GENERALIZED FC-GROUPS
IN FINITARY GROUPS

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**Terminology and notations**


- Given a group $G$ and an element $x \in G$, $x^G$ denotes the subgroup of $G$ generated by all conjugates of $x$ in $G$;

- A polycyclic group is a soluble group satisfying the maximal condition on its subgroups. A polycyclic-by-finite group is a group having a polycyclic normal subgroup of finite index.

- A Chernikov group is a soluble group satisfying the minimal condition on its subgroups.
- a group $G$ is called $FC$-group, or group with finite conjugacy classes, if $G/C_G(x^G)$ is a finite group for each $x \in G$. These groups have been introduced independently by R.Baer and B.H.Neumann in the 1950s;

- a group $G$ is called $PC$-group, or group with polycyclic-by-finite conjugacy classes, if $G/C_G(x^G)$ is a polycyclic-by-finite group for each $x \in G$. These groups generalize $FC$-groups. They have been introduced by S.Franciosi, F.de Giovanni and M.Tomkinson in 1990;

- a group $G$ is called $CC$-group, or group with polycyclic-by-finite conjugacy classes, if $G/C_G(x^G)$ is a Chernikov group for each $x \in G$. These groups generalize $FC$-groups. They have been introduced by Ya.D.Polovicki in 1962.
1. Generalized $FC$-groups

- Let $\mathfrak{X}$ be a class of groups which is closed with respect to forming homomorphic images and sub-direct products of its members, that is,
  - if $G \in \mathfrak{X}$ and $N \triangleleft G$, then $G/N \in \mathfrak{X}$;
  - if $N_1, N_2 \triangleleft G$ with $N_1 \cap N_2 = 1$ and $G/N_i \in \mathfrak{X}$ for $i = 1, 2$, then $G \in \mathfrak{X}$.

- From now, we will refer always to class of groups as $\mathfrak{X}$. Many authors call formations these classes of groups. W. Gaschütz seems to be the first who introduced them. There is a large branch in Finite Soluble Groups, investigating formations (see [1], [3]).

- An element $x$ of a group $G$ is said to be $XC$-central, or an $XC$-element, if $G/C_G(x^G) \in \mathfrak{X}$.

- If $\mathfrak{X} = \mathfrak{F}$ is the class of finite groups, then we have the notion of $FC$-element. Of course, an $FC$-group is a group in which each element is an $FC$-element.

- If $\mathfrak{X} = \mathfrak{PF}$ is the class of polycyclic-by-finite groups, then we have the notion of $PC$-element.

- If $\mathfrak{X} = \mathfrak{C}$ is the class of Chernikov groups, then we have the notion of $CC$-element.
Lemma 1 (R.Baer, 1950, [20], §4; R.Maier, 2002 [11]). The set $XC(G)$ of all $XC$-elements of a group $G$ is a characteristic subgroup of $G$.

Proof. If $x$ and $y$ are $XC$-elements of $G$, then both $G/C_G(x^G) \in \mathfrak{X}$ and $G/C_G(y^G) \in \mathfrak{X}$, so

$$G/(C_G(x^G) \cap C_G(y^G)) \in \mathfrak{X}.$$ 

But

$$C_G(x^G) \cap C_G(y^G) \leq C_G((xy^{-1})^G)$$

so $G/C_G((xy^{-1})^G)$ is isomorphic to a subgroup of the direct product of $G/C_G(x^G)$ and $G/C_G(y^G)$ and then $G/C_G((xy^{-1})^G) \in \mathfrak{X}$. This means that $xy^{-1}$ is an $XC$-element of $G$. Hence the $XC$-elements of $G$ form a subgroup $X(G)$. Of course, each automorphism of $G$ sends $XC$-elements in $XC$-elements. Then $XC(G)$ is characteristic subgroup of $G$. $\blacksquare$
Lemma 1 allows us to define inductively the following series in a group $G$.

