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Models and methods to redesign service networks  
in times of transition

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# Introduction

Digital transformation has been radically changing how public and private businesses operate and deliver value to customers. The introduction of new digital technologies into various aspects of organizations gives the opportunity to increase productivity, innovation and employment, and accessibility to services. To be effective, the transition toward digitalization of processes and services should not only integrate new technologies but also change the mindset of organizations and business models to support robust and sustainable corporate growth.

One of the most notable impacts of digital transformation is the radical transformation in how businesses and their final customers mutually communicate and transact. Traditional firms, who relied on face-to-face interaction with their customers through (solely) physical delivery channels in the past, have been constantly and dynamically expanding the spectrum of their delivery channels, integrating the digital ones, to respond to and anticipate the changing customers' expectations. However, in order to align and embed the gamma of physical and digital channels, it is necessary to integrate new technologies and delivery channels with the existing physical networks to increase overall performance.

In this thesis, we focus on the relationship between businesses and their customers (i.e., the delivery channel) within the retail sector. In particular, the research activities involve developing new tailored mathematical models to implement location-based actions toward a sustainable spatial reorganization (or restructuring) of retailers' facility networks. The models should be able to provide decision support and valuable insights for decision-makers in the aftermath of digitalization and customers' changing behaviour.

Location analysis is the literature field we refer to model our problems. Location problems aim to identify the optimal location to assign to one or more facilities to

satisfy a customer demand (actual or potential). The objectives may vary from cost-efficiency goals to guaranteeing equitable accessibility conditions to customers. In this thesis, we refer specifically to two classes of location problems: covering and cooperative. Covering location problems decide the optimal locations of facilities to “cover”, in fact, the demand; each customer is covered if at least a facility is within a given distance. Cooperative problems are an extension of covering problems where all the located facilities contribute to some extent to cover the demand.

The thesis aims to contribute to this field of research by introducing and investigating two problems; one consists of an application to a specific retail industry (banking sector), while the other is a generalization of the first problem by allowing cooperation to contribute to the existing body of location literature theoretically. The proposed models are applied to a real case study.

The thesis is organized as follows.

[Chapter 1](#) provides an overview of digital transformation, its main pillars and its impact on the retail sector’s service delivery. In particular, it is highlighted that digital channels are increasingly adopted and, nevertheless, they may not completely replace physical ones. By contrast, the physical networks of facilities should be reorganized to be still effective and meet the ever-changing customers’ needs.

In [Chapter 2](#), generalities on location analysis literature are provided by presenting its main classes of problems. Covering location problems are discussed along with their extensions, especially referring to cooperative problems. In addition, we investigate hierarchical problems within a covering framework. Such problems consider multiple types of facilities arranged in some way in a hierarchical structure (e.g., depending on the services they provide).

In [Chapter 3](#), we address the problem of *banking branch restructuring network*, which considers multiple and hierarchical facilities providing different interrelated services. The problem is formulated as a covering location problem. Although the branch network problem has been extensively analysed in the location literature, the novelty of our problem is mainly given by the combination of strategies to implement that make the branch network more complex due to the multifaceted and interrelated characteristics of banking services. We consider a real case study concerning one of the leading banking groups in Italy in a metropolitan city. Extensive experiments prove the models’ capability to obtain insightful managerial implications.

In [Chapter 4](#), the banking branch restructuring problem is then generalized to provide a methodological contribution to the location analysis. In particular, we introduce the *hierarchical nested cooperative location* problem. In this setting, all the facilities con-

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tribute to cover the demand to different extents depending on mutual relationships and interactions among facilities. The demand coverage is expressed using a probabilistic approach, recently introduced in a cooperative context and sparsely investigated. To the best of our knowledge, in the reference literature, there is no problem considering such a probabilistic approach applied to hierarchical facilities cooperating to cover the demand. The problem is tested on randomly generated instances to verify that the model provides meaningful solutions and to analyze cooperative mechanisms among facilities. Furthermore, the model is applied to the real-world case study used to test the branch restructuring problem.

Finally, some general conclusions and directions for further research are drawn.



# The impact of digitalization on industries and the service delivery in the retail sector

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## Summary

The introduction of new digital technologies into various aspects of organizations and businesses has radically changed how the latter operate and deliver value to customers. As a consequence, societies and economies, due to digitalization, have been experiencing the increasing easiness of access to services and ways to interact using a growing number of connected devices and data flows. In this chapter, an overview of the digital transformation phenomenon is provided by exploring its main characteristics and its general impact on business models and customers' behaviour. In order to adapt their business model to this changing landscape, public and private organizations are integrating their existing physical channels with digital ones to deliver their services to the final consumers. Referring to that, we discuss the adoption of digital channels by users across industries. We focus, in particular, on the retail sector, within which the evolution of business models is presented concerning the used physical and digital channels. Implications emerging from this analysis indicate how integrating digital channels with physical ones necessitates rethinking existing physical delivery networks.

## 1.1 Digital transformation: a disruptive phenomenon

In the literature, many different definitions of digital transformation have been proposed. [Stolterman and Fors \(2004\)](#) define it as “the changes that the digital technology causes or influences in all aspects of human life.” According to many researchers and practitioners, the phenomenon goes beyond merely digitizing processes and resources, representing “digital evolution of a company, business model, idea process, or methodology, both strategically and tactically” ([Mazzone, 2014](#)) where “the use of technology radically improves the performance or reach of enterprise” ([Westerman et al., 2011](#)) and “results in value and revenues being created from digital assets ([McDonald and Rowsell-Jones, 2012](#)). As [Del Rowe \(2017\)](#) argues, digital transformation is related to “the investment in people and technology to drive a business that is prepared to grow, adapt, scale, and change into the foreseeable future”.

[Gong and Ribiere \(2021\)](#) propose a unified definition as follows, “A fundamental change process enabled by digital technologies that aim to bring radical improvement and innovation to an entity (e.g., an organisation, a business network, an industry, or society) to create value for its stakeholders by strategically leveraging its key resources and capabilities.”

All these definitions converge in the same direction. Digital transformation has been widely changing how people live, connect, and exchange information as a result of the rapid advancement of technology information and communication.

Adopting, implementing, and integrating digital technologies into public and private businesses and processes has become an increasingly important condition for modern economies to thrive. It can potentially affect many sectors of the economy (including transport, energy, agri-food, telecommunications, financial services, factory production, and health care) and transform people’s lives ([European Parliament, 2019](#)). Indeed, digitalization is not merely limited to introducing new technologies and upgrading production processes and infrastructures but is a traversal necessity involving cultural and social change.

The introduction of new digital technologies into various aspects of organization and businesses have radically changed how the latter operate and deliver value to customers. As a consequence, societies and economies, due to digitalization, have been experiencing the increasing easiness of access to services and ways to interact using a growing number of connected devices and data flows. A radical and continuous reform of processes, business models, and customer behaviours is established. Introducing new technologies results in new requirements and expectations of demand. Consequently, marketplaces lead to great commitments to improve user value delivery. In order to remain competitive

and relevant, private and public organisations recognise the strategic requirement to adapt themselves, or even better anticipate, this changing landscape and push toward ever-more digitalisation of their processes and business models. Every function and level of organisation should be integrated into a digital framework, fostering business model renewal.

Since digital transformation has become an imperative for private and public organisations, an increasing interest has been erased in the research community to study the phenomenon, which is described as inherently disruptive (Karimi and Walter, 2015). Vial (2021) proposes a framework of the digital transformation process as in Fig. 1.1.

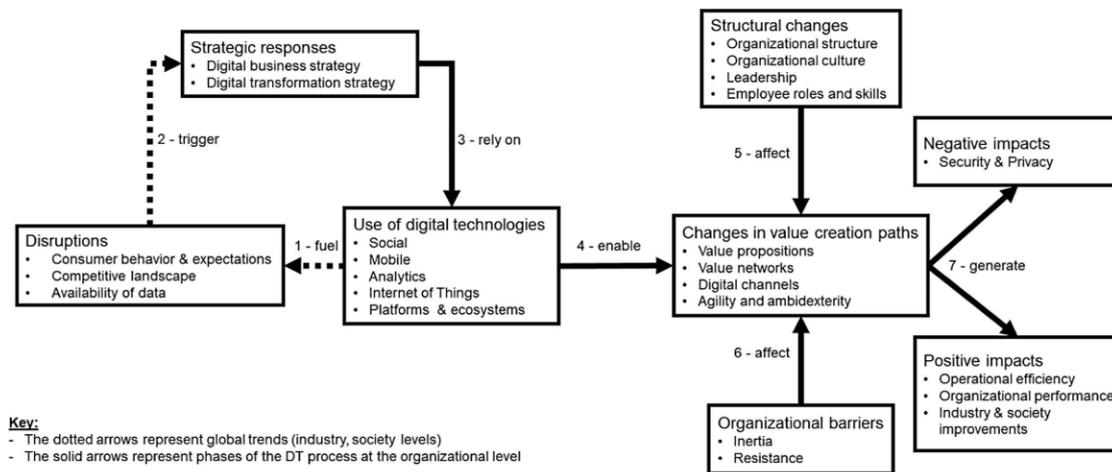


Fig. 1.1. Building blocks of the digital transformation according to Vial (2021).

The use of digital technologies fuels three types of disruptions (1) related to digital transformation at the societal and industry levels: the availability of data fostered by implementing new technologies, the competitive landscape, and the consumers' behaviour and expectations. These disruptions trigger strategic responses on the part of organizations (2). Organizations use digital technologies to alter the value creation paths they have previously relied upon to remain competitive (3). Indeed, digital technologies enable the creation of new value propositions (4) that rely increasingly on the provision of services. Moreover, organizations use digital technologies to implement changes to their delivery and sales channels, including digital channels to interact with final consumers. To that end, they should implement structural changes (5) and overcome barriers (6) that hinder their transformation effort. These changes lead to positive impacts for organizations, such as operational efficiency, and for individuals and society (e.g., improvement of the quality of life of individuals), although they can also be associated with undesirable outcomes primarily in the domain of security and privacy (7).

As already mentioned, digitalization has radically changed the way firms and their customers mutually communicate. Before the advent of the Internet, indeed, the firm-customer relationship was limited to face-to-face interaction at physical facilities. Afterwards, firms have started to include an increasing number of ever-more sophisticated digital channels to respond to the growing digital demand of customers and make easier access to services.

In particular, we focus on the retail sector and analyze the business model of retail businesses in delivering services to final consumers during the last decades. What emerges from this analysis is the easiness of accessing services through digital channels, which are increasingly adopted. Nevertheless, at the same time, physical channels are still relevant from businesses' and customers' points of view, and they are unlikely to be entirely overtaken by digital ones, at least in the foreseeable future. However, physical channels need to be rethought, and physical networks need to be redesigned in the light of digitalization to coexist with digital channels and still be effective. Several strategies are implemented by retail businesses to adapt their business model and their physical facilities networks to integrate digital and physical channels and respond to the changing demand.

## 1.2 Main pillars of digital transformation

According to [Brunetti et al. \(2020\)](#), digital transformation is a pervasive challenge of innovative systems that requires a multifaceted set of strategic actions falling into three main pillars: (i) "infrastructures and technologies" to collect and share information and facilitate interactions, (ii) "culture and skills" to improve digital education, and (iii) "ecosystems", including improving life quality, medium and long-term visions, investments, and partnerships.

In this context, new enabling technologies and increasing interactions in the organization infrastructure bridge the physical and digital worlds.

The *Internet of Things (IoT)* is one of the key technologies consisting in the interconnection via the Internet of "things" – e.g., devices, equipments, plants, engines, materials, products and services – to provide real-time data about their condition, performance, or location and make decisions by themselves ([Tan and Wang, 2010](#)). In the manufacturing industry, IoT involves the management and organisation of the entire value chain process to cost-efficiently realise a low-volume and high-mix production by avoiding machine failures on the factory floor, for instance, by using advanced analytics in predictive maintenance programs ([Othman et al., 2016](#)). Whereas, in the services sector (e.g., transportation and energy distribution), providers succeeded in boosting

customer satisfaction and making their business processes very transparent using IoT (Chin et al., 2023). Digital technologies and IoT represent a new paradigm of quality and effectiveness in managing assets through a massive utilisation of sensors monitoring and analysing critical parameters of infrastructures in real-time and over time.

Devices and IoT interconnection networks continuously transmit a large amount of data. In order to gain valuable insights to support the real-time decision-making process, *big data analytics* methods are applied. They consist of managing techniques to analyse vast amounts of data in order to make predictions and forecasts regarding, for instance, customer behaviour, market trends, and operational efficiencies. Raguseo (2018) discuss several benefits that may be achieved: efficiency goals and reduction of costs; the ability to provide better products and services; easily accessing, collecting and integrating structured and unstructured data from different sources over a short time frame. Moreover, according to Zhong et al. (2016), data analytics may be empowered by machine learning and artificial intelligence technologies to build up a knowledge base. It will guide performing in new and unknown cases to collect the required data.

However, such large amounts of data take up massive memory spaces, and it follows the need to manage and store them. To this end, businesses and organisations rely on *cloud computing* consisting of data migration and software applications to cloud-based services and platforms. This allows businesses to reduce infrastructure costs, increase collaboration and affordability, and easily scale their operations. In particular, Oliveira et al. (2014) identify and analyse the determinants of the adoption of cloud computing, in particular in the manufacturing and service sectors. The study suggests that firms recognise the relative advantages achieved by adopting cloud computing, especially concerning cost savings (e.g., decreased capital expenditures on Information Technologies and reduced maintenance and energy costs), reduction of environmental impacts and generating brand recognition. Moreover, the authors highlight how cloud computing can potentially reduce the complexity of IT services by introducing commonality across business functions and automation in the management process.

The mentioned technologies need to be supported by the development of *very high-capacity* infrastructures of communication (broadband and mobile) *networks* to be fully effective. The availability of a reliable and fast connection is essential to ensure real-time communication, data sharing and IoT performance.

In an ever-growing data-sharing environment, a resulting major issue is related to building a secure and resilient digital network infrastructure. As digital adoption increases, indeed, the need for robust *cybersecurity* measures becomes critical. Protecting sensitive data and ensuring customers' privacy has become a top priority for businesses

in the digital era (Sarker et al., 2020).

In order to be competitive and for digitalization to bring meaningful changes and not merely technological advances, it is necessary to progress also in the mindset, culture, and skills of: (i) top management, which defines the mission and vision of organizations and is responsible for boosting innovation, investments in high-tech infrastructures, process automation, and R&D; (ii) organizations' human capital, which is demanded to have new and more advanced digital capabilities, and (iii) customers who benefit from (physical and digital) products and services (Fitzgerald et al., 2014; Liu and Guo, 2020).

As regards 'ecosystems', the transition toward digitalization of processes and organizations is crucial in supporting a robust and sustainable corporate growth strategy. Moreover, it gives new impetus to competition in industries. Firms in public and private sectors implementing digitalization of processes and optimization of services are able to simplify corporate growth strategies and define new ways to interact and collaborate within them and with their customer base. For instance, it offers the opportunity to develop new physical and, especially, digital products and services and reinforce the business's profitability. In fact, according to Vial (2021), digital technologies facilitate the (re)combination of existing products and services to generate new forms of digital offerings, favouring services over products, lowering barriers to entry and hindering the sustainability of the competitive advantage of incumbent players. As competition moves from a physical plane to a virtual plane where information flows more freely, previous forms of barriers to entry become less significant.

### **1.3 Changing customers' behaviour and expectations**

Digital transformation and resultant business model innovation have fundamentally altered consumers' expectations and behaviours, putting immense pressure on traditional firms and disrupting numerous markets (Verhoef et al., 2021). Indeed, consumer behaviour is changing due to access to various technologies and devices both in the online and mobile contexts (Kannan et al., 2017).

Customers using technologies become active participants in a dialogue between an organization and its stakeholders, and, in turn, businesses need to incentivise customer engagement with digital technologies to drive the co-creation of value (Yeow et al., 2018).

Informative and transactional interactions among public and private organizations and their customers occur mainly through the delivery channels the first put at their disposal. Due to increasing and ongoing digitalization, organizations have constantly and dynamically expanded the spectrum of their delivery channels – mainly offline and physical before the 2000s and the spread of the Internet – to respond to and even antici-

pate customers' behaviour and expectations. Thus, more complex delivery systems have arisen with significant variations in the adoption and access to products and services.

A variety of factors, such as convenience, affordability, accessibility and customization, influence the adoption of digital technologies by customers (Meuter et al., 2000; Ananda et al., 2020; Fernandes and Oliveira, 2021; Anshu et al., 2022). Businesses consider these factors to implement digital strategies to thrive and tailor their offerings to meet their target audience's specific needs and preferences. Indeed, those factors may vary among different industries and customer segments according to their needs and individual digital skills.

Technology readiness refers to the propensity of people to embrace and use new technologies for accomplishing goals in home life and at work, such as information gathering, shopping or access services (Colby and Parasuraman, 2001).

Several theoretical models have been developed to study technology readiness. Among the most widely recognized theories, we mention the Technology Acceptance Model (TAM), designed to provide an explanation of the determinants of computer acceptance capable of explaining the user behaviour based on two core constructs, which are *perceived usefulness* and *perceive ease of use* (Davis et al., 1989).

More recently, the Unified Theory of Acceptance and Use of Technology (UTAUT) proposed by Venkatesh et al. (2003) and extended by Venkatesh et al. (2012) considers as core constructs of customers' technologies acceptance: (i) *performance expectancy*, i.e., benefits to consumers in performing certain activities by using technologies; (ii) *effort expectancy* or easiness of use, (iii) *social influence*, i.e., perception of customers that important others, e.g., family, believe they should use a particular technology; (iv) *facilitating conditions*, i.e., perceptions of the resources and support available to perform a behaviour, (v) *hedonic motivation*, defined as the fun or pleasure derived from using a technology; (vi) *price value* representing the consumers' cognitive trade-off between the perceived benefits and the monetary cost of using a technology; and (vii) *habit* on technology use deriving from prior experience. Individual differences - namely, age, gender, and experience - are hypothesized to moderate the effects of these constructs.

UTAUT has been applied in several contexts in order to evaluate the technology acceptance of customers in different industries, such as banking (Rahi and Ghani, 2019), online grocery shopping (Erjavec and Manfreda, 2022), educational sector (Mukred et al., 2019), and health care (Wang et al., 2020). For instance, the study by Rahi and Ghani (2019) showed that the user intention to adopt Internet banking is jointly predicted by performance expectancy, effort expectancy, website design, customer service, assurance and reliability. In particular, the study showed that customer service and website design

are the most important determinants and boost users' performance expectancy and effort expectancy toward the use of Internet banking technology.

In this context, anticipating rather than responding to changes in customer expectations has become a strategic imperative for firms in order to remain competitive (Vial, 2021). Therefore, businesses need to implement strategies to align and embed the gamma of physical and digital channels through integrating new technologies and new delivery channels with the existing ones, which need to be renewed and reorganized, in turn, to coexist with new ones and increase overall performance.

## 1.4 Digital transformation and digital divide

With the rise of digital technologies, a striking and alarming disparity in their access and usage among and within countries might hinder this transition and prevent it from achieving inclusive development (Bruno et al., 2023). In reference to this, the term "digital divide" has been coined to describe disparities in access and use of technologies and the ability of individuals and economic systems to participate in the global information society (Pethig and Kroenung, 2019). Since the end of the 1990s, consequences for economic well-being and life satisfaction due to the digital divide have been a significant concern for policymakers (Bartikowski et al., 2018) and central in the European debate.

Several papers have addressed the problem of defining measures and methods to assess the digital divide (see, e.g., Cruz-Jesus et al., 2012; Vicente and López, 2011; Reveiu et al., 2022) and identify the factors affecting the digital divide. For instance, the study by Lythreatis et al. (2022) classifies these factors into nine main categories (socio-demographic, socio-economic, personal elements, social support, type of technology, digital training, rights, infrastructure, and large-scale events).

Recently, the European Commission has proposed the Digital Economy and Society Index (DESI), which is a composite indicator to evaluate Europe's digital performance, overall and by countries, and monitor the digital divide. The data used to compute the index are collected by national authorities, such as Eurostat and Communications Committee. The indicator was first introduced in 2014, and since then, the European Commission has been committed to measuring DESI, publishing a yearly report and refining the methodology, modifying and modernizing the indicators included, and improving the assessment. The general aim is to identify and analyse policy areas for digital priority actions to deliver a comprehensive and sustainable digital transformation across all sectors of the economy (European Commission, 2022a). Nowadays, DESI has also become an exploitative tool to monitor countries' progress with respect to some targets.

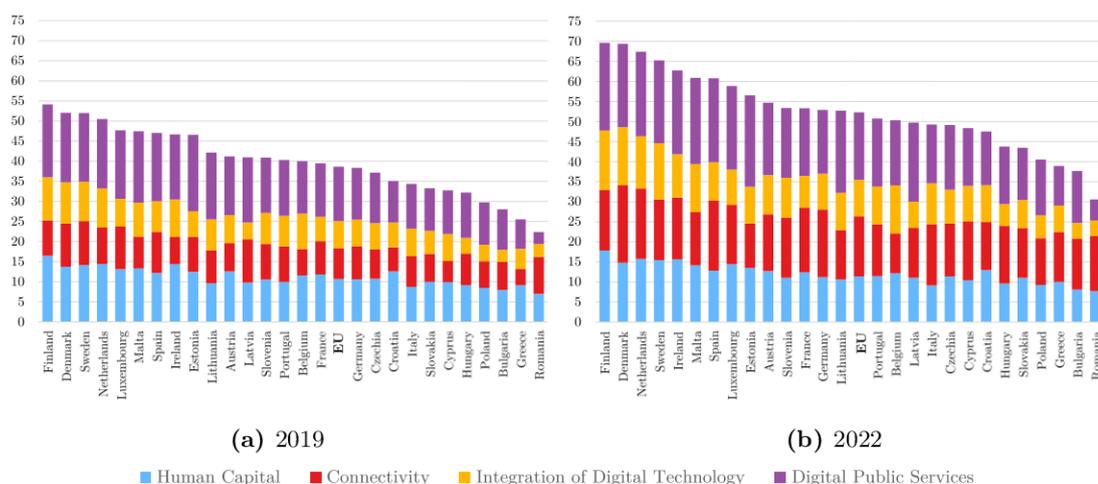
The DESI has a three-level structure of indicators that are progressively composed to

Digital Economy and Society Index	Dimension	Weight of dimension	Sub-dimension	Weight of sub-dimension	Indicator	
	1 Human capital	25%	1a Internet user skills	50%	1a1 At least basic digital skills 1a2 Above basic digital skills 1a3 At least basic digital content creation skills	
1b Advanced skills and development			50%	1b1 ICT specialists 1b2 Female ICT specialists 1b3 Enterprises providing ICT training 1b4 ICT graduates		
2 Connectivity		25%	2a Fixed broadband take-up	25%	2a1 Overall fixed broadband take-up 2a2 At least 100 Mbps fixed broadband take-up 2a3 At least 1 Gbps take-up	
			2b Fixed broadband coverage	25%	2b1 Fast broadband (NGA) coverage 2b2 Fixed Very High Capacity Network (VHCN) coverage	
		2c Mobile broadband	40%	2c1 5G spectrum 2c2 5G coverage 2c3 Mobile broadband take-up		
		2d Broadband prices	10%	2d1 Broadband price index		
3 Integration of digital technology		25%	3a Digital intensity	15%	3a1 SMEs with at least a basic level of digital intensity	
			3b Digital technologies for businesses	70%	3b1 Electronic information sharing 3b2 Social media 3b3 Big data 3b4 Cloud 3b5 AI 3b6 ICT for environmental sustainability 3b7 e-Invoices	
		3c e-Commerce	15%	3c1 SMEs selling online 3c2 e-Commerce turnover 3c3 Selling online cross-border		
		4 Digital public services	25%	4a e-Government	100%	4a1 e-Government users 4a2 Pre-filled forms 4a3 Digital public services for citizens 4a4 Digital public services for businesses 4a5 Open data

Fig. 1.2. Three-level structure of DESI and indicators that compose it.

obtain the final indicator, as shown in Fig. 1.2. At the first level, four main dimensions are considered: (i) *Human capital* assesses both the internet user skills of citizens and the advanced skills of specialists; (ii) *Connectivity* analyses fixed and mobile broadband; (iii) *Integration of digital technology* represents the adoption degree of new technologies by enterprises and e-commerce; (iv) *Digital public services* component describes the demand and supply of e-government. Readers may refer to the methodological note [European Commission \(2022b\)](#) for more details on the index composition.

Fig. 1.3 shows the scores of DESI for each EU country, stratified for each of the main dimensions in 2019 (pre-COVID) and 2022 (post-COVID) ([European Commission, 2023](#)). Comparing the data, some considerations can be drawn. The relative positions of the EU countries appear stable, with Northern countries as frontrunners (Finland, Denmark, the Netherlands and Sweden) and some Southern and Eastern countries in the lowest places. Nonetheless, DESI has shown an overall upward convergence trend in the last years across EU countries. Most of the EU members are advancing and making progress in their digital transformation, particularly those that started from lower levels



**Fig. 1.3.** DESI scores in 2019 and 2022 by country stratified by dimension.

and are gradually catching up by growing at a faster rate. This is the case of Italy, Poland and Greece, which improved their DESI scores substantially and implemented sustained investments in digital, supported by European funding.

On average, The EU-DESI score has increased by 55% – from 33.7 to 52.3 – over the last five years (2017-2022). All the dimensions present substantial improvement, although to an unequal degree. The dimension that improved the most is the connectivity (+141.9%), followed by the integration of digital technology (+65.1%) and digital public services (+44.2%). The human capital dimension presents the lowest increase, of (only) 10.1%. The boost toward such a sharp digitalization is definitely due to the COVID-19 pandemic, during and after which digital interaction had become the norm.

More in detail, as regards the connectivity dimension, 70% of EU households can benefit from fixed very high-capacity networks, on average, but unevenly between rural and urban areas and between countries.

The human capital component shows that insufficient digital skill levels hamper future growth prospects. Indeed, a large part of the EU population still lacks digital skills (54% of individuals possessed at least basic digital skills). This led to the digital divide deepening among and within countries. Moreover, the risks of digital exclusion increase as more and more services, including essential ones, are shifted online.

While human capital and connectivity were present in most of the previous indicators (see, e.g., [Lythreatis et al., 2022](#)), integration of digital technology into businesses and digital public services are also included in DESI, reflecting the importance given to the digitalization of private companies and public organizations and entities.

Businesses are getting increasingly digitalized, boosted by legislative proposals (see, e.g., the Digital Markets Act) to establish a level playing field to foster innovation, growth and competitiveness, both in the single market and globally. A substantial improvement has been recorded. However, the EU targets are still far from being reached. In 2021, the SMEs with at least a basic level in adopting digital technologies was 55% (on average).

Finally, the digitalization of key public services and services for businesses and citizens has been growing steadily over the last decade and accelerated by the COVID-19 pandemic. However, the progress is uneven across and within countries, also in this dimension. For instance, compared to services for businesses, those for citizens are less likely to be available online.

### 1.4.1 Initiatives and programmes to address digital divide

The digital transformation is a huge opportunity to increase productivity, innovation and employment, guarantee higher accessibility to services, and bridge the territorial divide. Therefore, coherent public policies are required.

The European Union is playing an active role in shaping the digital economy, with cross-policy initiatives that range from boosting investment to reforming EU laws, to non-legislative actions to improve coordination among EU-Members and exchange of best practices (Negreiro and Madiaga, 2019), such as promoting and training digital skills and creating enabling regulatory environments. The list of programmes and their timeline for the years 2020-2030 are available at the following links: <https://www.consilium.europa.eu/en/policies/?filters=76831> and <https://www.consilium.europa.eu/en/policies/a-digital-future-for-europe/timeline-digital-europe/>.

In particular, the “Path to the Digital Decade” (European Commission, 2021b) is the EU policy programme to encourage digital transformation to create a structured cooperation process between the European Commission and EU countries. The programme, adopted on 15 September 2021, recognizes the importance of digital transformation for the “recovery, prosperity, security and competitiveness and for the well-being of EU societies”.

In the communication “2030 Digital Compass: the European Way for the Digital Decade” (European Commission, 2021a), the targets expected to be reached by 2030 are defined with respect to four fundamental areas (citizens, business, public administrations and infrastructure). The programme aims to get a greater convergence between EU states and reinforce their collective competitiveness and resilience in the global context by promoting human-centred, inclusive and sustainable digital policies empowering citizens and businesses. (European Commission, 2021b, 2022a).

Afterwards, the Declaration on Digital Rights and Principles (European Commission, 2022c), proposed by the Commission on 26 January 2022, complements the targets presented in the 2030 Digital Compass with a shared reference framework in order to guide policymakers and private stakeholders in shaping the Digital Decade according to European values and the rights and freedoms enshrined in the EU’s legal framework. For instance, some of the rights and principles outlined in the declaration are “people at the centre” (i.e. digital technologies should protect people’s rights, support democracy, and ensure that all digital players act responsibly and safely) and “sustainability” (i.e. digital devices should support the green transition and should promote the awareness of people about the environmental impact and energy consumption of their devices).

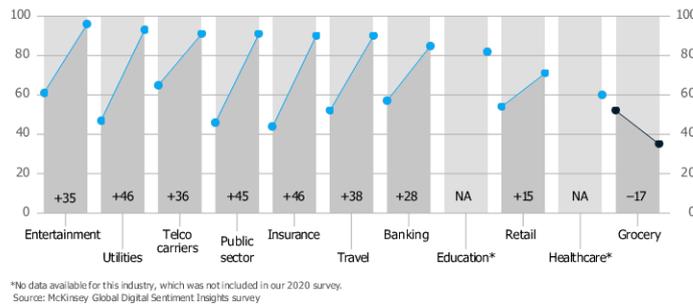
Finally, in order to promote post-COVID recovery and mitigate the socio-economic consequences of the pandemic, in 2020, the EU launched the “Next Generation EU” fund. Its centrepiece is the “Recovery and Resilience Facility”, a financial instrument offering grants and loans to support reforms and investments in the EU Member States for a total of 723.8€ billion. The flagship programmes of the plan are devoted to make EU societies and economies healthier, greener, more digital, resilient and inclusive. In particular, the objectives are: (i) *Power up* to promote clean and renewable technologies; (ii) *Renovate* to encourage energy efficiency of buildings; (iii) *Recharge and refuel* to spread sustainable transport systems and charging stations; (iv) *Connect* to cover the EU with rapid broadband services; (v) *Modernize* to digitalize public administrations; (vi) *Scale-up* to foster the adoption of data cloud and sustainable processors; (vii) *Reskill and upskill* to promote education and training to support the digital skills of citizens and businesses.

## 1.5 The adoption of digital delivery channels across industries

One of the most notable impacts of digital transformation has been the radical changes in how public and private organizations and their customers mutually communicate and transact, allowing more direct interactions through an increasingly varied spectrum of delivery channels or *touchpoints*. In particular, the development of delivery channel systems has involved mainly introducing more innovative digital channels and integrating them with existing ones. The number of digital touchpoints has increased, along with their adoption by customers, which has steadily grown during the last two decades after the spread of the Internet and more sophisticated devices.

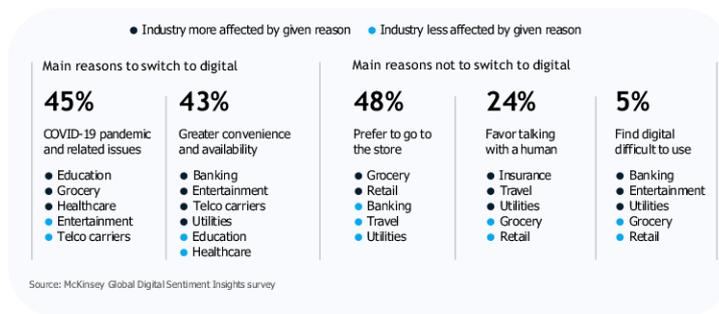
Lately, the COVID-19 pandemic has driven an extraordinary adoption of digital channels across countries and industries, as discussed by a recent survey by McKinsey

(Hajro et al., 2021) about the global consumer sentiment conducted in the six months ending in April 2021 on a sample of about 29,000 respondents in 24 countries from all over the world.



**Fig. 1.4.** Digital adoption (%) by industry in Europe and US in the six months ending in April 2021.

The study shows that, in the period under investigation, consumers continued to embrace digital channels in nearly every industry. Fig. 1.4 shows in detail the increase in the adoption of digital channels in terms of the percentage-point difference between the number of remote users in 2020 and 2021. Digital adoption grew fastest in the utilities, travel industries (+46%) as well as in the public sector (+45%). Consumers who had previously been limited to more local, physical-world offerings before the pandemic (for instance, in education and healthcare) have learned to access those services digitally and, given digital’s borderless nature, to access them globally. The adoption of digital channels in the grocery sector increased during the pandemic and experienced a decrease in fully digital behaviour in the post-pandemic period.

