GENERALIZED FC-GROUPS IN FINITARY GROUPS

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Terminology and notations

- The terminology is standard and follows D.J.Robinson, "Finiteness conditions and generalized soluble groups", vol. I and vol.II, Springer Verlag, 1972, Berlin.

- given a group G and an element $x \in G$, x^G denotes the subgroup of G generated by all conjugates of x in G;

- a polycyclic group is a soluble group satisfying the maximal condition on its subgroups. A polycyclic-by-finite group is a group having a polycyclic normal subgroup of finite index.

- a Chernikov group is a soluble group satisfying the minimal condition on its subgroups.

- a group G is called FC-group, or group with finite conjugacy classes, if $G/C_G\left(x^G\right)$ is a finite group for each $x \in G$. These groups have been introduced independently by R.Baer and B.H.Neumann in the 1950s;

- a group G is called PC-group, or group with polycyclic-by-finite conjugacy classes, if $G/C_G\left(x^G\right)$ is a polycyclic-by-finite group for each $x \in G$. These groups generalize FC-groups. They have been introduced by S.Franciosi, F.de Giovanni and M.Tomkinson in 1990;

- a group G is called CC-group, or group with polycyclic-by-finite conjugacy classes, if $G/C_G\left(x^G\right)$ is a Chernikov group for each $x \in G$. These groups generalize FC-groups. They have been introduced by Ya.D.Polovicki in 1962.

1. Generalized FC-groups

- Let $\mathfrak{X}$ be a class of groups which is closed with respect to forming homomorphic images and sub-direct products of its members, that is,
  - if $G \in \mathfrak{X}$ and $N \triangleleft G$, then $G/N \in \mathfrak{X}$;
  - if $N_1, N_2 \triangleleft G$ with $N_1 \cap N_2 = 1$ and $G/N_i \in \mathfrak{X}$ for $i = 1, 2$, then $G \in \mathfrak{X}$.

- From now, we will refer always to class of groups as $\mathfrak{X}$. Many authors call *formations* these classes of groups. W. Gaschütz seems to be the first who introduced them. There is a large branch in Finite Soluble Groups, investigating formations (see [[1], [3]]).

- An element x of a group G is said to be *XC-central*, or an *XC-element*, if $G/C_G(x^G) \in \mathfrak{X}$.

- If $\mathfrak{X} = \mathfrak{F}$ is the class of finite groups, then we have the notion of FC-element. Of course, an FC-group is a group in which each element is an FC-element.

- If $\mathfrak{X} = \mathfrak{P}\mathfrak{F}$ is the class of polycyclic-by-finite groups, then we have the notion of PC-element.

- If $\mathfrak{X} = \mathfrak{C}$ is the class of Chernikov groups, then we have the notion of CC-element.

Lemma 1 (R.Baer, 1950, [[20], §4]; R.Maier, 2002 [11]). *The set $XC(G)$ of all XC-elements of a group G is a characteristic subgroup of G.*

Proof. If x and y are XC-elements of G, then both $G/C_G(x^G) \in \mathfrak{X}$ and $G/C_G(y^G) \in \mathfrak{X}$, so

$$G/(C_G(x^G) \cap C_G(y^G)) \in \mathfrak{X}.$$

But

$$C_G(x^G) \cap C_G(y^G) \leq C_G((xy^{-1})^G)$$

so $G/C_G((xy^{-1})^G)$ is isomorphic to a subgroup of the direct product of $G/C_G(x^G)$ and $G/C_G(y^G)$ and then $G/C_G((xy^{-1})^G) \in \mathfrak{X}$. This means that xy^{-1} is an XC-element of G. Hence the XC-elements of G form a subgroup $X(G)$. Of course, each automorphism of G sends XC-elements in XC-elements. Then $XC(G)$ is characteristic subgroup of G. ■

Lemma 1 allows us to define inductively the following series in a group G.

