. For example, the equilibrium price schedule P_t^* may be such that traders observing $\underline{\theta}$ prefer to sell only large quantity and traders observing $\overline{\theta}$ optimally choose to buy with positive probability either small or large quantity. In the next sections we show how different outcomes can occur by changing the transaction cost c.

3.4.1 The no-trading equilibrium

We first analyze the no-trading equilibrium. Before to proceed, we need a more formal definition.

Definition 3.1 *The no-trading equilibrium is described by an equilibrium price schedule* $P^{ne} = \{B_L^{ne}, B_S^{ne}, A_S^{ne}, A_L^{ne}\}$ *and an equilibrium strategies correspondence* $\sigma^*(P)$ *such that:* $\sigma^*(P^{ne}) = \sigma^{ne}$*, where* $\sigma^{ne} = (\sigma_{\underline{\theta}, NT} = 1;\ \sigma_{\overline{\theta}, NT} = 1)$.[3]

In the no-trading equilibrium, no sell or buy order arises from informed traders. Since trades are not information based, they do not affect the probability, π, the market maker attaches to $\widetilde{V} = \overline{V}$, and the expected asset value, conditional on a trading order, is always equal to the unconditional expectation. Therefore, the competitive price schedule, given σ^{ne}, is $P^{ne} = \{B_L^{ne}, B_S^{ne}, A_S^{ne}, A_L^{ne}\}$ such that:

$$B_L^{ne} = B_S^{ne} = E[\widetilde{V}] - c = B^{ne}$$

$$A_S^{ne} = A_L^{ne} = E[\widetilde{V}] + c = A^{ne}.$$

P^{ne} is an equilibrium price schedule if $\sigma^*(P^{ne}) = \sigma^{ne}$. This happens only when the expected profit from trading of informed is strictly negative whatever quantity of asset they choose to sell or buy. In particular, the no-trading equilibrium prevails in the ask side of market when $E[\widetilde{V}\,|\overline{\theta}] - A^{ne} < 0$, that is true only if:

$$c > (\frac{\pi}{\pi + (1-\pi)\frac{1-p}{p}} - \pi) \cdot (\overline{V} - \underline{V}) = c^{ne}(\pi, \overline{\theta}).$$

$c^{ne}(\pi, \overline{\theta})$ is exactly equal to the informational advantage of traders observing $\overline{\theta}$. Hence, if the fixed transaction cost exceeds the informational advantage of traders endowed with the good signal, the no-trading equilibrium prevails in the ask side of market. Similarly, the bid side of market is in the no-trading equilibrium when $B^{ne} - E[\widetilde{V}\,|\underline{\theta}] < 0$, that is true only if:

$$c > (\pi - \frac{\pi}{\pi + (1-\pi)\frac{p}{1-p}}) \cdot (\overline{V} - \underline{V}) = c^{ne}(\pi, \underline{\theta}),$$

where $c^{ne}(\pi, \underline{\theta})$ is equal to the informational advantage of traders observing the bad signal. These results are summarized in the following proposition.

[3]For brevity we omit the time subscript.

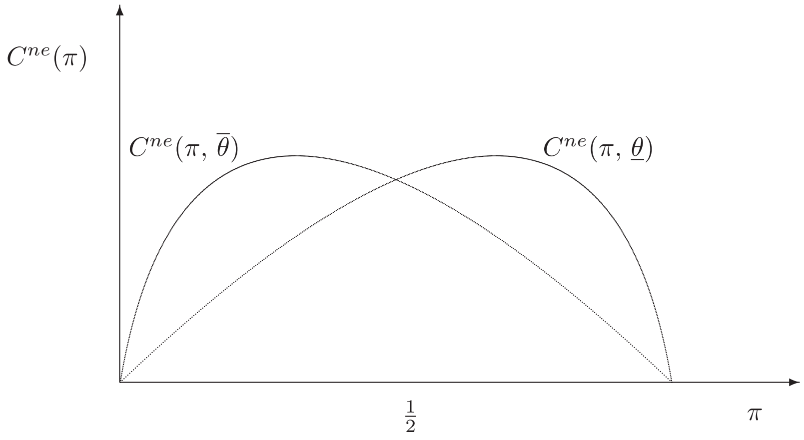

Figure 3.2: *Threshold costs $c^{ne}(\pi,\, \overline{\theta})$ and $c^{ne}(\pi,\, \underline{\theta})$ for no trading equilibrium, respectively, in the ask side and in the bid side of market as functions of public belief π.*

Proposition 3.4 *For any public belief $\pi \in (0,\ 1)$ on $\widetilde{V} = \overline{V}$, a no-trading equilibrium prevails in the bid side of market if, and only if, $c > c^{ne}(\pi,\, \underline{\theta})$. It prevails in the ask side of market if, and only if, $c > c^{ne}(\pi,\, \overline{\theta})$.*

Since $c^{ne}(\pi,\, \theta)$ is greater than zero for all $\pi \in (0,\, 1)$ and $\theta \in \Theta$, a no trading equilibrium never occurs in the absence of positive transaction costs. Figure 3.2 depicts $c^{ne}(\pi,\, \overline{\theta})$ and $c^{ne}(\pi,\, \underline{\theta})$ as functions of the public belief π. We can see immediately from the graph that the threshold cost for the no trading equilibrium in the ask side of market exceeds the threshold cost in the bid side of market when the market maker attaches a greater probability to $\underline{V}$ ($\pi < \frac{1}{2}$). The reverse is true when the unconditional probability of $\overline{V}$ is greater than $\frac{1}{2}$.