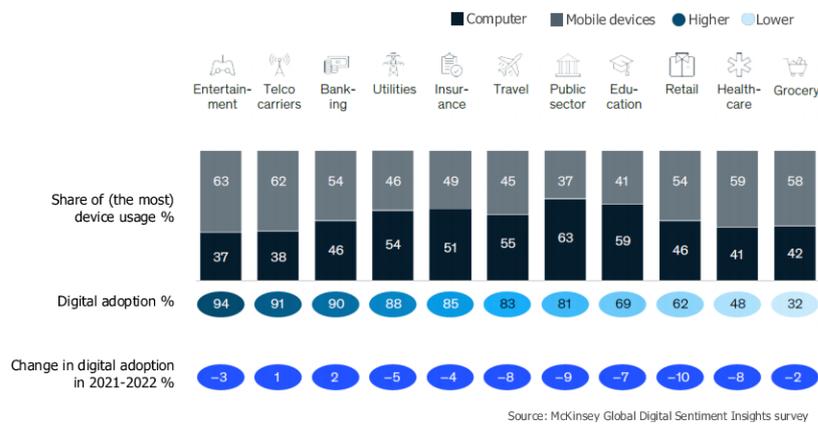


**Fig. 1.5.** Consumers (%) that decide to switch (or not) to digital channels and their motivations in the six months ending in April 2021.

Fig. 1.5, highlights that, although many users choose digital channels for their convenience and availability, physical channels and human interactions are still relevant for customers. According to the sample of respondents, one of the key elements to innovate

digital services is to create phygital (physical plus digital) interactions involving a human agent since many customers (20% of the sample) “want to see, feel, and try the goods” and, although recognizing the convenience of digital channels, they “like interacting with people”.

The subsequent report about the global consumer sentiment by McKinsey (Hajro et al., 2022) was conducted in the six months ending in June 2022. The report pointed out a great convergence in digital adoption among countries in the European Union, as evidenced by DESI (Section 1.4) as well.

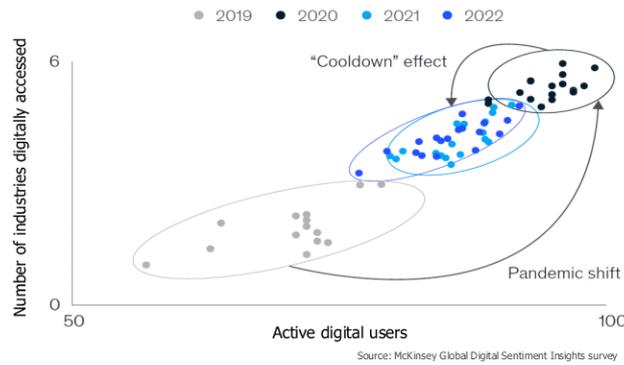


**Fig. 1.6.** Digital adoption (%) by industry and usage (computer and mobile) in the six months ending in June 2022.

Moreover, according to the report results, industries with higher mobile penetration are generally growing more in digital adoption than other sectors (Fig. 1.6). For instance, the banking industry, historically a digital leader, is among those with the highest percentage of customers using digital channels, with 90% of digitalized customers – of which 54% and 46% prefer mobile channels and the computer, respectively – and has continued to grow (+2% of digital users from 2021 to 2022), unlike almost all the other considered industries.

At a demographic level, the surveys report that high digital users are typically younger people, live in urban areas, and have a higher level of education and higher available incomes.

Fig. 1.7 depicts the trend in users’ digital adoption in the pre-pandemic year 2019 and during the pandemic years 2020-2022. The pandemic has pushed and forced almost all surveyed users to shift to digital channels in all considered industries. As the pandemic eases, instead, the digital adoption has settled down (cooldown effect), and consumers are returning to physical channels. However, in general, digital channels’ usage jumped



**Fig. 1.7.** Digital adoption 3 years into the pandemic: average number of industries accessed in a digital or digitally assisted way per user surveyed who have interacted with industries in digital or remote channels.

overall compared to the pre-pandemic period. Europe, for instance, has registered a net gain of 100 million digital users, and Europeans are interacting digitally with twice as many industries as they did before the pandemic.

## 1.6 The digital evolution of retail industry

The digital transformation has not only allowed businesses to reach global markets, breaking down geographical barriers and fostering a more interconnected and dynamic business ecosystem, but also improved efficiency and convenience for customers. The available touchpoints to interact and communicate with customers have been enriched over time and are always on the rise depending on the specific sector.

In this section, we focus on the retail industries – which is the sector under investigation in this thesis – and analyse the evolution of models of delivery channels implemented by retailers due to the introduction of digital channels.

With the emergence of new technologies, retailers have been applying new models to deliver their products and services to meet customer needs and increase marketing, sales and service effectiveness. Fig. 1.8 shows the key stages of the evolution of retailers' delivery channels. Such an evolution has had the ultimate goal of constantly improving *customer experience*, which represents the cumulative impact of multiple touchpoints over time that may be given by sensory, affective, creative-cognitive, physical, behavioural and lifestyle, and social-identity experiences (Schmitt, 1999; Zwilling, 2014). The customer experience is a marketing strategic asset that includes designing interactions with customers to satisfy their needs and overcome their expectations to increase their loyalty to the company.

In the pre-1990s era, the business models of retailers in handling interaction with cus-

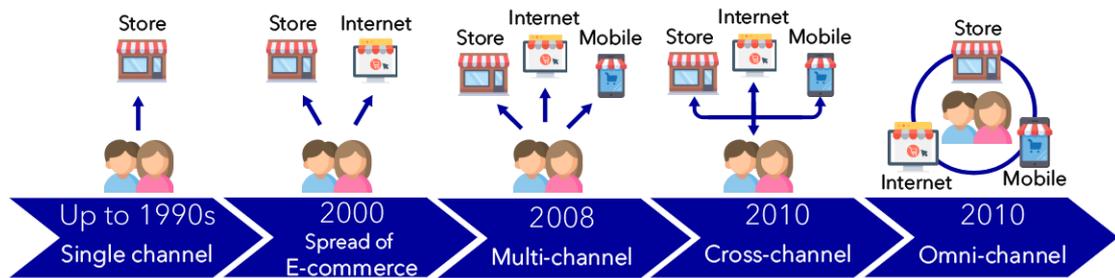


Fig. 1.8. The evolution of models of delivery channels (touchpoints) due to digitalization.

tomers and delivery of products and services primarily relied on *single-channel* delivery strategies. The relationship with their customers was based on face-to-face interactions at a physical store where customers travelled and could purchase products or benefit from services directly from the company. Customers' options were basically limited to visiting the physical store, mail-order catalogues, or making phone orders.

Starting from the 1990s, the spread and the permeation of new technologies and ongoing digitalization, especially the Internet, created a breakpoint in the retailer-customers relationship. New customer expectations and new types of firms' business models emerged. In particular, the first e-commerce websites and digital-only businesses were introduced between 1994 and 1995. For instance, it was during those years that Amazon and eBay were founded. The digital-only businesses, also in the present day, eliminate the need for physical stores or face-to-face interactions by showcasing their products, engaging with customers, and processing transactions through the Internet, which allows them to reach a global customer base anytime.

During the 2000s e-commerce started with its diffusion to dramatically change customer expectations and revolutionize how traditional businesses, relying on a single-channel model, operated. In those years, the phrase "brick-and-mortar" was coined to refer to traditional business models in contrast to digital-only ones. Thus, in order to remain competitive and further expand their customer bases, brick-and-mortar businesses have been pushed to adapt to this emerging context gradually. Specifically, they have adopted progressively more advanced and sophisticated delivery channel systems that fall under the broad umbrella of "click-and-mortar" models in which products and services are provided through both physical and digital channels.

The first click-and-mortar model that was introduced, approximately around 2008, afterwards the diffusion of smartphones, is the *multi-channel* approach. It allows customers to interact with the company through different alternative touchpoints, typically the physical store, the online website and the mobile apps. However, in this model, each

channel exists autonomously without integrated management of data, information and behaviour concerning customers. For instance, a customer purchasing a product from a retailer's website could not request assistance at the physical store since the staff would not have any information about that purchase.

Shortly thereafter, a further model to be introduced is the *cross-channel* one. In this case, the channels are connected and complementary to each other, providing a smooth customer experience. With the cross-channel approach, clients can use several channels for the same order. Online and offline channels work together directly to drive the purchase. A typical example is the "Click & collect" option, which allows clients to order online and get the product in-store.

The final stage is *the omni-channel* strategy, which is defined by [Cunnane and Permenter \(2012\)](#) as "a set of integrated processes and decisions that support a unified view of a brand from product purchase, return, and exchange standpoint irrespective of the channel (in-store, online, mobile, call center, or social)". Firms add the possibility of simultaneously sharing information so that customers can use several different (physical and digital) channels at the same time. Indeed, every type of communication by the firm, such as e-mail or chatbot, is designed to optimize the consumer experience based on consumer preferences. This model is thought to dissolve consumers' awareness of distinct channels and access to services wherever they want, blurring the lines between digital and physical experience. For example, they can use their mobile while they are in a physical store. All changes are updated in real-time across all channels, and firms can collect data on customer behaviour, and therefore understand their preferences and guarantee a worthy customer experience aiming at building loyalty and increasing their customer base.

Firms undertake the best strategy on the basis of their goals and business model. The move from single-channel to omni-channel strategies reflects a shift towards a more customer-centric approach, providing convenience, flexibility, and personalization in how customers interact with companies and access their products and services ([Akter et al., 2021](#)).

As a result, nowadays, customers expect seamless, personalized experiences across multiple digital touchpoints – the most common channels in the retail sector may be summarized as in [Fig. 1.9](#) – and they can make purchases or benefit from a given service by choosing from a large number of different channels to assess and compare products and services, including stores, websites, mail, e-mail and social media, each of which has its own features. For example, using offline channels, i.e. physical stores, customers can touch and feel what they are buying while using online channels, they can access more



**Fig. 1.9.** Main delivery channels used by retailers to interact with their customers.

choices and compare prices and other product information.

Moreover, customers directly or indirectly participate in value-creation activities, expanding the diversity of the service contents and facilitating the development of new business models. Today, customized digital services for customers are proliferating through activities such as customer experience monitoring that collects real-time data on customer behaviour (Chin et al., 2023).

### 1.6.1 From the customer experience to the customer journey

The relationship with customers has always been a crucial issue for companies to thrive and be competitive. The business models have become increasingly complex due to digitalization, the rising number of touchpoints, the evolving behaviour of customers and the way they perceive channels.

The concept of customer experience existed even before the disruption brought by digital technologies. Nevertheless, it has become an increasingly prominent issue during last years. Its definition has been extended and followed by the concept of the *customer journey*. Whether customer experience is the cumulative impact of multiple physical and digital touchpoints over time, the customer (end-to-end) journey is characterized by a holistic perspective as it represents the whole processual and experiential aspects of service processes as seen from the customer's viewpoint (Følstad and Kvale, 2018). In other words, the customer journey is the cognitive path leading a customer to purchase a product or a service, from the moment the company reaches the potential customer to the purchasing phase and, finally, building customer loyalty.

In order to design an effective customer journey in a customer-centric paradigm, it is essential to properly identify and design determinants and attracting factors, such as brand perception and recognition and customer service. The *accessibility* to products and services is one of the key factors contributing to establishing a fruitful customer experience. Neutens (2015) defines accessibility as a multi-faceted concept, which takes into account: (i) availability, which is the adequacy of the capacity to the demand volumes, (ii) geographical (or physical) accessibility, (iii) the appropriateness of provision, (iv) affordability, (v) and acceptability, i.e. compliance and satisfaction. In particular, physical accessibility represents the spatial relationship between customers and physical channels. In other words, it is the proximity the company guarantees to customers through its facilities (physical channels) providing its products and services. For instance, physical accessibility may be measured as the distance from the customers' residence to the nearest physical point or the diversity of physical points within a given distance from the place of residence (Haugen et al., 2012).

In the single-channel model, physical accessibility was, substantially, the main (and only) component to assess the company proximity. However, the introduction of digital channels and their merging with the physical touchpoints, beyond generating new products and services, has led to a shift toward more complex delivery models (e.g. multi or omni-channel strategies).

Consequently, accessibility has been taking on a new meaning with blurred outlines. According to Bolton et al. (2018) service research and practice are entering an era in which the digital and physical realms will become intertwined and blend into a holistic customer experience. Physical accessibility is joined by the digital feature, which has undoubtedly contributed to significantly changing the overall concept of accessibility. In general, physical accessibility has been extensively investigated in many sectors and estimated using several measures and methodologies (e.g., the distance to the closest facilities, and the two-step floating catchment area). Instead, digital accessibility is a relatively new issue. It should take into account many factors, also endogenous to the company, such as the digital skills of users or the degree of available connectivity. In order to gain a more comprehensive view of global proximity to customers, the interaction between physical and digital channels should be considered, as well as the way customers make joint use of them from an omni-channel perspective.

In conclusion, the retail industry, as well as other sectors, is witnessing a substantial and constant increase in the adoption of digital delivery channels by their customers, resulting, as a consequence, in an increase in overall users' accessibility. Nevertheless, researchers and practitioners question whether digital channels will completely replace

the physical and more traditional ones in the foreseeable future and to what extent physical channels will still be relevant for an effective customer journey.

## 1.7 Supply chain design in omnichannel retail

Since digital transformation has brought tangible changes to every aspect of people's lives, the phenomenon has been extensively analysed from different perspectives and in several contexts. Several studies investigated the impact of digitalization on productivity and employment (see, e.g., [Gaglio et al., 2022](#); [Anderton et al., 2023](#); [Du and Jiang, 2022](#); [Wengler et al., 2021](#)), business models (e.g., [Vaska et al., 2021](#); [Li, 2020](#)), marketing and customer-experience (e.g., [Tabrizi et al., 2019](#); [Caliskan et al., 2021](#)), supply chain and logistics (e.g., [Al Mashalah et al., 2022](#); [Cichosz et al., 2020](#)), and economies and societies (e.g., [Pappas et al., 2019](#); [Berman and Marshall, 2014](#)), ecological, economic, or social sustainability (e.g., [Brenner and Hartl, 2021](#)), both in private and public sectors, from manufacturing, retailing and financial services to transportation and healthcare.

In particular, supply chain management has become a strategic issue for any company to meet targets in terms of economic competitiveness, time and quality of service especially in an economic environment characterized by the globalization of trade and the acceleration of industrial cycles. These developments have been influenced by successive trends in the economy and society resulting from computerization, increased complexity of trade flows, increased competition and sustainable development ([Eskandarpour et al., 2015](#)).

The rise of digital channels such as websites, mobile apps, and social media has been revolutionizing the way consumers access information, make purchases, and seek support in the retail sector. Nevertheless, the physical channels are still a significant gateway from the customers' point of view, as highlighted in [Section 1.5](#).

As regards the adoption of delivery channels, customers may be broadly classified into three segments: *traditionalists* rely on only physical channels; *only-digital* customers exploit exclusively digital channels to benefit from a specific product or service; and *phygital* customers make a blended use of both physical and digital channels ([Herhausen et al., 2019](#)).

As a result, in this new emerging context, the digitalization of retailers involves not only the need to develop and enhance digital channels to interact with their customers. It should also consider integrating these new digital channels with the existing (physical) ones. To this end, physical channels should be rethought in the light of digitalization and, thus, reorganized as they are still a key determinant to guarantee a high service level and a comprehensive customer journey.

Therefore, in order to adapt their physical delivery networks (or facility networks) to new business models, retailers have been undertaking several and various strategic location-based actions aiming at restructuring their networks. Although these reorganization actions may be of many kinds depending on their specific application field and scope, we identify some of them as follows:

- *Closures* to downsize the facility network, usually in combination with or following the enhancement of digital channels;
- *Relocations*, consisting of closing existing facilities and opening new ones in other locations to capture different and more profitable markets;
- *Diversification/specialization* of existing facilities in terms of provided services to meet different segments of demand with changing behaviour;
- *Capacity replanning* to meet the changing volume of demand;
- *Outsourcing* aiming at entrusting the delivery of basic services to Third Party Providers (TPPs) having their network of facilities while devoting in-house facilities to more complex services.

The network configuration resulting from implementing such strategies is much more complex. The relationships within facilities and between customers and facilities are intertwined.

For instance, by implementing the diversification action, several types of facilities may be established, and quite often, they are arranged in a hierarchical structure. A typical example concerns the provision of banking services. Given the emergence of businesses other than banks that offer banking services and changes in customers' behaviour, it has become a necessity for banking institutions to modularize their services and be able to offer them at several diversified branches that range from fully staffed advanced branches to regional small-scale branches or even unstaffed branches aiming to attract new retail customers.

Moreover, such strategies may change the customers' perception of the service level provided by the network as a whole, the facilities they decide to patronise to benefit from a specific service, and their physical accessibility.

This thesis focuses on implementing one or more of these location-based actions toward a sustainable spatial reorganization of retailers' facility networks in the aftermath of digitalization and customers' changing behaviour. The reference literature to model such problems is the *facility location literature* whose generalities will be presented in

**Chapter 2.** Then, two problems will be presented and investigated; one consists of an application to the banking sector (**Chapter 3**), while the other is a generalization of the first problem aiming at theoretically contributing to the existing location analysis literature (**Chapter 4**).

## **1.8 Conclusions**

Nowadays, digitalization is a key strategic component for global economies to thrive. In this chapter, digital transformation has been discussed by considering three main pillars, i.e. technology, business models and competition, and customer behaviour.

The main initiatives undertaken by European institutions and policymakers, especially in the light of the COVID-19 pandemic, have been presented along with accreditation measures to assess the digital divide across European countries.

Then, the adoption of digital channels is addressed across industries and the impact of digitalization on the models of delivery channels is analyzed within the retail sector. Our analysis has shown that digital channels are increasingly adopted and have changed the concept of accessibility to products and services. However, digital channels may not completely replace physical ones. Nevertheless, to be still effective, the physical networks of facilities providing products and services should be reorganized to meet the ever-changing customers' needs by applying tailored strategic location-based actions.

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# Facility Location Problems

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## Summary

In this chapter, we discuss families of Facility Location Problems. We present an overview of the main features and the mathematical formulations of the most well-known models in the literature. Specific attention is given to the family of covering problems and their generalization, specifically to cooperative covering problems. Moreover, hierarchical problems are presented within the covering problem family. Elements presented in this chapter represent the reference background for developing the models and applications presented in the subsequent parts of the work.

## 2.1 Generalities

Defining the location of facilities is a crucial decision from a strategic and operational point of view for any public and private business. Indeed, such decisions may affect the whole supply chain of a business, from connections of plants and warehouses with suppliers to the relationship of front facilities and customers where the latter may access products and services.

Given the relevance of such problems, many researchers and practitioners have contributed to the creation of a considerable amount of knowledge – from theoretical, modelling and solution approach perspectives – since the 1960s, generating the research field of *location analysis*.

Seminal works of the modern location analysis are [Cooper \(1963\)](#) and [Hakimi \(1964\)](#),

1965), although the first evidence dates back to the 17th century, and the first location problem is attributable to Weber (1929). Since then, location analysis has become a very active field of research and gathers a wide variety of problems, also driven by interaction with other disciplines (e.g., economics, geography, and logistics). Recent and extensive collections have been published on the topic by Eiselt and Marianov (2011) and Laporte et al. (2019).

The core of location analysis consists of *Facility Location Problems* (FLPs). They aim to identify the best location for one or several facilities in a given space to serve a given demand (actual or potential) from a set of customers. The meaning of “best” strictly depends on the nature of the problem under study, the objectives to achieve and the constraints to which it is subjected (Laporte et al., 2019).

As reported by Eiselt and Laporte (1995) and ReVelle and Eiselt (2005), the fundamental elements of a FLP are:

- *Facilities* to locate providing services and/or products in order to satisfy the demand. They are typically characterized by some features, e.g., their costs, capacity and attractiveness, and the number and types of services they offer. In particular, the number of facilities to be located may be either a given parameter of the problem or a decision variable. Their costs can be related to their activation and/or operational activities. Moreover, facilities may differ in terms of the number of services or commodities they offer, e.g., *single-commodity* vs *multi-commodity* (Pirkul and Jayaraman, 1998). Also, they may differ in terms of products and services they provide. Facilities may also show some relation structure, e.g., it is possible to consider *hierarchical* (Farahani et al., 2014) or *multi-level problems* (Ortiz-Astorquiza et al., 2017). The capacity of a facility refers to the maximum number of customers that a facility can serve. Some facilities can have infinite or finite capacity. Related problems are known as *uncapacitated* and *capacitated problems*, respectively;
- *Location space* representing the space where facilities must be located to serve the demand. The location space can be *continuous*, a *network*, or *discrete*. In a continuous space, facilities may be positioned everywhere. There could be, at most, some forbidden zones where facilities cannot be located due to, e.g., geographical obstacles or technical constraints. In a network space, facilities can be located on nodes and/or edges of a network. Finally, in a discrete space, facilities can be chosen within a set of (discrete) candidate sites;
- *Customers* demanding products and services at the facilities located or to be located. Customers can be continuously spread over the location space or concen-

trated at specific points, namely *demand nodes*. Typically, each demand point is associated with a value of demand for some kind of services;

- *Objectives* to achieve. Several criteria may be involved in the definition of the *objective functions* to determine the optimal location decisions. In most applications, the latter can be driven by cost or covering objectives. In the first case, the typically considered costs are of location, i.e. the fixed cost to establish a facility, and/or of allocation, i.e. the variable cost needed for services and/or goods provision (Fernández and Landete, 2019). Instead, covering objectives refer to the total demand that can be served by a facility within a given distance from its location (García and Marín, 2019). Other objective functions may also depend on the specific nature of facilities and the services they provide. For instance, the location of undesirable facilities, namely *obnoxious facility problems* (Church and Drezner, 2022), may be targeted by dispersion objectives (Erkut, 1990). Multi-objective models may also be adopted when dealing with a multiplicity of objectives to optimize (Farahani et al., 2010);
- *Constraints* to satisfy. They could be extremely varied depending on the problem at hand. However, typical examples are economic and budget constraints, capacity constraints, topological constraints (e.g., minimum or maximum distances between facilities, zoning laws), and technical and technological restrictions;
- *Interactions* between customers and facilities and between facilities themselves. In particular, customers can be allocated to facilities or covered by them to satisfy their demand. The allocation can be compulsory or based on a preference system and utility functions. Accordingly, we distinguish between *location-allocation* and *location-choice models* (Drezner and Eiselt, 2002). The coverage, instead, is typically related to the presence of at least one located distance facility within a given distance from customers (Berman et al., 2010). Interaction between facilities refers to the way they compete to capture the largest market shares (Aboolian et al., 2007) or, conversely, to the way they cooperate in order to assure a certain service level to customers (Berman et al., 2011).

A wide range of mathematical models can be defined depending on the combinations of the above fundamental elements. Examples of classification schemes are proposed by Revelle et al. (2008) and Farahani et al. (2012).

In the remainder of the chapter, the basic facility location models will be presented in Section 2.2 with a more in-depth discussion in Section 2.3 of covering location problems, which are the reference problems of the research activities at the core of this thesis.

## 2.2 Basic Facility Location Problems

In the following, we refer to discrete problems, i.e., problems in which the location space is discrete and customers are concentrated in a finite number of demand nodes.

In order to present the basic facility location models, we introduce the following notation:

*Sets*

$I$  Set of demand nodes, where customers are located;

$J$  Set of candidate site for locating facilities;

*Parameters*

$p$  Number of facilities to locate;

$w_i$  Demand (weight) associated to node  $i \in I$ ;

$d_{ij}$  Distance between demand node  $i \in I$  and candidate site  $j \in J$ ;

*Decision variables*

$y_j$  Binary variable equal to 1 if a facility is located at candidate site  $j \in J$ , and 0 otherwise;

$x_{ij}$  Binary variable equal to 1 if customers from demand node  $i \in I$  are allocated to a facility located at candidate site  $j \in J$ , and 0 otherwise.

### 2.2.1 $p$ -Median problems

The  $p$ -median problem aims to find the optimal location of a fixed number of facilities  $p$ , usually called medians, among the candidates, so that the weighted (by the demand) sum of the distances between customers and their assigned facilities is minimized.

A  $p$ -median model is a *minisum* problem, in which the objective to minimize depends on the costs associated with each of the users in equal measure, in contrast with other concepts such as “center” and “equity” (see [Section 2.2.2](#)).

The first formulation of the  $p$ -median problem is attributable to [Hakimi \(1964\)](#) and is the following:

$$\min \sum_{i \in I} \sum_{j \in J} w_i d_{ij} x_{ij} \tag{2.1}$$

$$\text{s.t. } \sum_{j \in J} x_{ij} = 1 \quad \forall i \in I \tag{2.2}$$

$$x_{ij} \leq y_j \quad \forall i \in I, j \in J \tag{2.3}$$

$$\sum_{j \in J} y_j = p \tag{2.4}$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in I, j \in J \tag{2.5}$$

$$y_j \in \{0, 1\} \quad \forall j \in J \quad (2.6)$$

Constraints (2.2) ensure that the (whole) demand from each node is assigned to only one facility. According to constraints (2.3), customers can be assigned to candidate site  $j$  if there is an open facility in that location. Constraint (2.4) sets the number  $p$  of facilities that must be located. Finally, constraints (2.5) and (2.6) define the domain of the decision variables.

The minimum average weighted distance between customers and facilities is obtained by dividing the value of the objective function (2.1) by  $\sum_{i \in I} w_i$  (known).

Starting from the above formulation, several extensions have been proposed in the literature to include practical features or to target specific applications, showing a wide range of possibilities of use.

For instance, one of the first applications of  $p$ -median is clustering, which aims to gather a large set of elements into mutually exclusive smaller groups based on their similarities. [Klastorin \(1985\)](#) showed that the  $p$ -median formulation is effective in providing good clusterings.

In order to replicate an elastic demand with respect to the distance, [Holmes et al. \(1972\)](#) presented a formulation considering that people would not travel beyond a given threshold distance. They applied the model to the real case of daycare facilities in Columbus, Ohio. In the same work, the authors also introduced the *capacitated  $p$ -median* problem by imposing a maximum capacity level for facilities to locate.

Moreover, in order to avoid strong inefficiencies or obtain a more balanced distribution of customers between facilities, *balancing requirements* may be introduced, as in [Carreras and Serra \(1999\)](#). To this end, the authors imposed in a capacitated  $p$ -median formulation also a lower bound on the capacity level of the facilities. In the paper, the model is applied to the location of pharmacies in a rural region of Spain.

We refer to [Marín and Pelegrín \(2019\)](#) for a recent review on this family of optimization problems.

### 2.2.2 $p$ -Center problems

In  $p$ -center problems, the aim is to find the optimal location of exactly  $p$  facilities to minimize the maximum distance between each demand node and its closest facility. Unlike median (minisum) problems, which tend to favour customers concentrated in more dense population areas, center models belong to the class of *minimax* problems and tend to limit the detriment of spatially more dispersed customers. The 1-center location problem on a network was formally introduced by [Hakimi \(1964\)](#). Herein, we report the discrete formulation of the  $p$ -center problem by [Daskin \(2011\)](#).

Let us introduce the continuous decision variable  $C$ , denoting the maximum distance between a demand node and its assigned facility. By referring to the notation previously introduced, the problem can be formalized:

$$\min C \tag{2.7}$$

$$\text{s.t. } \sum_{j \in J} d_{ij} x_{ij} \leq C \quad \forall i \in I \tag{2.8}$$

$$(2.2), (2.3), (2.4), (2.5), (2.6)$$

Objective function (2.7) together with constraints (2.8) ensure that the objective value is equal to the maximum of the distances between demand points and the active facilities. The basic formulation may be modified by including demands  $w_i$  as weighting factors in the objective function.

Hakimi (1964) introduced the *absolute center problems* to locate exactly one facility that may be, for instance, a police station or a hospital. Whereas, capacity restrictions on the facilities are considered by Özsoy and Pınar (2006).

Given their fairness scope, center location problems are commonly applied in emergency service applications, where the goal of quick response times is significantly more important than any efficiency target, related to the cost of delivering that service.

For an overview of formulations, solution methods and fields of applications, we refer to Çalık et al. (2019).

### 2.2.3 Fixed-charge facility location problems

Fixed-charge facility location problems aim to locate at the minimum cost facilities offering a certain service to allocate customers demanding such service. From the two types of decisions, i.e. location of facilities and allocation of customers, derive fixed-charge costs, which are costs for establishing the facilities and assignment costs for customers.

Let us introduce  $q_j$  and  $f_j$  as the maximum capacity and the fixed-charge (setup) cost for establishing facility  $j$ , respectively, and  $c_{ij}$  as the assignment cost incurred for serving all the demand from demand node  $i$  at facility  $j$ . Making use of the notation previously introduced, a standard formulation for the fixed-charge facility location problems is as follows (Cornuéjols et al., 1991):

$$\min \sum_{j \in J} f_j y_j + \sum_{j \in J} \sum_{i \in I} c_{ij} x_{ij} \tag{2.9}$$

$$\text{s.t. } \sum_{i \in I} d_{ij} x_{ij} \leq q_j y_j \quad \forall j \in J \quad (2.10)$$

(2.2), (2.5), (2.6)

Objective function (2.9) minimizes the sum of location and allocation costs. Constraints (2.10) ensure that the overall demand served by facility  $j$  does not exceed the latter's capacity and prevent, simultaneously, from being allocated to non-open facilities.

In the above formulation, the total demand generated from a certain node is served by the same facility (*single allocation*). However, in several applications, the demand from the same node may be split among several facilities (*multiple allocation*) by relaxing  $x_{ij}$  variables, i.e. by substituting constraints (2.5) with constraints:

$$0 \leq x_{ij} \leq 1 \quad \forall i \in I, j \in J \quad (2.11)$$

An important particular case is the fixed-charged problem with uncapacitated facilities (Erlenkotter, 1978), known as *uncapacitated facility location problem*. This problem assumes that the capacity of any open facility is sufficient to satisfy the demand of all customers, and thus capacity constraints (2.10) are not needed. Moreover, the multiple allocation setting could be allowed by considering (2.11) instead of (2.5).

Given their wide applicability, extensive literature on fixed-charge facility location problems is available in terms of modelling and applications as well as solution approaches. Indeed, applications arise in several contexts, such as supply chain management (see Melo et al., 2009, for a survey), distributed systems (Klose and Drexl, 2005), humanitarian relief and emergency system (Balcik and Beamon, 2008). Moreover, several heuristics and exact methods have been developed (see, e.g., Nozick, 2001; Yang et al., 2012; Fischetti et al., 2017). Interested readers on the topic may refer to the review by Fernández and Landete (2019).

## 2.3 Covering location problems

Covering location problems, initially introduced by Toregas et al. (1971), constitute a major class of problems in facility location analysis, with numerous applications in a wide range of domains. The “coverage”, in fact, is one of the classical objectives within location modelling. The goal is to ensure that customers are covered, i.e. served by (at least) one facility whose distance is lower than a certain value  $r$ , typically indicated as *covering radius*.

Two major types of covering models were proposed in the early relevant literature:

- The *Location Set Covering Problem* (LSCP) seeking to determine the minimum

number of facilities and their location to fully cover all the demand nodes in a given location space. The seminal papers are [Toregas et al. \(1971\)](#) and [ReVelle et al. \(1976\)](#);

- The *Maximum Covering Location Problem* (MCLP) aiming to maximize the covered demand, given a limited number ( $p$ ) of facilities to locate or a budget ( $B$ ). It was formally introduced by [Church and ReVelle \(1974\)](#)

In order to formulate the basic LSCP and MCLP models, let us consider the notation proposed in [Section 2.2](#) and denote the covering set  $N_i$  as the set of all candidate sites  $j$  within the covering radius  $r$  from demand node  $i$ , i.e.,  $N_i = \{j \in J : d_{ij} \leq r\}$ .

The LSCP can be formulated as follows:

$$\min \sum_{j \in J} y_j \tag{2.12}$$

$$\text{s.t. } \sum_{j \in N_i} y_j \geq 1 \quad \forall i \in I \tag{2.13}$$

$$(2.6)$$

Objective function [\(2.12\)](#) aims to minimize the number of facilities to locate. Constraints [\(2.13\)](#) guarantee that each demand node  $i$  finds at least one located facility  $j$  within distance  $r$ ; thus, every node is covered within radius  $r$ . Constraints [\(2.6\)](#) explain the binary nature of decision variables.