Definition 2 (R.Baer, 1950, [[20], §4] - D.H.McLain, 1956, [[20], §4]). *Let G be a group. The ascending characteristic series of G*

$$1 = X_0 \lhd X_1 \lhd \ldots \lhd X_\alpha \lhd X_{\alpha+1} \lhd \ldots,$$

where $X_1 = XC(G)$, $X_{\alpha+1}/X_\alpha = XC(G/X_\alpha)$ and

$$X_\lambda = \bigcup_{\alpha<\lambda} X_\alpha,$$

with α ordinal and λ limit ordinal, is called upper XC-central series of G.

- The first term $XC(G)$ of the upper XC-central series of G is called

$$XC - center$$

of G and the α-th term X_α of the upper XC-central series of G is called

$$XC - center\, of\, length\, \alpha$$

of G.

- The last term $\overline{XC}(G)$ of the upper XC-central series of G is called

$$XC - hypercenter$$

of G.

If $G = X_\beta$, for some ordinal β, we say that G is an XC-*hypercentral* group of type at most β.

- The

$$XC - length$$

of an XC-hypercentral group is defined to be the least ordinal β such that $G = X_\beta$.

- If $G = X_c$ for some positive integer c, we say that G is

$$XC - nilpotent\, of\, length\, c.$$

- It is easy to see that

$$Z(G) \leq XC(G).$$

- Many authors call

$$generalized\ FC - groups$$

those groups having a nontrivial upper XC-central series.

Remark 3. A group G is an XC-group if all its elements are XC-elements, that is, if G has XC-length at most 1. Roughly speaking, the upper XC-central series of G measures the distance of G to be an XC-group.

Remark 4.

- If $\mathfrak{X} = \mathfrak{F}$, we have the notion of FC-hypercentral group and classical results can be found in [[20], §4]. In this circumstance, we specialize the previous symbol X in the symbol F, for introducing XC-hypercentral series and related definitions.

- If $\mathfrak{X} = \mathfrak{P}\mathfrak{F}$, we have the notion of PC-hypercentral group and a complete description of such groups can be found in [4]. In this circumstance, we specialize the previous symbol X in the symbol P, for introducing PC-hypercentral series and related definitions.

- If $\mathfrak{X} = \mathfrak{C}$, we have the notion of CC-hypercentral group and a complete description of such groups can be found in [5]. In this circumstance, we specialize the previous symbol X in the symbol C, for introducing CC-hypercentral series and related definitions.

Theorem 5(McLain's Theorem [[20], Theorem 4.38], 1964). *In a locally nilpotent group G it is true the following relation between the terms Z_α of the upper central series of G and the terms F_α of the upper FC-central series of G:*

$$Z_\alpha \leq F_\alpha \leq Z_{\omega\alpha}$$

for each ordinal α.

Remark 6. Our approach to generalized central series of a group has been introduced by D.H.McLain and S.Dixmier, obtaining classical results in Theory of Generalized FC-groups.

- See [[20], Theorem 4.37, Theorem 4.38] for McLain's Theorem.
- See [6], [[20], §4] for FC-hypercentral groups.
- See [[2], Theorem A, Theorem B, Theorem C] for PC-hypercentral groups and CC-hypercentral groups.
- See [8] for XC-hypercentral groups.

Remark 7. On the other hand hypercentral groups, FC-hypercentral groups, PC- hypercentral groups and CC-hypercentral groups can be much different between themselves as it is shown either by the consideration of the infinite dihedral group or by means of examples in [2] and [8].

Example 8. The infinite dihedral group

$$G = \mathbb{D}_\infty =< a, x : a^x = a^{-1}, x^2 = 1 >$$

has $PC(G) = \mathbb{D}_\infty$, $CC(G) = FC(G) = Z(G) = 1$. Then G is PC-nilpotent, but neither FC-hypercentral, nor CC-hypercentral, nor hypercentral.

Remark 9. A *linear group* of degree n over a field K is a subgroup of the general linear group $GL(n, K)$. Our results are devoted to linear PC-hypercentral groups and linear CC-hypercentral groups.

Remark 10. Classical literature on linear groups is given by

- [14], [18], [19], [22], [24];
- more recent is [17], where [[17], Chapter 5] gives a survey on chains conditions in linear groups;
- these chains conditions have been first introduced in Russian literature (see [14], [18], [22]).