To investigate the link between public belief and threshold costs, let consider the limit case $p = 1$, and suppose that $\widetilde{V} = \overline{V}$. Since signals are perfect, the expected asset value of informed traders is always $\overline{V}$, whatever public belief. The informational advantage of informed traders is equal to $(\overline{V} - \underline{V})(1 - \pi)$. It is low if the probability the market maker attaches to $\widetilde{V} = \overline{V}$ is high, and it grows as the public belief about $\widetilde{V} = \overline{V}$ approaches to 0. Then, the more the valuation of the market differs from $\overline{V}$, the higher the cost that informed are willing to pay in order to transact.

In a similar way, when signals are not perfect ($p < 1$), the informational advantage of traders observing θ is low if the public belief is consistent with θ, and it grows as the valuation of the market maker moves in the opposite direction respect to the signal. But, unlike the case of perfect signals, if the market maker attaches a very low probability to the asset value consistent with θ, then the informational advantage of traders observing θ reduces. This occurs because, when signals are not perfect, the expectation of informed traders depends not only on the private signal, but also on the public belief. If the past history of trades strictly indicates a value inconsistent with θ, traders observing this signal attach a low weight to their private information respect to the public belief. However, if the market attaches a greater probability to $\underline{V}$ ($\pi < \frac{1}{2}$), the informational advantage of traders observing $\underline{\theta}$ is bigger than that of traders observing $\overline{\theta}$; if the market attaches a greater probability to $\overline{V}$ ($\pi > \frac{1}{2}$), the reverse is true.

Proposition 3.5 *If the public belief π is greater than $\frac{1}{2}$, then $c^{ne}(\pi, \underline{\theta}) > c^{ne}(\pi, \overline{\theta})$. Otherwise, $c^{ne}(\pi, \underline{\theta}) \leq c^{ne}(\pi, \overline{\theta})$.*

From proposition 3.5, it follows that, when the market attaches a greater probability to $\underline{V}$, the no-trading equilibrium prevails in the market if the fixed transaction cost exceeds the threshold cost of traders observing the good signal; when the market attaches a greater probability to $\overline{V}$, there is a no-trading equilibrium if c is bigger than the threshold cost of traders observing the bad signal.

In the no-trading equilibrium, no new information reaches the market because all informed traders choose to refrain from trading. Therefore, the economy is in an informational cascade.

It is interesting to notice that, the occurrence of an informational cascade does not depend on the asset quantities that agents can sell or buy. This result follows from the interaction between the risk neutrality of agents and the assumption of costs proportional to the quantity of asset that traders choose to buy or sell.

3.4.2 The separating equilibrium

In this section, we analyze the market in the separating equilibrium. The following definition gives a more formal description of this equilibrium.

Definition 3.2 *The separating equilibrium is described by an equilibrium price schedule $P^{se} = \{B_L^{se},\, B_S^{se},\, A_S^{pe},\, A_L^{se}\}$ and an equilibrium strategies correspondence $\sigma^*(P)$ such that: $\sigma^*(P^{se}) = \sigma^{se}$, where:*

$$\sigma^{se} = (\sigma^{se}_{\underline{\theta},\, SL} = 1,\; \sigma^{se}_{\overline{\theta},\, BL} = 1).$$

When a separating equilibrium prevails, the price schedule is such that, the trader who enters the market places a large sell order with probability 1, if he observes the bad signal, and he places a large buy order with probability 1, if he observes the good signal. This implies that small trades are not information-based and then they do not affect the public belief about the true asset value.

Given the informed strategy σ^{se}, the asset valuation conditional on a small trade is the same as the unconditional expectation. This means that, in the separating equilibrium, competitive bid and ask prices for the small quantity are given by:

$$\begin{aligned} B_S^{se} &= E[\widetilde{V}] - c \\ A_S^{se} &= E[\widetilde{V}] + c. \end{aligned}$$

This is not the case for large trades. The conditional expectation of $\widetilde{V}$ conditional on a large sell order is:

$$E[\,\widetilde{V} \mid SOQ_L,\, \sigma^{se}] = \underline{V} + \frac{\pi}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)p}{\mu\frac{\gamma}{4}+(1-\mu)(1-p)}} \cdot (\overline{V} - \underline{V}),$$

which is lower than the unconditional expectation, because a large selling order could be submitted by a trader endowed with bad signal. Similarly, the conditional expectation of $\widetilde{V}$, given a large buy order, is:

$$E[\widetilde{V} \mid BOQ_L, \sigma^{se}] = \underline{V} + \frac{\pi}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)(1-p)}{\mu\frac{\gamma}{4}+(1-\mu)p}} \cdot (\overline{V} - \underline{V}),$$

which exceeds the unconditional expectation to reflect the probability of trading with a trader observing a good signal.