According to the above notation and considering binary decision variables  $z_i$ ,  $i \in I$  equal to 1 if customers at demand node  $i$  are covered, and 0 otherwise, the MCLP formulation is formulated as follows:

$$\max \sum_{i \in I} w_i z_i \tag{2.14}$$

$$\text{s.t. } \sum_{j \in N_i} y_j \geq z_i \quad \forall i \in I \tag{2.15}$$

$$z_i \in \{0, 1\} \quad \forall i \in I \tag{2.16}$$

$$(2.4), (2.6)$$

Objective function [\(2.14\)](#) maximizes the total demand covered by a fixed number  $p$  of facilities to locate [\(2.4\)](#). Constraints [\(2.15\)](#) guarantee that a demand node  $i$  is considered as covered whether at least one facility is located within covering radius  $r$ . Finally, constraints [\(2.6\)](#) and [\(2.16\)](#) define the domain of decision variables.

Both the above formulations can be derived from the more general formulation proposed by [García and Marín \(2019\)](#). Therein, the authors also surveyed the theoretical properties of covering models as well as several solution methods proposed in the literature. Nevertheless, reviews on the topic have been regularly published due to the importance of covering problems and the rich related literature. Readers may also refer, e.g., to [Schilling \(1993\)](#), or [Plastria \(2002\)](#) who provide a review of continuous covering models, or the most recent [Farahani et al. \(2012\)](#). Moreover, many contributions in the literature deal with solution approaches. For instance, [Daskin \(1997\)](#) applied the branch-and-bound method to the LSCP and some heuristic methods, e.g. greedy adding algorithm and Lagrangian relaxation, to the MCLP. More recently, [Cordeau et al. \(2019\)](#) developed a tailored Benders' decomposition to solve LSCPs and MCLPs of very large scale, obtaining optimal solutions very quickly for instances with 100 candidate facility locations and up to 15 (MCLP) and 40 (LSCP) million demand nodes.

The wide applicability of covering models stimulated strong interest in the research community, testified by a huge number of publications over the last 50 years.

Covering location problems has been initially (and most widely) applied to emergency services and in the public sector. Indeed, the coverage concept allows for modelling problems involving ordinary services whose access by consumers should be guaranteed within a maximum distance.

In the seminal paper, in fact, [Toregas et al. \(1971\)](#) introduced the SCLP to model the problem of locating emergency service facilities where the time response depending on the distance is a crucial feature to guarantee the effectiveness of the service. In this context, several papers deal with the problem of locating fire stations (see, e.g., [Badri et al., 1998](#); [Degel et al., 2014](#)) and ambulances (see, e.g., [Gendreau et al., 1997](#); [Rajagopalan et al., 2008](#); [Erdemir et al., 2010](#)). [Li et al. \(2011\)](#) provided a review focused on covering models and optimization techniques for emergency response facility location and planning.

Applications in the public sector vary from healthcare (e.g. [Oztekin et al., 2010](#)) to schools (e.g. [Doerner et al., 2009](#)), post offices (e.g. [Xu et al., 2020](#)) and police stations (e.g., [Curtin et al., 2010](#)).

Furthermore, covering location problems have also been extensively applied to locate facilities in the private sector, such as bank branches, shopping centres, restaurants, and retail shops ([Alexandris and Giannikos, 2010](#); [Küçükaydın and Aras, 2020](#); [Méndez-Vogel et al., 2023](#)), where the covering radius may represent the catchment area of a facility or the distance customers are willing to travel to access the services attract.

Covering problems also apply in many other areas, such as radar installation or loca-

tion of telecommunications antennas and designing of sensor networks (e.g., Karabulut et al., 2017).

Readers may refer to Farahani et al. (2012) and García and Marín (2019) for comprehensive reviews on covering location problems.

### 2.3.1 Hierarchical Facility Location Problems

*Hierarchical* FLPs deal with the location of interacting facilities at different levels of a hierarchical system. In this class of problems, the key features are the different types of involved facilities, the services they provide, and the (possible) raking or (implicit or explicit) relationship among facilities (Contreras and Ortiz-Astorquiza, 2019).

Hierarchical facility systems are present in several contexts, such as supply chains and production-distribution systems (Hinojosa et al., 2000), health care (Marianov and Taborga, 2001), education systems (Teixeira and Antunes, 2008), banking industry (Min and Melachrinoudis, 2001), and emergency medical services (Şahin et al., 2007).

According to the recent survey on hierarchical FLPs by Farahani et al. (2014), they can be classified according to the following characteristics:

- *Flow pattern* refers to the way in which network flows are routed through the various levels of a hierarchical system. In a single-flow pattern, customers visit facilities in each of the levels until they reach the highest level, while in a multi-flow pattern, customers visit facilities in a subset of levels, possibly skipping some levels (Şahin and Süral, 2007). Regardless of the flow type (single or multi), the flow pattern can also be classified into referral, if lower-level facilities refer a fraction of customers to higher-level facilities, or non-referral;
- *Service availability* refers to the customer accessibility to a service at different levels of the hierarchy. The latter may be nested (or successively inclusive) if a higher-level facility provides all services that are also provided by a lower-level facility and at least one additional service, or non-nested (or successively exclusive) if facilities at each level offer different service (Narula, 1984);
- *Spatial configuration* of levels in the hierarchy, which may be coherent if demands that are allocated to a specific lower-level facility are assigned to the same higher-level facility or, conversely, non-coherent (Şahin and Süral, 2007);
- *Other features*, such as network levels and types of services, capacity, presence of uncertainty (deterministic, probabilistic, or stochastic models) or time (static or multi-period models), and application;

- *Objective function* including median, fixed charge, center, and coverage objectives.

As highlighted by the last point, hierarchical FLPs are closely related to classical facility location problems. In particular, in the following, we focus on the hierarchical problems with coverage objectives and present the formulations for variants of LSCP and MCLP. Moore and ReVelle (1982) firstly investigated an MCLP with hierarchical facilities. The majority of papers in location literature introduce hierarchical models with maximal covering objectives (see, e.g., Marianov and Taborga, 2001; Çınar and Yaman, 2011), while a few with set covering objectives (Daskin and Stern, 1981; Marianov and Serra, 2001).

In order to present hierarchical formulations of basic covering problems as presented by Daskin (2011), the notation introduced in previous sections is extended and modified as follows:

#### *Sets*

- $I$  Set of demand nodes, where customers are located;
- $J$  Set of candidate site for locating facilities;
- $K$  Set of hierarchical levels/facility type/services. Level 1 is the lowest order of the hierarchy and level  $|K|$  is the highest order;

#### *Parameters*

- $p_q$  Number of facilities of type  $q \in K$  to locate;
- $w_{ik}$  Demand of associated to node  $i \in I$  for service  $k \in K$ ;
- $d_{ij}$  Distance between demand node  $i \in I$  and candidate location  $j \in J$ ;
- $c_{qj}$  Cost for establishing a facility of level  $q \in K$  at candidate site  $j \in J$  with  $c_{qj} \leq c_{q(j+1)} \forall j \in \{1, \dots, k, \dots, |K| - 1\}$ ;
- $a_{ij}^{kq}$  Covering parameter equal to 1 if a facility of type  $q \in K$  at candidate site  $j$  can cover node  $i \in I$  for demand of service  $k \in K$ ;

#### *Decision variables*

- $y_{jq}$  Binary variable equal to 1 if a facility of level  $q \in K$  is located in  $j \in J$ , and 0 otherwise;
- $z_{ik}$  Binary variable equal to 1 if demand for service  $k \in K$  from demand node  $i \in I$  is covered, and 0 otherwise.

Depending on the service availability of the hierarchical system (i.e., nested or non-nested) and the covering radius of each service, the parameter  $a_{ij}^{kq}$  is differently configured. Specifically, in a nested configuration, a facility of type  $k$  offers services from 1 to  $k$  (Daskin, 2011). Therefore, if the distance  $r^k$  represents the coverage radius for satisfying the demand for service of level  $k$ , a facility of level  $k$  covers all the demand

nodes within  $r^k$  for services from level 1 to  $k$ ; moreover, the covering radius  $r^k$  should increase as the corresponding hierarchical level  $k$  increases. (Moore and ReVelle, 1982).

A general formulation of the hierarchical variant of the MCLP as presented by Daskin (2011) is the following:

$$\max \sum_{k \in K} \sum_{i \in I} w_{ik} z_{ik} \quad (2.17)$$

$$\text{s.t.} \sum_{j \in J} \sum_{q \in K} a_{ij}^{kq} y_{jq} \geq z_{ik} \quad \forall i \in I, k \in K \quad (2.18)$$

$$\sum_{j \in J} y_{jq} = p_q \quad \forall q \in K \quad (2.19)$$

$$z_{ij} \in \{0, 1\} \quad \forall i \in I, k \in K \quad (2.20)$$

$$y_{jq} \in \{0, 1\} \quad \forall j \in J, q \in K \quad (2.21)$$

Objective function (2.17) maximizes the covered demand for each type of service. Constraints (2.18) guarantee that demand from node  $i$  for service  $k$  is covered if and only if at least one facility able to provide service  $k$  to demand node  $i$  is located. Constraints (2.19) stipulate that exactly  $p_q$  facilities of type  $q$  must be located. Finally, constraints (2.20) and (2.21) define the domain of the decision variables.

Using the same notation and referring to Marianov and Serra (2001), the LSCP can be formulated as follows:

$$\min \sum_{j \in J} \sum_{p \in K} c_{jp} y_{jp} \quad (2.22)$$

$$\text{s.t.} \sum_{j \in J} \sum_{q \in K} a_{ij}^{kq} y_{jq} \geq 1 \quad \forall i \in I, k \in K \quad (2.23)$$

$$(2.21)$$

Objective function (2.12) aims to minimize the total cost of the network. The full satisfaction of the demand is guaranteed by constraints (2.23), which stipulate that each demand node  $i$  must be covered for each service  $k$  by at least one located facility able to provide service  $k$  to demand node  $i$ . Introducing costs in the LSCP could be essential to avoid locating only (few) facilities of higher-level with larger covering radius.

## 2.4 Generalization of Covering Problems

As the range of demand covering applications expanded over the years, it was recognized that some assumptions were somewhat unrealistic and that the original models

could not adequately cater for realistic situations.

According to [Berman et al. \(2010\)](#), the three main (implicit) assumptions that can be relaxed are:

**Ass.1** *Binary (all or nothing) coverage*, i.e. a customer is either fully covered if the distance to the nearest operating facility does not exceed a predetermined limit or not covered at all if the nearest facility is beyond that limit.

**Ass.2** *Individual coverage*, i.e. the coverage of each customer depends only on the nearest established facility. In other words, demand sharing is not allowed among multiple facilities.

**Ass.3** *Fixed covering radius*, i.e. the covering radius  $r$  is a fixed exogenous parameter.

In particular, intuitively, one can imagine that the concept of coverage provided by a facility tends to decrease over distance (relaxation of Ass.1), or customers may patronize more than one facility (relaxation of Ass.2). Moreover, the covering radius may vary depending on some attributes of the facility itself (relaxation of Ass.3).

Generalizing these aspects, [Berman et al. \(2010\)](#) surveyed classes of covering models derived by relaxing the above assumptions, namely:

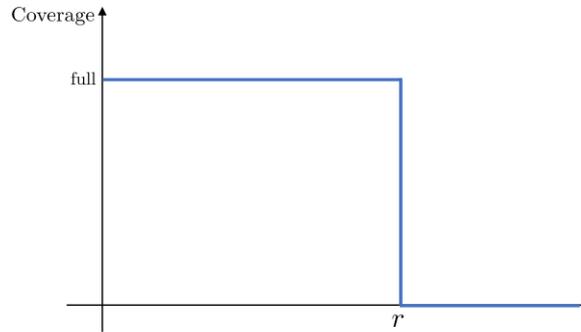
**Rel.1** *Gradual (or partial) coverage* relaxing the binary coverage assumption. This class of problems was introduced to model situations where a customer is fully covered by a facility up to a certain distance, beyond which coverage starts to decrease and the demand results to be partially covered;

**Rel.2** *Cooperative coverage* relaxing the individual coverage assumption. It is assumed, instead, that all located facilities contribute to covering the demand, not only the closest one;

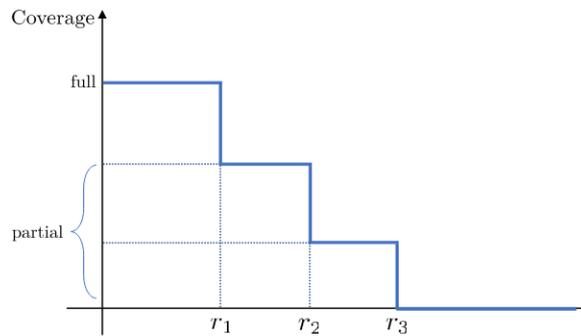
**Rel.3** *Variable radius* relaxing the fixed radius assumption. In these problems, the coverage radius becomes a decision variable rather than a parameter, and it may be modified (increased or decreased) with a corresponding effect on the system performance and cost.

In further detail, gradual problems have been the first class to be introduced by [Church and Roberts \(1983\)](#) and have been more extensively analysed ([Eiselt and Marianov, 2009](#); [Berman and Wang, 2011](#); [Peker and Kara, 2015](#), e.g., ). In particular, the coverage is modelled by a decay function that may represent the proportion of covered

demand or the probability of covering a demand node. For instance, in the seminal paper, the authors generalized the MCLP by defining the coverage as a stepwise function representing multiple coverage radii (Fig. 2.1b) instead of a single one (Fig. 2.1a).



(a) Single covering radius



(b) Multiple covering radii

**Fig. 2.1.** Example of functions representing coverage

Several shapes for the coverage decay function have been introduced, such as linear decreasing function [Berman et al. \(2009c\)](#); [Verter and Zhang \(2015\)](#) or trapezoidal-shaped function ([Berman and Wang, 2011](#); [Drezner et al., 2004](#)), in several contexts.

The ability of facilities to cooperate in providing coverage applies in many situations where the individual coverage assumption may not be valid. In cooperative covering problems, it is assumed that each facility provides a coverage level that decays over distance according to some coverage decay function; these individual coverages are somehow aggregated through a specified aggregate coverage function to determine the overall coverage of services offered to the customer. In this case, the coverage of a demand node is achieved when the total (aggregated) coverage received by the customers exceeds a

certain acceptable threshold.

Gradual and cooperative coverages share some similarities. Indeed, cooperative problems use a decay function to depict facilities' coverage as well as gradual problems. However, gradual covering problems allow the possibility of *partial coverage* of demand nodes (Berman and Krass, 2002), unlike cooperative covering problems where the threshold states if a demand node is (fully) covered or not. On the other hand, gradual problems consider the (possibly partial) coverage from just the closest located facility, while cooperative problems consider the simultaneous presence of all located facilities, representing the peculiarity of such problems. A more detailed discussion on the existing cooperative literature is provided in Section 2.5, being at the core of the present work.

As far as the coverage radius is concerned, in the early literature, it was assumed to be a fixed parameter under the direct control of the decision-maker. However, in many applications, this radius may be modified (increased or decreased) with a corresponding effect on the system performance and cost. For instance, in the retail sector, as the size or the attractiveness of a retail store increases, so does the size of its trading area and the customers that can be attracted. Berman et al. (2009b) extended the LSCP by considering variable covering radii; the cost of establishing a facility rises as its covering radius increases.

Some papers in the literature consider a combination of the above relaxations. Berman et al. (2019) provided a generalized version of MCLP in a gradual and cooperative framework. According to the gradual coverage, demand nodes may be either fully or partially covered or uncovered; the degree of coverage, i.e. the proportion of covered demand, is provided by a decay function associated with facilities. Then, assuming a cooperative behaviour of facilities, a further function is introduced to represent how facilities interact to provide the combined value of partial coverage. In particular, the authors adopted a probabilistic approach to define demand covered by all located facilities. Other contributions considering a gradual cooperative coverage are Drezner and Wesolowsky (2014), Karabulut et al. (2017) and Bagherinejad et al. (2018). Few contributions consider all these three assumptions. Karatas and Dasci (2020) proposed a MCLP considering two-level facilities by assuming gradual and cooperative coverage. The authors also considered a variable radius representing the size of the facility to locate. Two non-linear formulations for the problem are presented and their linearizations are developed.

## 2.5 Cooperative Covering Location Problems

A Cooperative Covering Location Problems (CCLP) assumes that all established facilities may contribute to the coverage of the demand and not only the closest one as

in classical covering problems (Berman et al., 2010).

By referring to the notation proposed in Sections 2.2 and 2.3, the cooperative coverage mechanism may be described as follows:

- The coverage provided by a (single) facility  $j$  to a demand node  $i$  is modelled by a *coverage decay function*  $\phi_{ij}$  decreasing over the distance. The (single) coverage is expressed as:

$$\phi_{ij} = f(d_{ij}) \quad \forall i \in I, j \in J \quad (2.24)$$

where  $f()$  is left continuous, and non-negative and non-increasing with respect to the distance  $d_{ij}$ . This way, it is assumed gradualness in the coverage by facilities avoiding the situation in which a demand node is fully covered within the coverage radius and not covered at all beyond such a distance (classical coverage);

- The overall coverage level received by each customer at node  $i$  is given by combining the coverage  $\phi_{ij}$  provided by all facilities  $j$  that are established ( $y_j = 1$ ) through a specific *aggregate function* as follows:

$$\Phi_i = g(\phi_{ij}y_j, j \in J) \quad \forall i \in I \quad (2.25)$$

where  $g()$  is a non-negative and non-decreasing function with respect to  $\phi_{ij}$ ;

- A demand node  $i$  is covered if the total aggregated coverage  $\Phi_i$  exceeds a certain *threshold*  $T$  otherwise, it is not covered.

The coverage decay function has to be modelled to be able to represent a specific situation according to its nature (e.g., a physical phenomenon or facility's attractiveness). Several shapes have been proposed within CCLPs literature to model the coverage decay function, many of which were borrowed from gradual covering problems, as already mentioned. Table 2.1 summarizes the coverage decay functions (2.24) used in papers dealing with CCLPs. The most widely used functions are inverse and trapezoidal-shaped ones. However, other functions have been introduced in cooperative literature tailored to represent specific service systems based on the application at hand and on what the coverage, in fact, represents and how it is interpreted.

Typical examples of applications in which cooperation is established include the case of facilities emitting physical signals (light, sound, etc.), where each customer receives signals from all facilities, not only one within the covering radius. The inverse and linear functions are used in such applications (e.g., Berman et al., 2009a; Drezner and Drezner,

**Table 2.1**

Expression of coverage decay functions (2.24) used in CCLPs literature. Where present,  $\bar{d}$  is the distances within which the single coverage level  $\phi_{ij}$  is equal to 1 (full coverage); whilst  $\bar{\bar{d}}$  is the distances within beyond which the coverage level is equal to 0.

Type	Expression	Used by
Inverse	$\phi(d_{ij}) = \frac{1}{d_{ij}^2}$	Berman et al. (2009a); Berman et al. (2011); Berman et al. (2013); Karabulut et al. (2017); Li et al. (2018); Drezner et al. (2019).
Trapezoidal-shaped	$\phi(d_{ij}) = \begin{cases} 1 & \text{if } d_{ij} \leq \bar{d} \\ \frac{\bar{\bar{d}} - d_{ij}}{\bar{\bar{d}} - \bar{d}} & \text{if } \bar{d} < d_{ij} \leq \bar{\bar{d}} \\ 0 & \text{if } d_{ij} > \bar{\bar{d}} \end{cases}$	Berman et al. (2009a); Tabrizi et al. (2011); Zhang et al. (2017); Ashtiani et al. (2018); Rezazadeh et al. (2018); Bababeik et al. (2018); Wang et al. (2021)
Linear	$\phi(d_{ij}) = \max\{0, 1 - d_{ij}/\bar{\bar{d}}\}$	Averbakh et al. (2014); Drezner and Drezner (2014); Jayalakshmi and Singh (2017); Berman et al. (2019); Álvarez-Miranda and Sinnl (2019).
Sigmoid	$\phi(d_{ij}) = \begin{cases} 1 & \text{if } d_{ij} \leq \bar{d} \\ \frac{1}{1 + e^{A(d_{ij} - \frac{\bar{\bar{d}} - \bar{d}}{2})}} & \text{if } \bar{d} < d_{ij} \leq \bar{\bar{d}} \\ 0 & \text{if } d_{ij} > \bar{\bar{d}} \end{cases}$ where $A > 0$ is a parameter controlling the shape of the curve.	Wang et al. (2016); Ming et al. (2021).
Fermi-type model	$\phi(d_{ij}) = \begin{cases} 1 & \text{if } d_{ij} \leq \bar{d} \\ \frac{1}{1 + 10^{[(d_{ij} - \bar{d})/(\rho - \bar{d}) - 1]/b}} & \text{if } \bar{d} < d_{ij} \leq \bar{\bar{d}} \\ 0 & \text{if } d_{ij} > \bar{\bar{d}} \end{cases}$ where $\rho$ is the range at which the coverage probability is equal to 0.5 and $b$ describes the function 'tailing' character.	Karatas (2017); Karatas (2018); Karatas and Dasci (2020); Karatas (2020); Karatas and Eriskin (2021).
Exponential	$\phi_{ij} = e^{-\eta d_{ij}}$ where $\eta > 0$ is a parameter controlling the shape of the curve.	Karabulut et al. (2017); Davari (2019).

2014). Other similar applications concern the optimal location of sensor network systems (e.g., for surveillance). Common functions are fermi-type model (e.g., Karatas, 2018) or exponential (e.g., Karabulut et al., 2017).

Moreover, there exist many other application areas in which the coverage provided by facilities is given by non-physical signals. Examples are emergency response systems (e.g., fire stations), where a single facility within a given distance may not guarantee

adequate coverage – trapezoidal-shaped and sigmoid functions are applied in this context by (Bababeik et al., 2018) and (Wang et al., 2016), respectively – or retail services, where  $\phi_{ij}$  may represent the attractiveness of facility  $j$  and customers' perception of the service is given by the simultaneous presence of all available facilities nearby. However, cooperative retail-focused applications are still missing in the literature.

A critical role is also played by the aggregate function, generally expressed by (2.25), which has to be carefully and realistically defined since it ultimately states whether a customer is covered or not. In cooperative literature, the coverage by facilities is typically meant as deterministic or probabilistic. The aggregation of coverages represents one of the peculiar features of cooperative covering mechanism, and its functional expression affects the whole formulation of the problem. By adopting a deterministic approach, the aggregate coverage is usually obtained as the sum of individual coverage levels provided by located facilities, i.e.:

$$\Phi_i = \sum_{j \in J} \phi_{ij} y_j \quad (2.26)$$

Conversely, if the coverage decay function  $\phi_{ij}$  represents the probability that demand node  $i$  is covered by facility  $j$ , the coverage could be assumed based on events independent or, more in general, linked by a certain degree of correlation. Based on this assumption, the aggregate coverage function should be modelled accordingly, generally non-linearly. In particular, whether the coverage by facilities is assumed to be based on independent events, the aggregate function is modelled as follows:

$$\Phi_i = 1 - \prod_{j \in J} (1 - \phi_{ij} y_j) \quad (2.27)$$

where  $\prod_{j \in J} (1 - \phi_{ij} y_j)$  represent the probability that demand node  $i$  is not covered.

Both the functions (2.26) and (2.27) have been first presented in the seminal paper by Berman et al. (2011).

A more general probabilistic approach is presented by Drezner and Drezner (2014), who proposed the *joint coverage function* to aggregate the coverage levels  $\phi_{ij}$  provided by located facilities by including a possible (positive) correlation among coverage events expressed by a parameter  $\theta$ , bounded between 0 and 1. The joint coverage function is formulated as follows:

$$\Phi_i = \theta \max_{j \in J} \{\phi_{ij} y_j\} + (1 - \theta) \left[ 1 - \prod_{j \in J} (1 - \phi_{ij} y_j) \right] \quad (2.28)$$

The aggregate coverage  $\Phi_i$  is formulated as a convex combination of two extreme probabilities that captures the whole spectrum of dependency. In particular, if  $\theta = 1$ , the coverage of  $i$  is achieved by the closest facility, which is the one with the maximum probability (perfect dependence of coverage events). In contrast, if  $\theta = 0$ , the cooperation is the maximum possible, being the coverage events independent. By varying the parameter  $\theta$ , the correlation among events may be properly calibrated.

**Table 2.2**

Functional expression of coverage aggregation (2.25) used in CCLPs literature.

Function	Proposed by	Used by
Sum (2.26)	Berman et al. (2009a)	Berman et al. (2011); Tabrizi et al. (2011); Berman et al. (2013); Averbakh et al. (2014); Wang et al. (2016); Jayalakshmi and Singh (2017); Karabulut et al. (2017); Ash-tiani et al. (2018); Rezazadeh et al. (2018); Bagherinejad et al. (2018); Bababeik et al. (2018); Li et al. (2018); Davari (2019); Drezner and Drezner (2019); Wang et al. (2021); Ming et al. (2021).
Product (2.27)	Berman et al. (2009a)	Wang et al. (2016); Zhang et al. (2017); Karatas (2017); Karabulut et al. (2017); Karatas (2018); Karatas (2020); Karatas and Dasci (2020); Karatas and Eriskin (2021).
Joint Coverage (2.28)	Drezner and Drezner (2014)	Berman et al. (2019); Álvarez-Miranda and Sinnl (2019).

Table 2.2 lists the specific functions used in papers dealing with CCLPs. Most of the papers adopted a deterministic approach, expressed by the sum (2.26), to model the coverage relationship of facilities in covering the demand. The probabilistic approach is less frequently applied, and, in most cases, coverage is based on independent events through the product function (2.27). The more general case where some degree of correlation of coverage events may exist applying joint coverage (2.28) has not been extensively analysed.

Three may be considered as the seminal papers of CCLPs, each applied a different location space: (i) planar case (Berman et al., 2009a); (ii) discrete case (Berman et al., 2011); (iii) network case (Berman et al., 2013; Averbakh et al., 2014). In the following, we focus on discrete versions and present the basic cooperative counterparts of LSCP and MCLP models as in Berman et al. (2011).

The Cooperative Location Set Covering Problem (CLSCP) minimizes the number of facilities to be located in order to attain at least the percentage  $\alpha$  of demand receiving a coverage level greater than the threshold  $T$ . The CLSCP may be formulated as follows:

$$\min (2.12) \tag{2.29}$$

$$\text{s.t. } \sum_{i \in I} w_i z_i \geq \alpha \sum_{i \in I} w_i \quad \forall i \in I \quad (2.30)$$

$$\Phi_i \geq T z_i \quad \forall i \in I \quad (2.31)$$

$$(2.25), (2.6), (2.16)$$

The Cooperative Maximal Covering Location Problem (CMCLP) aims to maximize the weight of demand resulting covered if their aggregate coverage  $\Phi_i$  from the  $p$  located facilities exceeds the threshold  $T$ . The CMCLP may be formulated as follows:

$$\max (2.14) \quad (2.32)$$

$$\text{s.t. } (2.4), (2.6), (2.16), (2.25), (2.31)$$

Since cooperative location problems has been introduced for a decade and several modeling and theoretically contributions, solutions approaches and specific applications, a systematic review on the topic is missing.

## 2.6 Reorganization of services in location analysis

In this thesis, we consider digital transformation as the primary motivation that led to the need to reorganize facility networks in order to integrate digital channels into the existing delivery channel system. We refer to FLPs to model the reorganization network problems under investigation.

Traditionally, FLPs have been a valuable decision support tool for public and private organizations planning to open new facilities in a given region (Drezner and Hamacher, 2001). Indeed, most of the methods proposed in location literature, including many of those discussed in the previous sections of this chapter, address problems in which services have to be organized ex-novo.

In many contexts, however, it may be necessary to reorganize an existing facility network. Motivations for such decisions can be various; the most discussed in the literature is related to the variability of parameters over time, such as demand, costs, market structure, and competition. Due to this variability, decisions taken on the basis of certain conditions could reveal inefficiency later and require some adjustments in the organization of the system.

A research stream in facility location theory dealing with the territorial reorganisation of networks has become increasingly rich. The approaches may be classified, as in (Bruno et al., 2016), into ex-ante and ex-post reorganization models, which respectively take decisions before and after the occurrence of changes in the system. Within the first class, multi-period and stochastic models address the problem of designing facility networks

on the basis of forecasting future conditions over a planning horizon or by considering the probability of future trends of input parameters, respectively.

For instance, in order to reflect tolerable levels of disruption organization and deal with changing demand patterns, [Wesolowsky and Truscott \(1975\)](#) proposed a multi-period p-median problem, including opening and closing costs for facilities, while [Galvão and Santibañez-Gonzalez \(1992\)](#) assumed the number of facilities varying at each period of the planning horizon. In a fixed-change location setting, [Roodman and Schwarz \(1977\)](#) considered for the first time a set of already located facilities at the beginning of the planning horizon that could be then removed. The model has been extended by [Albareda-Sambola et al. \(2009\)](#) in order to serve an increasing demand over the periods.

In the class of ex-post re-organization decisions, models consider existing configurations of the facilities and aim at modifying the current spatial organization of a given service, in order to react to some occurred circumstances (e.g. changes in the distribution of the demand, financial need) and improve the efficiency of the system. [ReVelle et al. \(2007\)](#) present the “Planned Shrinkage Model” formulated as a covering problem aiming to address the reorganization problem by reducing the number of operating facilities due to emerging financial circumstances that impose to contain costs. Also, reorganization problems have recently involved reshaping districting problems in several contexts, such as schools ([Eagleson et al., 2002](#)), police ([D’Amico et al., 2002](#)), and postal services ([Bruno et al., 2021](#)).

Herein, the territorial reorganization is intended as a process aimed at modifying the set of facilities operating in a given region in ex-post framework, by applying one or more strategies presented in [Section 1.7](#). In particular, recalling them, we have presented as main reorganization strategies (i) closing existing facilities, (ii) resizing the remaining ones in term capacity, (iii) relocating part of them to meet new customers’ segment, (iv) diversifying/specializing facilities in terms of provided services, and (v) outsourcing basic services. The novelty aspects of the proposed models will be highlighted in the dedicated chapters by positioning the contributions in the existing literature.

## 2.7 Conclusions

In this chapter, we presented generalities on Facility Location Problems. We introduced characteristics and formulations of the basic mathematical models known in the literature. We placed a particular emphasis on Covering Location Problems and Cooperative Covering Location Problems, representing the referring problems for this thesis. Also, Hierarchical Problems within the covering framework have been more in-depth analyzed. In the next chapters, we will present a novel mathematical model to address the

problem of reorganizing (or restructuring) banks' branch networks due to digitalization. Then, a second model problem will be introduced by generalizing the first problem in a cooperative covering fashion. Therefore, in each chapter, specific literature streams will be verticalized, and the gaps we aim to fill will be accordingly outlined.

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# Redesigning facility networks: the branch network restructuring problem

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## Summary

In this chapter, we address a real problem faced by banking groups concerning the restructuring of branch networks due to the digital transition of banking services. We first discuss how banking institutions are dealing with digital transformation and leveraging digital channels to deliver basic services while reducing the number of physical facilities to dedicate to more complex and added-value operations. In order to implement the new business model, banking groups are shrinking their internal branch networks and outsourcing basic services to maintain physical proximity to customers unwilling to adopt digital channels. To tackle this problem, a mathematical programming model is presented to support the decision-making process concerning the branch network restructuring. The model is formulated as a covering location problem and considers the possibility of arranging the network in a (nested) hierarchical structure where different branch typologies provide services and external facilities support branches in delivering basic services. The objective is to identify the network structure able to serve all the demand and minimize the total network costs. A specific parameter is introduced to regulate the outsourcing level that the bank is willing to achieve. The model is tested by considering a real case study concerning one of the main banking groups in Italy. The obtained results show the capability of the model to provide interesting scenarios and

fruitful managerial implications.

### 3.1 The reorganization of branch networks in the light of digitalization

Banking groups across Europe are facing the challenge of restructuring their branch networks. This trend is due to different phenomena.

Firstly, the market strategies pursued during the late 1990s and early 2000s, aimed at improving proximity to customers and gaining market share from competitors, led to a significant proliferation of branches over the territories. This phenomenon was exacerbated, in the following years, by the bank mergers and acquisitions, which produced financial conglomerates with overlapping branch networks (Berger et al., 1999).