**Definition 2** (R. Baer, 1950, [[20], §4] - D. H. McLain, 1956, [[20], §4]). Let $G$ be a group. The ascending characteristic series of $G$

\[ 1 = X_0 \triangleleft X_1 \triangleleft \ldots \triangleleft X_\alpha \triangleleft X_{\alpha+1} \triangleleft \ldots, \]

where $X_1 = XC(G)$, $X_{\alpha+1}/X_\alpha = XC(G/X_\alpha)$ and

\[ X_\lambda = \bigcup_{\alpha<\lambda} X_\alpha, \]

with $\alpha$ ordinal and $\lambda$ limit ordinal, is called upper $XC$-central series of $G$. 
- The first term $XC(G)$ of the upper $XC$-central series of $G$ is called

\[ XC \text{- center} \]

of $G$ and the $\alpha$-th term $X_\alpha$ of the upper $XC$-central series of $G$ is called

\[ XC \text{- center of length } \alpha \]

of $G$.

- The last term $\overline{XC}(G)$ of the upper $XC$-central series of $G$ is called

\[ XC \text{- hypercenter} \]

of $G$.

If $G = X_\beta$, for some ordinal $\beta$, we say that $G$ is an $XC$-hypercentral group of type at most $\beta$. 
- The
  \[XC - length\]
  of an \(XC\)-hypercentral group is defined to be the least ordinal \(\beta\) such that \(G = X_\beta\).

- If \(G = X_c\) for some positive integer \(c\), we say that \(G\) is
  \[XC - nilpotent of length c.\]

- It is easy to see that
  \[Z(G) \leq XC'(G).\]

- Many authors call
  \[generalized FC - groups\]
  those groups having a nontrivial upper \(XC\)-central series.
Remark 3. A group $G$ is an $XC$-group if all its elements are $XC$-elements, that is, if $G$ has $XC$-length at most 1. Roughly speaking, the upper $XC$-central series of $G$ measures the distance of $G$ to be an $XC$-group.

Remark 4.

- If $X = F$, we have the notion of $FC$-hypercentral group and classical results can be found in [20], §4. In this circumstance, we specialize the previous symbol $X$ in the symbol $F$, for introducing $XC$-hypercentral series and related definitions.

- If $X = PF$, we have the notion of $PC$-hypercentral group and a complete description of such groups can be found in [4]. In this circumstance, we specialize the previous symbol $X$ in the symbol $P$, for introducing $PC$-hypercentral series and related definitions.

- If $X = C$, we have the notion of $CC$-hypercentral group and a complete description of such groups can be found in [5]. In this circumstance, we specialize the previous symbol $X$ in the symbol $C$, for introducing $CC$-hypercentral series and related definitions.
**Theorem 5** (McLain’s Theorem [[20], Theorem 4.38], 1964). In a locally nilpotent group $G$ it is true the following relation between the terms $Z_{\alpha}$ of the upper central series of $G$ and the terms $F_{\alpha}$ of the upper $FC$-central series of $G$:

$$Z_{\alpha} \leq F_{\alpha} \leq Z_{\omega \alpha}$$

for each ordinal $\alpha$.

**Remark 6.** Our approach to generalized central series of a group has been introduced by D.H.McLain and S.Dixmier, obtaining classical results in Theory of Generalized $FC$-groups.

- See [[20], Theorem 4.37, Theorem 4.38] for McLain’s Theorem.
- See [6], [[20], §4] for $FC$-hypercentral groups.
- See [[2], Theorem A, Theorem B, Theorem C] for $PC$-hypercentral groups and $CC$-hypercentral groups.
- See [8] for $XC$-hypercentral groups.
Remark 7. On the other hand hypercentral groups, $FC$-hypercentral groups, $PC$-hypercentral groups and $CC$-hypercentral groups can be much different between themselves as it is shown either by the consideration of the infinite dihedral group or by means of examples in [2] and [8].

Example 8. The infinite dihedral group

$$G = \mathbb{D}_\infty = \langle a, x : a^x = a^{-1}, x^2 = 1 \rangle$$

has $PC(G) = \mathbb{D}_\infty$, $CC(G) = FC(G) = Z(G) = 1$. Then $G$ is $PC$-nilpotent, but neither $FC$-hypercentral, nor $CC$-hypercentral, nor hypercentral.
Remark 9. A linear group of degree $n$ over a field $K$ is a subgroup of the general linear group $GL(n, K)$. Our results are devoted to linear $PC$-hypercentral groups and linear $CC$-hypercentral groups.

Remark 10. Classical literature on linear groups is given by

- [14], [18], [19], [22], [24];
- more recent is [17], where [[17], Chapter 5] gives a survey on chains conditions in linear groups;
- these chains conditions have been first introduced in Russian literature (see [14], [18], [22]).

