Achieving Isolation in Mixed-Criticality Industrial Edge Systems with Real-Time Containers *Appendix*

Marco Barletta
Università degli Studi di Napoli Federico II, Italy

Marcello Cinque
Università degli Studi di Napoli Federico II, Italy

Luigi De Simone
Università degli Studi di Napoli Federico II, Italy

Raffaele Della Corte
Università degli Studi di Napoli Federico II, Italy

A Schedulability Test Proof

This is the appendix to the paper[1]. Here, we provide the analytical proof of our simplification of the schedulability test presented in Subsection 3.2. In particular, the response time analysis is described by equations (1) and (2).

$$L_i(w_i) = C_i + \sum_{\forall j \in hp(i)} \left\lceil \frac{w_i + J_j}{T_j} \right\rceil C_j \qquad with\ w_i^0 = C_i + \left(\left\lceil \frac{C_i}{C_S} \right\rceil - 1\right)(T_S - C_S) \tag{1}$$

$$w_i^{n+1} = L(w_i^n) + \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil - 1\right)(T_s - C_s) + \sum_{\substack{\forall X \in hp(S)\\ servers}} \left\lceil \frac{max\left(0, w_i^n - \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil - 1\right)T_s\right) + J_X}{T_X} \right\rceil C_X \tag{2}$$

According to analysis in Subsection 3.2, the third term in Equation 2 can be rewritten as:

$$\sum_{\substack{\forall X \in hp(S)\\ servers}} C_X \tag{3}$$

We provide in the following the proof about Equation 2 and Equation 3.

From previous studies[2,3,4], the interference for a deferrable server is made up of the load that can be generated by a higher priority server x, that is up to:

$$I(t) = \left\lceil \frac{t + J_x}{T_x} \right\rceil C_x = \left\lceil \frac{t - C_x}{T_x} \right\rceil C_x + C_x \tag{4}$$

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[1] M. Barletta, M. Cinque, L. De Simone, R. Della Corte. *Achieving Isolation in Mixed-Criticality Industrial Edge Systems with Real-Time Containers.* Proceedings 34rd Euromicro Conference on Real-Time Systems (ECRTS 2022).

[2] R. Davis and A. Burns. *An investigation into server parameter selection for hierarchical fixed priority pre-emptive systems.* 16th International Conference on Real-Time and Network Systems (RTNS 2008).

[3] R. Davis and A. Burns. *Hierarchical fixed priority preemptive scheduling.* 26th IEEE International Real-Time Systems Symposium (RTSS'05). IEEE, 2005.

[4] G. Bernat and A. Burns. *New results on fixed priority aperiodic servers.* Proceedings 20th IEEE Real-Time Systems Symposium (Cat. No. 99CB37054). IEEE, 1999.

However, this is the worst case of a more generic formula that can be expressed in this way:

$$I(t) = \begin{cases} \left\lceil \frac{t-\phi_x}{T_x} \right\rceil C_x + \phi_x & \text{if } \phi_x < C_x \\ \left\lceil \frac{t-\phi_x}{T_x} \right\rceil C_x + C_x & \text{if } \phi_x \geq C_x \end{cases} = \left\lceil \frac{t-\phi_x}{T_x} \right\rceil C_x + min(\phi_x, C_x) \qquad \forall \phi_x \in [0, T_x] \tag{5}$$

To demonstrate Equation 5, we rewrite a generic time instant t as:

$$t = \phi_x + k * T_x + \alpha \qquad \phi_x \in [0, T_x] \quad \alpha \in [0, T_x] \tag{6}$$

Where ϕ_x is the initial phasing of the server, $k * T_x$ is a multiple of the server period and α is the exceeding. In ϕ_x, at most $min(C_x, \phi_x)$ load is provided by the server. In $k * T_x$ at most $k * C_x$ load is provided and finally in α the load is at most $min(\alpha, C_x)$.
Thus, the load provided by the server in [0,t] is: $L(t) = min(C_x, \phi_x) + k * C_x + min(\alpha, C_x)$.
Then, for a lower priority server the preemption time is: $I(t) = (k + 1) * C_x + min(\phi, C_x)$, since if there is any exceeding of the period, the higher priority server must complete the execution. The formula can be rewritten as:

$$I(t) = \left\lceil \frac{t-\phi_x}{T_x} \right\rceil C_x + min(\phi_x, C_x) \tag{7}$$

Being t the extent in the last period[3], and T the common period, if we prove that $t \leq T$, then

$$I(t) \leq \left\lceil 1 - \frac{\phi_x}{T} \right\rceil C_x + min(\phi_x, C_x) = C_x + min(\phi_x, C_x) \tag{8}$$

Since the periods are lockstep, $\phi_x = 0$ (i.e., all servers have the same phasing).
We prove that $t \leq T$, even if it is trivial due to the extent in the last period is of course not greater than the period. From [3]:

$$t = w_i^n - \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil - 1 \right) T \qquad L_i(w_i^n) = C_j + \sum_{\forall i \in hp(j)} \left\lceil \frac{w_i + J_i}{T} \right\rceil C_i \tag{9}$$

If t value were greater than T (i.e., $t > T$) then $w_i^n > \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil \right) T$.
However, this is not possible since:

$$w_i^n = L(w_i^{n-1}) + \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil - 1 \right) (T - C_s) + \sum_{\substack{\forall X \in hp(S) \\ servers}} \left\lceil \frac{max\left(0, w_i^{n-1} - \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil - 1 \right) T \right) + J_X}{T} \right\rceil C_X \tag{10}$$

In order to have schedulable servers, the third term should be less than $T_s - C_s$ (see note[3]):

$$\begin{aligned} & w_i^n \leq L(w_i^{n-1}) + \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) T - \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) C_s - T + C_s + T - C_s \\ & L(w_i^{n-1}) - \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) C_s + \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) T \geq w_i^n \end{aligned} \tag{11}$$

If we suppose $w_i^n > \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil \right) T$ then:

$$L(w_i^{n-1}) - \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) C_s + \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) T \geq w_i^n > \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil \right) T \tag{12}$$

The first two terms are not positive because of the definition of ceiling function. Thus:

$$\eta + \left(\left\lceil \frac{L(w_i^{n-1})}{C_s} \right\rceil \right) T \geq w_i^n > \left(\left\lceil \frac{L(w_i^n)}{C_s} \right\rceil \right) T \qquad with\, \eta \leq 0 \tag{13}$$

But this is not possible since the last term is a non-decreasing succession.