On the other hand, the diffusion of digital technologies has completely disrupted customers' habits and behaviours, as discussed in Chapter 1. As a result, the traditional “bricks-and-mortar” model, based on the use of physical facilities and face-to-face interactions, is progressively evolving toward the “click-and-mortar” model, in which traditional channels are integrated with digital ones (e.g., internet and mobile banking). Accordingly, banks are rethinking the role of physical proximity and investing in digitalization as a strategic mechanism for achieving competitive advantage (Pennathur, 2001). In the EU-27, individuals using Internet banking have doubled from 2008 to 2021. Denmark, Finland and the Netherlands lead the ranking with more than 91% of internet users, while the lowest-performing countries are Romania and Bulgaria (15%), Greece (42%) and Italy (45%) (Eurostat, 2021).

Although digital channels are expected to increase their diffusion and penetration, the physical branches will keep playing a crucial role. In the new delivery model, they will remain the primary and highly valued point of interaction, providing high-level services that cannot be easily delivered with digital alternatives (Vera, 2017). Indeed, a recent survey shows that digital channels are mainly used for informative and transactional needs (e.g., money transfers and payments), while branches are preferred to handle complex advisory services, such as personal loans (61%) and mortgages (69%) (Deloitte Insight, 2018). Moreover, “traditional clients” (or “branch lovers”) who do not adopt digital channels and prefer to patronise physical facilities also to perform basic services still represent 28% of the total demand.

In the above context, European banking groups are undertaking various restructuring actions with the final aim of reducing network costs and guaranteeing, at the same time, good access to physical and digital channels to customers for service provision.

Among the strategies presented in Section 1.7, the main ones being implemented in

the banking industry are:

- *Branch closure* to eliminate redundant facilities. In Europe, the downward trend in the number of active branches started in 2008, with a pace of 2.97% closures per year, and it reached the maximal contraction between 2019 and 2020, with a peak of 8.21% ([European Central Bank, 2021](#));
- *Branch specialization* to evolve toward a multi-format system, where different types of branches exist, delivering different categories of services and adopting multiple delivery modes;
- *Outsourcing* to supplement branches in the physical provision of basic banking services (e.g., deposits, withdrawals and bill payments). The goal is to use external facilities, usually located at regularly visited shops (e.g., supermarkets or tobacco shops), to maintain a capillary presence over the territory and guarantee proximity to those customers who prefer physical channels. In this context, outsourcing is mainly intended as leverage to cut costs, and refocus banks on their core/strategic activities, and a catalyst for digital transformation ([Gunasekaran et al., 2015](#)).

By mixing the above strategies, banking groups aim to shrink the network and progressively transform the remaining branches into digital facilities in which staff is dedicated to more complex and added-value operations. In the new model, the physical provision of basic services is consolidated in a lower number of branches or outsourced to external facilities.

To tackle this problem, in this chapter, we present a mathematical programming model to support location-based actions to restructure banking branch networks.

Facility location problems have been traditionally applied in the banking sector to address the problem of optimally locating new branches. This is in line with the trend denoting the sector until the early 2000s, characterised by a significant proliferation of branches.

However, as a response to the transformation and the new emerging conditions described above, recently, contributions started to focus on the branch restructuring problem. The actions that are usually taken into account regard facility relocation, closure and capacity resizing (more details will be provided in the following section).

The model proposed in this chapter aims to contribute to this literature by introducing a novel hierarchical branch restructuring problem that considers more complex actions. In particular, in our framework, restructuring decisions involve closing down existing facilities, diversifying branches in terms of provided services, and outsourcing a

subset of banking services to third-party providers (TPPs). Four different types of facilities are considered. Specifically, the internal network involves three types of branches arranged in a nested hierarchical structure (Farahani et al., 2014). Furthermore, such a network can be integrated by activating some external points for the provision of basic services. Hence, the model decides (i) how many and which existing branches to close, (ii) which services are offered at the active branches, and (iii) how many and which external facilities to include in the network. Each facility is associated with a covering radius representing a service level to guarantee, and the objective consists of minimizing the total network costs to cover all the demand. The proposed model is applied to the real-world case of an Italian banking group, which owns a very dense network in most regions. The provided results show how the model is capable of providing interesting scenarios and supporting decision-makers in restructuring decisions.

The rest of the chapter is organized as follows. First, a verticalization of the banking applications in the location literature is provided in Section 3.2. Section 3.3 presents the problem description and the formulation of the mathematical model. The case study is presented in Section 3.4, while the design of the experiments and the results deriving from the application of the model are presented and discussed in Section 3.5. Finally, Section 3.6 draws some concluding remarks and ideas for future research.

## 3.2 The location of banking branches: a review of the literature

According to a recent survey on service facility location problems by Celik Turkoglu and Erol Genevois (2020), the banking sector is one of the most investigated application fields.

In particular, the scientific literature has initially focused on the problem of locating new facilities and expanding the existing networks, consistently with the expansion policies undertaken by banking groups in the late 1990s and early 2000s. Min and Melachrinoudis (2001) propose a chance-constrained goal programming model to locate three types of capacitated facilities arranged in a nested hierarchical network, i.e. Automated Teller Machines (ATMs), branches, and main branches, providing increasingly complex services. The model allocates all the demand considering conflicting goals, i.e., profit and budget availability. Miliotis et al. (2002) propose a two-step methodology combined with GIS techniques to design an effective branch network taking into account the demand area's factors (e.g. geographical, social, and economic) and competition in each area by using appropriate demand-covering models. Xia et al. (2010) formulate an enhanced version of the maximal covering location problem to open new branches

that maximize the profit of sited facilities. They consider three types of branches according to their size with different coverage functions. [Cinar and Ahiska \(2010\)](#) use a combination of multi-criteria methods (FAHP, TOPSIS) to select the most appropriate city for opening a branch among six alternatives in Turkey. The authors investigate five main criteria (e.g., demographic, socio-economic) and twenty-one sub-criteria associated with the bank's mission and strategy. [Mimis \(2012\)](#) develop a methodology to find the optimal location for a number of branches and ATMs to expand an existing nested hierarchical network. The methodology integrates GIS, used to define catchment areas through Voronoi diagrams, and directed tabu search to minimise the average distance travelled by the users. Moreover, many works address the problem of expanding banking networks by locating ATMs ([Wang et al., 2002](#); [Aldajani and Alfares, 2009](#); [Byers et al., 2012](#); [Kisore and Koteswaraiah, 2017](#)).

Contributions dealing with the branch network restructuring problem can be found starting from the early 2000s due to the transformation that occurred in the reference sector. [Morrison and O'Brien \(2001\)](#) propose a GIS-based spatial interaction model to estimate the impact of removing one or more branches from the network, on the basis of the expected number of transactions and customers. [Wang et al. \(2003\)](#) propose a budget-constrained location model for relocating branches in an urban area, with the aim of minimizing the total weighted travel distance for customers. [Zhang and Rushton \(2008\)](#) present a location-allocation model for both opening and closing branches in a competitive environment, maximizing a utility function that considers the branches' size and their distance to customers. [Basar et al. \(2017\)](#) propose a multi-objective mathematical programming model aiming at maximising the transaction volume of located branches while minimizing the cost of relocating branches and the penalties incurred for opening the same type of branches (e.g. retail, entrepreneur, corporate, and commercial) near each other.

Given the increased use of electronic payment methods and card transactions, the ATM location problem is discussed by [Denstad et al. \(2019\)](#). The authors propose a multi-objective linear integer mathematical programming model for relocating ATMs in an existing network.

Besides facility relocation and shrinking action, recent literature focuses on more complex restructuring actions, including facility sizing and resizing. [Monteiro and Fontes \(2006\)](#) propose a branch network restructuring problem, where facilities can be closed, relocated and/or resized. They formulate a non-linear model aimed at satisfying client needs for banking services at a minimum cost. [Ruiz-Hernández et al. \(2015\)](#) propose the Capacitated Branch Restructuring Problem (CBRP), which deals with the problem

of closing redundant capacitated branches and resizing remaining branches in order to maintain a constant service level. Ceding market share to competitors is allowed whenever the restructuring costs are prohibitively expensive. [Ruiz-Hernández and Delgado-Gómez \(2016\)](#) analyze a similar problem assuming uncertainty on the reaction of those clients whose preferred facility gets closed, and the likelihood that some of them will migrate to a competitor. Finally, [Ruiz-Hernández et al. \(2017\)](#) address the branch restructuring problem from a game theoretical point of view, where two competing companies are simultaneously engaged in restructuring processes. The authors explore the existence of different types of equilibrium. [Yavari and Mousavi-Saleh \(2019\)](#) present a hierarchical problem for restructuring banking networks; they consider two types of capacitated facilities – main and auxiliary – that serve two different types of customers. The model’s objective is to satisfy customers’ demand while minimising the total cost of closing and/or resizing existing facilities and locating auxiliary facilities.

The model proposed in this chapter aims to contribute to the body of literature presented above by considering novel restructuring actions. In particular, besides branch closure, we consider branch diversification and outsourcing. Diversifying internal branches leads to restructure the internal network in a nested hierarchical structure, where a higher-level facility offers services provided by lower levels in addition to some other exclusive services ([Farahani et al., 2014](#)). Such a structure is then complicated by the outsourcing strategy, which opens the possibility of integrating the internal network with external facilities. As the external facilities do not offer a subset of services provided by the lowest level internal facilities, the emerging structure cannot be configured as a four-level nested hierarchical structure. Instead, they offer a subset of services provided by the facilities at the top of the internal hierarchy; hence, the final emerging structure can be considered partially nested (more details will be provided in the following section). In this scenario, the addressed hierarchical covering problem is more complex due to the multifaceted and interrelated characteristics of banking services.

In the following section, we describe the branch network restructuring problem and its assumption ([Section 3.3.1](#)); then, the mathematical formulation is presented in ([Section 3.3.2](#)).

## 3.3 A model to restructure branch facility networks

### 3.3.1 Problem description

As anticipated in [Section 3.1](#), we consider three main actions: (i) the shrinking of the existing network, (ii) the diversification of remaining branches, and (iii) the outsourcing of a subset of banking services. The final aim of banking groups is to completely

transform the service delivery model by relying on the opportunities provided by digital technologies and the new habits of clients. In particular, banks intend to push the transition toward a new concept of the branch, where basic services (i.e., deposits, withdrawals, receipt payments) are delivered solely through the support of automatic devices while the staff is dedicated to more complex and added value services. However, in order to meet the demand of *traditional clients*, who still need staff assistance to perform basic services, the physical provision of such services will be consolidated in a lower number of facilities or outsourced to external (non-banking) facilities. The latter, by nature, are regularly visited by clients and there they may receive all the support from operators.

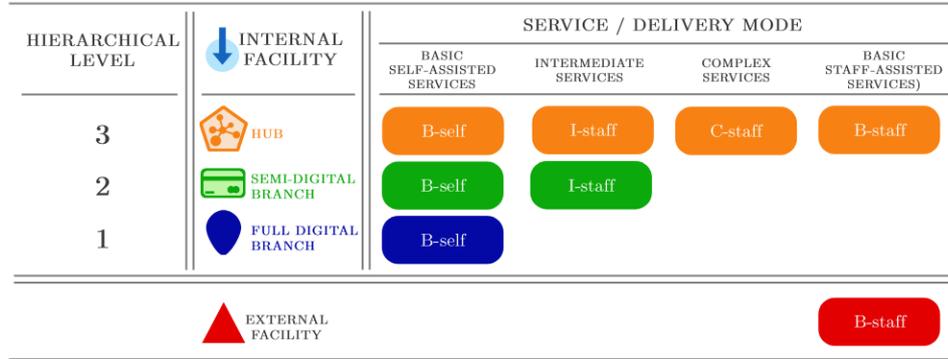
In order to introduce the branch typologies, let us consider three broad categories of financial services: (i) *basic services* (B), e.g., deposits, withdrawals, receipt payments, pay bills, (ii) *intermediate services* (I), e.g., loans and mortgages, safe deposit boxes, financial advice, investments and credits and (iii) *complex services* (C), e.g., dedicated to enterprises and corporations, including other banks, loans beyond a large amount. While intermediate and complex services always require physical interaction with the staff (staff-assisted), the basic services may be delivered with (B-staff) or without (B-self) staff assistance.

In the final configuration of the network, we distinguish three main types of *internal facilities*, on the basis of the delivered services and the adopted delivery channels:

- *Full digital branches*, providing basic services solely through digital channels (B-self). In other words, they are points equipped with automatic devices that allow clients to receive basic services without staff assistance;
- *Semi-digital branches*, providing intermediate services (I-staff) beside the services provided by full digital branches (B-self);
- *Traditional Branches or Hubs*, providing the full gamma of services and delivery modes. In addition to services provided by semi-digital branches (B-self and I-staff), they provide complex services (C-staff). Moreover, clients may receive basic services assisted by staff (B-staff) or automatic machines (B-self).

The internal facility network is arranged according to a 3-level nested hierarchical structure, as shown in [Fig. 3.1](#). The hierarchy is nested in the sense that a higher-level facility provides all the services provided at the lower level plus (at least) one additional service.

As the consolidation of the physical provision of basic services (B-staff) risks to leave uncovered a significant portion of demand (traditional clients), banking groups are



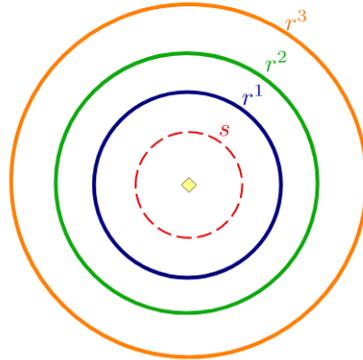
**Fig. 3.1.** Partially nested hierarchy of internal and external facilities.

adopting the outsourcing strategy to guarantee their proximity to the network. Such a strategy consists of selecting existing facilities within the network of brick-and-mortar retailers to be integrated into the network. The inclusion of external facilities in the network configures the final hierarchy as a *partially nested hierarchy*, as shown in Fig. 3.1. Indeed, they provide a subset of services provided by the top-level facilities of the internal hierarchy, i.e. the hubs.

The proposed branch network restructuring problem is formulated as a hierarchical covering model aimed at deciding (i) the location and number of the internal facilities that will be kept open; (ii) the type of service they will provide and the delivery mode (i.e. full digital branches, semi-digital branches, or hubs); and (iii) the number and the location of external facilities to be activated. The objective is to minimise the total costs. The outsourcing level is constrained by the model, leaving banking groups the possibility to fix an upper bound on the level of services to outsource.

Each hierarchical level  $k$  is associated with a covering radius  $r^k$ , representing the maximum distance customers are willing to travel to receive the services provided by a facility of level  $k$  or higher (nested hierarchy). It holds that  $r^1 < r^2 < r^3$ . In addition, a fourth covering radius  $s$  is introduced to represent the distance customers are willing to commute to reach a facility providing basic staff-assisted services (i.e., hubs or external facilities). This parameter can be considered influenced by the propensity of customers to adopt digital channels. The lower the propensity, the higher should be the proximity to guarantee to customers. Hence, in this transitional phase,  $s$  may be considered smaller than the other covering radii, as shown in Fig. 3.2 (i.e.  $s < r^1 < r^2 < r^3$ ), but we may expect that it increases as the digitalization of banking users reach a more mature phase.

A demand node can be covered for basic staff-assisted services within  $s$  either by a hub or by an external facility. In the first case, the demand node is fully covered and



**Fig. 3.2.** Arrangement of covering radii.

may receive all the services within  $s$  (Fig. 3.3a). In the second case, basic services are provided by external facilities within  $s$  whilst internal facilities of appropriate level must be located within adequate distances to satisfy the demand for other banking services (Fig. 3.3b).

Demand node  
  Hub  
  Semi-digital branch  
  Full digital branch  
  External facility



(a) Full coverage within  $s$  by a hub.

(b) Coverage within  $s$  by an external facility and within  $r^k \forall k$  by internal facilities.

**Fig. 3.3.** Coverage examples of a demand node.

As concerns the other problem parameters, we assume that the cost  $\omega_j^k$  of a given internal facilities  $j$  depends on the hierarchical level and that it rises as the gamma of provided services increases ( $\omega_j^1 < \omega_j^2 < \omega_j^3$ ). The outsourcing cost is mainly related to the agreement with external facilities. Moreover, we assume that the latter facilities are capacitated as banking services are not their primary activity, and it is reasonable to consider that they have limited operability.

The outsourcing mechanism is regulated by the parameter  $\alpha^{ext}$  (*maximum outsourcing degree*) representing the maximum fraction of demand the bank is willing to outsource for B-staff services. The decision-maker may calibrate such a parameter as leverage to restructure the internal network. For instance, a bank interested in keeping its own physical presence on the territories would set a low  $\alpha^{ext}$  in combination with a small  $s$ . Conversely, by allowing a high outsourcing degree (i.e., a larger value of  $\alpha^{ext}$ ), the bank may push the consolidation of its internal network to a greater extent. Moreover, the parameter  $\alpha^{ext}$  may somehow also represent the risk the bank is willing to run by outsourcing services to third parties for the sake of consolidation. Indeed, besides great opportunities, outsourcing may increase exposure to operational risks, resulting in financial losses and reputation damages (Gewald and Dibbern, 2009). Hence, banks are required to define transparent outsourcing policies and adopt strategies to monitor and prevent risks (European Banking Authority, 2019). In this sense, an appropriate calibration of  $\alpha^{ext}$  may be useful to find trade-off solutions between cost-efficiency goals and the minimization of outsourcing risks.

### 3.3.2 Mathematical formulation

On the basis of the above assumptions, we introduce the following notation that will be used through this and subsequent sections:

#### Sets

- $I$  set of demand nodes;
- $K$  set of hierarchical level of internal facilities.  $K = \{1, 2, 3\}$ ;
- $J^{int}$  set of internal facilities;
- $J^{ext}$  set of external facilities that are eligible for activation.

#### Parameters

- $\omega_j^k$  cost of internal facility  $j \in J^{int}$  of level  $k$ ;
- $\gamma_j$  cost of external facility  $j \in J^{ext}$ ;
- $d_{ij}$  distance between node  $i$  and facility  $j \in J^{int} \cup J^{ext}$ ;
- $r^k$  covering radius of level  $k$ , with  $r^k < r^{k+1}$ ,  $k \in \{1, 2\}$ ;
- $s$  covering radius for basic staff-assisted services;
- $C_j$  capacity of external facility  $j \in J^{ext}$ ;
- $\tau_i$  demand for basic staff-assisted services from node  $i \in I$ ;
- $\alpha^{ext}$  maximum fraction of demand the bank is willing to outsource for basic staff-assisted services.

#### Covering sets

- $N_i^k$  set of internal facilities accessible to demand node  $i$  within  $r^k$  ( $N_i^k = \{j \in J^{int} : d_{ij} \leq r^k\}$ );
- $M_i^{ext}$  set of external facilities  $j \in J^{ext}$  accessible to demand node  $i$  within  $s$  ( $M_i^{ext} = \{j \in J^{ext} : d_{ij} \leq s\}$ );
- $M_i^3$  set of internal facilities  $j \in J^{int}$  accessible to demand node  $i$  within  $s$  ( $M_i^3 = \{j \in J^{int} : d_{ij} \leq s\}$ ).

*Decision variables*

- $y_j^k$  binary variable equal to 1 if an internal facility of level  $k$  is located at  $j \in J^{int}$  and 0 otherwise;
- $z_j$  binary variable equal to 1 if external facility  $j \in J^{ext}$  is integrated into the network and 0 otherwise;
- $v_i$  binary variable equal to 1 if demand node  $i$  is internally covered, i.e. by hubs, within  $s$  for basic staff-assisted services and 0 otherwise;
- $x_{ij}$  fraction of demand from  $i \in I$  that is covered by the external facility  $j \in M_i^{ext}$  to receive basic staff-assisted services.

With this notation, we can write the MILP formulation of the hierarchical branch restructuring problem as follows:

$$\min \sum_{k \in K} \sum_{j \in J^{int}} \omega_j^k y_j^k + \sum_{j \in J^{ext}} \gamma_j z_j \quad (3.1)$$

$$\text{s.t.} \quad \sum_{j \in N_i^k} \sum_{t=k}^3 y_j^t \geq 1 \quad \forall i \in I, k \in K \quad (3.2)$$

$$\sum_{j \in M_i^3} y_j^3 \geq v_i \quad \forall i \in I \quad (3.3)$$

$$\sum_{j \in M_i^3} y_j^3 \leq |M_i^3| v_i \quad \forall i \in I \quad (3.4)$$

$$\sum_{j \in M_i^{ext}} x_{ij} = 1 - v_i \quad \forall i \in I \quad (3.5)$$

$$x_{ij} \leq z_j \quad \forall i \in I, j \in M_i^{ext} \quad (3.6)$$

$$\sum_{k \in K} y_j^k \leq 1 \quad j \in J^{int} \quad (3.7)$$

$$\sum_{i \in I} \tau_i v_i \geq (1 - \alpha^{ext}) \sum_{i \in I} \tau_i \quad (3.8)$$

$$\sum_{i \in I} \tau_i x_{ij} \leq C_j \quad \forall j \in J^{ext} \quad (3.9)$$

$$y_j^k \in \{0, 1\} \quad \forall k \in K, j \in J^{int} \quad (3.10)$$

$$z_j \in \{0, 1\} \quad \forall j \in J^{ext} \quad (3.11)$$

$$v_i \in \{0, 1\} \quad \forall i \in I \quad (3.12)$$

$$x_{ij} \geq 0 \quad \forall i \in I, \forall j \in M_i^{ext} \quad (3.13)$$

The objective function (3.1) minimizes the costs of the facility network. The first term is associated with the internal network; the second term accounts for the contribution of external facilities.

Constraints (3.2) guarantee that each demand node  $i \in I$  is covered within distance  $r^k$  by a facility of at least level  $k$ . Constraints (3.3)-(3.6) guarantee that all demand nodes are covered for basic staff-assisted services. In particular, a demand node  $i \in I$  is internally uncovered when no hubs are located within  $s$  (constraints (3.3)); conversely, a demand node  $i$  is internally covered if at least one hub is located within  $s$  (constraints (3.4)). If no hubs are located within  $s$  from node  $i \in I$ , the demand generated from  $i$  for basic staff-assisted services must be spread among activated external facilities located within distance  $s$  from node  $i$  (constraints (3.5) and (3.6)). Constraints (3.7) allow internal facilities  $j \in J^{int}$  to assume only one level. Constraint (3.8) defines the lower bound on the portion of demand for basic staff-assisted services that the bank wants to manage internally through hubs. External facilities' capacity is assigned by constraints (3.9). Lastly, expressions (3.10)-(3.13) describe the nature of the decision variables.

In the following sections we present the results obtained when applying the branch network restructuring problem to a local network of a large banking institution in Italy.

### 3.4 Italian scenario and case study

The Italian banking scenario was chosen as an illustrative example to implement and test the branch network restructuring problem proposed in Section 3.3. In the last decades, Italian banking groups have launched strong restructuring policies to resize their branch networks and closed almost 37% of branches from 2008 to 2021 (Banca d'Italia, 2022). Despite the high number of closures, the Italian branch network remains oversized. Indeed, the average number of inhabitants per branch is 2,719 against 3,227 for EU-27 and 24,230 for the Netherlands in 2021 (European Central Bank, 2021). Moreover, as anticipated in Section 3.1, Italy, especially in the Southern regions, presents a significant digital divide compared to the other European and Western countries (Bruno et al., 2023). As a result, in Italy, the branch restructuring process is at an early stage, and a strong effort has still be done in this direction.

In the described context, we consider one of the leading Italian banking groups

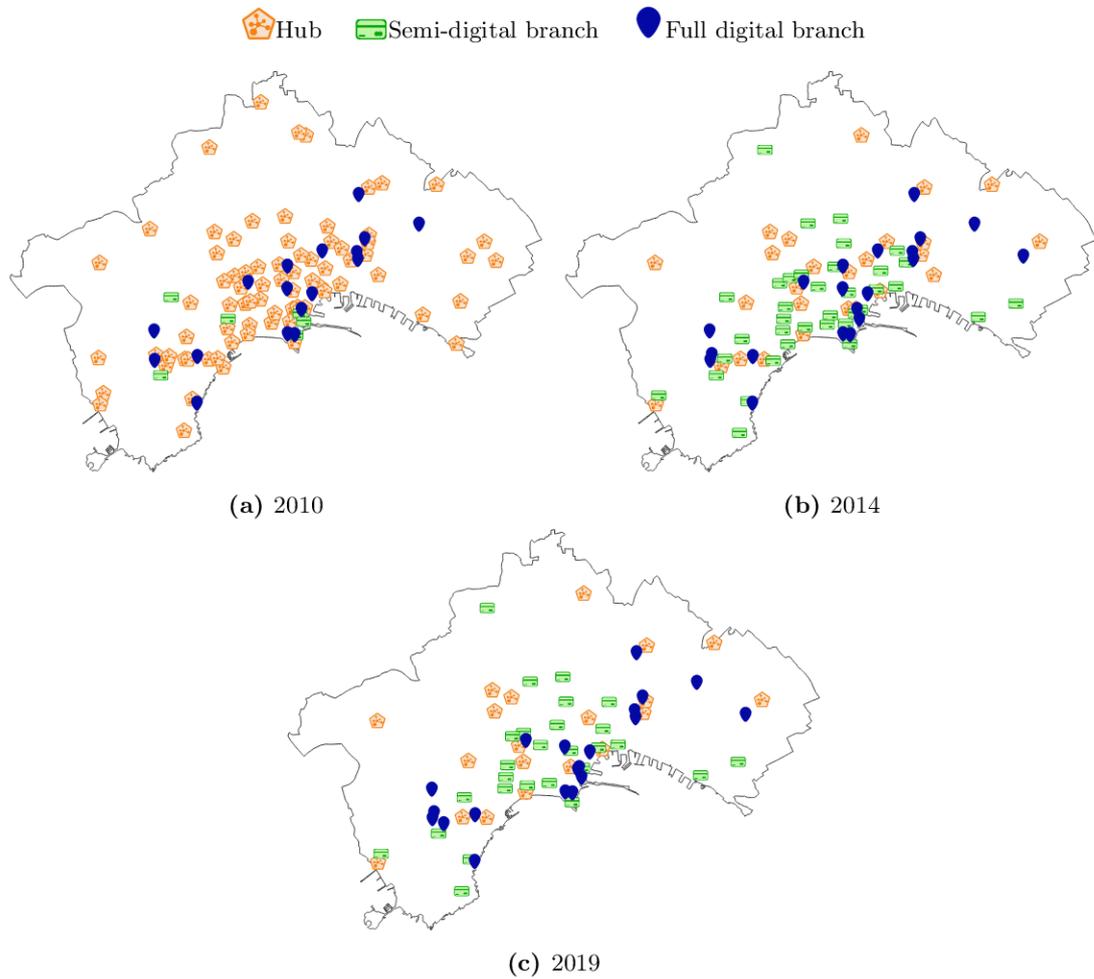
owning the largest branch network on the national territory. The region of interest is the city of Naples (Southern Italy), where the bank holds the densest branch network compared to competitors. Such a bank has undertaken an intensive restructuring during the last years, based on closures and diversification of branches according to different degrees of digitalization with a resulting classification of internal facilities similar to the one we propose in Fig. 3.1.

In order to show the evolution of the branch network, we report in Fig. 3.4 the spatial distribution of the bank's internal facilities from 2010 to 2019, considering 2014 as a midterm year. In addition, Table 3.1 reports the transition matrices describing the closures and the transformations of internal facilities with respect to each pair of the considered years (2010-2014, 2014-2019). The diversification strategy allows solely to downgrade facilities in terms of provided services, i.e., hubs can be transformed into semi-digital or full digital branches, and semi-digital branches into full digital ones.

**Table 3.1**

Bank's restructuring from 2010 to 2019 in the city of Naples.

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				●	■	■								
				●	■	■	×	2019						
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**Fig. 3.4.** Bank's network form 2010 to 2019 in the city of Naples.

development and require additional efforts. Thus, the mathematical model we propose is suitable for providing scenarios for future restructuring.

### 3.4.1 Test instances

As mentioned above, we tested the model on a real-world case study in an urban context. The study area is represented by the city of Naples, where the bank holds 19 full digital branches, 27 semi-digital branches and 20 hubs in 2019 (*network as is*). Naples is the third city in the country for population and one of the most densely populated, with 962,003 resident inhabitants in 2011 on an area of 117.27 km<sup>2</sup>. In order to apply the model to the study area, we discretised the demand space into 3,836 nodes corresponding to the centroids of the populated census tracts of the city (set of demand

nodes  $I$ ). We considered the census tracts' since they are the lowest level of territorial aggregation adopted for statistical purposes by the Italian Statistical Institute (ISTAT), and, therefore, they represent a trade-off between the held of a realistic representation of demand and its manageability issue (ISTAT, 2011).

All the distances we refer to hereafter, including the distance  $d_{ij}$  between the demand node  $i \in I$  and the generic facility  $j \in J^{int} \cup J^{ext}$ , are network distances.

In order to estimate the demand for basic staff-assisted services, we used data provided confidentially by another banking institution operating in a similar context. In particular, assuming a dependence between the number of clients and the distance to the closest branch, such data were used to calibrate a regression model that was used to simulate the spatial distribution of clients in our study area (Fig. 3.5).

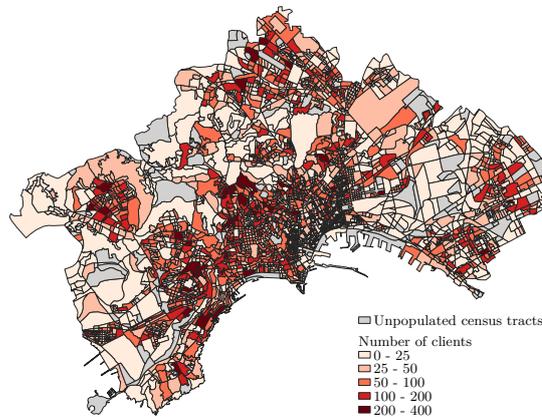
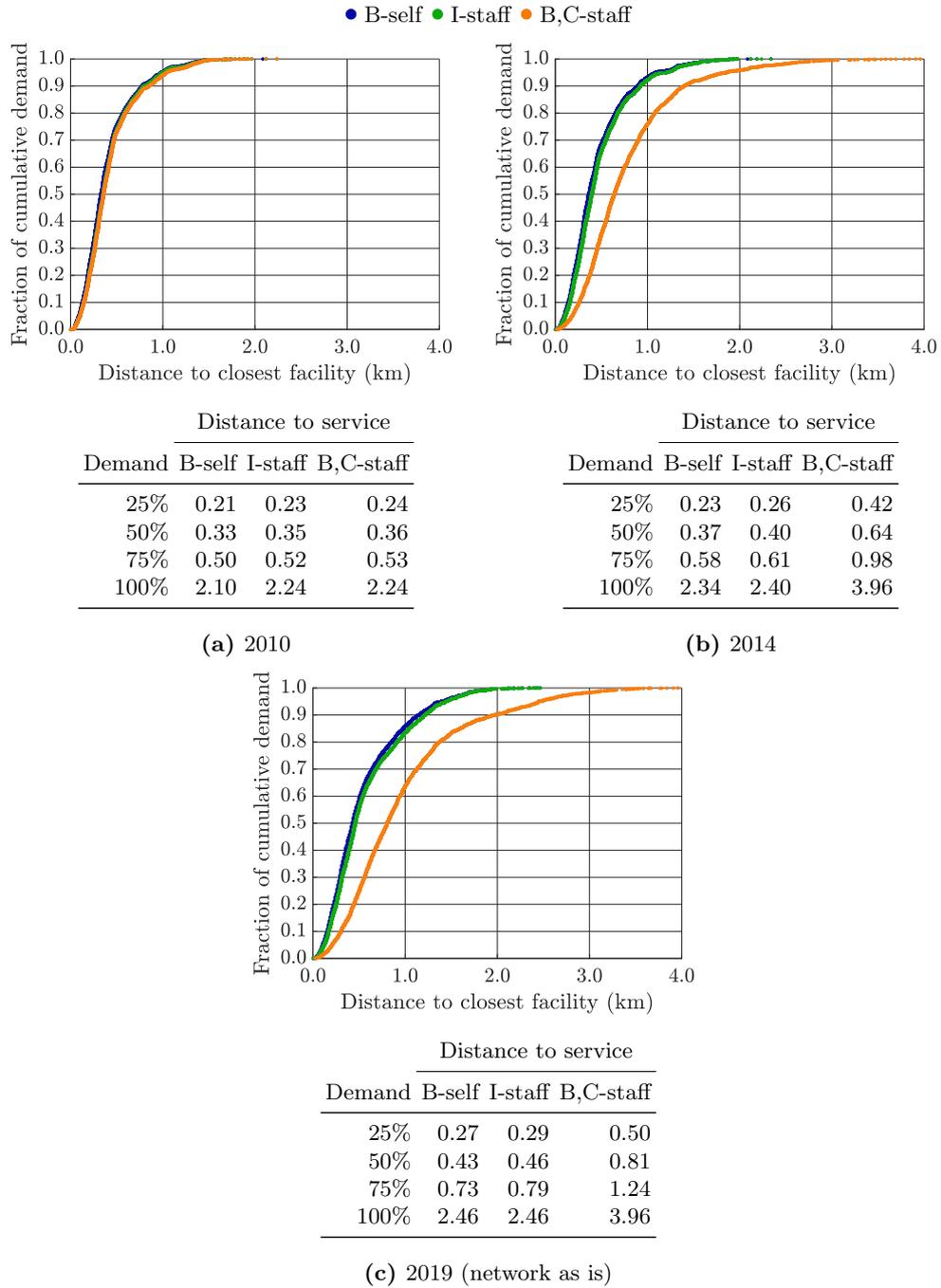


Fig. 3.5. Simulated spatial distribution of clients in Naples.