Remark 11. We extend [[15], Theorem 2]. This result says that a linear group of degree $n$ over an arbitrary field $K$ is $FC$-hypercentral if and only if it has a normal nilpotent subgroup of finite index.
2. Main results

**Theorem A** (F.Russo, 2007). Let $G$ be a linear group of degree $n$ over an arbitrary field $K$. Then the following conditions are equivalent

(i) $G$ is PC-nilpotent;
(ii) $G$ is PC-hypcentral;
(iii) $G$ contains a normal nilpotent subgroup $N$ such that $G/N$ is a polycyclic-by-finite group.

**Theorem B** (F.Russo, 2007). Let $G$ be a linear group of degree $n$ over an arbitrary field $K$. If $\text{char } K = 0$ and $G/Z(G)$ does not contain subgroups of infinite exponent, then the following conditions are equivalent

(i) $G$ is FC-nilpotent;
(ii) $G$ is FC-hypcentral;
(iii) $G$ contains a normal nilpotent subgroup $N$ such that $G/N$ is a finite group;
(iv) $G$ is $CC$-nilpotent;
(v) $G$ is $CC$-hypcentral.
3. An auxiliary result

Lemma 12. Let $G$ be a linear group. If $G$ is a PC-group, then $G/Z(G)$ is a polycyclic-by-finite group.

Proof. We denote by $[G]$ the envelope of the group $G$ in the vector space of all matrices of degree $n$ over the field $K$. The group $G$ can be naturally regarded as a group of operators on the space $[G]$ relative to similarity transformations by matrices. Let $g_1, g_2, \ldots, g_r$ a maximal linearly independent system of matrices in the group $G$, where $r \leq n$. Obviously $G \leq [G]$ and each element of the space $[G]$ can be represented in the form $k_1 g_1 + k_2 g_2 + \ldots + k_r g_r$, where $k_1, \ldots, k_r \in K$. Then the centralizer $[C]$ of an element of the space $[G]$ in $[G]$ coincides with the centralizer $C$ of an element of the group $G$ in $G$. Since $[G]$ admits a finite basis, $G$ is a PC-group of finite rank (see [10, p.33] for the notion of finite rank). In particular

$$Z(G) = C_G(g_1^G) \cap C_G(g_2^G) \cap \ldots \cap C_G(g_r^G),$$

so that $G/Z(G) = G/(C_G(g_1^G) \cap C_G(g_2^G) \cap \ldots \cap C_G(g_r^G))$. For each $i \in \{1, \ldots, r\}$, the quotient $G/C(g_i^G)$ is a polycyclic-by-finite group. $G/Z(G)$ is isomorphic to a subgroup of a direct product of finitely many polycyclic-by-finite groups and so it is a polycyclic-by-finite group. ■
Remark 13. The proof of Lemma 12 shows that we may exclude from our investigation all the groups which are direct product of infinitely many distinct polycyclic-by-finite groups without center.

Let $D$ be the infinite dihedral group and consider the direct product $G = Dr_{i \geq 0}D_i$ of countably many distinct copies of $D$.

We note that $G$ is a group with $Z(G) = 1$, $G$ is not linear, is a $PC$-group, $G/Z(G) = G$ is not a polycyclic-by-finite group.

Lemma 12 is not true for $G$. 
4. The argument of M.Murach

Proof of Theorem A. The condition (i) implies obviously the condition (ii).
Assume that (ii) holds. We want to prove the condition (iii).
Let $G \leq GL(n, K)$ and

$$1 = P_0 \triangleleft P(G) = P_1 \triangleleft P_2 \ldots \triangleleft P_\beta = PC(G) = G$$

be an upper $PC$-central series of the group $G$ , where $\beta$ is an ordinal.

We denote by $[G]$ the envelope of the group $G$ in the vector space of all matrices of degree $n$ over the field $K$. Similarly by $[P(G)]$ we denote the linear envelope of all matrices in the group $P_1 = PC(G)$.

We construct a basis of the space $[G]$ as follows. Let $p_1, p_2, \ldots, p_{r_1}$ be a basis of the space $[P_1]$, where $r_1$ is a positive integer which denotes the dimension of $[P_1]$ as $K$-vectorial space.

We amplify it to a basis of the space $[G]$ by adding some matrices of the group $G$. We obtain a basis $p_1, p_2 \ldots, p_{r_1}, p_{r_1+1}, p_{r_1+2}, \ldots, p_r$ of the space $[G]$, where $r \leq n$.