Remark 11. We extend [[15], Theorem 2]. This result says that a linear group of degree n over an arbitrary field K is FC-hypercentral if and only if it has a normal nilpotent subgroup of finite index.

2. Main results

Theorem A (F.Russo, 2007). *Let G be a linear group of degree n over an arbitrary field K. Then the following conditions are equivalent*

(i) *G is PC-nilpotent;*

(ii) *G is PC-hypercentral;*

(iii) *G contains a normal nilpotent subgroup N such that G/N is a polycyclic-by-finite group.*

Theorem B (F.Russo, 2007). *Let G be a linear group of degree n over an arbitrary field K.*

If $char K = 0$ and $G/Z(G)$ does not contain subgroups of infinite exponent, then the following conditions are equivalent

(i) *G is FC-nilpotent;*

(ii) *G is FC-hypercentral;*

(iii) *G contains a normal nilpotent subgroup N such that G/N is a finite group;*

(iv) *G is CC-nilpotent;*

(v) *G is CC-hypercentral.*

3. An auxiliary result

Lemma 12. *Let G be a linear group. If G is a PC-group, then $G/Z(G)$ is a polycyclic-by-finite group.*

Proof. We denote by $[G]$ the envelope of the group G in the vector space of all matrices of degree n over the field K. The group G can be naturally regarded as a group of operators on the space $[G]$ relative to similarity transformations by matrices. Let $g_1, g_2, \ldots, g_r$ a maximal linearly independent system of matrices in the group G, where $r \leq n$. Obviously $G \leq [G]$ and each element of the space $[G]$ can be represented in the form $k_1 g_1 + k_2 g_2 + \ldots + k_r g_r$, where $k_1, \ldots, k_r \in K$. Then the centralizer $[C]$ of an element of the space $[G]$ in $[G]$ coincides with the centralizer C of an element of the group G in G. Since $[G]$ admits a finite basis, G is a PC-group of finite rank (see [10, p.33] for the notion of finite rank). In particular

$$Z(G) = C_G(g_1^G) \cap C_G(g_2^G) \cap \ldots \cap C_G(g_r^G),$$

so that $G/Z(G) = G/(C_G(g_1^G) \cap C_G(g_2^G) \cap \ldots \cap C_G(g_r^G))$. For each $i \in \{1, \ldots, r\}$, the quotient $G/C(g_i^G)$ is a polycyclic-by-finite group. $G/Z(G)$ is isomorphic to a subgroup of a direct product of finitely many polycyclic-by-finite groups and so it is a polycyclic-by-finite group. ■

Remark 13. The proof of Lemma 12 shows that we may exclude from our investigation all the groups which are direct product of infinitely many distinct polycyclic-by-finite groups without center.

Let D be the infinite dihedral group and consider the direct product $G = Dr_{i \geq 0} D_i$ of countably many distinct copies of D.

We note that G is a group with $Z(G) = 1$, G is not linear, is a PC-group, $G/Z(G) = G$ is not a polycyclic-by-finite group.

Lemma 12 is not true for G.

4. The argument of M.Murach

Proof of Theorem A. The condition (i) implies obviously the condition (ii).

Assume that (ii) holds. We want to prove the condition (iii).

Let $G \leq GL(n, K)$ and

$$1 = P_0 \triangleleft P(G) = P_1 \triangleleft P_2 \ldots \triangleleft P_\beta = \overline{PC}(G) = G$$

be an upper PC-central series of the group , where β is an ordinal.

We denote by $[G]$ the envelope of the group G in the vector space of all matrices of degree n over the field K. Similarly by $[P(G)]$ we denote the linear envelope of all matrices in the group $P_1 = PC(G)$.

We construct a basis of the space $[G]$ as follows. Let $p_1, p_2, \ldots, p_{r_1}$ be a basis of the space $[P_1]$, where r_1 is a positive integer which denotes the dimension of $[P_1]$ as K-vectorial space.