Before applying the mathematical model to the selected case, we are interested in performing a preliminary analysis of clients' accessibility to banking services, i.e., basic services that are self-assisted (B-self) or staff-assisted (B-staff), intermediate services (I-staff), and complex services (C-staff) as in Fig. 3.1. In particular, we assume that each client patronizes the closest facility providing the required service and compute the accessibility as in Bruno et al. (2021). Our aim is to analyze how accessibility conditions to the banking network have changed after the restructuring implemented between 2010 and 2019 and to define the baseline to calibrate the covering radii.

Fig. 3.6 shows the distributions of the accessibility in 2010, 2014 and 2019, also reporting accessibility distances for each quartile. The three accessibility curves coincide almost entirely overlap in 2010 (Fig. 3.6a) – as also demonstrated by the table below showing accessibility distances for each quartile of demand – given the predominant presence of hubs (77 over 100 internal facilities). The closure of 17 hubs during 2010-



**Fig. 3.6.** Evolution of clients' accessibility to the bank's services form 2010 to 2019 in Naples.

2014 leads to a deterioration in clients' accessibility to B-staff and C-staff –from 0.36 km to 0.64 km for 50% of the demand and from 2.24km to 3.96 km for the total–

while accessibility to B-self and I-staff are mostly unaffected due to the stable number of the other facilities (Fig. 3.6b). After the network adjustments made in 2014-2019, the accessibility conditions to the different services have generally slightly worsened in the light of a more widespread action of closures (Fig. 3.6c). The higher deterioration is +0.26 occurs at the third quartile (75% of the demand) for B,C-staff (from 0.98 km to 1.24 km) while the overall maximum accessibility distances (100% of the demand) – equal to 2.34 km for B-self and I-staff, and 3.96 km for B,C-staff – does not suffer an actual decline. Based on the actual accessibility conditions in 2019, covering radii are calibrated (see Section 3.5).

**Table 3.2**

Facilities' cost from uniform distribution per facility type.

Facility type	Uniform distribution bounds
Hub	[800, 1000]
Semi-digital branch	[480, 600]
Full digital branch	[80, 100]
External facility	[8, 10]

In addition to the internal facilities, we considered 623 additional points in the network of retail shops as potential locations for external facilities ( $J^{ext}$ ). Selecting possible external points to be included in the network represents a crucial issue to address, as it involves multiple arrangements with different retailers with different characteristics. In our test instance, we considered an enlarged and not-constrained set of external points. Either way, the decision-maker has complete control over filling this set based on the outsourcing policies to undertake.

As concerns facilities' costs, we randomly generated them from different uniform distributions. The bounds for each distribution increase according to the hierarchical levels, as shown in Table 3.2. Finally, we assume that the capacity of each external point is equal to the average of the basic staff-assisted transactions performed per month by a branch with only one bank counter operator ( $C_j = 1, 200 \forall j \in J^{ext}$ ). In the following, we apply the model proposed in Section 3.3.2 to the 2019 network and produce several restructuring scenarios by varying some crucial parameters.

### 3.5 Application of the model to a real-case study

In this section, we discuss the results obtained by applying the proposed model to the case study described in Section 3.4. As the problem under investigation involves a network that has been partially restructured, we assumed that a facility already downgraded cannot be upgraded anymore; i.e., for instance, a full digital branch cannot be

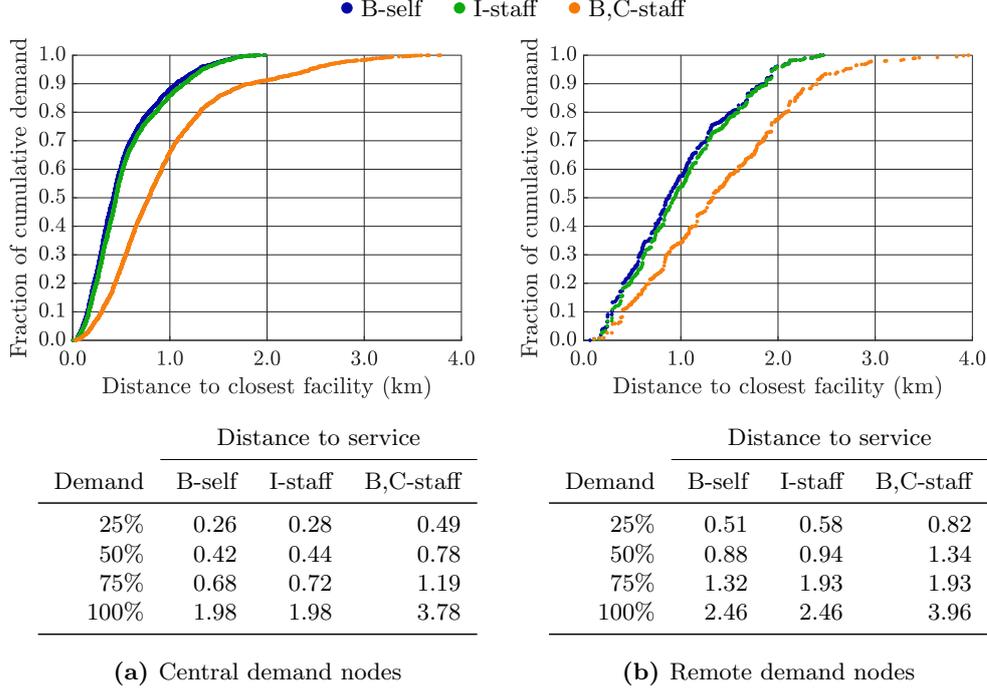
converted into a semi-digital branch. At this aim, we adapted the model presented in [Section 3.3](#) to prevent internal facilities to be upgraded.

We applied the proposed model by varying some crucial parameters in order to present a comparative analysis of several restructuring scenarios that may provide helpful insights to the decision maker.

The restructuring actions considered by the model are: (i) the closure and (ii) the diversification of internal facilities, and (iii) the outsourcing of basic banking services. The decision maker may be interested in pushing one action rather than another and controlling their extent. To this end, we identified the covering radii ( $s$  and  $r^k$ ,  $k \in K$ ) and the maximum outsourcing degree ( $\alpha^{ext}$ ) as the calibrating parameters to implement an action-driven restructuring. For instance, the bank may be more conservative and preserve a high level of internalization by fixing a low value of  $\alpha^{ext}$  or, conversely, to massively shrink the internal network by increasing it. Furthermore, we decided to vary  $s$  and  $r^3$  to analyze the model response to the conflicting targets of complex services consolidation ( $r^3$ ) and basic services capillarity ( $s$ ).

As regards the covering radii  $r^k$ , we can reasonably assume that clients' distance perception and travelling willingness to access banking services depends on their residential area. Indeed, clients living in the suburbs and isolated areas are usually willing to travel longer distances than those who live in urban neighbourhoods, where services and activities are more concentrated. Therefore, we defined an agglomeration index to classify the demand nodes into *central* and *remote*, depending on mutual distances between them. Specifically, for each demand node, we computed its average distances from the closest ten nodes; those demand nodes within the 90th percentile of the global distribution were considered central (3,467) and the remaining ones remote (369). Accordingly, we associated to each of these groups two distinct covering radii  $r^k$  for each hierarchical level  $k$ , namely  $r_{central}^k$  and  $r_{remote}^k$ . In order to properly calibrate these covering radii, we referred to the accessibility conditions of clients to the different sets of banking services in the *as is* scenario. In [Fig. 3.7](#), we report the accessibility distributions stratified by the demand node classification into central ([Fig. 3.7a](#)) and remote ([Fig. 3.7b](#)). As expected, clients in urban nodes have better access to services than those in remote nodes. As the restructuring aims to shrink the existing internal network, the current accessibility conditions are expected to worsen. Therefore, we set the values of covering radii above the corresponding maximum accessibility distances to different levels of services, as reported in [Table 3.3](#).

As concerns  $s$ , different considerations have been made. Since the outsourcing action aims to guarantee physical proximity to clients and build customer loyalty, we set  $s$



**Fig. 3.7.** Clients' accessibility to the bank's services in the network as is.

smaller than the actual maximum accessibility to BS-self ( $s < r_{central}^1$ ). In particular, we tested  $s = 1.0$  km and  $s = 1.5$  km.

The maximum outsourcing degree  $\alpha^{ext}$  may not be chosen regardless of the covering radii setting. Indeed, to obtain a feasible solution,  $\alpha^{ext}$  should be greater than a certain lower bound ( $LB(\alpha^{ext})$ ). This lower bound is computed as the fraction of basic staff-assisted services required by clients who find the closest hub beyond  $s$  in the initial configuration of the network:

$$LB(\alpha^{ext}) = f(s) = \frac{\sum_{i \in \hat{I}} \tau_i}{\sum_{i \in I} \tau_i} \quad (3.14)$$

where  $\hat{I} = \{i \in I : \min_{j \in J^3} d_{ij} > s\}$ .

On the other hand, the bank will always internally handle a certain percentage of basic staff-assisted services, as the model locates at least one hub. Hence, an upper bound on the maximum outsourcing degree  $UB(\alpha^{ext})$  can be found. At this aim, we solve a relaxed problem that does not allow outsourcing and locates only internal facilities to cover clients at the minimum cost ( $\Omega = (3.1), (3.2), (3.7), (3.10)$ ). The solution obtained, namely  $\sigma(\Omega)$ , defines the located hubs ( $y_j^{3*}$ ). The upper bound of  $\alpha^{ext}$  is computed as the

**Table 3.3**

Maximum accessibility distances ( $d_{acc}$ , in km) to services by levels and demand classification in the network as is and corresponding setting of covering radii ( $r^k$ , in km).

Demand nodes	B-self		I-staff		C-staff			
	$d_{acc}$	$r^1$	$d_{acc}$	$r^2$	$d_{acc}$	$r^3$		
Central	1.98	2.00	1.98	3.00	3.78	5.00	7.00	9.00
Remote	2.46	3.00	2.46	4.00	3.96	6.00	8.00	10.00

fraction of basic staff-assisted services required by clients who find within  $s$  the closest hub located by the relaxed problem:

$$UB(\alpha^{ext}) = f(s, r_{central}^k, r_{remote}^k) = 1 - \frac{\sum_{i \in \tilde{I}} \tau_i}{\sum_{i \in I} \tau_i} \quad (3.15)$$

where  $\tilde{I} = \{i \in I : \min_{y_j^{3*} \in \sigma(\Omega)} d_{ij} \leq s\}$ .

Based on the lower and upper bounds, listed in Table 3.4, we varied  $\alpha^{ext}$  starting from  $LB(\alpha^{ext})$  and increasing it with a step 0.1 until reaching the step in correspondence of  $UB(\alpha^{ext})$  or immediately higher. Indeed, all the solutions obtained with  $\alpha^{ext} \geq UB(\alpha^{ext})$  are identical. For instance, if  $s = 1.0$  km,  $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$  km, setting  $\alpha^{ext} = 1$  is precisely the same of setting  $\alpha^{ext} = 0.92$  ( $UB(\alpha^{ext})$ ).

**Table 3.4**

Bounds of  $\alpha^{ext}$  (covering radii in km)

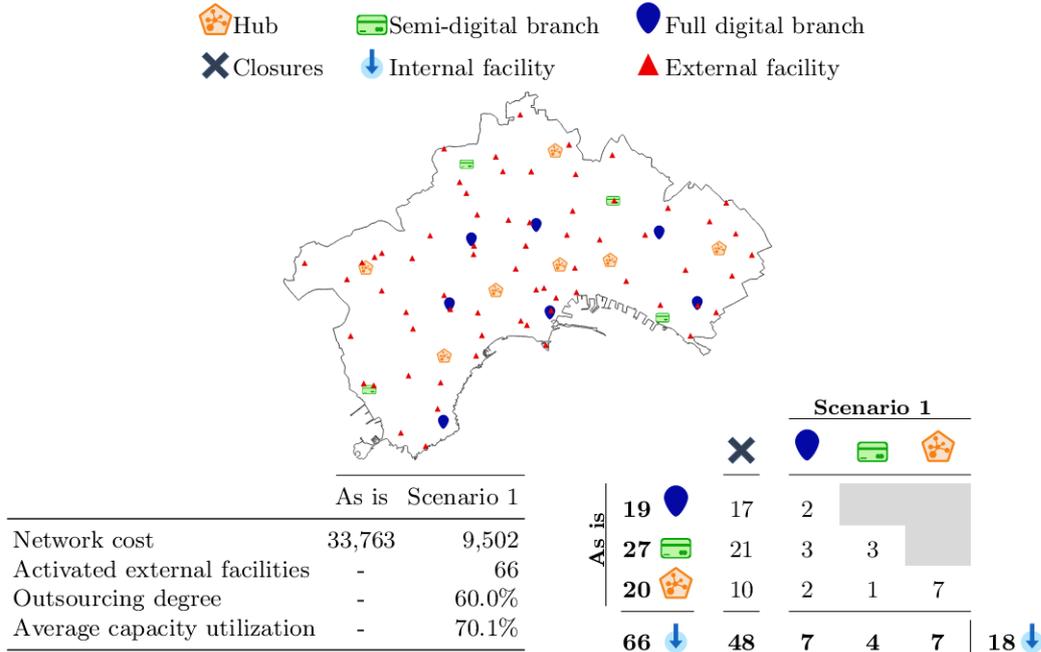
	$LB(\alpha^{ext}) = f(s)$	$UB(\alpha^{ext}) = f(s, r_{central}^k, r_{remote}^k)$		
		$r_{central}^3 = 5.00$ $r_{remote}^3 = 6.00$	$r_{central}^3 = 7.00$ $r_{remote}^3 = 8.00$	$r_{central}^3 = 9.00$ $r_{remote}^3 = 10.00$
$s = 1.00$	0.40	0.89	0.95	0.92
$s = 1.50$	0.20	0.77	0.92	0.90

We generated 44 test problems for each combination of the above parameters that have been solved using CPLEX in limited computational time. This is advantageous for all practitioners interested in solving the model with a commercial solver within a limited time frame.

### 3.5.1 Illustrative results

In this section, illustrative scenarios are compared by varying the parameters  $s$  and  $\alpha^{ext}$  to analyze the emerging balance between internal and external networks. For each

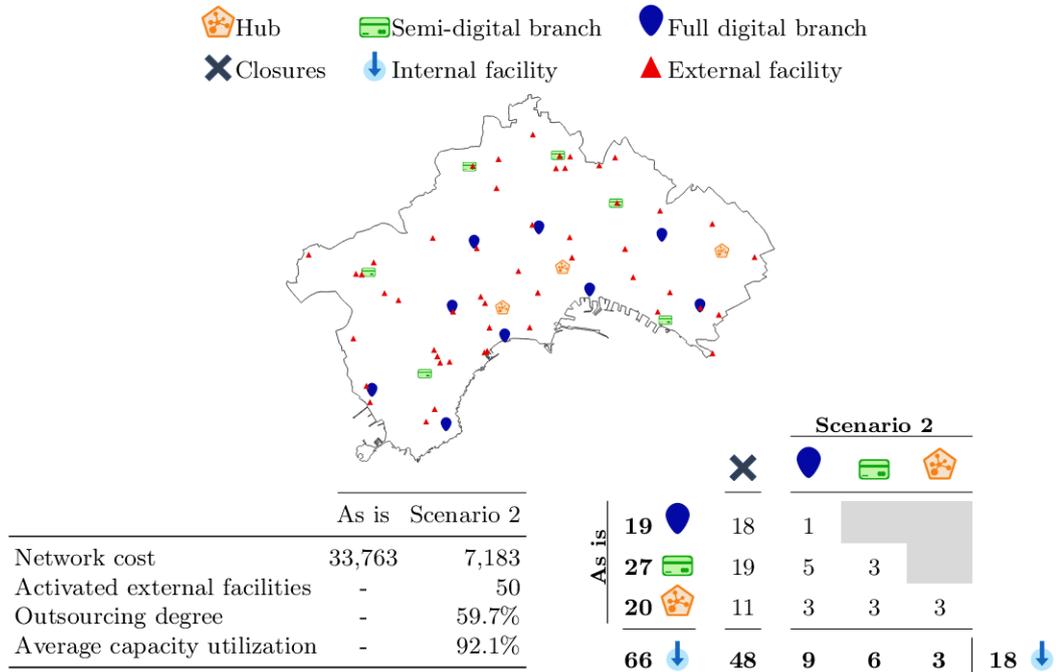
scenario, we report the network configuration map, the transition matrix explaining the transformation of internal facilities, the number of activated external facilities, and some key indicators. The latter are: (i) the cost of the overall network (internal and external); (ii) the outsourcing degree, i.e. the percentage of basic staff-assisted services handled by external facilities,  $(\sum_{i \in I} \tau_i (1 - v_i) / \sum_{i \in I} \tau_i)$ ; (iii) the average capacity utilization, i.e. the average percentage of basic staff-assisted services handled by external facilities  $(\sum_{i \in I, j \in M_i^{ext}} \tau_i x_{ij} / \sum_{j \in J^{ext}} C_j z_j)$ .



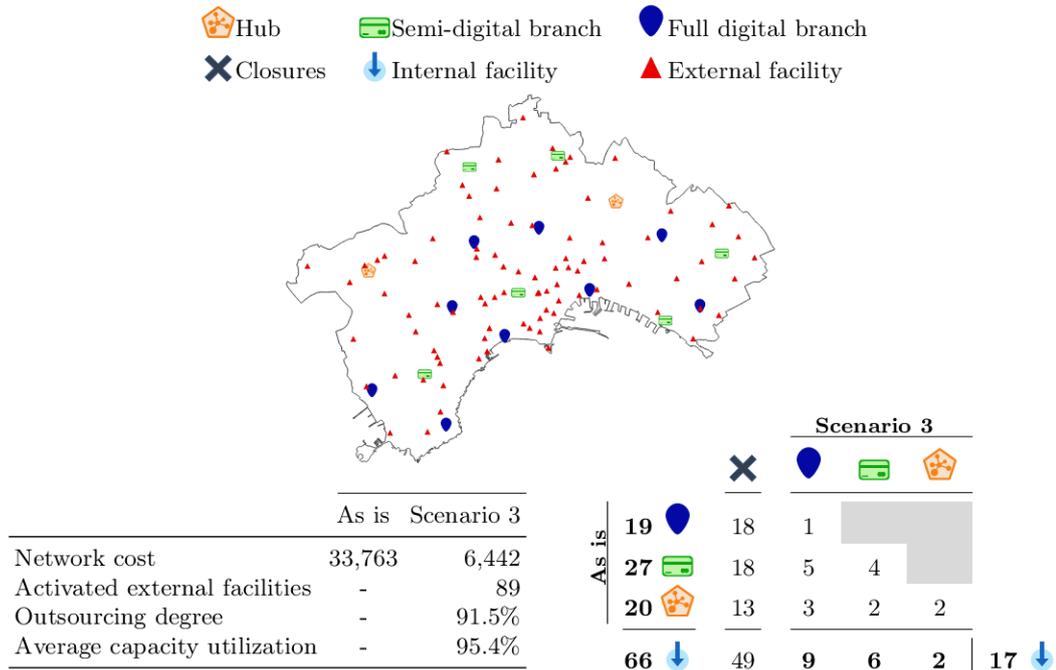
**Fig. 3.8.** Restructuring scenario 1:  $s = 1.0$  km,  $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$  km,  $\alpha^{ext} = 0.6$ .

Fig. 3.8 shows a first scenario produced by the model by setting  $s = 1.0$  km,  $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$  km, and  $\alpha^{ext} = 0.6$ . The internal network is significantly shrunk, with the closure of 48 out of 66 facilities, with a substantial reduction of costs. The new scenario includes 7 full digital branches, 4 semi-digital branches, and 7 hubs, while 66 external facilities are activated.

Fig. 3.9 shows a second scenario obtained by increasing the covering radius  $s$  from 1.0 km to 1.5 km. Although the total number of internal facilities compared to scenario 1 is the same (18), the network configuration is completely different. Indeed, fewer hubs remain open (3 instead of 7), and, as a consequence, more semi-digital (6 instead of 4) and full digital branches (9 instead of 7) have been located. Furthermore, fewer external facilities are activated (50 instead of 66), which are more saturated in terms of capacity



**Fig. 3.9.** Restructuring scenario 2:  $s = 1.5$  km,  $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$  km,  $\alpha^{ext} = 0.6$ .



**Fig. 3.10.** Restructuring scenario 3:  $s = 1.0$  km,  $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$  km,  $\alpha^{ext} = 1$ .

utilization (92.1% instead of 70.1%).

A last illustrative scenario is provided in Fig. 3.10 using  $\alpha^{ext} = 1$  and  $s = 1.0$  km. With respect to scenario 1, the model activates 23 additional external facilities (89 instead of 66) and leaves open only 2 hubs instead of 7 at a lower overall cost. As in the previous scenario, this leads to a different internal configuration with 6 semi-digital branches and 9 full digital branches, differently located. The activated external facilities absorb almost the total demand of basic staff-assisted services with an outsourcing degree equal to 91.5%, slightly below the  $UB(\alpha^{ext})$ ; they are also exploited almost entirely with a capacity utilization equal to 95.4% on average.

Besides the spatial configuration of the network, a comparative analysis of clients' accessibility in the illustrative scenarios is provided (Fig. 3.11). The accessibility to services provided (exclusively) by internal facilities is obtained as described in Section 3.4.1. Instead, in order to compute the accessibility to basic staff-assisted services, we assume that clients are covered by a hub (if present within  $s$ ) or by the closest activated external facility according to the capacity.

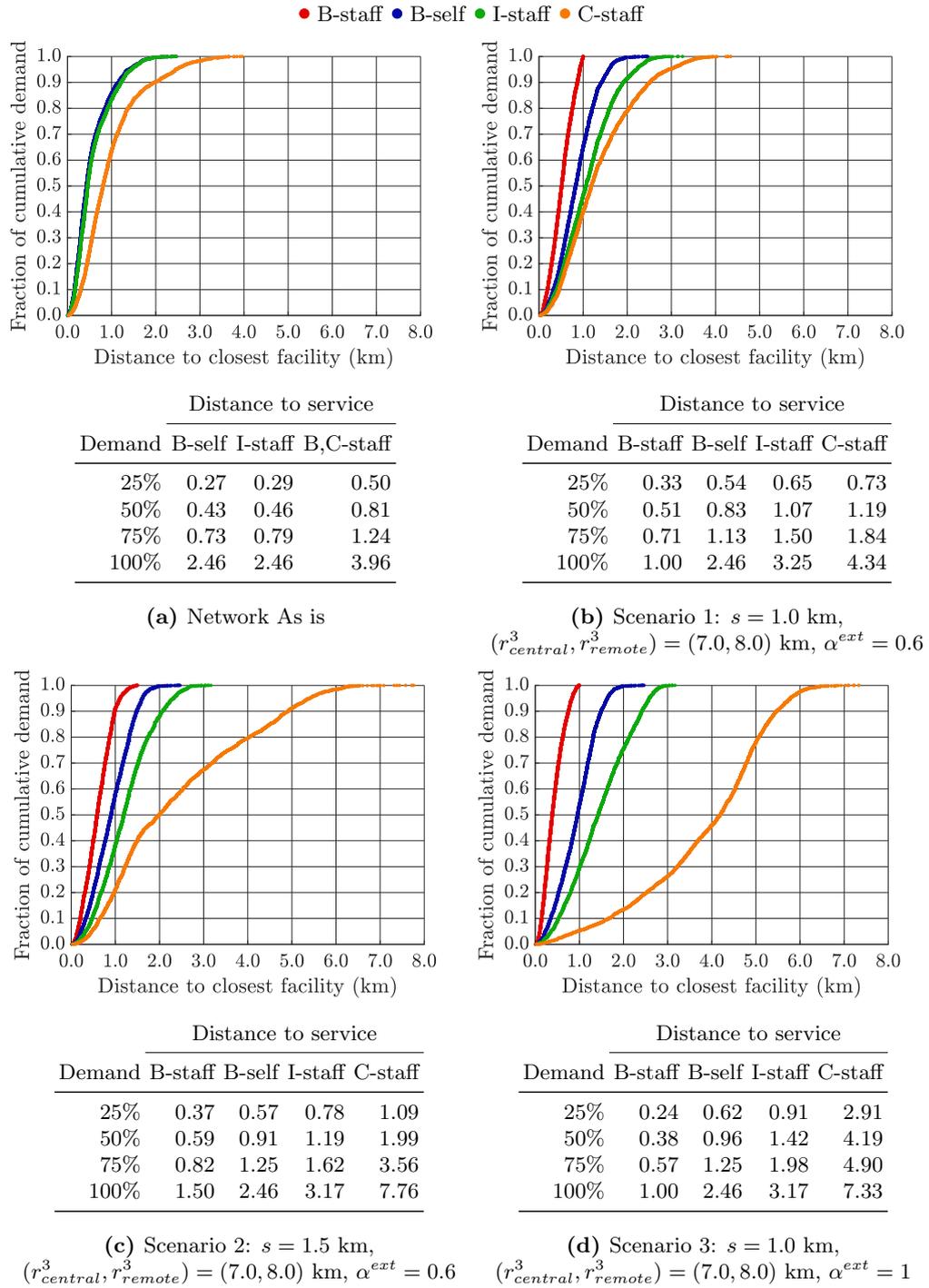
Comparing accessibility to the network as is (Fig. 3.11a) of scenario 1 (Fig. 3.11b), each client has access to B-staff within 1 km instead of 4 km – and 50% of demand within half such a distance (0.51 km) – thanks to external facilities. Moreover, the accessibility to I-staff is the one that gets worse the most, recording an increase of 0.79 km at the fourth quartile. In contrast, the accessibility to C-staff does not deteriorate substantially, given the strict value of  $\alpha^{ext} = 0.6$ . Clients, indeed, are covered within just over 4 km, albeit  $r_i^3$  is (much) larger.

Considering scenario 2 in Fig. 3.11c, we can notice that, as expected, the increase of  $s$  worsens the accessibility to B-staff. However, the most significant effect is on the accessibility curve to C-staff provided by hubs, which is smooth and shows a decrease in slope at 1.5 km ( $s$ ). Indeed, 50% of demand is covered within about 2 km (1.19 km in scenario 1), while each client is covered within slightly below 8 km (equal to  $r_{remote}^3$ ).

An even more significant worsening to C-staff occurs by increasing  $\alpha^{ext}$  (Fig. 3.11d), where the lower number of hubs leads to a very slow-growing curve. For instance, the accessibility distance of 50% of demand is more than double, equal to 4.19 km.

### 3.5.2 Summary results and managerial implications

The comparative analysis of all produced scenarios (Tables 3.5 and 3.6) may provide valuable restructuring insights to the decision-maker. Among others, we aim to demonstrate how decision-makers may evaluate the effects of various outsourcing policies by leveraging the identified critical parameters.



**Fig. 3.11.** Restructured network: clients' accessibility to services.

**Table 3.5**Bank's network restructuring:  $s = 1.0$  km

(a) $(r_{central}^3, r_{remote}^3) = (5.0, 6.0)$ km									
	$\alpha^{ext}=0.2$	$\alpha^{ext}=0.3$	$\alpha^{ext}=0.4$	$\alpha^{ext}=0.5$	$\alpha^{ext}=0.6$	$\alpha^{ext}=0.7$	$\alpha^{ext}=0.8$	$\alpha^{ext}=0.9$	$\alpha^{ext}=1$
 Hubs	n.a.	n.a.	14	9	7	5	4	4	4
 Semi-digital branches	n.a.	n.a.	2	4	4	5	6	5	5
 Full digital branches	n.a.	n.a.	5	5	7	8	7	8	8
 Internal facilities	n.a.	n.a.	21	18	18	18	17	17	17
 Closures	n.a.	n.a.	45	48	48	48	49	49	49
 External facilities	n.a.	n.a.	56	60	66	69	77	83	83
Network cost	n.a.	n.a.	14,275	10,997	9,502	8,505	8,040	7,659	7,659
Outsourcing degree	n.a.	n.a.	40.0%	49.7%	60.0%	69.7%	79.1%	88.6%	88.6%
Av. Cap. utilization	n.a.	n.a.	55.0%	63.8%	70.1%	77.9%	79.3%	82.4%	82.4%

(b) $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$ km									
	$\alpha^{ext}=0.2$	$\alpha^{ext}=0.3$	$\alpha^{ext}=0.4$	$\alpha^{ext}=0.5$	$\alpha^{ext}=0.6$	$\alpha^{ext}=0.7$	$\alpha^{ext}=0.8$	$\alpha^{ext}=0.9$	$\alpha^{ext}=1$
 Hubs	n.a.	n.a.	14	9	7	5	3	2	2
 Semi-digital branches	n.a.	n.a.	2	4	4	4	6	7	6
 Full digital branches	n.a.	n.a.	5	5	7	9	9	9	9
 Internal facilities	n.a.	n.a.	21	18	18	18	18	18	17
 Closures	n.a.	n.a.	45	48	48	48	48	48	49
 External facilities	n.a.	n.a.	56	60	66	73	75	80	89
Network cost	n.a.	n.a.	14,275	10,997	9,502	8,091	7,410	6,927	6,577
Outsourcing degree	n.a.	n.a.	40.0%	49.7%	60.0%	69.8%	78.8%	86.4%	94.7%
Av. Cap. utilization	n.a.	n.a.	55.0%	63.8%	70.1%	73.7%	81.0%	83.3%	82.1%

(c) $(r_{central}^3, r_{remote}^3) = (9.0, 10.0)$ km									
	$\alpha^{ext}=0.2$	$\alpha^{ext}=0.3$	$\alpha^{ext}=0.4$	$\alpha^{ext}=0.5$	$\alpha^{ext}=0.6$	$\alpha^{ext}=0.7$	$\alpha^{ext}=0.8$	$\alpha^{ext}=0.9$	$\alpha^{ext}=1$
 Hubs	n.a.	n.a.	14	9	6	5	3	3	2
 Semi-digital branches	n.a.	n.a.	2	4	5	4	6	5	6
 Full digital branches	n.a.	n.a.	5	5	7	9	9	9	9
 Internal facilities	n.a.	n.a.	21	18	18	18	18	17	17
 Closures	n.a.	n.a.	45	48	48	48	48	49	49
 External facilities	n.a.	n.a.	56	60	65	73	77	87	89
Network cost	n.a.	n.a.	14,275	10,997	9,188	8,091	7,307	6,914	6,567
Outsourcing degree	n.a.	n.a.	40.0%	49.7%	59.5%	69.8%	78.3%	88.2%	92.4%
Av. Cap. utilization	n.a.	n.a.	55.0%	63.8%	70.6%	73.7%	78.5%	78.2%	80.1%

n.a.: Infeasible solution ( $\alpha^{ext} < LB(\alpha^{ext})$ );: Solution with  $\alpha^{ext} > UB(\alpha^{ext})$ ;

Outsourcing degree: Percentage of basic staff-assisted services handled by activated external facilities.

Av. Cap. utilization: Percentage of employed capacity of activated external facilities on average.