The group $G$ is in a natural way a group of operators on the space $[G]$ and its subspace $[P_1]$ relative to similarity transformations by matrices.
By definition of upper $PC$-central series, every element of the subgroup $P_1$ is a $PC$-element and therefore every element of the subspace $[P_1]$ also is also a $PC$-element.

It follows that the centralizer in $[G]$ of an element $p_i$ of a basis of $[P_1]$ coincides with $C_G(p^G_i)$ for each $i \leq r_1$. Then the centralizer

$$C_1 = C_G(p_1^G) \cap C_G(p_2^G) \cap \ldots \cap C_G(p_{r_1}^G)$$

has $G/(C_G(p_1^G) \cap C_G(p_2^G) \cap \ldots \cap C_G(p_{r_1}^G))$ which is a polycyclic-by-finite group. Clearly $C_1$ is normal in $G$ so that the subspace $[P_1]$ is $G$-invariant as group of operators.

The group $G$ induces on the space $[G]$ of dimension $r$ a matrix group which also has an upper $PC$-central series.

Matrices of this induced group have degree $r \leq n^2$, where $n^2$ is the dimension of the space of all matrices of degree $n$ over the field $K$.

The centralizer $C_1$ of the subspace $[P_1]$ in the group of operators $G$ induces as a group of operators on the subspace $[P_1]$ a unitary group of matrices of degree $r_1$ and on the factor space $[G]/[P_1]$ a matrix group, having an upper $PC$-central series, whose matrices are of degree $n_1 = r - r_1$.

We denote this group with $G_1$. For $G_1 \leq GL(n_1, K)$
we may repeat all the above considerations.

Hence the finiteness of the decreasing sequence of natural numbers \(n^2 > n_1^2 > \ldots\) implies the finiteness of an upper PC-central series of the group \(G\).

Consequently every matrix group having such series is PC-nilpotent of class \(c \leq n^2\).

The preceding argument allows us to suppose that the group \(G\) can be assumed to be PC-nilpotent of class \(c\) without loss of generality.

Therefore there exists a series in \(G\)
\[
1 = P_0 \triangleleft P(G) = P_1 \triangleleft P_2 \ldots \triangleleft P_c = G
\]
with obvious meaning of symbols.

We proceed by induction on \(c\). If \(c = 1\), then \(G\) is a linear PC-group and the result follows by Lemma 10.

Let \(c > 1\) and suppose the result is true for each linear PC-nilpotent group of class at most \(c - 1\).

It was shown above that the centralizer \(C_1\) has the factor group \(G/C_1\) which is a polycyclic-by-finite group, moreover \(C_1\) is PC-nilpotent as subgroup of the PC-nilpotent group \(G\).

Then we may suppose \(C_1\) of PC-nilpotence class ei-
ther $c - 1$ or $c$.

If $C_1$ has $PC$-nilpotence class $c - 1$, then by induction hypothesis the result follows.

Assume that $C_1$ has $PC$-nilpotence class $c$.

If $C$ denotes the centralizer of all elements of a basis of the space $[C_1]$ in its group of operators $C_1$, then the group $C_1$ induces on the space $[C_1]$ a $PC$-nilpotent matrix group, isomorphic to $C_1/C$ and of $PC$-nilpotent class $c - 1$.

Indeed $C_1 \leq [C_1]$ implies that $C$ is the center of the group $C_1$, that is, $C = Z(C_1)$.

Since $C_1$ coincides with the centralizer of the subgroup $P_1$ in the group $G$,

$$C \geq Z(P_1) = C_1 \cap P_1$$

and the quotient $P_1/Z(P_1)$ is a polycyclic-by-finite group.

Now the group $P_2/Z(P_1)$, being an extension of the polycyclic-by-finite group $P_1/Z(P_1)$ by the $PC$-group

$$(P_1/Z(P_1))/(P_2/Z(P_1)) \cong P_2/P_1,$$

is again a $PC$-group.

Moreover $P_2/Z(P_1)$ is a $PC$-subgroup of the group $G/Z(P_1)$, because each element of $P_2/Z(P_1)$ is a $PC$-element of $G/Z(P_1)$. 
It follows that $P_2/Z(P_1)$, and therefore its subgroup $C_1/Z(P_1)$, is $PC$-nilpotent of class $c - 1$.