We amplify it to a basis of the space $[G]$ by adding some matrices of the group G. We obtain a basis $p_1, p_2 \ldots, p_{r_1}, p_{r_1+1}, p_{r_1+2}, \ldots, p_r$ of the space $[G]$, where $r \leq n$.

The group G is in a natural way a group of operators on the space $[G]$ and its subspace $[P_1]$ relative to similarity transformations by matrices.

By definition of upper PC-central series, every element of the subgroup P_1 is a PC-elements and therefore every element of the subspace $[P_1]$ also is also a PC-element.

It follows that the centralizer in $[G]$ of an element p_i of a basis of $[P_1]$ coincides with $C_G(p_i^G)$ for each $i \leq r_1$. Then the centralizer

$$C_1 = C_G(p_1^G) \cap C_G(p_2^G) \cap \ldots \cap C_G(p_{r_1}^G)$$

has $G/(C_G(p_1^G) \cap C_G(p_2^G) \cap \ldots \cap C_G(p_{r_1}^G))$ which is a polycyclic-by-finite group. Clearly C_1 is normal in G so that the subspace $[P_1]$ is G-invariant as group of operators.

The group G induces on the space $[G]$ of dimension r a matrix group which also has an upper PC-central series.

Matrices of this induced group have degree $r \leq n^2$, where n^2 is the dimension of the space of all matrices of degree n over the field K.

The centralizer C_1 of the subspace $[P_1]$ in the group of operators G induces as a group of operators on the subspace $[P_1]$ a unitary group of matrices of degree r_1 and on the factor space $[G]/[P_1]$ a matrix group, having an upper PC-central series, whose matrices are of degree $n_1 = r - r_1$.

We denote this group with G_1. For $G_1 \leq GL(n_1, K)$

we may repeat all the above considerations.

Hence the finiteness of the decreasing sequence of natural numbers $n^2 > n_1^2 > \dots$ implies the finiteness of an upper PC-central series of the group G.

Consequently every matrix group having such series is PC-nilpotent of class $c \leq n^2$.

The preceding argument allows us to suppose that the group G can be assumed to be PC-nilpotent of class c without loss of generality.

Therefore there exists a series in G

$$1 = P_0 \triangleleft P(G) = P_1 \triangleleft P_2 \dots \triangleleft P_c = G$$

with obvious meaning of symbols.

We proceed by induction on c. If $c = 1$, then G is a linear PC-group and the result follows by Lemma 10.

Let $c > 1$ and suppose the result is true for each linear PC-nilpotent group of class at most $c - 1$.

It was shown above that the centralizer C_1 has the factor group G/C_1 which is a polycyclic-by-finite group, moreover C_1 is PC-nilpotent as subgroup of the PC-nilpotent group G.

Then we may suppose C_1 of PC-nilpotence class either $c-1$ or c.

If C_1 has PC-nilpotence class $c-1$, then by induction hypothesis the result follows.

Assume that C_1 has PC-nilpotence class c.

If C denotes the centralizer of all elements of a basis of the space $[C_1]$ in its group of operators C_1, then the group C_1 induces on the space $[C_1]$ a PC-nilpotent matrix group, isomorphic to C_1/C and of PC-nilpotent class $c-1$.

Indeed $C_1 \leq [C_1]$ implies that C is the center of the group C_1, that is, $C = Z(C_1)$.

Since C_1 coincides with the centralizer of the subgroup P_1 in the group G,

$$C \geq Z(P_1) = C_1 \cap P_1$$

and the quotient $P_1/Z(P_1)$ is a polycyclic-by-finite group.

Now the group $P_2/Z(P_1)$, being an extension of the polycyclic-by-finite group $P_1/Z(P_1)$ by the PC-group

$$(P_1/Z(P_1))/(P_2/Z(P_1)) \simeq P_2/P_1,$$

is again a PC-group.

Moreover $P_2/Z(P_1)$ is a PC-subgroup of the group $G/Z(P_1)$, because each element of $P_2/Z(P_1)$ is a PC-element of $G/Z(P_1)$.