Let be $\lambda^{se}_{SOQ_L} = \frac{\mu\frac{\gamma}{4}+(1-\mu)p}{\mu\frac{\gamma}{4}+(1-\mu)(1-p)} > 1$ and $\lambda^{se}_{BOQ_L} = \frac{\mu\frac{\gamma}{4}+(1-\mu)(1-p)}{\mu\frac{\gamma}{4}+(1-\mu)p} < 1$, respectively, the likelihood ratio of a large sell and buy order in the separating equilibrium. The condition of zero expected profit for the market maker implies that, given the informed traders strategy σ^{se}, equilibrium bid and ask prices for the large quantity are equal to:

$$\begin{aligned} B^{se}_L = &\ \underline{V} + \tfrac{\pi}{\pi+(1-\pi)\lambda^{se}_{SOQ_L}} \cdot (\overline{V} - \underline{V}) - c \\ A^{se}_L = &\ \underline{V} + \tfrac{\pi}{\pi+(1-\pi)\lambda^{se}_{BOQ_L}} \cdot (\overline{V} - \underline{V}) + c. \end{aligned}$$

The price schedule P^{se} determines a separating equilibrium if an informed traders entering the market chooses to submit a large trading order, that is, if $\sigma^*(P^{se}) = \sigma^{se}$. So, a separating equilibrium prevails in the market only if, given the price schedule P^{se}, the expected profit of informed traders is not lower trading the large quantity, Q_L, than trading the small quantity, Q_S, and it is strictly positive. For the the bid side of the market this means:

$$(B^{se}_L - E[\widetilde{V} \mid \underline{\theta}])Q_L \geq (B^{se}_S - E[\widetilde{V} \mid \underline{\theta}])Q_S \geq 0, \tag{3.5}$$

and for the ask side:

$$(E[\widetilde{V} \mid \overline{\theta}] - A^{se}_L)Q_L \geq (E[\widetilde{V} \mid \overline{\theta}] - A^{se}_S)Q_S \geq 0. \tag{3.6}$$

A separating equilibrium occurs in the market when the expected profit from trading of informed traders is strictly positive, and the advantage due to the large quantity exceeds the better price available for the small trades. Consider the ask side of market and suppose that, given the price schedule P^{se}, the expected profit from buying the large quantity of a trader observing $\overline{\theta}$ is strictly positive. The difference between the expected profit from buying the large and the small quantity can be written as follows: $\Delta(\Pi) = [(E[\widetilde{V} \mid \overline{\theta}] - A^{se}_L)(Q_L - Q_S)] - [(A^{se}_L - A^{se}_S)Q_S]$. The first term represents the expected gain due to the greater quantity of asset brought, and the second term is the loss due to the higher price payed to purchase the first Q_S units of asset. An informed trader endowed with $\overline{\theta}$ chooses to place a large trading order if this difference is positive. Clearly, the separating equilibrium is more likely to prevail when the distance between the large and the small quantity is greater. The following proposition states a necessary condition on Q_L and Q_S for a separating equilibrium to occur on each side of market. Denote with $\lambda_{\underline{\theta}}$ and $\lambda_{\overline{\theta}}$ the informational content of signals $\underline{\theta}$ and $\overline{\theta}$, where: $\lambda_{\underline{\theta}} = \frac{p}{1-p}$ and $\lambda_{\overline{\theta}} = \frac{1-p}{p}$.

Proposition 3.6 *Given the public belief π, if a separating equilibrium prevails in the bid side of market, then:*

$$\frac{Q_L - Q_S}{Q_S} > \frac{\lambda^{se}_{SOQ_L} - 1}{\lambda_{\underline{\theta}} - \lambda^{se}_{SOQ_L}}(\pi + (1-\pi)\lambda_{\underline{\theta}}), \tag{3.7}$$

and if it prevails in the ask side of market, then:

$$\frac{Q_L - Q_S}{Q_S} > \frac{1 - \lambda^{se}_{BOQ_L}}{\lambda^{se}_{BOQ_L} - \lambda_{\overline{\theta}}}(\pi + (1-\pi)\lambda_{\overline{\theta}}). \tag{3.8}$$

Proof: See Appendix.

A separating equilibrium can prevail in the market only if Q_L is sufficiently larger than Q_S. From proposition 3.6, it follows that the minimum distance between the large and the small quantity to observe a separating equilibrium reduces as signals become more informative. Moreover, it is easy to verify that in the absence of transaction costs, if Q_S and Q_L satisfy conditions 3.7 and 3.8, the market equilibrium is separating. On the other hand, if the distance between Q_L and Q_S is too small in relation to the informational advantage of informed traders, a separating equilibrium never prevails in the market whatever π and apart from the existence of the transaction cost c. This result is illustrated by the following proposition.

Proposition 3.7 *If $\frac{Q_L - Q_S}{Q_S} < \frac{\lambda^{se}_{SOQ_L} - 1}{\lambda_{\underline{\theta}} - \lambda^{se}_{SOQ_L}}$, a separating equilibrium never occurs in the market.*

Proof: See Appendix.