The induction hypothesis implies that the result is true for $C/C_1$.

Since $C = Z(C_1)$, $C_1$ has a normal nilpotent subgroup $L$ such that $C_1/L$ is a polycyclic-by-finite group.

Then $G$ has the subgroup $C_1$ such that, $C_1$ has a normal nilpotent subgroup $L$ such that $C_1/L$ is a polycyclic-by-finite group, the quotient $G/C_1$ is a polycyclic-by-finite group.

The fact that the class of polycyclic-by-finite groups is closed with respect the extension of two of its members allows us to conclude that $G$ has a normal nilpotent subgroup $M$ such that $G/M$ is a polycyclic-by-finite group.

Then the statement (iii) is proved.

Now assume that (iii) holds. We will prove the condition (i).

Let $N$ be a normal nilpotent subgroup of $G$ with class of nilpotence $c$ such that the quotient $G/N$ is a polycyclic-by-finite group.

From the definitions and the fact that $N$ is nilpotent,
we have

\[ N = Z_c(N) \leq F_c = F_c(N) \leq P_c = P_c(N). \]

This can be found also in [5].

Of course, the class of $PC$-nilpotent groups is closed with respect to extensions by polycyclic-by-finite groups.

This situation happens for $G$, which is an extension of $P_c(N)$ by a group $H$ isomorphic to $G/N$. Then $G$ is $PC$-nilpotent of class at most $c + 1$. ■
5. Examples

Example 14. Let $Q$ be the additive group of the rational numbers and $Q$ be a finitely generated infinite subgroup of $U(Q)$, the group of units of $Q$. Then $Q$ generates a subring $Q_\pi$ for some finite non-empty set of primes $\pi$; here $\pi$ is the set of primes dividing numerators or denominators of elements of $Q$. Under these circumstances we shall say that $Q$ is a $\pi$-generating subgroup of $U(Q)$.

We write $A = Q_\pi$ and

$$Q = \langle x_0 \rangle \times \langle x_1 \rangle \times \cdots \times \langle x_n \rangle,$$

where $n$ is a positive integer, $x_0 \in \{-1, 1\}$ and $x_1, \ldots, x_n$ generate the free abelian group

$$\langle x_1 \rangle \times \cdots \times \langle x_n \rangle.$$

If $-1 \not\in Q$, then $Q = \langle x_1 \rangle \times \cdots \times \langle x_n \rangle$ and $n = r$ is the Prüfer rank of $Q$. $G$ is generated by $A$ together with elements $y_1, \ldots y_r$, where $y_i$ is a pre-image of $x_i$ under the epimorphism $G \to Q$, for all $i \in \{1, \ldots, r\}$. Here $y_i$ acts on $Q$ via multiplication by $x_i$. Also $[y_i, y_j] = c_{ij} \in A$, where $c_{ij}$ satisfy the system of linear equations over $Q$:

$$\forall i, j, k \in \{1, \ldots, r\}$$

$$c_{ij} = -c_{ji}, \quad c_{jk}(x_i - 1) + c_{ki}(x_j - 1) + c_{ij}(x_k - 1) = 0.$$

The second equation is the famous Hall-Witt identity. This construction is in [21], p.205-206].
$G$ has a unique minimal normal subgroup $A$ and we note that $G$ is a split extension of $A$ by $Q$. For each $g \in G$, $C_G(g^G) \geq A$ so that $G/C_G(g^G)$ is torsion-free polycyclic and $G$ is a $PC$-group. We conclude that $G = PC(G)$. On the other hand $G/C_G(g^G)$ is not finite, $Z(G) = 1$ and $CC(G) = FC(G) = 1$. We conclude that $G$ is neither a $CC$-group nor nilpotent nor an $FC$-group.

It is not hard to see that $G$ can be embedded into $GL(2, \mathbb{Q})$. 
6. Finitary groups

- Let $V$ be a vector space over the field $K$ and let $g$ be a $K$-automorphism of $V$. In the affine general linear group $GL_K(V) \rtimes V$ on $V$, we can consider commutators, centralizers and normalizers as in any group. We shall adopt these notation in the following. Thus we write $[V, g]$ for $V(g - 1)$ and $C_V(g)$ for the fixed-point stabilizer of $g$ in $V$.