It follows that $P_2/Z(P_1)$, and therefore its subgroup $C_1/Z(P_1)$, is PC-nilpotent of class $c-1$.

The induction hypothesis implies that the result is true for C/C_1.

Since $C = Z(C_1)$, C_1 has a normal nilpotent subgroup L such that C_1/L is a polycyclic-by-finite group.

Then G has the subgroup C_1 such that, C_1 has a normal nilpotent subgroup L such that C_1/L is a polycyclic-by-finite group, the quotient G/C_1 is a polycyclic-by-finite group.

The fact that the class of polycyclic-by-finite groups is closed with respect the extension of two of its members allows us to conclude that G has a normal nilpotent subgroup M such that G/M is a polycyclic-by-finite group.

Then the statement (iii) is proved.

Now assume that (iii) holds. We will prove the condition (i).

Let N be a normal nilpotent subgroup of G with class of nilpotence c such that the quotient G/N is a polycyclic-by-finite group.

From the definitions and the fact that N is nilpotent, we have

$$N = Z_c(N) \leq F_c = F_c(N) \leq P_c = P_c(N).$$

This can be found also in [5].

Of course, the class of PC-nilpotent groups is closed with respect to extensions by polycyclic-by-finite groups.

This situation happens for G, which is an extension of $P_c(N)$ by a group H isomorphic to G/N. Then G is PC-nilpotent of class at most $c + 1$. ■

5. Examples

Example 14. Let $\mathbb{Q}$ be the additive group of the rational numbers and Q be a finitely generated infinite subgroup of $U(\mathbb{Q})$, the group of units of $\mathbb{Q}$. Then Q generates a subring $\mathbb{Q}_\pi$ for some finite non-empty set of primes π ; here π is the set of primes dividing numerators or denominators of elements of Q. Under these circumstances we shall say that Q is a π-generating subgroup of $U(\mathbb{Q})$.

We write $A = \mathbb{Q}_\pi$ and

$$Q =< x_0 > \times < x_1 > \times \ldots \times < x_n >,$$

where n is a positive integer, $x_0 \in \{-1, 1\}$ and $x_1, \ldots, x_n$ generate the free abelian group

$$< x_1 > \times \ldots \times < x_n > .$$

If $-1 \notin Q$, then $Q =< x_1 > \times \ldots \times < x_n >$ and $n = r$ is the Prüfer rank of Q. G is generated by A together with elements $y_1, \ldots y_r$, where y_i is a pre-image of x_i under the epimorphism $G \to Q$, for all $i \in \{1, \ldots, r\}$. Here y_i acts on Q via multiplication by x_i. Also $[y_i, y_j] = c_{ij} \in A$, where c_{ij} satisfy the system of linear equations over $\mathbb{Q}$:

$$\forall i, j, k \in \{1, \ldots, r\}$$

$$c_{ij} = -c_{ji}, \quad c_{jk}(x_i - 1) + c_{ki}(x_j - 1) + c_{ij}(x_k - 1) = 0.$$

The second equation is the famous Hall-Witt identity. This construction is in [[21], p.205-206].

G has a unique minimal normal subgroup A and we note that G is a split extension of A by Q. For each $g \in G$, $C_G(g^G) \geq A$ so that $G/C_G(g^G)$ is torsion-free polycyclic and G is a PC-group. We conclude that $G = PC(G)$. On the other hand $G/C_G(g^G)$ is not finite, $Z(G) = 1$ and $CC(G) = FC(G) = 1$. We conclude that G is neither a CC-group nor nilpotent nor an FC-group.

It is not hard to see that G can be embedded into $GL(2, \mathbb{Q})$.

6. Finitary groups

- Let V be a vector space over the field K and let g be a K-automorphism of V. In the affine general linear group $GL_K(V) \ltimes V$ on V, we can consider commutators, centralizers and normalizers as in any group. We shall adopt these notation in the following. Thus we write $[V, g]$ for $V(g-1)$ and $C_V(g)$ for the fixed-point stabilizer of g in V.