The minimum size of the relative distance between the large and the small quantity depends on the probability of a noise trading order. If μ is low, the information that the market maker can infer from a large trading order is very accurate ($\lambda^{se}_{SOQ_L}$ is closer to $\lambda_{\underline{\theta}}$ than to 1). This, in tune, implies that the difference between B^{se}_S and B^{se}_L and the difference between A^{se}_L and A^{se}_S are significant and then the losses due to the worse prices in trading Q_L rather than Q_S are high. Hence, Q_L has to be very large with respect to Q_S in order to encourage informed traders to separate from small noise traders. In the following we assume $\frac{Q_L - Q_S}{Q_S} > \frac{\lambda^{se}_{SOQ_L} - 1}{\lambda_{\underline{\theta}} - \lambda^{se}_{SOQ_L}}$. Proposition 3.8 illustrates the conditions on c for the occurrence of a separating equilibrium on the bid and on the ask side of market.

Proposition 3.8 *For any public belief π such that condition 3.7 is satisfied, a separating equilibrium prevails in the bid side of market if, and only if: $c \leq c^{se}(\pi, \underline{\theta})$, where $c^{se}(\pi, \underline{\theta}) = c^{ne}(\pi, \underline{\theta}) - \frac{Q_L}{Q_L - Q_S}(\pi - \frac{\pi}{\pi + (1-\pi)\lambda^{se}_{SOQ_L}})(\overline{V} - \underline{V})$. Analogously, for any public belief π such that condition 3.8 is satisfied, a separating equilibrium prevails in the ask side of market if, and only if: $c \leq c^{se}(\pi, \overline{\theta})$, where $c^{se}(\pi, \overline{\theta}) = c^{ne}(\pi, \overline{\theta}) - \frac{Q_L}{Q_L - Q_S}(\frac{\pi}{\pi + (1-\pi)\lambda^{se}_{BOQ_L}} - \pi)(\overline{V} - \underline{V})$*

Proof: See Appendix.

We can conclude that a separating equilibrium prevails in the market only if the large quantity is big enough with respect to the small quantity, so that both $c^{se}(\pi, \underline{\theta})$ and $c^{se}(\pi, \overline{\theta})$ are positive, and the transaction cost is very low.

3.4.3 The pooling equilibrium

In this section we study a market in the pooling equilibrium. This equilibrium can be formalized in the following definition.

Definition 3.3 *The pooling equilibrium is described by an equilibrium price schedule* $P^{pe} = \{B_L^{pe},\, B_S^{pe},\, A_S^{pe},\, A_L^{pe}\}$ *and an equilibrium strategies correspondence* $\sigma^*(P)$ *such that:* $\sigma^*(P^{pe}) = \sigma^{pe}$, *where*

$$\sigma^{pe} = (\sigma^{pe}_{\underline{\theta},SL},\, \sigma^{pe}_{\underline{\theta},SS},\, \sigma^{pe}_{\underline{\theta},RT};\, \sigma^{pe}_{\overline{\theta},BS},\, \sigma^{pe}_{\overline{\theta},BL},\, \sigma^{pe}_{\overline{\theta},RT}),$$

with

$$\sigma^{pe}_{\underline{\theta},SL}, +\sigma^{pe}_{\underline{\theta},SS}, +\sigma^{pe}_{\underline{\theta},RT} = 1 \text{ and } \sigma^{pe}_{\underline{\theta},RT} < 1,$$

and

$$\sigma^{pe}_{\overline{\theta},BL}, +\sigma^{pe}_{\overline{\theta},BS}, +\sigma^{pe}_{\overline{\theta},RT} = 1 \text{ and } \sigma^{pe}_{\overline{\theta},RT} < 1.$$

In a market in the pooling equilibrium, informed traders submit both small and large orders. More precisely, the competitive price schedule $P^{pe} = \{B_L^{pe},\, B_S^{pe},\, A_S^{pe},\, A_L^{pe}\}$ is such that:

- the trader who enters the market chooses to sell the large quantity with probability $\sigma^{pe}_{\underline{\theta},SL} > 0$ and the small quantity with probability $\sigma^{pe}_{\underline{\theta},SS} > 0$, if he observes $\underline{\theta}$;
- the trader who enters the market chooses to buy the large quantity with probability $\sigma^{pe}_{\overline{\theta},BL} > 0$ and the small quantity with probability $\sigma^{pe}_{\overline{\theta},BS} > 0$, if he observes $\overline{\theta}$.

For $(P^{pe},\, \sigma^*(P))$ to describe a pooling equilibrium, two conditions must be satisfied. First, informed traders must be indifferent between trading the large and the small quantity, given the price schedule P^{pe}. Second, the market maker must anticipate zero expected profit from each trade, given $\sigma^*(P^{pe}) = \sigma^{pe}$.

First condition requires:

$$(B_L^{pe} - E[\widetilde{V}\,|\,\underline{\theta}])Q_L = (B_S^{pe} - E[\widetilde{V}\,|\,\underline{\theta}])Q_S \geq 0 \tag{3.9}$$

$$(E[\widetilde{V}\,|\,\overline{\theta}] - A_L^{pc})Q_L = (E[\widetilde{V}\,|\,\overline{\theta}] - A_S^{pc})Q_S \geq 0. \tag{3.10}$$

For every price schedule which satisfies condition 3.9, the optimal strategy of traders observing the bad signal, is any mixed strategy defined on the simplex $\Delta(SL,\, SS)$ if the expected profit from selling is strictly positive, or any mixed strategy defined on the simplex $\Delta(SL,\, SS,\, NT)$ in the case of zero expected profit. Analogously, condition 3.10 implies that the optimal strategy of traders observing the good signal is any mixed strategy defined on the simplex

$\Delta(BL,\, BS)$ if the expected profit from buying is strictly positive, or any mixed strategy defined on the simplex $\Delta(BL,\, BS,\, NT)$ in the case of zero expected profit.