**Table 3.6**Bank's network restructuring:  $s = 1.5$  km

		(a) $(r_{central}^3, r_{remote}^3) = (5.0, 6.0)$ km								
		$\alpha^{ext}=0.2$	$\alpha^{ext}=0.3$	$\alpha^{ext}=0.4$	$\alpha^{ext}=0.5$	$\alpha^{ext}=0.6$	$\alpha^{ext}=0.7$	$\alpha^{ext}=0.8$	$\alpha^{ext}=0.9$	$\alpha^{ext}=1$
	Hubs	12	9	6	5	5	4	4	4	4
	Semi-digital branches	2	2	4	5	5	6	5	5	5
	Full digital branches	5	7	8	8	7	7	8	8	8
	Internal facilities	19	18	18	18	17	17	17	17	17
	Closures	47	48	48	48	49	49	49	49	49
	External facilities	23	29	34	40	48	58	63	63	63
	Network cost	12,238	9,035	8,471	8,216	8,042	7,757	7,470	7,470	7,470
	Outsourcing degree	19.0%	29.3%	38.2%	46.8%	57.3%	69.7%	76.4%	76.4%	76.4%
	Av. Cap. utilization	63.8%	78.0%	86.7%	90.2%	92.0%	92.7%	93.5%	93.5%	93.5%

		(b) $(r_{central}^3, r_{remote}^3) = (7.0, 8.0)$ km								
		$\alpha^{ext}=0.2$	$\alpha^{ext}=0.3$	$\alpha^{ext}=0.4$	$\alpha^{ext}=0.5$	$\alpha^{ext}=0.6$	$\alpha^{ext}=0.7$	$\alpha^{ext}=0.8$	$\alpha^{ext}=0.9$	$\alpha^{ext}=1$
	Hubs	12	9	6	5	3	3	2	2	2
	Semi-digital branches	2	2	4	4	6	6	7	7	6
	Full digital branches	5	7	8	9	9	9	9	9	9
	Internal facilities	19	18	18	18	18	18	18	18	17
	Closures	47	48	48	48	48	48	48	48	49
	External facilities	23	29	34	40	50	56	60	60	74
	Network cost	12,238	9,936	8,471	7,800	7,183	7,036	6,736	6,736	6,442
	Outsourcing degree	19.0%	29.3%	38.2%	47.0%	59.7%	68.4%	74.0%	74.0%	91.5%
	Av. Cap. utilization	63.8%	78.0%	86.7%	90.6%	92.1%	94.1%	95.1%	95.1%	95.4%

		(c) $(r_{central}^3, r_{remote}^3) = (9.0, 10.0)$ km								
		$\alpha^{ext}=0.2$	$\alpha^{ext}=0.3$	$\alpha^{ext}=0.4$	$\alpha^{ext}=0.5$	$\alpha^{ext}=0.6$	$\alpha^{ext}=0.7$	$\alpha^{ext}=0.8$	$\alpha^{ext}=0.9$	$\alpha^{ext}=1$
	Hubs	12	9	6	4	3	2	2	2	2
	Semi-digital branches	2	2	4	6	6	7	7	6	6
	Full digital branches	5	7	8	8	9	9	9	9	9
	Internal facilities	19	18	18	18	18	18	18	17	17
	Closures	47	48	48	48	48	48	48	49	49
	External facilities	23	29	34	41	48	52	60	75	75
	Network cost	12,238	9,935	8,471	7,782	7,117	6,818	6,736	6,431	6,431
	Outsourcing degree	19.0%	29.3%	38.2%	49.8%	58.6%	64.2%	74.0%	89.9%	89.9%
	Av. Cap. utilization	63.8%	78.0%	86.7%	93.7%	94.1%	95.3%	95.1%	92.4%	92.4%

n.a.: Infeasible solution ( $\alpha^{ext} < LB(\alpha^{ext})$ );: Solution with  $\alpha^{ext} > UB(\alpha^{ext})$ ;

Outsourcing degree: Percentage of basic staff-assisted services handled by activated external facilities.

Av. Cap. utilization: Percentage of employed capacity of activated external facilities on average.

The direct outcome of increasing  $\alpha^{ext}$  is the reduction of the network cost and the number of hubs. The provision of B-staff services is delegated to an increasing of external facilities. For instance, let note how in [Table 3.5a](#), increasing  $\alpha^{ext}$  from its minimum value 0.4 to the maximum 1.0, the presence of external facilities becomes more significant, passing from 56 to 83. Contextually, the number of hubs drops from 14 to 4. Consequently, the overall layout of the internal facility network also changes. Indeed, the number of semi-digital and full digital branches adapts to the hub network, increasing from 2 to 5 and from 5 to 8, respectively.

The covering radius  $s$  rules the proximity extent the bank desires to establish with their clients through hubs and external facilities. By increasing  $s$ , the recourse to outsourcing becomes less significant. In [Table 3.6a](#), where  $s = 1.5$  km, we can see how the maximum outsourcing level ( $\alpha^{ext} = 1.0$ ) can be reached through a lower number of external facilities, which are 63 instead of 83 in the previous ([Table 3.5a](#)), where  $s = 1.0$  km.

Moreover, the number of external facilities is also affected by the covering radius  $r^3$ . Indeed, if we push toward a strong consolidation of the hubs as we constrain clients to access complex services within larger distances. Hence, the banks need to outsource B-staff further and activate more external facilities. For example, by considering  $\alpha^{ext} = 1.0$  and  $s = 1.0$  km, 2 hubs are kept open ([Table 3.5c](#)) instead of 4 ([Table 3.5a](#)) with the activation of 6 additional external facilities (from 83 to 89).

The presented results show that banks interested in restructuring their networks by adopting the proposed strategies have to find a trade-off between savings and risks.

On the one hand, consolidating the internal network may result in a substantial increase in savings. Indeed, the outsourcing strategies represent a great opportunity to cut costs and refocus banks toward core activities. Nevertheless, it has to be carefully evaluated as it may expose the bank to security and operational risks ([Gewald and Dibern, 2009](#); [Gunasekaran et al., 2015](#)). Moreover, consolidation comes to the detriment of proximity to clients, which can be maintained by relying on external facilities instead of solely on the bank's branches. Beyond the security and operational risks, entrusting banking services to third-party providers may compromise the reputation of the bank and threaten customers' loyalty.

On the other hand, banks may reduce the exposure to such risks by keeping a capillary presence with their own branches. Nonetheless, following the ongoing trend of branch closures and investing in digital channels in the sector, banks have to progressively consolidate the internal network in order to remain competitive and position themselves for sustainable growth and succeed in a highly competitive marketplace.

Finally, it is worth highlighting that banks willing to adopt outsourcing strategies are called to develop robust risk management frameworks, select their partners carefully and define appropriate strategies to monitor and supervise them (e.g., implement effective security protocols) to prevent potential risks, such as personal data misuse. At this end, the optimal location of potential external points to be activated remains a relevant problem to be addressed. Indeed, beyond the discussed critical parameters ( $\alpha^{ext}$ ,  $s$  and  $r^3$ ), the outsourcing policies may constrain the scenarios produced by our model as it takes as an input the set of potential external points ( $J^{ext}$ ). Nonetheless, this does not infringe on the generality of the model, which provides a valuable decision-support aid. Indeed, banks may first decide which points are eligible for outsourcing according to their internal policies (*ex-ante decision*) and then use the model to determine which arrangements activate among the eligible ones. Otherwise, they may first use the model to identify the best locations among an enlarged and not-constrained set of external points and then decide their eligibility for outsourcing through a benefit-cost analysis (*ex-post decision*).

### 3.6 Conclusions

In this chapter, we analyzed the digital disruption in the delivery channel systems of the banking sector. In particular, the delivery channels that customers use to interact with banking groups have evolved, leading the latter to redefine their business models and the configuration of their facility networks to consolidate them while maintaining an adequate proximity level, especially to less digitalized users.

The branch network restructuring problem has been presented, aiming at restructuring banking branch networks to downsize them and meet the emerging needs of customers. In coherence with the most adopted strategies by banks to address the restructuring problem, the model considers the closure of existing branches, diversifying branches in terms of provided services, and outsourcing basic services as possible strategies. Different types of facilities are considered and arranged in a partially nested hierarchy according to the services they provide. The strategies we implemented have not previously been addressed in the literature. In our framework, the restructuring problem turns out to be more complex due to the multifaceted and interrelated characteristics of banking services.

A specific parameter ( $\alpha^{ext}$ ) is introduced to regulate the outsourcing degree that the bank is willing to achieve. The objective is to identify the configuration of the facility network able to serve all the demand within given distances and minimize the total costs.

The model was tested on a real-world case study of a banking group in the Italian

scenario. Three critical parameters – the covering radii  $r^3$  and  $s$ , and the maximum outsourcing degree  $\alpha^{ext}$  – have been identified to manage and control the restructuring strategies. In order to support the decision maker in the decision-making process, we conducted extensive experimentation by varying these parameters to present different scenarios. The obtained results show the capability of the model to provide fruitful managerial implications depending on the goals and service levels the bank intends to guarantee.

As regards the case study under investigation, more accurate results could be obtained by considering real facilities' costs. Therefore, despite the unavailability of data, a project should be seeded to evaluate such costs (e.g., fixed costs, employees, and the type of agreement between the bank and external facilities).

Moreover, an interesting perspective and research direction may involve a multi-period version of the problem to allow the bank to gradually consolidate the internal network while outsourcing services according to the proximity constraints ( $s$ ) that can progressively be relaxed according to users' digitalization.

Although the model has been developed for the banking sector, the proposed approach can be generalised and adapted to other business sectors. Therefore, the managerial implications could be more significant.

An additional improvement in the problem definition could involve the contextual presence of different facilities, also of the same hierarchical level, within the covering radii that cooperate to cover the demand (Berman et al., 2009a). Such an extension will be the focus of the next chapter.



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# Redesigning facility networks: a two-level hierarchical nested cooperative location problem

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## Summary

In this chapter, we introduce the two-level Hierarchical Nested Cooperative Location (HNCL) model, extending the literature on discrete cooperative covering problems presented in [Section 2.5](#) and generalizing the branch restructuring problem presented in [Chapter 3](#). Herein, we analyse the problem of locating facilities arranged in a two-level nested hierarchy and cooperating to maximize the covered demand. Facilities are characterised by different coverage decay functions according to the hierarchical level they belong to (upper and lower). Cooperation occurs between facilities at the same level (intra-level cooperation) and different levels (inter-level cooperation) of the hierarchy according to mechanisms modelled through joint coverage functions. Two budget constraints are introduced for the total cost the decision-maker is willing to incur for locating facilities at each level. The HNCL problem is first formulated as a mixed-integer non-linear programming (MINLP) model; then, an equivalent mixed-integer linear program (MILP) is developed. Moreover, a classification framework is presented to evaluate the contribution of each cooperation type in covering the demand. We tested the HNCL model on instances randomly generated and extracted from the real-world network pre-

sented in [Chapter 3](#). The results are provided and discussed, highlighting the contribution of cooperative coverage compared to the individual one.

## 4.1 Context and motivations

As discussed in [Chapter 1](#), the delivery of services by modern businesses and organizations has become much more dynamic to cater to the customers' rapidly changing needs due to digital transformation.

The trend to customize services, either individually or collectively, implies that firms need to cover their customers in an adaptable, flexible and, at the same time, cost-efficient manner. As a result, firms develop their networks by establishing facilities that offer multiple and different services according to customers' needs. Quite often, such facilities may be organized in a hierarchical structure, where facilities at each level may offer a specific set of services. A typical example concerns the provision of banking services, which has been the focus of [Chapter 3](#). As in the banking sector, there are many situations where facilities typically need to cooperate with each other to ensure that customers are appropriately covered, in the sense that their different types of needs are provided for in a cost-efficient manner that guarantees the firms' sustainability.

In this chapter, we analyse the problem of multiple types of facilities arranged in a nested hierarchical structure and cooperating to cover the demand. Specifically, we propose the two-level Hierarchical Nested Cooperative Location (HNCL) model.

The hierarchy is nested (or successively inclusive) in the sense that facilities of the upper level offer all services available at facilities of the lower level, plus at least one additional service ([Narula, 1984](#)). In particular, two sets of services are defined: (i) *exclusive services*, provided solely by upper-level facilities, and (ii) *shared services*, provided by facilities of both hierarchical levels.

As regards the exclusive services of upper-level facilities, the model intends to guarantee the coverage of all the demand according to the traditional coverage paradigm (binary and individual). This means that each demand point needs to find at least a facility within a fixed radius.

Instead, as concerns the shared services, i.e. those provided by facilities at upper and lower levels, cooperation is allowed. In particular, we consider two distinct coverage decay functions, each associated with a hierarchical level representing the probability of coverage by facilities. Cooperation may occur between facilities at the same level (*intra-level cooperation*) and at different levels (*inter-level cooperation*). Each cooperation is modelled through a joint coverage function.

The aim is to design a network by locating facilities at each hierarchical level to

cover all the demand for exclusive services and maximize the covered demand for shared services. We assume a specific available budget for the installation of facilities at each level.

In [Chapter 3](#), we proposed a hierarchical covering to restructure the branch networks by closing and diversifying internal branches and outsourcing services to external facilities. In this chapter, we generalise the above problem to be applied in several contexts by introducing cooperation among upper-level facilities (branches) and lower-level facilities (external facilities).

The proposed HNCL problem is first formulated as a Mixed Integer Non-Linear Programming (MINLP). Then, a linearizing approach is developed to provide an equivalent Mixed Integer Non-Linear Programming (MILP) formulation.

We perform an extensive series of computational experiments to compare the performance of MINLP and MILP formulations and investigate the effect of certain parameters on the optimal solution to instances of various sizes. The results are analysed, highlighting the contributions of the coverage mechanisms.

The aim of the chapter is threefold. First, The HNCL aims to extend the available literature on cooperative problems, presented in [Section 2.5](#). Indeed, to the best of our knowledge, no hierarchical cooperative problems have been previously introduced considering a nested hierarchy. Therefore, inter-level cooperation has not been previously taken into account.

A second contribution is given by the linearization method developed to provide an equivalent MILP formulation of the HNCL.

Finally, a further novelty is the proposed classification framework to measure the contribution of cooperation to the coverage, representing a valuable tool to explore the produced scenarios in order to investigate how they vary according to the values of the parameters.

The problem under investigation may also open to many research perspectives as the problem can be effectively used to represent further real-world applications, especially in the logistics, retail and healthcare sectors.

The rest of the chapter is organized as follows. In [Section 4.2](#), we analyse the cooperative literature with respect to HNCL by highlighting the gaps the latter aims to bridge. In [Section 4.3](#), we formulate the proposed HNCL model as a non-linear programming model and explain a process for transforming it into an equivalent linear formulation. In [Section 4.4](#), the classification framework we developed is presented. We then explain our experimentation process in [Section 4.5](#) and report our findings on a set of real instances as well as an extended range of randomly generated ones in [Section 4.6](#). Some

managerial implications are then drawn in [section 4.7](#). Finally, [Section 4.8](#) summarize the work in the concluding section.

## 4.2 Literature review

Cooperative location problems in a discrete location space were formally introduced by [Berman et al.](#) in 2011 and have received increasing attention in the following years, leading to a worthy literature body.

The majority of papers contributing to this class of covering problems consider a single type of facilities (S) (see, e.g., [Zhang et al., 2017](#); [Davari, 2019](#); [Drezner and Drezner, 2019](#); [Wang et al., 2021](#)).

Those considering multiple types of facilities (M) in a cooperative context (e.g., [Wang et al., 2016](#); [Li et al., 2018](#); [Karatas, 2018](#)) generally introduced distinct coverage decay functions depending on facilities' features (e.g., size) or provided services. For instance, [Karatas and Eriskin \(2021\)](#) analyzed the minimal covering location problem of undesirable capacitated facilities by assuming gradual and cooperative coverage. Facilities differ on the basis of their size. The coverage decay function is characterized by a parameter representing the size of a facility; on equal distances, as the size grows, the coverage increases.

In some cases, the different types of facilities may be arranged in a hierarchical structure, which could be nested (Ne) or non-nested (NNe). [Karatas and Dasci \(2020\)](#) proposed a two-level non-nested hierarchical location and sizing problem for maximizing expected demand coverage while satisfying a budget constraint. Facilities of the upper and lower levels provide disjoint sets of services, and cooperation occurs only among lower-level facilities. A non-nested hierarchy is also considered by [Karatas \(2020\)](#) to address the two-level location problem for heterogeneous sensor networks in a cooperative setting. In particular, the hierarchy is modelled as a hub and spoke topology where spokes cooperate to provide a certain level of coverage to critical facilities that need protection against intruders. Hubs, on the other hand, serve as communication centres and collect data from assigned spokes.

In a cooperative framework, the individual coverages have been modelled through various decay functions, many of which borrowed from gradual covering literature (see [Sections 2.4](#) and [2.5](#)). Such coverages may assume different meanings. They may represent the *intensities* of physical signals (e.g., light or sound) dissipating over the distance according to given physical laws ([Berman et al., 2011](#)), or *probabilities* when they model the spatial interaction among demand and facilities ([Zhang et al., 2017](#)).

The mechanism ruling the aggregation of single coverages depends on the nature of

the coverages themselves. When dealing with intensities, the aggregation mechanism is typically deterministic (D), and the overall coverage is usually obtained as the sum of individual coverages (see, e.g. [Ashtiani et al., 2018](#); [Ming et al., 2021](#)). In contrast, when dealing with probabilities, the aggregation mechanism is probabilistic (P) and differently computed if the coverage events are assumed independent (I) or somehow correlated (C). On the one hand, when the events are perfectly correlated, the demand perceives only the maximum coverage among those by located facilities. Contrariwise, if the events are independent, all located facilities contribute to a demand node coverage, which is obtained through a function of the product of the single coverages. The case of partial correlation has been modelled through the (non-linear) *joint coverage function* by [Drezner and Drezner \(2014\)](#) and [Berman et al. \(2019\)](#), which is defined as a convex combination of the two extreme cases. The weight  $\theta$  ruling the combination represents a correlation degree among coverage events. By varying  $\theta$ , it is possible to capture the whole spectrum of dependencies.

The aggregate coverage function represents one of the peculiar features of the cooperative mechanism and its mathematical definition deeply affects the whole formulation of the problem. In a deterministic framework, the model is typically linear (L) in contrast to the probabilistic one, where the aggregate function leads to the insurgence of non-linearities (NL). Some approaches have been developed to linearize them. For instance, [Karatas \(2017\)](#) developed an approach to linearize the aggregate coverage function defined under conditions of independent coverage events by mapping the problem to a network-like structure. [Álvarez-Miranda and Sinnl \(2019\)](#) exploited the submodularity features of the joint coverage function to linearize it in a single facility type setting.

In some models, the cooperative assumption is jointly considered with the other relaxations mentioned in [Section 2.4](#), which are gradual coverage (G) and variable coverage radius (V). [Bagherinejad et al. \(2018\)](#) proposed a non-linear maximal covering location problem, assuming gradual and cooperative coverage.

As regards solution approaches, heuristics and metaheuristics (H), and exact methods (E) have been developed to solve cooperative problems. For instance, [Bababeik et al. \(2018\)](#) proposed a combined e-constraint method (AUGMECON) with a fuzzy-logic approach for solving a bi-objective model related to the problem of locating relief trains. [Karabulut et al. \(2017\)](#) propose three Tabu search heuristics for solving the optimal cooperating sensor deployment problem.

In order to compare the HNCL with existing cooperative models, we summarized the main features of the cited papers dealing with cooperative coverage in [Table 4.1](#). It can be noticed that no previous studies consider multiple facilities that (i) are arranged

**Table 4.1**  
Positioning HNCL in cooperative literature.

Paper	Facility type	Hierarchy	Cooperative mechanism	Coverage events	Model formulation	Additional relaxations	Solution approach
Álvarez-Miranda and Sinnl (2019)	S	–	P	C	NL, L	G	E
Ashtiani et al. (2018)	M	–	D	–	L	–	–
Bababeik et al. (2018)	S	–	D	–	L	–	E
Bagherinejad et al. (2018)	S	–	D	–	NL	G	H
Berman et al. (2009a)	S	–	D	–	L	–	E, H
Berman et al. (2011)	S	–	D	–	L	–	H
Berman et al. (2019)	S	–	P	C	NL	G	H
Davari (2019)	S	–	D	–	NL, L	–	H
Drezner and Drezner (2014)	S	–	P	C	NL	G	H
Drezner and Drezner (2019)	S	–	D	–	NL	G	H
Karabulut et al. (2017)	M	–	D, P	I	NL, L	G	H
Karatas (2017)	S	–	P	I	NL, L	G, V	E
Karatas (2018)	M	–	P	I	NL, L	–	H
Karatas (2020)	M	Nn	P	I	NL, L	–	–
Karatas and Dasci (2020)	M	Nn	P	I	NL, L	G, V	–
Karatas and Eriskin (2021)	M	–	P	I	NL, L	G, V	–
Li et al. (2018)	M	–	D	–	L	–	–
Ming et al. (2021)	M	–	D	–	L	G	–
Wang et al. (2016)	M	–	D	–	NL	G	–
Wang et al. (2021)	S	–	D	–	L	–	–
Zhang et al. (2017)	S	–	P	I	NL	–	H
HNCL	M	Ne	P	C	NL, L	–	–

The features are: Facility type (S: Single, M: Multiple), Hierarchy (Ne: Nested, Nn: Non-Nested, –: Not present), Cooperative mechanism (D: Deterministic, P: Probabilistic), Coverage events (I: based on Independent events, C: based on possibly Correlated events, –: Not applicable), Model formulation (L: Linear, NL: Non-linear), Additional relaxations (G: Gradual coverage, V: Variable radius, –: None), Solution approach (E: Exact, H: Heuristic and Metaheuristic, –: Not presented).

in a nested hierarchy and (ii) cooperate (also among levels) to cover the demand while (iii) assuming possibly correlated coverage events.

The HNCL problem aims to bridge this gap by considering two-level nested hierarchical facilities where cooperation is between not only facilities at the same hierarchical level (intra-level cooperation) but also at different hierarchical levels (inter-level cooperation). Such cooperations are (non-linearly) modelled through the joint coverage function.

It is worth highlighting that assuming a nested hierarchy is a key feature of our problem. Indeed, whether considering a non-nested hierarchy, facilities at different levels would provide disjoint sets of services. Thus, facilities at the same level could cooperate to cover the demand for the services they (exclusively) provide, but they could not cooperate with other levels.

Including cooperation among facilities at different levels implies further non-linearities in the model formulation compared to those already investigated in the literature. Therefore, another contribution of the present work concerns the development of a linearizing approach for intra-level cooperation and, then, inter-level cooperation by extending the linearization proposed by Karatas (2017) to provide an equivalent MILP of the HNCL.

In the following section, we present the HNCL problem, its MINLP formulation and the development of the equivalent MILP.

### 4.3 The Hierarchical Nested Cooperative Location (HNCL) problem

In this section, we introduce the Hierarchical Nested Cooperative Location (HNCL) model. In the following, we define the problem framework and its assumptions (Section 4.3.1); then, in Section 4.3.2 we provide the MINLP formulation of the problem while the equivalent MILP formulation is presented in Section 4.3.3.

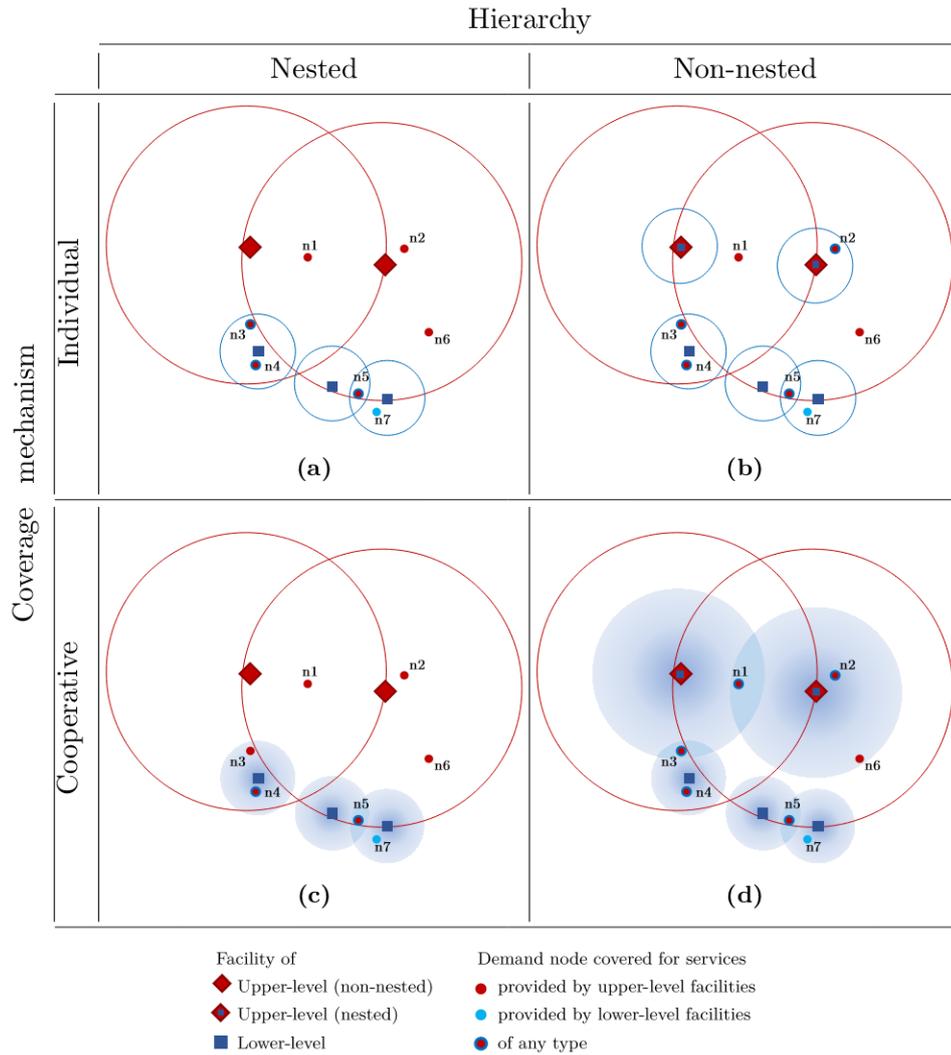
#### 4.3.1 Problem description

We assume a two-level nested hierarchy involving upper and lower-level facilities and two distinct sets of services. Specifically, exclusive services are complex services provided solely by upper-level facilities, and customers may be willing to travel a relatively long distance to access them. Whilst, shared services, provided by facilities of both hierarchical levels, are basic services that customers are expected to find within short distances.

While the coverage mechanism for exclusive services is binary and individual, in the sense that a demand node is covered if it finds at least one upper-level facility within a predefined (large) radius  $R$ , the cooperation may occur among facilities at the same level (intra-level cooperation) and at different levels (inter-level cooperation) to cover the demand for shared services.

In order to provide a clear picture of the HNCL problem setting, in Fig. 4.1, we compare different versions of hierarchical covering problems by referring to an example with 7 demand nodes and 5 facilities. In particular, we classify the problem according to (a) the typology of hierarchy (nested vs non-nested) and (b) the coverage mechanism for the lower-level services (individual vs cooperative). By combining the criteria, we consider four problem settings:

1. The *two-level hierarchical non-nested problem* (Fig. 4.1a), where facilities at the upper level provide, besides exclusive services, those provided by the lower level (shared services). Compared to Fig. 4.1a, in Fig. 4.1b, a different symbol has been



**Fig. 4.1.** Hierarchical covering problems: a classification framework.

used to represent upper-level facilities. Moreover, each of them is characterized by two radii, the blue one for the shared services and the red one for exclusive ones. Here, a demand node can be covered for shared services by an upper-level facility. See node  $n_2$  that in the previous case remained uncovered.

2. The *two-level hierarchical non-nested cooperative problem*. Lower-level facilities provide decay coverages (as shown in Fig. 4.1c through the gradient colour around them) that can be aggregated to cover the demand. In this case, the node  $n_5$  is covered thanks to cooperation.

3. The *two-level HNCL* (our problem) that is nested and cooperative (Fig. 4.1d). The main novelty here is that the coverage for shared services can be provided thanks to cooperation among facilities at the same level (lower-lower or upper-upper) and at different levels (lower-upper). See node n1 covered by cooperation between the two upper-level facilities and n3 by cooperation between one upper and one lower-level facility.

This last case has not been explored in the literature, as shown in section 4.2, and is the focus of the chapter. Given this rough picture of the problem, let us formalize it.

Let  $I$  denote the set of demand nodes,  $K = \{U, L\}$  the set of hierarchical levels of facilities – where  $U$  and  $L$  define the upper and the lower levels, respectively – and  $J^k$  the set of candidate sites for facilities of level  $k \in K$ .

We introduce two distinct *coverage decay functions*  $\phi_{ij}^k$  for shared services, each associated with a hierarchical level  $k$ , that represent the coverage level provided by a  $k$ -level facility  $j \in J^k$  to a demand node  $i \in I$ . The functions  $\phi_{ij}^k$  are left-continuous and non-increasing over the distance  $d_{ij}$  from  $i$  to  $j$  ( $\phi_{ij}^k = f_k(d_{ij})$ ), and explain the coverage probability ( $0 \leq \phi_{ij}^k \leq 1$ ). We assume that, for each given distance, the coverage level of an upper-level facility is higher than that provided by a lower-level one.

All located facilities contribute to the definition of the coverage level of each demand node for shared services. We use the joint coverage function (Drezner and Drezner, 2014) both for aggregating coverages from the same level and from different levels. Let us remind that the joint coverage is defined as a convex combination of two contributions: the first depicting the case of a *perfect dependency* in which the customers perceive only the maximum coverage, and the second one representing the case of *perfect independence* in which a function of the product of the single coverages gives the aggregate coverage. By varying the weights of the two contributions, it is possible to capture the whole spectrum of event dependencies. In our problem, we introduce three different aggregate coverage functions associated with intra-lower, intra-upper and inter-level cooperations:

- (i) The (two) intra-level aggregate coverages  $\Phi_i^k$ . Each of these functions is associated with a hierarchical level  $k$  and combines the coverage level provided by located facilities of level  $k$  to demand node  $i \in I$ :

$$\Phi_i^k = \theta^k \max_{j \in J^k} \{ \phi_{ij}^k x_j^k \} + (1 - \theta^k) (1 - Q_i^k) \quad \forall i \in I, k \in K \quad (4.1)$$

where  $x_j^k$  is equal to 1 when a facility of level  $k$  is located at candidate site  $j \in J^k$  and 0 otherwise, and  $Q_i^k$  represents the overall non-coverage probability of a demand node  $i$  by all located facilities of level  $k$  under the assumption of indepen-

dent coverage events, i.e.  $Q_i^k = \prod_{j \in J^k} (1 - \phi_{ij}^k x_j^k)$ . If  $\theta^k = 1$ , the coverage of  $i$  is achieved by the closest facility that is the one with the maximum probability (perfect dependence of coverage events). In contrast, if  $\theta^k = 0$ , the cooperation is the maximum possible, being the coverage events independent. See [Drezner and Drezner, 2014](#) and [Berman et al., 2019](#) for a complete description. Therefore, the degree of cooperation among facilities of level  $k$  may be controlled by setting the parameter  $\theta^k$  depending on the specific application at hand;

- (ii) Inter-level aggregate coverage  $\Phi_i$ . This function combines the coverage level jointly from facilities of each hierarchical level to demand node  $i \in I$ :

$$\begin{aligned} \Phi_i &= \theta \max \{ \Phi_i^U; \Phi_i^L \} + (1 - \theta) [1 - (1 - \Phi_i^U) (1 - \Phi_i^L)] \\ &= \theta \max \{ \Phi_i^U; \Phi_i^L \} + (1 - \theta) (\Phi_i^U + \Phi_i^L - \Phi_i^U \Phi_i^L) \quad \forall i \in I \end{aligned} \quad (4.2)$$

Similarly, the parameter  $\theta$  regulates the cooperation degree among facilities at distinct hierarchical levels. Therefore,  $\theta = 1$  means that the coverage of demand node  $i$  is provided by facilities of that hierarchical level providing the maximum; conversely, the cooperation among facilities at different hierarchical levels is maximum if  $\theta = 0$ .

We assume that a demand node  $i$  is covered if the (global) inter-level aggregate function  $\Phi_i$  exceeds a certain *threshold*  $T$  (binary coverage, [Berman et al. \(2010\)](#)).

The HNCL model aims to locate two types of facilities arranged in a nested hierarchy such that the demand covered for shared services is maximized whilst the coverage for exclusive services is guaranteed to each customer. within a (large) fixed covering radius  $R$ , according to a traditional (set) covering definition. We introduce two budgets, each available to locate facilities at a specific level. Thereby, a decision-maker implementing the HNCL model may calibrate budgets depending on the investment they are willing to undertake for locating facilities of a specific hierarchical level, which are characterised by different features.