- A group $G$ is said to be *finitary skew linear*, if it is a subgroup of
  
  $$FGL_K(V) = \{ g \in GL_K(V) | \dim_K [V, g] < \infty \}.$$

- Clearly, finitary skew linear groups are a generalization of the well-known linear groups. When $n = \dim_K V$ is a positive integer, we have
  
  $$FGL_K(V) \cong GL(n, K)$$

  so that we find the well-known linear groups.
- A finitary skew linear group $G$ is called **unipotent** if for every $g \in G$, the endomorphism $g - 1$ is nilpotent.

- A subgroup $G$ of $FGL_K(V)$ is said to be a **stability group**, if it stabilizes a series (of arbitrary order-type) in $V$. Such groups are locally nilpotent and unipotent (see [[25], 2.1b] and [[17], Theorem 1.2.6] ).

- The references [9], [10], [12], [13], [16], [17], [25] describe those finitary skew linear groups which possess a chain of subgroups.
Let $G$ be a stability subgroup of $FGL_K(V)$. For all $v \in V$ and $g, h \in G$, we have

$$[v, g, h] = [v, h, g] + [vgh, [g, h]].$$  \hfill (1)

For every $g \in G$, let $d(g) = \dim_K[V, g]$ be the degree of $g$. For each positive integer $i$ and $j$ such that $i \geq j$, we recursively define

$$G_{i,0} = 1, G_{i,j} = G_{i,j-1} \cdot \langle g \in Z_j(G)|d(g) \leq 2^{i-j}\rangle. \hfill (2)$$

Every subgroup $G_{i,j}$ is normal in $G$, and the $G_{i,i}$ form an ascending chain with union $Z_\omega(G)$.

Moreover, if $g \in Z_j(G)$ with $d(g) \leq 2^{i-j}$, and if $h \in G$, then $[g, h] \in Z_{j-1}(G)$ with

$$d([g, h]) \leq 2 \cdot d(g) \leq 2^{i-(j-1)}$$

by [[16], Lemma 1, (iv)]. This shows that

$$[G_{i,j}, G] \leq G_{i,j-1} \hfill (3)$$

for each positive integer $j \leq i$. 

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Lemma 15 (O. Puglisi, F. Leinen, U. Meierfrankenfeld in [9], [12], [13]). Let $G$ be a finitary skew linear group and $i$ be a positive integer.

(i) If $G$ is a stability group, then it stabilizes a finite chain in $[V, G_{i,i}]$.

(ii) If $G$ is a stability group, then it stabilizes an ascending chain of length at most $\omega$ in

$$\bigcup_{i \geq 1}[V, G_{i,i}] = [V, Z_\omega(G)].$$

(iii) If $G$ is a stability group, then it stabilizes a finite chain in $V/C_V(G_{i,i})$.

(iv) If $G$ is a stability group, then it stabilizes an ascending chain of length at most $\omega$ in

$$V/\bigcap_{i \geq 1} C_V(G_{i,i}) = V/C_V(Z_\omega(G)).$$

Proof. This follows from [[9], Lemma 2.1, Lemma 2.2]. $\blacksquare$
Lemma 16 (F. Russo, 2007). Let $G$ be a unipotent finitary skew linear group.

(i) If $G$ is a periodic FC-group, then $G$ has normal nilpotent subgroup $N$ such that $G/N$ is a residually finite group.

(ii) If $G$ is a PC-group, then $G$ has a normal nilpotent subgroup $N$ such that $G/N$ is a residually polycyclic-by-finite group.

(iii) If $G$ is a periodic CC-group, then $G$ has a normal nilpotent subgroup $N$ such that $G/N$ is a residually Chernikov group.

Proof. (Sketch of the proof) We modify the argument in Lemma 12, using Lemma 15. ■
7. A partial result

**Proposition 17.** (F.Russo, 2007) Let $G$ be a unipotent finitary skew linear group.

(i) If $G$ is a periodic FC-hypercentral group, then $G$ has a normal nilpotent subgroup $N$ such that $G/N$ is a residually finite group.

(ii) If $G$ is a PC-hypercentral group, then $G$ has a normal nilpotent subgroup $N$ such that $G/N$ is a residually polycyclic-by-finite group.

(iii) If $G$ is a periodic CC-hypercentral group, then $G$ has a normal nilpotent subgroup $N$ such that $G/N$ is a residually Chernikov group.

**Proof.** (Sketch of the proof) We variety the argument of M.Murach. ■
References