Second condition requires, on the bid side of market:

$$
\begin{aligned}
B_L^{pe} &= \; \underline{V} + \frac{\pi}{\pi+(1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\underline{\theta},\,SL}p}{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\underline{\theta},\,SL}(1-p)}} \cdot (\overline{V}-\underline{V}) - c \\
B_S^{pe} &= \; \underline{V} + \frac{\pi}{\pi+(1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\underline{\theta},\,SS}p}{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\underline{\theta},\,SS}(1-p)}} \cdot (\overline{V}-\underline{V}) - c
\end{aligned}
\tag{3.11}
$$

for some $\sigma^{pe}_{\underline{\theta},SL}$ and $\sigma^{pe}_{\underline{\theta},SS}$ positive and such that $\sigma^{pe}_{\underline{\theta},SL} + \sigma^{pe}_{\underline{\theta},SS} \leq 1$, and on the ask side of market:

$$
\begin{aligned}
A_S^{pe} &= \; \underline{V} + \frac{\pi}{\pi+(1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\overline{\theta},\,BS}(1-p)}{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\overline{\theta},\,BS}p}} \cdot (\overline{V}-\underline{V}) + c \\
A_L^{pe} &= \; \underline{V} + \frac{\pi}{\pi+(1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\overline{\theta},\,BL}(1-p)}{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\overline{\theta},\,BL}p}} \cdot (\overline{V}-\underline{V}) + c
\end{aligned}
\tag{3.12}
$$

for some $\sigma^{pe}_{\overline{\theta},BS}$ and $\sigma^{pe}_{\overline{\theta},BL}$ positive and such that $\sigma^{pe}_{\overline{\theta},BS} + \sigma^{pe}_{\overline{\theta},BL} \leq 1$.

It is immediate to see that if $B_L > B_S$, traders endowed with $\underline{\theta}$ never choose the small sale, and if $A_L < A_S$, traders observing $\overline{\theta}$ never buy the small quantity. Conditions 3.9 and 3.10 can be satisfied only if the price schedule is such that $B_L^{pe} \leq B_S^{pe}$ and $A_L^{pe} \geq A_S^{pe}$. More precisely, a pooling equilibrium with positive probability of no trading for traders observing $\underline{\theta}$ occurs only if: $E[\widetilde{V}\,|\,\underline{\theta}] = B_L^{pe} = B_S^{pe}$, and the pooling equilibrium with positive probability of no trading for traders observing $\overline{\theta}$ occurs only if: $E[\widetilde{V}\,|\,\overline{\theta}] = A_L^{pe} = A_S^{pe}$. A pooling equilibrium with zero probability of no trading for informed traders requires on the bid side of market: $B_L^{pe} < B_S^{pe}$, and on the ask side of market: $A_L^{pe} > A_S^{pe}$. This, in tune, implies that $\sigma^{pe}_{\underline{\theta},SL} \geq \sigma^{pe}_{\underline{\theta},SS}$ and $\sigma^{pe}_{\overline{\theta},BL} \geq \sigma^{pe}_{\overline{\theta},BS}$.

The next proposition dictates conditions on c for the occurrence of a pooling equilibrium on each side of market.

Proposition 3.9 *For any public belief $\pi \in (0,\ 1)$, a pooling equilibrium prevails in the bid side of market if, and only if, $c \in (c^{se}(\pi,\, \underline{\theta}),\; c^{ne}(\pi,\, \underline{\theta})]$ and $c \geq 0$. It prevails in the ask side of market if, and only if, $c \in (c^{se}(\pi,\, \overline{\theta}),\; c^{ne}(\pi,\, \overline{\theta})]$ and $c \geq 0$.*

Proof: See Appendix.

A pooling equilibrium prevails in the market when the transaction cost is lower than the informational advantage of traders observing a private signal, but it is not low enough to induce informed to separate from small noise traders. In particular, if conditions 3.7 and 3.8 are not satisfied, $c^{ne}(\pi, \underline{\theta})$ and $c^{ne}(\pi, \overline{\theta})$ are negative and the market equilibrium is pooling every time the informational advantage of informed traders exceeds the transaction cost.

In a market in the pooling equilibrium, the learning process about the true asset value can be very slow. If the informational advantage of traders endowed with a private signal only exceeds little the transaction cost, then at the equilibrium the probability of an information based trading order is low. Moreover, in a pooling equilibrium with positive probability of no trading for informed traders, the informational content of a large trading order is the same as the informational content of a small trading order. The information that the market can infer from a trading order increases if the transaction cost is small with respect to the informational advantage of traders observing a private signal. In this case, the probability that an informed trader refrains from trading goes to zero and the large orders become more informative than the small ones.