### 4.3.2 HNCL: a mixed-integer non-linear programming (MINLP) formulation

In order to formulate the two-level HNCL model, we introduced the following notation:

*Sets*

$I$  Set of demand nodes;

$K$  Set of hierarchical levels.  $K = \{U, L\}$  where  $U$  and  $L$  denote the upper and the lower levels, respectively;

$J^k$  Set of candidate sites of facilities of level  $k \in K$ ;

*Parameters*

$w_i$  Demand associated with node  $i \in I$ ;

$c_j^k$  Cost for locating a facility of level  $k \in K$  at location  $j \in J^k$ ;

$B^k$  Budget available for locating facilities of level  $k \in K$ ;

$d_{ij}$  Distance between demand node  $i \in I$  and candidate facility location  $j \in \bigcup_{k \in K} J^k$ ;

$R$  Covering radius for exclusive services, solely provided by upper-level facilities;

$\phi_{ij}^k$  Coverage level for shared services provided to demand node  $i \in I$  by facility of level  $k \in K$  located at  $j \in J^k$ ;

$\theta^k$  Intra-level correlation factor depicting the dependency among facilities at level  $k \in K$ ;

$\theta$  Inter-level correlation factor depicting the dependency in coverage events at facilities of different levels;

$T$  The threshold for coverage of shared services.

*Covering sets*

$N_i^U$  Set of candidate upper-level facilities  $j \in J^U$  that are within the distance  $R$  from the demand node  $i \in I$ , i.e.  $N_i^U = \{j \in J^U : d_{ij} \leq R\}$ .

*Decision variables*

$x_j^k$  Binary decision variable equal to 1 if a facility of level  $k \in K$  is located at  $j \in J^k$ ;

$y_i$  Binary decision variable equal to 1 if demand node  $i \in I$  is covered for the shared services;

$\Phi_i^k$  Aggregate coverage provided to node  $i \in I$  by facilities located at the same level  $k \in K$ ;

$\Phi_i$  Aggregate coverage provided to node  $i \in I$  by facilities located at any level.

With this notation, we can formulate the Hierarchical Nested Cooperative Location (HNCL) model as follows:

$$\max \sum_{i \in I} w_i y_i \quad (4.3)$$

$$\text{s.t.} \quad \sum_{j \in N_i^U} x_j^U \geq 1 \quad \forall i \in I \quad (4.4)$$

$$(4.1), (4.2)$$

$$\Phi_i \geq T y_i \quad \forall i \in I \quad (4.5)$$

$$\sum_{j \in J^k} c_j^k x_j^k \leq B^k \quad \forall k \in K \quad (4.6)$$

$$x_j^k \in \{0, 1\} \quad \forall k \in K, j \in J^k \quad (4.7)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (4.8)$$

$$\Phi_i^U, \Phi_i^L, \Phi_i \geq 0 \quad \forall i \in I \quad (4.9)$$

The objective function (4.3) maximizes the total covered demand for shared services provided by facilities of both upper and lower levels. Constraints (4.4) guarantee that each demand node is covered within the distance  $R$  by at least one located upper-level facility. Constraints (4.1) functionally define, for each demand node  $i \in I$ , the aggregate coverage level from facilities located at the same level  $k$ . The global coverage of each demand node from all located facilities is defined by constraints (4.2). Constraints (4.5) guarantee that a demand node is covered for shared services if the global coverage level exceeds the threshold  $T$ . The maximum budget to spend for locating facilities of each hierarchical level  $k$  is assigned by constraints (4.6). Finally, constraints (4.7)-(4.9) define the nature of decision variables.

### 4.3.3 HNCL: a mixed-integer linear programming (MILP) formulation

In this section, we develop a linear reformulation of the MINLP presented in Section 4.3.2. The non-linearities occur both in aggregating coverage at the same hierarchical level (constraints (4.1)) and in aggregating coverage among the hierarchical levels (constraints (4.2)). We first describe our linearizing approach; then, the equivalent MILP reformulation is provided in Section 4.3.3.1.

The max-parts in (4.1) and (4.2) are linearized by introducing the following auxiliary decision variables:

$z_{ij}^k$  Binary decision variable equal to 1 if and only if the maximum coverage level at demand node  $i$  provided by located facilities of level  $k$  is achieved at  $j \in J^k$  (and is equal to  $\phi_{ij}^k$ );

$\Phi_i^{max}$  Maximum aggregate coverage provided to demand node  $i \in I$  by located facilities of each hierarchical level, i.e.  $\Phi_i^{max} = \max \{ \Phi_i^U; \Phi_i^L \}$ ;

$s_i^k$  Binary decision variable equal to 1 if  $\Phi_i^k \neq \Phi_i^{max}$ ,  $i \in I, k \in K$ ;

and adding the following constraints to the model:

$$\sum_{j \in J^k} z_{ij}^k \leq 1 \quad \forall i \in I, k \in K \quad (4.10)$$

$$z_{ij}^k \leq x_j^k \quad \forall i \in I, k \in K, j \in J^k \quad (4.11)$$

$$\sum_{j \in J^k} \phi_{ij}^k z_{ij}^k \geq \phi_{ij}^k x_j^k \quad \forall i \in I, k \in K, j \in J^k \quad (4.12)$$

$$\Phi_i^{max} \geq \Phi_i^k \quad \forall i \in I, k \in K \quad (4.13)$$

$$\Phi_i^{max} \leq \Phi_i^k + s_i^k \quad \forall i \in I, k \in K \quad (4.14)$$

$$\sum_{k \in K} s_i^k \leq 1 \quad \forall i \in I \quad (4.15)$$

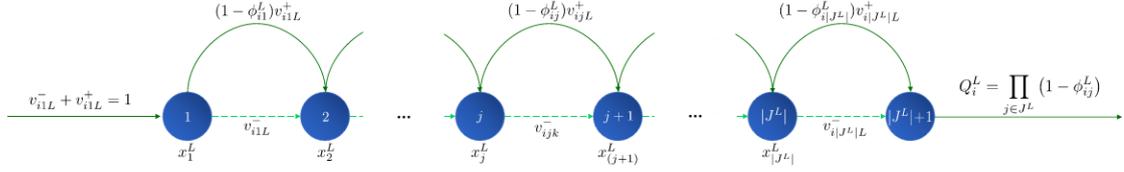
$$z_{ij}^k \in \{0, 1\} \quad \forall i \in I, j \in J^k, k \in K \quad (4.16)$$

$$s_i^k \in \{0, 1\} \quad \forall i \in I, k \in K \quad (4.17)$$

Constraints (4.10)-(4.12) assign each demand node to the located facility of level  $k$  providing the maximum coverage level. Specifically, each demand node must be assigned to at most one (constraints (4.10)), located (constraints (4.11)) facility of hierarchical level  $k$  providing the highest coverage level among all the located ones (constraints (4.12)). Then,  $\max_{j \in J^k} \{\phi_{ij}^k x_j^k\} = \sum_{j \in J^k} \phi_{ij}^k z_{ij}^k$ . Constraints (4.13)-(4.15) guarantee that the variable  $\Phi_i^{max}$  is equal to the maximum between  $\Phi_i^U$  and  $\Phi_i^L$  for each demand node. Indeed, constraints (4.13) set the lower bound of  $\Phi_i^{max}$  to the larger between  $\Phi_i^U$  and  $\Phi_i^L$ . In contrast, constraints (4.14) set the upper bound of  $\Phi_i^{max}$  to the smaller between  $\Phi_i^U$  and  $\Phi_i^L$  plus the decision variable  $s_i^k$ . The variable  $s_i^k$  is equal to 1 only for at most one (constraints (4.15)) inequality among those obtained by varying  $k$  in (4.14) for each demand node. For constraints (4.13)-(4.15) to be simultaneously verified, the upper and the lower bound of  $\Phi_i^{max}$  are identical and correspond to the maximum.

The prod-parts included in (4.1) and (4.2) are represented by  $Q_i^k$  and  $\Phi_i^U \Phi_i^L$ , respectively. In order to linearize  $Q_i^k$ , we implement the linearization process introduced by Karatas (2017) and the references therein (Morton et al., 2007; Salmerón, 2012). The main idea is to reduce each  $Q_i^k$  to a network flow problem. For the sake of simplicity, let us consider the overall non-coverage probability  $Q_i^L$  of node  $i$  by facilities of the lower level  $L$ . The linearizing approach maps a network crossed by a flow, as shown in Fig. 4.2.

By referring to Fig. 4.2, the network is designed as follows: (i)  $|J^L|+1$  nodes (depicted by blue bullets in the figure) representing the candidate locations for facilities of level  $L$  plus a pseudo location (the  $(|J^L|+1)$ -th); (ii) each adjacent node pair, the  $j$ -th and  $(j+1)$ -th is linked through two arcs – one labelled positive (in dark green) and one negative (in light green) associated with auxiliary decision variables  $v_{ijL}^+$  and  $v_{ijL}^-$ , respectively – that describe the flow; (iii) the labelled positive arc connecting the  $j$ -th and  $(j+1)$ -th arc is associated with a weight equal to  $(1 - \phi_{ij}^L)$ ; (iv) the variable  $x_j^L$  rules the flow outgoing from node  $j$ . The flow crossing the network is initialized to 1,



**Fig. 4.2.** Network related to demand node  $i$  and crossed by the flow – scaled by the located facilities of level  $L$  – representing the overall non-coverage probability  $Q_i^L$ . Each blue node represents a candidate location  $j$  of level  $L$ ; labelled positive and labelled negative arcs are depicted by solid dark green lines and dashed light green lines, respectively. A generic node  $j$  is related to variable  $x_j^L$ , whose value directs the flow outgoing from  $j$  on the labelled positive arc ( $x_j^L = 1$ ) and the labelled negative arc ( $x_j^L = 0$ ).

i.e. the value of  $Q_i^L$  if no facilities of level  $L$  are located. Suppose a facility is located at  $j$  and, then,  $x_j^L = 1$ . In this case, the flow crosses the labelled positive arc from  $j$ -th and  $(j + 1)$ -th nodes and is scaled of a factor equal to  $(1 - \phi_{ij}^L)$ . Otherwise, the flow crosses the labelled negative arc, remaining unchanged. Therefore, the flow entering the  $(j + 1)$ -th node of the network is equal to  $\prod_{t=1}^j (1 - \phi_{it}^L x_t^L)$ . Finally, the flow entering the pseudo location  $(|J^L| + 1)$  and thus outgoing the network equals the non-coverage probability  $Q_i^L$ . See Karatas (2017) for a detailed description.

The approach by Karatas (2017) allows to linearize  $Q_i^k$  of a single hierarchical level  $k$ , while linearizing the prod-part of the aggregation of coverages results in a further and more challenging task when more levels of facilities interact to cooperate. Indeed, additional products among decision variables are involved, as will be shown. Our main contribution to the linearizing process is extending the approach proposed by Karatas (2017) when cooperation occurs in a two-level hierarchy of facilities. The prod-part  $\Phi_i^U \Phi_i^L$  is exploded, recalling their definition in (4.1), and replacing  $\max_{j \in J^k} \{\phi_{ij}^k x_j^k\}$  with its equivalent  $\sum_{j \in J^k} \phi_{ij}^k z_{ij}^k$ , as follows:

$$\Phi_i^U \Phi_i^L = \theta^U (1 - \theta^L) \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + (1 - \theta^U) \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L \quad (4.18)$$

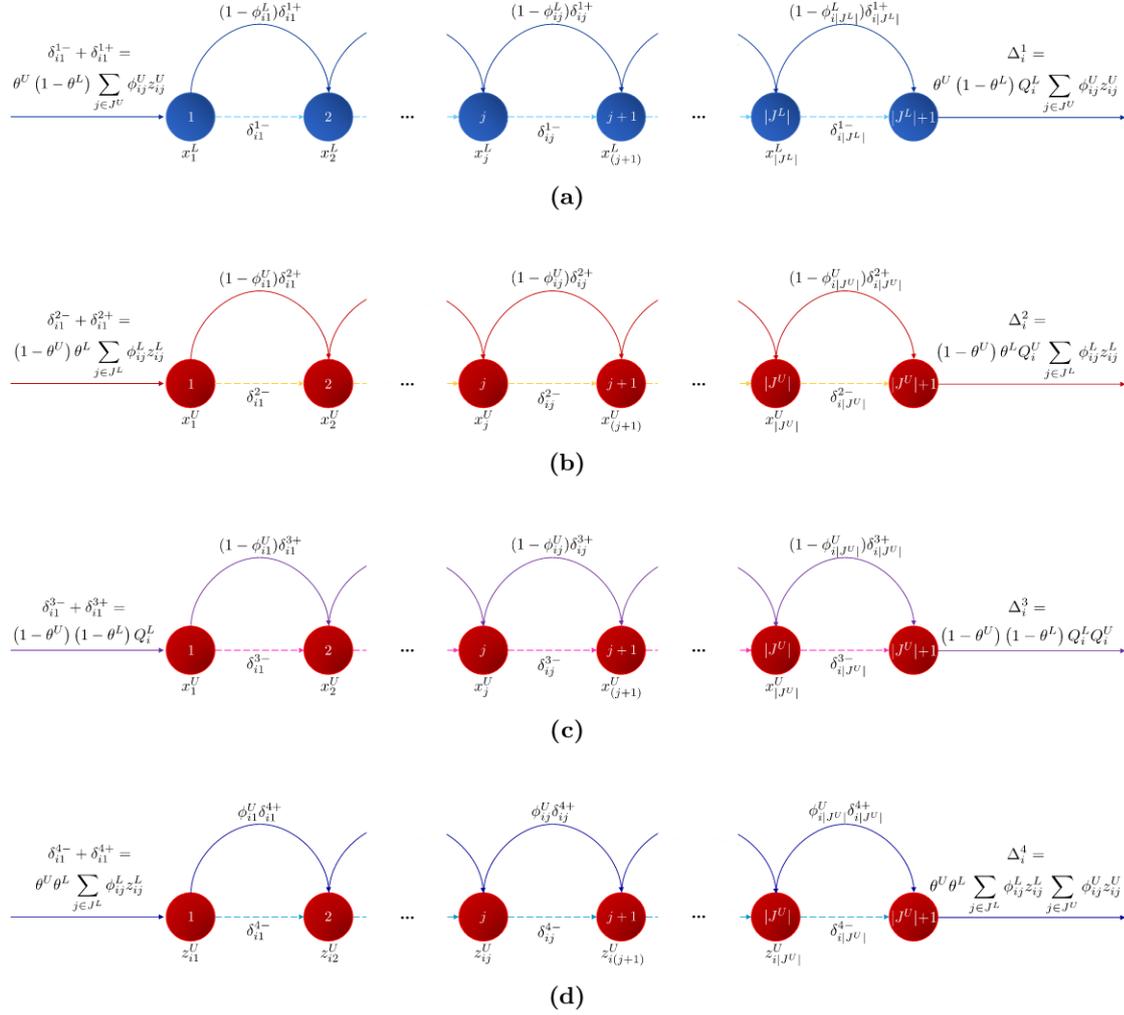
$$+ (1 - \theta^U) (1 - \theta^L) (1 - Q_i^U - Q_i^L) - \theta^U (1 - \theta^L) Q_i^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + \quad (4.19)$$

$$- (1 - \theta^U) \theta^L Q_i^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + \quad (4.20)$$

$$+ (1 - \theta^U) (1 - \theta^L) Q_i^L Q_i^U + \quad (4.21)$$

$$+ \theta^U \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U \quad (4.22)$$

See Section 4.8 for a comprehensive description.



**Fig. 4.3.** The four networks related to demand node  $i$  and crossed by the flow representing the non-linear contributions: (a)  $\Delta_i^1$  equal to the reverse sign of (4.19); (b)  $\Delta_i^2$  equal to the reverse sign of (4.20); (c)  $\Delta_i^3$  equal to (4.21); (d)  $\Delta_i^4$  equal to (4.22). Blue and red nodes depict the candidate sites of facilities of the lower and the upper level, respectively. Labelled positive and labelled negative arcs are depicted by solid and dashed lines, respectively. The variable below each node directs the flow across the arcs.

The extended expression of  $\Phi_i^U \Phi_i^L$  is made up of the right-hand side of (4.18), which is linear and will be defined by  $\Lambda_i$  for shortness in the following, and four non-linear contributions, which are:  $\Delta_i^1$  equal to the reverse sign of (4.19),  $\Delta_i^2$  to the reverse sign of (4.20),  $\Delta_i^3$  to (4.21), and  $\Delta_i^4$  to (4.22). Then, (4.18)-(4.22) can be equivalently as:

$$\Phi_i^U \Phi_i^L = \Lambda_i - \Delta_i^1 - \Delta_i^2 + \Delta_i^3 + \Delta_i^4$$

Each non-linear contribution is given by a product of several factors. Our approach consists of introducing for each of the non-linearities  $\Delta_i^n, n \in \{1, 2, 3, 4\}$  a new network flow problem that linearizes one of that factors, which can be handled in a Karatas (2017) fashion (e.g.,  $Q_i^L$ ), and initializing the flow crossing the network to the rest of the non-linear contribution. The networks we introduce are depicted in Fig. 4.3. Additional auxiliary variables  $\delta_{ij}^{n+}$  and  $\delta_{ij}^{n-}$  describe the flow crossing the arcs.

The network in Fig. 4.3a is used for linearizing  $\Delta_i^1$ , which contains  $Q_i^L$ . The network is designed to linearize  $Q_i^L$ , as in the previous case: the nodes represent the candidate sites of lower-level facilities; variables  $x_j^L$  direct the flow outgoing from  $j$ ; the labelled positive arc from facility candidate sites  $j$  to  $(j+1)$  is weighted by  $(1 - \phi_{ij}^L)$ . However, in this case, the initial flow is set to the rest of the expression of  $\Delta_i^1$ , i.e.  $\theta^U (1 - \theta^L) \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U$ . The outgoing flow after having crossed the whole network will equal  $\Delta_i^1 = \theta^U (1 - \theta^L) Q_i^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U$ .

The contribution  $\Delta_i^2$  is a mirror-like case of  $\Delta_i^1$  where however  $Q_i^U$  is linearized through the network depicted in Fig. 4.3b, which is made by: red nodes representing candidate sites of upper-level facilities;  $(1 - \phi_{ij}^U)$  is the weight associated with the labelled positive arc linking nodes  $j$  and  $(j+1)$ ; the flow is initialized to  $(1 - \theta^U) \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L$  and is direct from  $j$  by the value of  $x_j^U$ .

The network related to  $\Delta_i^3$  is still designed to linearize  $Q_i^U$  whilst the flow is initialized to  $Q_i^L$ , representing the output of the network in Fig. 4.2, multiplied by  $(1 - \theta^U) (1 - \theta^L)$ .

The contribution  $\Delta_i^4$  is dissimilar since any  $Q_i^k$  does not appear. However, a modified network flow problem may nevertheless be implemented, as shown in Fig. 4.3d. Indeed, the network can be designed with: nodes representing, for instance, candidates sites of upper-level facilities to handle  $\sum_{j \in J^U} \phi_{ij}^U z_{ij}^U$ ; the weight associated with the labelled positive arc linking  $j$  and  $j+1$  is equal to  $\phi_{ij}^U$ ; the variable  $z_{ij}^U$  is responsible for directing the flow outgoing from  $j$ . The flow entering the network is  $\theta^U \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L$ . Then, the flow will cross only one labelled positive arc corresponding to the located upper-level facility  $j$  that provides the maximum coverage such that  $z_{ij}^U = 1$  and all the others labelled negative arcs of the network. Therefore, our linearization approach holds since only one addend is positive in  $\sum_{j \in J^U} \phi_{ij}^U z_{ij}^U$ .

It is worth noticing that we preferred to design the network on the set  $J^U$  instead  $J^L$ , whenever possible. Indeed, reasonably assuming  $|J^U| \ll |J^L|$  in real-case applications, we reduce the number of auxiliary decision variables.

#### 4.3.3.1 HNCL: the MILP mathematical model

In order to linearize the model presented in Section 4.3.2, we introduce the following *auxiliary decision variables*:

- $Q_i^k$  Probability of demand node  $i \in I$  to be not covered by all located facilities of hierarchical level  $k$  when their coverage probabilities are based on independent events, i.e.  $Q_i^k = \prod_{j \in J^k} (1 - \phi_{ij}^k x_j^k)$ ,  $k \in K$ ;
- $\Delta_i^n$   $n$ -th non-linear contribution in the definition of  $\Phi_i^U \Phi_i^L$ , where  $n \in \{1, 2, 3, 4\}$ .  $\Delta_i^1$  corresponds to the reverse sign of (4.19),  $\Delta_i^2$  to the reverse sign of (4.20),  $\Delta_i^3$  to (4.21), and  $\Delta_i^4$  to (4.22);
- $v_{ijk}^+$  Labelled positive flow of demand node  $i$  and facility candidate site  $j$  of hierarchical level  $k$  related to  $Q_i^k$ ;
- $v_{ijk}^-$  Labelled negative flow of demand node  $i$  and facility candidate site  $j$  of hierarchical level  $k$  related to  $Q_i^k$ ;
- $\delta_{ij}^{n+}$  Labelled positive flow of demand node  $i$  and facility candidate site  $j$  related to  $\Delta_i^n$ , where  $n = 1$  if  $j \in J^L$  and  $n \in \{2, 3, 4\}$  if  $j \in J^U$ ;
- $\delta_{ij}^{n-}$  Labelled positive flow of demand node  $i$  and facility candidate site  $j$  related to  $\Delta_i^n$ , where  $n = 1$  is  $j \in J^L$  and  $n \in \{2, 3, 4\}$  if  $j \in J^U$ .

The MILP alternative formulation of HNCL model is:

$$\max (4.3)$$

$$\text{s.t. } v_{i,1,k}^- + v_{i,1,k}^+ = 1 \quad \forall i \in I, k \in K \quad (4.23)$$

$$v_{i,j,k}^- + (1 - \phi_{ij}^k) v_{i,j,k}^+ = v_{i,(j+1),k}^- + v_{i,(j+1),k}^+ \quad \forall i \in I, k \in K, j \in \{1, \dots, |J^k| - 1\} \quad (4.24)$$

$$v_{i,|J^k|,k}^- + (1 - \phi_{i,|J^k|}^k) v_{i,|J^k|,k}^+ = Q_i^k \quad \forall i \in I, k \in K \quad (4.25)$$

$$x_j^k \geq v_{ij}^+ \quad \forall i \in I, k \in K, j \in J^k \quad (4.26)$$

$$1 - x_j^k \geq v_{ijk}^- \quad \forall i \in I, k \in K, j \in J^k \quad (4.27)$$

$$\Phi_i^k = \theta^k \sum_{j \in J^k} \phi_{ij}^k z_{ij}^k + (1 - \theta^k)(1 - Q_i^k) \quad \forall i \in I, k \in K \quad (4.28)$$

$$\delta_{i1}^{1-} + \delta_{i1}^{1+} = \theta^U (1 - \theta^L) \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U \quad \forall i \in I \quad (4.29)$$

$$\delta_{i1}^{2-} + \delta_{i1}^{2+} = \theta^L (1 - \theta^U) \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L \quad \forall i \in I \quad (4.30)$$

$$\delta_{i1}^{3-} + \delta_{i1}^{3+} = (1 - \theta^U) (1 - \theta^L) Q_i^L \quad \forall i \in I \quad (4.31)$$

$$\delta_{i1}^{4-} + \delta_{i1}^{4+} = \theta^U \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L \quad \forall i \in I \quad (4.32)$$

$$\delta_{ij}^{1-} + (1 - \phi_{ij}^L) \delta_{ij}^{1+} = \delta_{i(j+1)}^{1-} + \delta_{i(j+1)}^{1+} \quad \forall i \in I, j \in \{1, \dots, |J^L| - 1\} \quad (4.33)$$

$$\delta_{ij}^{n-1} + (1 - \phi_{ij}^U) \delta_{ij}^{n+} = \delta_{i(j+1)}^{n-} + \delta_{i(j+1)}^{n+} \quad \forall i \in I, n \in \{2, 3\}, j \in \{1, \dots, |J^U| - 1\} \quad (4.34)$$

$$\delta_{ij}^{4-} + \phi_{ij}^U \delta_{ij}^{4+} = \delta_{i(j+1)}^{4-} + \delta_{i(j+1)}^{4+} \quad \forall i \in I, j \in \{1, \dots, |J^U| - 1\} \quad (4.35)$$

$$\delta_{i|J^U|}^{1-} + (1 - \phi_{i|J^U|}^L) \delta_{i|J^U|}^{1-} = \Delta_i^1 \quad \forall i \in I \quad (4.36)$$

$$\delta_{i|J^U|}^{n-} + (1 - \phi_{i|J^U|}^U) \delta_{i|J^U|}^{n+} = \Delta_i^n \quad \forall i \in I, n \in \{2, 3\} \quad (4.37)$$

$$\delta_{i|J^U|}^{4-} + \phi_{i|J^U|}^U \delta_{i|J^U|}^{4+} = \Delta_i^4 \quad \forall i \in I \quad (4.38)$$

$$x_j^L \geq \delta_{ij}^{1+} \quad \forall i \in I, j \in J^L \quad (4.39)$$

$$x_j^U \geq \delta_{ij}^{n+} \quad \forall i \in I, j \in J^U, n \in \{2, 3\} \quad (4.40)$$

$$z_{ij}^U \geq \delta_{ij}^{4+} \quad \forall i \in I, j \in J^U \quad (4.41)$$

$$1 - x_j^L \geq \delta_{ij}^{1-} \quad \forall i \in I, j \in J^L \quad (4.42)$$

$$1 - x_j^U \geq \delta_{ij}^{n-} \quad \forall i \in I, n \in \{2, 3\}, j \in J^U \quad (4.43)$$

$$1 - z_{ij}^U \geq \delta_{ij}^{4-} \quad \forall i \in I, j \in J^U \quad (4.44)$$

$$\Phi_i = \theta \Phi_i^{max} + (1 - \theta) (\Phi_i^U + \Phi_i^L + \Lambda_i + \Delta_i^1 + \Delta_i^2 - \Delta_i^3 - \Delta_i^4) \quad \forall i \in I \quad (4.45)$$

$$v_{ijU}^+, v_{ijU}^-, \delta_{ij1}^+, \delta_{ij2}^+, \delta_{ij3}^+, \delta_{ij1}^-, \delta_{ij2}^-, \delta_{ij3}^- \geq 0 \quad \forall i \in I, j \in J^U \quad (4.46)$$

$$v_{ijL}^+, v_{ijL}^-, \delta_{ij4}^+, \delta_{ij4}^- \geq 0 \quad \forall i \in I, j \in J^L \quad (4.47)$$

$$Q_i^U, Q_i^L, \Phi_i^{max}, \Delta_i^1, \Delta_i^2, \Delta_i^3, \Delta_i^4 \geq 0 \quad \forall i \in I \quad (4.48)$$

(4.4) – (4.17)

In order to compute the non-coverage probability  $Q_i^k$  for each demand node  $i$  and each hierarchical level  $k$ : constraints (4.23) initialize the flow by setting the sum of outgoing flows from the first node equal to 1; constraints (4.24) balance the outgoing scaled flows from  $j \in J^k$  to the unscaled incoming flow to  $j + 1$ ; finally,  $Q_i^k$  is computed by (4.25) as the sum of the scaled flows outgoing from the network, i.e. from  $|J^k|$ . Furthermore, technical constraints (4.26) ensure that a facility of hierarchical level  $k$  is located at candidate site  $j \in J^k$  when  $v_{ijk}^+$  is positive. Conversely, if  $v_{ijk}^-$  is positive, a facility must not be located at  $j$  by constraints (4.27).

The equivalent formulation of the intra-level aggregate coverage of facilities of level  $k$  expressed by (4.1) is provided by constraints (4.28).

In order to compute  $\Phi_i^U \Phi_i^L$ , the four flows, each established for each demand node and corresponding to a non-linear contribution in  $\Phi_i^U \Phi_i^L$ , are initialized by constraints (4.29)-(4.32). Then, for each demand node, the flow balance crossing the four networks

is defined by constraints (4.33)-(4.35). Finally, the four non-linear contributions for each demand node are computed as the outgoing flow from the networks by constraints (4.36)-(4.38). Moreover, if  $\delta_{ij}^{n+}$  is positive, a facility must be located at the candidate site  $j \in J^L$  when  $n = 1$  (constraints (4.39)) or at the candidate site  $j \in J^U$  when  $n \in \{2, 3\}$  (constraints (4.40)) or the latter provides the maximum coverage level among located upper-level facilities when  $n = 4$  (constraints (4.41)). Instead, a facility is not located at the candidate site  $j$  when  $\delta_{ijn}^-$  is positive (constraints (4.42) and (4.43)), neither it can provide the maximum coverage level (constraints (4.44)).

The equivalent linear formulation of the global inter-level aggregate coverage defined by (4.2) for each demand node is provided by the set of constraints (4.45).

Finally, constraints (4.46)-(4.48) declare the nature of the additional decision variables.

## 4.4 The value of the cooperation

In order to measure the contribution of intra-level and inter-level cooperations, we introduced the following framework classifying covered demand nodes according to the mechanism responsible for the coverage, as summarised in Table 4.2.

**Table 4.2**

Coverage classification of covered demand nodes, where  $x_j^{k*}, \Phi_i^{k*}, \Phi_i^*$  is the optimal solution of an HNCL model, and  $\phi_i^{max} = \max_{k \in K, j \in J^k} \phi_{ij}^k x_j^{k*}$ .

Covered demand	Set of nodes covered	Percentage of covered demand
by Individual coverage	$I^{Ind} = \{i \in I : \phi_i^{max*} \geq T\}$	$\frac{\sum_{i \in I^{Ind}} w_i}{\sum_{i \in I} w_i} (\%)$
by Cooperation	$I^{Coop} = \{i \in I : \phi_i^{max*} < T \wedge \Phi_i^* \geq T\}$	$\frac{\sum_{i \in I^{Coop}} w_i}{\sum_{i \in I} w_i} (\%)$
Intra-Upper Level	$I^{UC} = \{i \in I^{Coop} : \Phi_i^{U*} \geq T \wedge \Phi_i^{L*} < T\}$	$\frac{\sum_{i \in I^{UC}} w_i}{\sum_{i \in I} w_i} (\%)$
Intra-Lower Level	$I^{LC} = \{i \in I^{Coop} : \Phi_i^{U*} < T \wedge \Phi_i^{L*} \geq T\}$	$\frac{\sum_{i \in I^{LC}} w_i}{\sum_{i \in I} w_i} (\%)$
Intra-Upper/Lower Level	$I^{ULC} = \{i \in I^{Coop} : \Phi_i^{U*} \geq T \wedge \Phi_i^{L*} \geq T\}$	$\frac{\sum_{i \in I^{ULC}} w_i}{\sum_{i \in I} w_i} (\%)$
Inter-Level	$I^{IC} = \{i \in I^{Coop} : \Phi_i^{U*} < T \wedge \Phi_i^{L*} < T\}$	$\frac{\sum_{i \in I^{IC}} w_i}{\sum_{i \in I} w_i} (\%)$

Let  $x_j^{k*}, \Phi_i^{k*}, \Phi_i^*$  be the optimal solution of a HNCL model and  $\phi_i^{max*}$  the maximum coverage provided to demand node  $i$  by a single located facility, regardless of its hierarchical level, i.e.  $\phi_i^{max} = \max_{k \in K, j \in J^k} \phi_{ij}^k x_j^{k*}, i \in I$ . If  $\phi_i^{max*}$  exceeds the threshold  $T$ , the demand node  $i$  is nevertheless covered, regardless of any established cooperation. In this case, since locating (just) such a facility guarantees the covering condition  $\Phi_i^* \geq T$ , we can state that the coverage mechanism involved in covering  $i$  is the *individual coverage*.

Instead, if  $\phi_i^{max*}$  is lower than the threshold  $T$  albeit the demand node  $i$  is covered, i.e.  $\Phi_i^* \geq T$ , the *cooperation* among more than one facility is responsible for covering the demand node  $i$ . Additionally, we assessed the type of cooperation involved (intra-level, inter-level) by investigating the specific values of  $\Phi_i^k$  in relation to the threshold  $T$ , as reported in Table 4.2. For instance, a demand node  $i$  is covered by intra-lower level cooperation if  $\phi_i^{max*}$  and  $\Phi_i^{U*}$  are below the threshold  $T$ , which is, instead, exceeded by  $\Phi_i^{L*}$ . Thus, the installation of lower-level facilities and their resulting cooperation is responsible for the coverage of  $i$ . Instead, if  $\Phi_i^{U*}$  and  $\Phi_i^{L*}$  are less than  $T$  while  $\Phi_i^*$  exceeds  $T$ , the demand node  $i$  is covered by the cooperation among all types of facilities (inter-level cooperation).

Finally, based on the above node classification, the demand is aggregated to compute the percentage of covered demand by each mechanism, as shown in Table 4.2. Such percentages represent their contribution to the coverage of the demand.