- A group G is said to be $finitary\ skew\ linear$, if it is a subgroup of

$$FGL_K(V) = \{g \in GL_K(V) | \dim_K[V, g] < \infty\}.$$

- Clearly, finitary skew linear groups are a generalization of the well-known linear groups. When $n = \dim_K V$ is a positive integer, we have

$$FGL_K(V) \simeq GL(n, K)$$

so that we find the well-known linear groups.

- A finitary skew linear group G is called *unipotent* if for every $g \in G$, the endomorphism $g - 1$ is nilpotent.

- A subgroup G of $FGL_K(V)$ is said to be a *stability group*, if it stabilizes a series (of arbitrary order-type) in V. Such groups are locally nilpotent and unipotent (see [[25], 2.1b] and [[17], Theorem 1.2.6]).

- The references [9], [10], [12], [13], [16], [17], [25] describe those finitary skew linear groups which possess a chain of subgroups.

Let G be a stability subgroup of $FGL_K(V)$. For all $v \in V$ and $g, h \in G$, we have

$$[v, g, h] = [v, h, g] + [vhg, [g, h]]. \quad (1)$$

For every $g \in G$, let $d(g) = \dim_K [V, g]$ be the *degree* of g. For each positive integer i and j such that $i \geq j$, we recursively define

$$G_{i,0} = 1, G_{i,j} = G_{i,j-1} \cdot \langle g \in Z_j(G) | d(g) \leq 2^{i-j} \rangle. \quad (2)$$

Every subgroup $G_{i,j}$ is normal in G, and the $G_{i,i}$ form an ascending chain with union $Z_\omega(G)$.

Moreover, if $g \in Z_j(G)$ with $d(g) \leq 2^{i-j}$, and if $h \in G$, then $[g, h] \in Z_{j-1}(G)$ with

$$d([g, h]) \leq 2 \cdot d(g) \leq 2^{i-(j-1)}$$

by [[16], Lemma 1, (iv)]. This shows that

$$[G_{i,j}, G] \leq G_{i,j-1} \quad (3)$$

for each positive integer $j \leq i$.

Lemma 15 (O.Puglisi, F.Leinen, U.Meierfrankenfeld in [9], [12], [13]). *Let G be a finitary skew linear group and i be a positive integer.*

(i) *If G is a stability group, then it stabilizes a finite chain in $[V, G_{i,i}]$.*

(ii) *If G is a stability group, then it stabilizes an ascending chain of length at most ω in*

$$\bigcup_{i\geq 1}[V, G_{i,i}] = [V, Z_\omega(G)].$$

(iii) *If G is a stability group, then it stabilizes a finite chain in $V/C_V(G_{i,i})$.*

(iv) *If G is a stability group, then it stabilizes an ascending chain of length at most ω in*

$$V/\bigcap_{i\geq 1} C_V(G_{i,i}) = V/C_V(Z_\omega(G)).$$

Proof. This follows from [[9], Lemma 2.1, Lemma 2.2]. ■

Lemma 16(F.Russo, 2007). *Let G be a unipotent finitary skew linear group.*

(i) *If G is a periodic FC-group, then G has normal nilpotent subgroup N such that G/N is a residually finite group.*

(ii) *If G is a PC-group, then G has a normal nilpotent subgroup N such that G/N is a residually polycyclic-by-finite group.*

(iii) *If G is a periodic CC-group, then G has a normal nilpotent subgroup N such that G/N is a residually Chernikov group.*

Proof. (Sketch of the proof) We modify the argument in Lemma 12, using Lemma 15. ∎

7. A partial result

Proposition 17. (F.Russo, 2007) *Let G be a unipotent finitary skew linear group.*

(i) *If G is a periodic FC-hypercentral group, then G has a normal nilpotent subgroup N such that G/N is a residually finite group.*

(ii) *If G is a PC-hypercentral group, then G has a normal nilpotent subgroup N such that G/N is a residually polycyclic-by-finite group.*

(iii) *If G is a periodic CC-hypercentral group, then G has a normal nilpotent subgroup N such that G/N is a residually Chernikov group.*

Proof. (Sketch of the proof) We variety the argument of M.Murach. ■

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