3.5 Comments and concluding remarks

In this chapter we have examined the impact of proportional trading costs on price discovery. Our goal in doing so is to demonstrate the robustness of herd behavior due to informational externalities in financial markets with trading frictions. To this aim, we have investigated, with two different models, the effect on the informational content of orders of exogenous transaction costs proportional to the asset price and to the orders size. The analysis shows that in both cases the market is not informationally efficient and an informational cascade develops with positive probability.

In the case of transaction costs proportional to the asset price, we find that the probability of an informational cascade is greater in a context of high prices. In particular, if the true asset value in the bad state is close to zero, an informational cascade cannot develop when the asset prices are low.

As a result, cascades will tend to be asymmetric. They will almost never emerge in depressed markets, while they are more likely to be present in bull markets. Therefore, they are more likely to result in market crashes than in price jumps.

In the case of transaction costs proportional to the order size, we show that the occurrence if an informational cascade does not depend on the amount of the asset traded. It develops as the transaction cost exceeds the informational advantage of traders endowed with private information. Moreover, as in the previous models, during a cascade no informed trader chooses to trade at the equilibrium.

If the transaction cost is not too big with respect to the informational advantage of informed traders, two equilibria can arise. The separating equilibrium, where informed choose to trade only the large quantity. The pooling equilibrium, where informed choose to submit both large and small orders with positive probability. The first outcome prevails in the market if two conditions are satisfied: the transaction cost has to be very low, and the difference between the size of large and small orders must be sufficiently large. Since in the separating equilibrium the spread at the large quantity exceeds the spread at the small quantity, if the large quantity is not large enough to offset the better prices for small orders, a separating equilibrium never prevails whatever transaction cost.

Finally, the analysis shows that as the public belief gets concentrated in the extreme tails of the asset value distribution, the upper threshold costs for both separating and trading equilibria reduce. Consequently, before an informational cascade develops, the trading volume gradually decreases.

Appendix

Proof of Proposition 3.1

We prove the proposition for the ask price; the proof for the bid price is symmetric. First we assume that $A_t^* = \frac{E_t[\widetilde{V}]}{1-c}$ and we prove that this implies $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c$. Suppose by way of obtaining a contradiction that $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} > c$. This means that $E_t[\widetilde{V}|\theta_1] > \frac{E_t[\widetilde{V}]}{1-c}$. Hence, at the least the traders observing θ_1 prefer to buy the asset when the ask price is $\frac{E_t[\widetilde{V}]}{1-c}$. As a consequence, the expected asset value conditional to a buy order at $\frac{E_t[\widetilde{V}]}{1-c}$ is greater than the unconditional expected asset value. This implies that the market maker's expected profit is negative, which contradict the fact that $\frac{E_t[\widetilde{V}]}{1-c}$ is the equilibrium ask price.

On the other hand, if $\frac{E_t[\widetilde{V}|\theta_1]-E_t[\widetilde{V}]}{E_t[\widetilde{V}|\theta_1]} \leq c$ then no informed trader prefers to buy the asset.[4] Hence $\inf\{\frac{E_t[\widetilde{V}|BO\ at\ \frac{E_t[\widetilde{V}]}{1-c}, \sigma^*(P_t^*|t)]}{1-c}\} = \frac{E_t[\widetilde{V}]}{1-c} = A_t^*$. $\square$

Proof of Proposition 3.2

First we prove that there exist unique $\underline{\pi}$ and $\overline{\pi}$ such that when $\pi \in [0,\ \underline{\pi}) \bigcup (\overline{\pi},\ 1]$, all informed traders prefer to refrain from trading.

Consider the function $g_\theta(\pi)$ defined at page 66. Notice that:

- $g_\theta(0) = g_\theta(1) = 0$
- $\max g_\theta(\pi) = \frac{\overline{V}-\underline{V}}{(\sqrt{\overline{V}}+\sqrt{\lambda_\theta \underline{V}})^2} \cdot |\lambda_\theta - 1|$
- $g_\theta^{''}(\pi) = -\frac{2\lambda_\theta \underline{V}\overline{V}}{(\pi\overline{V}+(1-\pi)\lambda_\theta\underline{V})^3} \cdot |1-\lambda_\theta| \cdot (\overline{V} - \underline{V}) < 0\ \forall \pi \in [0,\ 1]$.

Since by assumption $0 < c \leq \overline{c}^p$, from the strictly concavity of $g_\theta(\pi)$ it follows that there exist unique $\underline{\pi}_\theta$ and $\overline{\pi}_\theta$, with $\underline{\pi}_\theta < \overline{\pi}_\theta$, such that

[4] The relative informational advantage of informed traders increases with the signal precision.

$g_\theta(\underline{\pi}_\theta) = g_\theta(\overline{\pi}_\theta) = c$ and $g_\theta(\pi) > c$ if, and only if, $\pi \in (\underline{\pi}_\theta,\ \overline{\pi}_\theta)$. Thus, $\underline{\pi} = \inf\{\underline{\pi}_\theta\}_{\theta\in\Theta}$ and $\overline{\pi} = \sup\{\overline{\pi}_\theta\}_{\theta\in\Theta}$.

In order to prove the second part of the theorem, we first notice that $g_{\theta_1}(\frac{\underline{V}}{\overline{V}+\underline{V}}) = g_{\theta_N}(\frac{\underline{V}}{\overline{V}+\underline{V}}) = \underline{c}_p$, and $g_{\theta_1}(\pi) < g_{\theta_N}(\pi)$ for all $\pi > \frac{\underline{V}}{\overline{V}+\underline{V}}$. Moreover, if $\overline{V} > \underline{V}\lambda_1$ then $\frac{\underline{V}}{\overline{V}+\underline{V}} < \arg\max g_{\theta_1}(\pi) < \overline{\pi}_{\theta_1}$. As a consequence, if $\overline{V} > \underline{V}\lambda_1$ then the relative informational advantage of traders endowed with θ_N is strictly positive for all $\pi \in [\overline{\pi}_{\theta_1},\ \overline{\pi}_{\theta_N})$ whatever $c \in (0,\ \overline{c}_p)$. In contrast, if $\overline{V} < \underline{V}\lambda_1$ and c is greater than $\underline{c}_p$, then $\overline{\pi}_{\theta_1}$ is lower than $\underline{\pi}_{\theta_N}$ and both $g_{\theta_1}(\pi)$ and $g_{\theta_N}(\pi)$ are strictly negative for all $\pi \in (\overline{\pi}_{\theta_1},\ \underline{\pi}_{\theta_N})$. Hence an informational cascade occurs also when $\pi \in (\overline{\pi}_{\theta_1},\ \underline{\pi}_{\theta_N})$. $\square$

Proof of Proposition 3.3

If $\underline{V} = 0$, the relative informational advantage of traders endowed with the signal θ as a function of π is given by $g_\theta(\pi) = (1-\pi)\cdot|\lambda_\theta - 1|$. Since by assumption $0 < c \leq \overline{c}^p$, it follows that for any $\theta \in \Theta$ there exists an unique $\overline{\pi}_\theta$ such that $g_\theta(\overline{\pi}_\theta) = c$, and $g_\theta(\pi) > c$ if, and only if, $\pi < \overline{\pi}_\theta$. Thus, $\overline{\pi} = \sup\{\overline{\pi}_\theta\}_{\theta\in\Theta}$. $\square$

Proof of Proposition 3.6

Assume that a separating equilibrium prevails in the ask side of market and suppose, by way of obtaining a contradiction, that

$$\frac{Q_L - Q_S}{Q_S} \leq \frac{\lambda^{se}_{SOQ_L} - 1}{\lambda_{\underline{\theta}} - \lambda^{se}_{SOQ_L}}(\pi + (1-\pi)\lambda_{\underline{\theta}}). \tag{3.13}$$

With simple algebraic calculus 3.13 becomes:

$$(E[\widetilde{V}\,|\,\overline{\theta}] - E[\widetilde{V}\mid BOQ_L,\,\sigma^{se}])Q_L \leq (E[\widetilde{V}\,|\,\overline{\theta}] - E[\widetilde{V}])Q_S. \tag{3.14}$$

Since $A^{se}_L = E[\widetilde{V}\mid BOQ_L,\,\sigma^{se}] + c$ and $A^{se}_S = E[\widetilde{V}] + c$, 3.14 implies:

$$(E[\widetilde{V}\,|\,\overline{\theta}] - A^{se}_L)Q_L < (E[\widetilde{V}\,|\,\overline{\theta}] - A^{se}_S)Q_S$$

which contradicts the assumption that a separating equilibrium prevails in the ask side of market. The part of the proof concerning the bid side of market is analogous and it is omitted. $\square$

Proof of Proposition 3.7

Let be $f_{\underline{\theta}}(\pi) = \frac{\lambda^{se}_{SOQ_L}-1}{\lambda_{\underline{\theta}}-\lambda^{se}_{SOQ_L}}(\pi+(1-\pi)\lambda_{\underline{\theta}})$ and $f_{\overline{\theta}}(\pi) = \frac{1-\lambda^{se}_{BOQ_L}}{\lambda^{se}_{BOQ_L}-\lambda_{\overline{\theta}}}(\pi+(1-\pi)\lambda_{\overline{\theta}})$. The proposition is proved immediately by noting that $\min_{\pi\in[0,\,1]} f_{\overline{\theta}}(\pi) = \min_{\pi\in[0,\,1]} f_{\underline{\theta}}(\pi) = \frac{\lambda^{se}_{SOQ_L}-1}{\lambda_{\underline{\theta}}-\lambda^{se}_{SOQ_L}}$. $\square$

Proof of Proposition 3.8

The proof is straightforward by plugging P^{se} into conditions 3.5 and 3.6, and by noting that condition 3.7 implies $c^{se}(\pi,\,\underline{\theta}) > 0$ and condition 3.8 implies $c^{se}(\pi,\,\overline{\theta}) > 0$. $\square$

Proof of Proposition 3.9

We prove the proposition for the ask side of market. The proof for the bid side is similar. First we consider the pooling equilibrium with positive probability of no trading for traders observing $\overline{\theta}$. This equilibrium requires that the ask price for both the large and the small quantity equates the expected asset value conditional to $\overline{\theta}$.

By combining $E[\widetilde{V}\,|\,\overline{\theta}] = A^{pe}_L = A^{pe}_S$ with condition 3.12 we obtain: $\sigma^{pe}_{\overline{\theta},\,BS} = \sigma^{pe}_{\overline{\theta},\,BL} = \frac{1-\sigma^{pe}_{\overline{\theta},\,NT}}{2} < \frac{1}{2}$. Then, the ask side of market is characterized by a pooling equilibrium with no trading if there exists a $\sigma^{pe}_{\overline{\theta},\,Buy} < \frac{1}{2}$ such that:

$$E[\widetilde{V}\,|\,\overline{\theta}] = \underline{V} + \frac{\pi}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\overline{\theta},\,Buy}(1-p)}{\mu\frac{\gamma}{4}+(1-\mu)\sigma^{pe}_{\overline{\theta},\,Buy}p}} \cdot (\overline{V} - \underline{V}) + c.$$

We define the function $f(x)$ defined on $[0,\,\frac{1}{2}]$ as follows:

$$f(x) = E[\widetilde{V}\,|\,\overline{\theta}] - \underline{V} + \frac{\pi}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4}+(1-\mu)x(1-p)}{\mu\frac{\gamma}{4}+(1-\mu)xp}} \cdot (\overline{V} - \underline{V}).$$

It is easy to see that:

- $f'(x) < 0$ for all $x \in [0,\,\frac{1}{2}]$
- $\min_{x\in[0,\,\frac{1}{2}]} f(x) = f(\frac{1}{2})$
- $\max_{x\in[0,\,\frac{1}{2}]} f(x) = f(0)$

We can conclude that there exists a $\sigma^{pe}_{\overline{\theta},\,Buy} < \frac{1}{2}$ such that $f(\sigma^{pe}_{\overline{\theta},\,Buy}) = c$, if and only if $c \in (f(\frac{1}{2}),\ f(0)]$. This part of the proof is complete by noting that $f(0) = c^{ne}(\pi,\,\overline{\theta})$ and $f(\frac{1}{2}) > c^{se}(\pi,\,\overline{\theta})$.

Let's move to consider the pooling equilibrium with zero probability of no trading for traders observing $\overline{\theta}$. This equilibrium requires that $E[\widetilde{V}\,|\,\overline{\theta}] > A^{pe}_L > A^{pe}_S$. By combining $A^{pe}_L > A^{pe}_S$ with condition 3.12 we obtain: $\sigma^{pe}_{\overline{\theta},\,BL} = 1 - \sigma^{pe}_{\overline{\theta},\,BS} \in (\frac{1}{2},\ 1)$. So, the ask side of market is characterized by a pooling equilibrium with $\sigma^{pe}_{\overline{\theta},\,NT} = 0$, if there exists a $\sigma^{pe}_{\overline{\theta},\,BL} \in (\frac{1}{2},\ 1)$ such that:

$$[(\frac{\pi}{\pi + (1-\pi)\lambda_{\overline{\theta}}} - \frac{\pi}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4} + (1-\mu)\sigma^{pe}_{\overline{\theta},\,BL}(1-p)}{\mu\frac{\gamma}{4} + (1-\mu)\sigma^{pe}_{\overline{\theta},\,BL}p}}) \cdot (\overline{V} - \underline{V}) - c] \cdot Q_L =$$

$$[(\frac{\pi}{\pi + (1-\pi)\lambda_{\overline{\theta}}} - \frac{\pi}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4} + (1-\mu)(1-\sigma^{pe}_{\overline{\theta},\,BL})(1-p)}{\mu\frac{\gamma}{4} + (1-\mu)(1-\sigma^{pe}_{\overline{\theta},\,BL})p}}) \cdot (\overline{V} - \underline{V}) - c] \cdot Q_S. \tag{3.15}$$

We define the function $g(\alpha)$ defined on $[\frac{1}{2},\ 1]$ as follows:

$$g(\alpha) = \frac{\pi}{\pi + (1-\pi)\lambda_{\overline{\theta}}} - \frac{1}{Q_L - Q_S}\cdot$$

$$(\frac{\pi \cdot Q_L}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4} + (1-\mu)\alpha(1-p)}{\mu\frac{\gamma}{4} + (1-\mu)\alpha p}} - \frac{\pi \cdot Q_S}{\pi + (1-\pi)\frac{\mu\frac{\gamma}{4} + (1-\mu)(1-\alpha)(1-p)}{\mu\frac{\gamma}{4} + (1-\mu)(1-\alpha)p}})$$

Condition 3.15 is satisfied if and only if there exists $\alpha \in (\frac{1}{2},\ 1)$ such that $g(\alpha)(\overline{V} - \underline{V}) = c$. Since $g(\alpha)$ is a strictly decreasing function, this is true when $c \in (g(1)(\overline{V} - \underline{V}),\ g(\frac{1}{2})(\overline{V} - \underline{V}))$. The proposition for the ask side of market is proved by noting that $g(1)(\overline{V} - \underline{V}) = c^{se}(\pi,\,\overline{\theta})$, and $g(\frac{1}{2})(\overline{V} - \underline{V}) = f(\frac{1}{2})$. $\square$

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