## 4.5 Test instances and experimental design

In order to prove the capability of the model to provide meaningful solutions, the MINLP and MILP formulations of the HNCL model presented have been tested both on random and real instances.

**Table 4.3**  
Setting of the 5 randomly generated instances.

Random instances	
Set cardinalities	$ I  = 100$ $ J^U  =  J^L  = 50$
Fixed parameters	$d_{ij}$ is the Euclidean distance between $i \in I$ and $j \in J^U \cup J^L$ $R = 1,000$ $\phi_{ij}^U, \phi_{ij}^L$ given by (4.49) with $\alpha_1 = 1/6, \alpha_2 = 1/2$ $T = 0.7$ $w_i \sim U(10, 500)$ $c_j^U \sim U(800, 1000), c_j^L \sim U(8, 10)$ $B^U = \min \left\{ \sum_{j \in J^U} c_j^U x_j^U : \sum_{j \in N_i^U} x_j^U \geq 1, i \in I, x_j^U \in \{0, 1\}, j \in J^U \right\}$

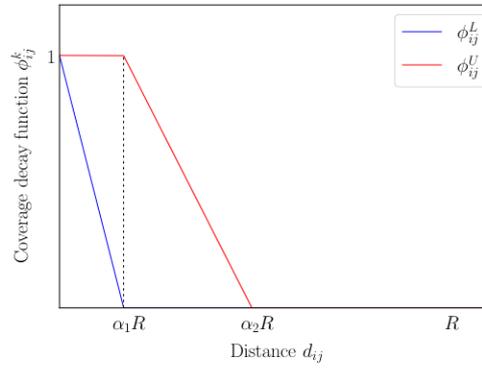
The setting of the random instances are listed in Table 4.3. Specifically, for each instance, we randomly generated 100 demand nodes and 50 candidate location sites, assuming  $J^U = J^L$ , from a uniform distribution on a two-dimensional  $1.000 \times 1.000$  square. For each pair of demand node and candidate location site, we computed  $d_{ij}$  using the Euclidean distance metric. The covering radius  $R$  representing the (large) distance within which each demand node must have access to exclusive services provided by upper-level facilities is fixed at 1.000. In contrast, the coverage levels  $\phi_{ij}^U$  and  $\phi_{ij}^L$ ,

related to shared services provided by facilities of the upper and lower levels, are modelled into a cooperative perspective to guarantee higher proximity to the demand.

According to the different natures of the considered facilities, we defined two distinct coverage decay functions, each associated with a hierarchical level, expressed by (4.49) and depicted in Fig. 4.4. Specifically, we defined a *trapezoidal-shaped function* and a *linear function* for the upper and lower facilities, respectively. We assume upper-level facilities to have a stronger coverage capability on an equal distance compared to lower-level ones. The parameters characterising both functions are defined as proportional (of a value between 0 and 1) to the covering radius  $R$ . The threshold of coverage is set equal to 0.7.

$$\phi_{ij}^U = \begin{cases} 1 & d_{ij} \leq \alpha_1 R \\ \frac{\alpha_2 R - d_{ij}}{(\alpha_2 - \alpha_1)R} & \alpha_1 R < d_{ij} \leq \alpha_2 R \\ 0 & d_{ij} > \alpha_2 R \end{cases} \quad 0 < \alpha_1 < \alpha_2 < 1 \quad (4.49)$$

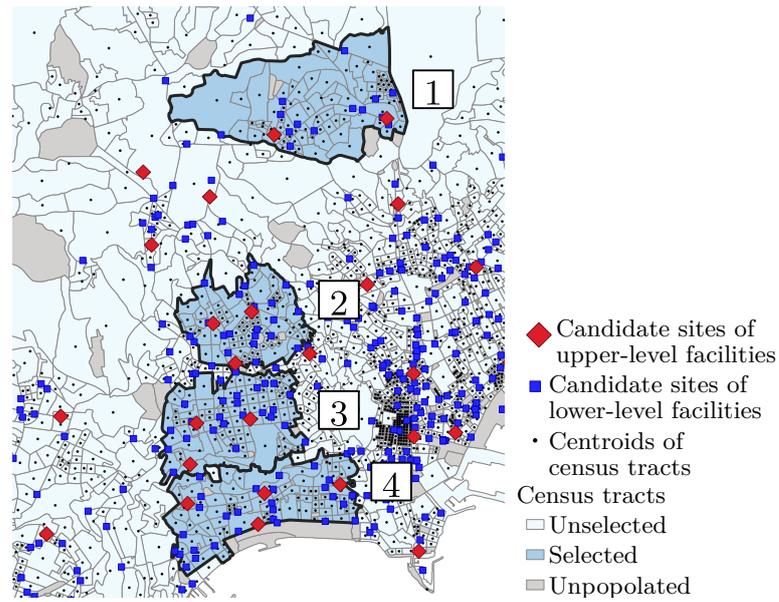
$$\phi_{ij}^L = \max \left\{ 0; \frac{\alpha_1 R - d_{ij}}{\alpha_1 R} \right\}$$



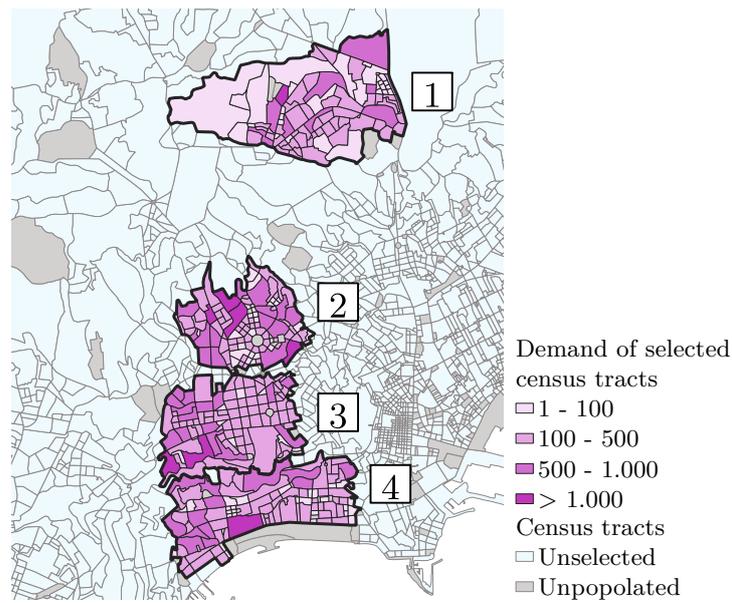
**Fig. 4.4.** Coverage decay functions  $\phi_{ij}^U, \phi_{ij}^L$  where  $0 < \alpha_1 < \alpha_2 < 1$ .

We have drawn the weight of each demand node  $w_i$  and the costs for locating facilities of the upper and lower levels,  $c_j^U$  and  $c_j^L$  from uniform distributions, whose parameters are reported in Table 4.3. Finally, the budget  $B^U$  has been set by solving a set covering problem aiming at locating upper-level facilities within  $R$  from each demand node at the minimum cost.

Furthermore, we applied the HNCL model to the real-world case study presented in



(a) Location and demand space



(b) Distribution of the demand

Real Instance	1	2	3	4
Set cardinality	$ I  = 86$	$ I  = 102$	$ I  = 97$	$ I  = 99$
	$ J^U  = 2$	$ J^U  = 3$	$ J^U  = 3$	$ J^U  = 4$
	$ J^L  = 13$	$ J^L  = 21$	$ J^L  = 23$	$ J^L  = 27$

Fig. 4.5. Representation and setting of the real instances.

**Section 3.4.** Specifically, we considered a banking institution outsourcing basic banking services by integrating the network of internal branches (upper-level facilities  $J^U$ ) with existing external facilities (lower-level facilities  $J^L$ ). We defined four real instances taken from four neighbourhoods in a city, as shown in Fig. 4.5 that reports the cardinality of the sets of demand nodes and candidate sites. We discretised for each instance the demand space into nodes corresponding to the centroids of the populated census tracts of the neighbourhood (set of demand nodes  $I$ ). The location and the demand nodes are depicted in Fig. 4.5a. The set of upper-level facilities  $J^U$  (red diamonds) consists of existing branches; the lower-level facilities representing set  $J^L$  are chosen within the network of retail shops (blue squares). The weight of each demand node  $w_i$  represents the resident population of the census tract (Fig. 4.5b). The remaining parameters are set up as for the random instances.

For each real and random instance, we varied the intra-level correlation factors assuming them identical,  $\theta^U = \theta^L$ , and the inter-level correlation factor  $\theta$  in order to compare solutions obtained with different degrees of cooperation allowed to facilities at the same hierarchical level (by the value of  $\theta^U, \theta^L$ ) and at different hierarchical levels (by the value of  $\theta$ ). In particular, we tested the correlation factors at their extreme values, i.e. 0 when maximum cooperation is allowed and 1 when no cooperation is allowed, and at intermediate values 0.3 and 0.7.

Furthermore, we are interested in investigating how lower-level facilities support the upper-level ones and how they interact in covering the demand for shared services. For this reason, we tested the HNCL on the random and real instance by varying the budget for locating lower-level facilities  $B^L$ , starting from 0 with step 10 until obtaining a value of objective function  $z^* = \bar{z}$ . Such  $\bar{z}$  represents the maximum achievable covered demand with a given combination of  $\theta^U, \theta^L, \theta$ , and it has been obtained as the value of the objective function of a relaxed HNCL where  $B^L$  is not assigned and a lower-level facility is located in every candidate site.

We conducted a total of 2.509 experiments, of which 1.713 for random instances and 796 for real instances.

## 4.6 Results

In this section, the results obtained by solving the HNCL for the test instances are presented and analysed. We first compare computational results obtained by solving the MINLP and the MILP formulations in In Section 4.6.2, we present some illustrative solutions showing the contribution of cooperation in covering the demand. Finally, in Section 4.6.3 the obtained results for random instances are summarized and discussed.

### 4.6.1 Computational experiments

In order to evaluate the proposed linearization approach, we compare the computational results obtained by using both the MINLP and MILP formulations on all the random and real instances.

All experiments were conducted on a PC with an Intel Core i7 processor at 2.30 GHz with 16 GB RAM. The MILP model was solved using CPLEX 20.1 within the AIMMS modelling environment, whereas the original non-linear model (MINLP) was solved using the Knitro Solver within AMPL. A time limit of 10 minutes (600 seconds) was set for each solver.

Table 4.4 shows the computational results in terms of the minimum, the maximum and the average value of CPU time (in seconds) and optimality gap (%) for each formulation, also distinguishing for each set of instances (real and random).

**Table 4.4**  
Computational results of MILP and MINLP formulations.

		CPU time (s)			Optimality gap (%)		
		Min	Max	Average	Min	Max	Average
MINLP	All Instances	0.02	600.00*	251.90	0.00%	53.70%	12.04%
	Real Instances	0.02	300.00	27.79	0.00%	20.00%	1.02%
	Random Instances	14.64	600.00*	400.48	0.00%	53.70%	19.35%
MILP	All Instances	0.11	455.92	23.27	0.00%	0.00%	0.00%
	Real Instances	0.11	5.64	1.52	0.00%	0.00%	0.00%
	Random Instances	0.12	455.92	37.68	0.00%	0.00%	0.00%

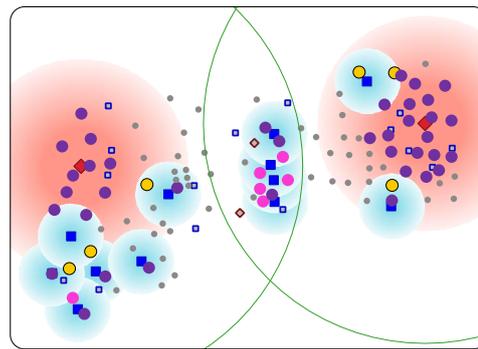
\*reached the time limit

Clearly, the linearized model produces much better solutions both in terms of quality and computation time. In the real instances, the non-linear model required, on average, nearly 28 seconds of CPU time, whereas the linear model took only around 1.5 seconds. With respect to solution quality, CPLEX was able to solve all real instances optimally, whereas Knitro returned optimality gaps up to 20%. The differences in the random instances are even more extreme. While CPLEX took an average of 37.68 seconds and managed to solve all instances to optimality, Knitro required an average of 400 seconds per instance and gave optimality gaps as large as 53.70% as it exceeded the maximum time limit (600 seconds) on several occasions.

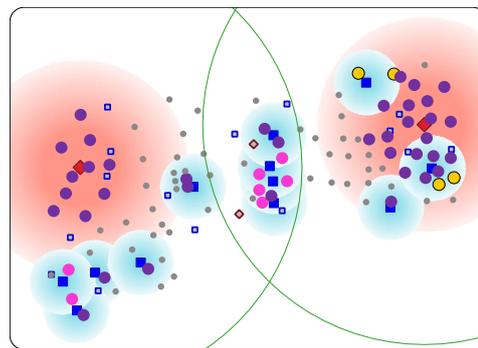
### 4.6.2 Illustrative solutions: measuring the cooperation

In this section, illustrative solutions are presented, showing the contribution of each coverage mechanism according to the classification provided in Section 4.4.

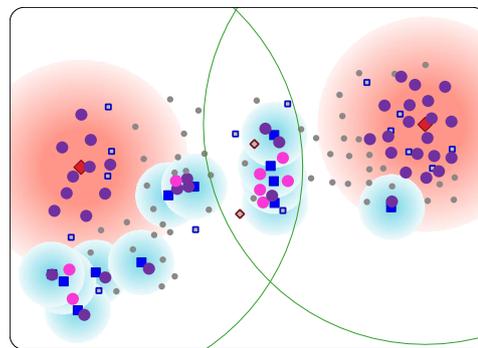
Fig. 4.6 depicts the maps of three illustrative solutions obtained for Real Instance 4 by varying the inter-level correlation factor  $\theta \in \{0, 0.7, 1\}$  with fixed  $B^L = 110$  and



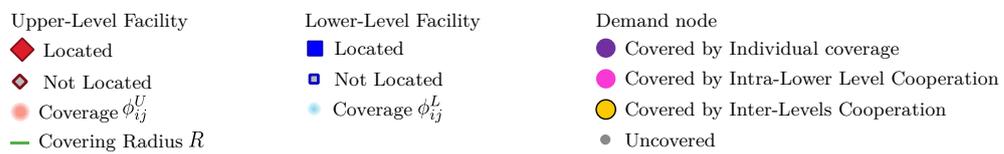
(a)  $\theta = 0$ ,  
Covered demand 50.26%



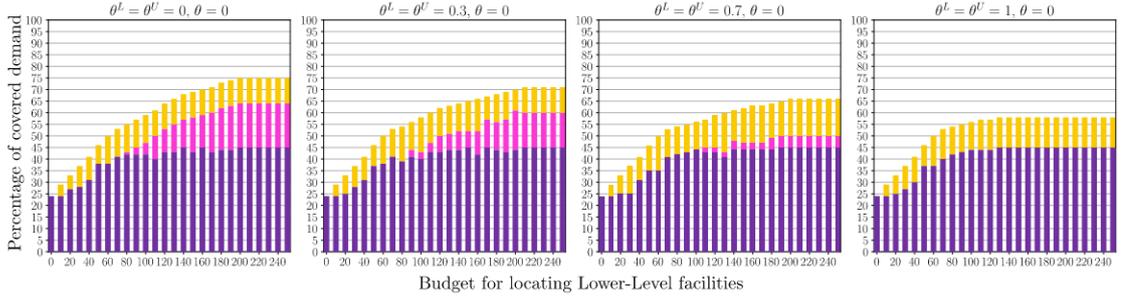
(b)  $\theta = 0.7$ ,  
Covered demand 45.62%



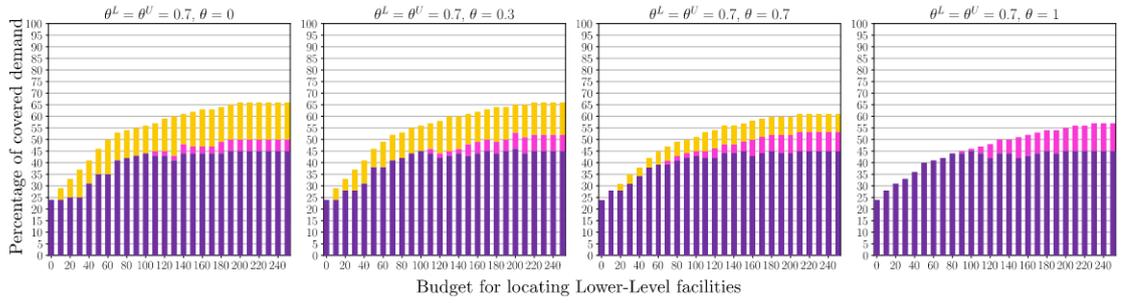
(c)  $\theta = 1$ ,  
Covered demand 45.19%



**Fig. 4.6.** Illustrative solutions for Real Instance 4 by varying  $\theta$  with  $B^L = 110, \theta^U = \theta^L = 0$ .



$\bar{z}^{\%} = \bar{z} / \sum_{i \in I} w_i (\%)$	75%	71%	66%	58%
$B^L(\bar{z})$	200	210	200	130

(a) Fixed  $\theta = 0$ , varied  $\theta^U = \theta^L$ 

$\bar{z}^{\%} = \bar{z} / \sum_{i \in I} w_i (\%)$	66%	66%	61%	57%
$B^L(\bar{z})$	200	220	210	230

(b) Fixed  $\theta^L = \theta^U = 0.7$ , varied  $\theta$ 

■ Individual coverage    ■ Intra-Lower Level cooperation    ■ Inter-Level cooperation

Fig. 4.7. Random instance 3: Contribution of coverages by varying  $B^L, \theta^L = \theta^U, \theta$ 

$\theta^U = \theta^L = 0$  (maximum intra-level cooperation). The demand nodes are coloured according to the mechanism responsible to their coverage.

Firstly, it can be noticed that by fixing the budget for locating upper-level facilities  $B^U$  in a set covering fashion, the individual coverages  $\phi_{i_j}^U$  provided by the 2 located upper-level facilities do not overlap; thus, intra-upper level cooperation does not take place and  $\theta^U$  does not affect the solutions.

Secondly, the effect produced by increasing  $\theta$ , i.e., restricting inter-level cooperation, may be appreciated by comparing maps in Figs. 4.6a, 4.6b, and 4.6c. Demand nodes covered by inter-level cooperation (yellow dots) progressively decrease, from 6 to 4 to 0, as well as the overall covered demand, from 50.26% to 45.62% to 45.19%.

In Fig. 4.7, we compare the percentage of covered demand by varying the correlation factors  $\theta^U = \theta^L, \theta$  and the budget for locating lower-level facilities  $B^L$ . The percentage

of covered demand is split according to our coverage classification scheme: individual coverage is represented in purple, intra-lower level in pink, and inter-level in yellow. As in the solutions shown in Fig. 4.6, the cooperation among upper-level facilities does not occur, and the value of  $\theta^U$  does not affect the solution. For each setting of  $\theta^U = \theta^L$  and  $\theta$ , we report the upper bound of the percentage of covered demand  $\bar{z}^{\%} = \bar{z} / \sum_{i \in I} w_i(\%)$ , and the minimum budget for lower-level facilities to be made available to achieve that percentage,  $B^L(\bar{z})$ . All solutions corresponding to larger budgets than  $B^L(\bar{z})$  are identical to the solution obtained with  $B^L = B^L(\bar{z})$ .

By analysing the graphs in Fig. 4.7, the percentage of covered demand progressively increases by making available a larger budget  $B^L$  and thus locating additional lower-level facilities until reaching  $B^L(\bar{z})$ . However, different coverage mechanisms of various extents are involved. In particular, in Fig. 4.7a, by fixing  $\theta$  and increasing  $\theta^L$ , thus limiting intra-lower level cooperation, the pink stripe (intra-lower level cooperation) tends to thin until it disappears for  $\theta^U = \theta^L = 1$ . Conversely, more restricted cooperation among facilities at different hierarchical levels (higher  $\theta$ ) means a decreasing fraction of covered demand by such a type of cooperation, depicted by the yellow stripe in the graphs in Fig. 4.7b. In both cases, a restriction in the cooperation degree, by increasing  $\theta^L$  or  $\theta$ , results in an overall covered demand for a fixed budget  $B^L$ , and upper bound  $\bar{z}^{\%}$  in the covered demand that are increasingly lower. It is worth noticing in Fig. 4.7b (fixed  $\theta^L = 0.7$ ) that the upper bound of covered demand is invariant for an increased  $\theta$  from 0 to 0.3. However, such coverage can be reached only through an increment of budget  $B^L$  equal to 20. In conclusion, Fig. 4.7 highlights how cooperation has a significant impact in covering the demand and that the correlation factors can be leveraged to rule the cooperation degree.

### 4.6.3 Summary results

In this section, summary results obtained for random instances are reported in Table 4.5 and discussed, referring to some specific representative combinations of correlation factors  $\theta^L, \theta$ . Since intra-upper level cooperation is not established,  $\theta^U$  is irrelevant, and thus they are not reported. For each combination, we selected three representative budgets, i.e. small ( $B^L = 50$ ), medium ( $B^L = 100$ ) and large ( $B^L = 300$ ). In correspondence with each budget, the minimum and maximum percentage of covered demand among random instances are indicated as well as the average among all random instances. Then, according to what is illustrated in Section 4.4, the percentage of covered demand is split into the contributions given by the coverage mechanisms, individual or cooperative. The latter is further subdivided according to the cooperation types.

**Table 4.5**  
Summary results for random instances.

$\theta^L = \theta = 1$	Min			Max			Average		
	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$
<b>Covered demand%</b>	32%	40%	45%	41%	50%	55%	37%	46%	50%
<b>by Individual coverage%</b>	32%	40%	45%	41%	50%	55%	37%	46%	50%
<b>by Cooperation%</b>	0%	0%	0%	0%	0%	0%	0%	0%	0%
Intra-Lower Level	0%	0%	0%	0%	0%	0%	0%	0%	0%
Inter-Level	0%	0%	0%	0%	0%	0%	0%	0%	0%
$\theta^L = \theta = 0$	Min			Max			Average		
	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$
<b>Covered demand%</b>	40%	55%	69%	53%	67%	87%	46%	60%	77%
<b>by Individual coverage%</b>	32%	44%	48%	40%	44%	49%	34%	41%	45%
<b>by Cooperation%</b>	8%	11%	21%	13%	23%	38%	12%	19%	32%
Intra-Lower Level	0%	4%	13%	0%	7%	17%	1%	6%	17%
Inter-Level	8%	7%	8%	13%	16%	21%	11%	13%	15%
$\theta^L = 1, \theta = 0$	Min			Max			Average		
	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$
<b>Covered demand%</b>	40%	52%	56%	52%	64%	71%	45%	57%	62%
<b>by Individual coverage%</b>	32%	40%	44%	40%	46%	52%	35%	43%	47%
<b>by Cooperation%</b>	8%	12%	12%	12%	18%	19%	10%	14%	14%
Intra-Lower Level	0%	0%	0%	0%	0%	0%	0%	0%	0%
Inter-Level	8%	12%	12%	12%	18%	19%	10%	14%	14%
$\theta^L = \theta = 0.7$	Min			Max			Average		
	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$
<b>Covered demand%</b>	36%	46%	58%	45%	57%	65%	40%	52%	62%
<b>by Individual coverage%</b>	30%	38%	49%	41%	49%	54%	36%	44%	48%
<b>by Cooperation%</b>	6%	8%	9%	4%	8%	11%	3%	8%	14%
Intra-Lower Level	2%	3%	4%	0%	1%	4%	1%	2%	7%
Inter-Level	4%	5%	5%	4%	7%	7%	2%	6%	7%
$\theta^L = 0.7, \theta = 0.3$	Min			Max			Average		
	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$
<b>Covered demand%</b>	38%	52%	61%	49%	61%	74%	43%	55%	66%
<b>by Individual coverage%</b>	33%	44%	48%	39%	48%	53%	35%	43%	47%
<b>by Cooperation%</b>	5%	8%	13%	10%	13%	21%	7%	12%	19%
Intra-Lower Level	0%	2%	0%	0%	0%	3%	0%	1%	6%
Inter-Level	5%	6%	13%	10%	13%	18%	7%	11%	13%
$\theta^L = 0.3, \theta = 0.7$	Min			Max			Average		
	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$	$B^L=50$	$B^L=100$	$B^L=300$
<b>Covered demand%</b>	37%	50%	61%	46%	61%	80%	41%	53%	70%
<b>by Individual coverage%</b>	37%	41%	46%	38%	46%	53%	35%	41%	47%
<b>by Cooperation%</b>	0%	9%	15%	8%	15%	27%	6%	12%	23%
Intra-Lower Level	0%	4%	10%	3%	10%	20%	2%	7%	17%
Inter-Level	0%	5%	5%	5%	5%	7%	3%	5%	5%

If  $\theta^L = \theta = 1$ , no cooperation is allowed. The HNCL turns into a non-cooperative model, which nevertheless includes the decay in the coverage level provided by different hierarchical facilities and a minimum threshold for coverage. In this case, each demand node perceives only the maximum coverage by a single facility (individual coverage), regardless of its hierarchical level. If the budget  $B^L = 300$ , the covered demand is on average equal to 50% with an increase of 13% with respect to the case of  $B^L = 50$ . This slight improvement shows that lower-level facilities may contribute to a small extent when they cannot cooperate, given their individual coverage dissipates rapidly.

On the contrary, when the maximum intra and inter-level cooperations are allowed ( $\theta^L = \theta = 0$ ), the coverage achieves the maximum value. For example, the average covered demand for  $B^L = 300$  reaches 77%, +27% compared to the previous case, given the substantial contribution given by intra-lower level (17%) and inter-level (15%) cooperations that partially replaces the individual coverage, which goes from 50% to 45%.

The analysed settings represent the extreme coverage cases from the minimum ( $\theta^L = \theta = 1$ ) to the maximum achievable covered demand ( $\theta^L = \theta = 0$ ), and any other solution obtained with any combination of  $\theta^L$  and  $\theta$ , presents a percentage of covered demand within the range thus defined. For instance, setting  $\theta^L = \theta = 0.7$  and  $B^L = 100$ , on average, the demand covered by cooperation is 8%, of which 2% is intra-lower level and 6% is inter-level, compared to 19% if  $\theta^L = \theta = 0$ . The overall covered demand equals 52%, between 46% (if  $\theta^L = \theta = 1$ ) and 60% (if  $\theta^L = \theta = 0$ ).

A case of interest is the combination  $\theta^L = 1$  and  $\theta = 0$ . In this case, each demand node perceives the maximal coverage provided by a single facility at the lower level ( $\theta^L = 1$  and 0% of covered demand by intra-lower level cooperation) and a single facility at the upper level (intra-upper level cooperation is not established). These two coverages are combined to define the overall coverage with the maximum freedom of cooperation ( $\theta = 0$ ). The inter-level cooperation gives discrete support to the overall demand coverage, i.e., on average, 10% with a small budget  $B^L$  and 14% with a medium or large budget  $B^L$ ; individual coverage, in comparison, covers 35%, 43%, and 47% of demand, respectively.

Comparing the contributions of the cooperation mechanisms, if  $\theta^L \geq \theta$ , the inter-level cooperation generally covers a more extensive amount of demand than the cooperation among lower-level facilities. See, for instance, the results obtained by choosing  $\theta^L = 0.7$  and  $\theta = 0.3$ . Considering the instance showing the maximum covered demand, the only contributing cooperation type with a small or medium budget ( $B^L = 50$  and  $B^L = 100$ ) is the inter-level one, with 10% and 13% of covered demand, respectively; intra-lower level cooperation does not occur. Even considering a large budget ( $B^L = 300$ ), the inter-

level cooperation reaches 18% of covered demand, whilst the intra-lower level cooperation contributes with a 3%.

Conversely, if  $\theta^L < \theta$ , the coverage provided by intra-level cooperation is typically more significant, as shown by results with  $\theta^L = 0.3$  and  $\theta = 0.7$ . In this setting with a large budget, regarding the solution presenting the maximum covered demand, the intra-level cooperation provides 20% of covered demand, opposing the inter-level cooperation that contributes with a 7%.

As expected, the presented results show that solutions are highly sensitive to the correlation factors ruling the degree of cooperation and how, by establishing additional lower-level facilities, a greater extent of demand can be covered thanks to intra and inter-level cooperations.

## 4.7 Managerial implications

The formulated model can be effectively used to represent many real-world decision-making problems concerning the design of service networks.

Indeed, there are several application contexts where users' accessibility to a given service does not depend solely on the distance from their closest facility but on the number, density and characteristics of all the facilities located in their neighbourhood.

From a methodological point of view, this is testified by well-established approaches in the spatial accessibility-related literature, considering the simultaneous presence of facilities within a given distance from users. It is the case of the Floating Catchment Area (FCA) or the gravity-based methods (Luo and Wang, 2003). Also, in the location literature, many problems consider sophisticated spatial interaction models to determine the demand allocation or the attraction degree of located facilities.

From a practical perspective, many examples of interests involving cooperative and hierarchical systems may be provided.

New trends in location-based actions in the banking sector to reorganise the branch networks, investigated in Chapter 3, have been the starting point of the problem analysed in this chapter. In this system, external facilities may cooperate to provide coverage to users among them and also with the internal branches themselves.

Another example may be found in the logistics sector. Optimizing last-mile delivery is gaining momentum after the e-commerce boom, and new delivery solutions have been explored to meet this growing demand. In this context, the "Click & Collect" is a relatively new method of selling products according to which customers place their order online and then collect it at a specific location, such as parcel lockers, manned collection points, or shops. It allows consolidating deliveries (for providers) and saving fees on

shipping or delivery (for customers). The Click & Collect network must be expanded to encourage customers to opt for self-collection instead of home delivery (Mangiaracina et al., 2019). For this purpose, a cooperative paradigm may effectively adapt. Indeed, the customers' acceptance of self-collection depends on the set of attractive options they may patronize within a limited distance. Such options are heterogeneous and can be organized according to hierarchical networks. For example, shops and collection points can be arranged in a nested hierarchy as both can be used to collect parcels, but shops also provide additional services.

Lastly, in the healthcare sector, users interact with various types of facilities to receive services and health facilities are often arranged in nested hierarchical structures (Mitropoulos et al., 2006). For example, diagnostic centres, primary-care centres, and hospitals that offer progressively complex services. All these facilities cooperate in a hierarchical fashion to ensure that their patients have adequate and fair availability and accessibility to the healthcare services they commonly provide.

In the above-described settings, the HCNL may be suitable for effectively designing nested hierarchical facilities networks, where cooperation at each level and between levels can be exploited as a mechanism to cover the demand better.

In the design of service networks, the coverage level provided by a single facility to a given demand node represents its ability to attract that node and depends on its characteristics and the distance from the demand (coverage decay function). The way users simultaneously perceive the coverage levels from all facilities in terms of overall attraction depends on the type of services under consideration and the users' preferences. Despite more options within the covering radius, there are cases where users perceive only that facility with the strongest attraction (individual coverage) and others where they start experiencing the effects of other facilities in their neighbourhood. This aspect is modelled through the aggregation function. The choice of adopting the joint coverage function allows us to represent a wide spectrum of situations characterized by different degrees of cooperation. Additionally, by introducing the concept of inter-level cooperation, we assume a distinct function to aggregate coverages from different levels of facilities. This is significant as it enables us to represent situations where users perceived coverages differently from the two levels.

As demonstrated by the results, the scenarios produced by the model are significantly affected by the calibration of parameters ruling the aggregations (i.e.,  $\theta^U$ ,  $\theta^L$  and  $\theta$ ). Although tuning these parameters in real-world applications can be challenging, as they represent how users perceive the simultaneous presence of multiple facilities, it could be very insightful for the decision-maker to analyse the scenarios generated by the model

by varying them. By doing so, indeed, the decision-maker may assess the robustness of the obtained solutions in terms of the network configuration. High robustness levels can increase the decision-maker's confidence in activating specific locations in contrast to solutions highly sensitive to the parameters mentioned. Moreover, the decision-maker may appreciate the value of cooperation and how this contribution is dependent on the hypothesis made on the aggregation mechanisms.

Finally, the model may help decision-makers to allocate the available budget optimally. Generally, the higher-level facilities have a stronger attraction but a higher activation cost, while lower-level facilities have lower costs and less attractiveness. Hence, the model helps design the network to find a trade-off between location costs and user coverage. By varying the budget for activating lower-level facilities, decision-makers can assess the marginal benefit in terms of additional covered demand achievable with an extra investment. Once the marginal benefit becomes low, the decision-maker may decide that it is not convenient to expand the lower-level facility network further.

## 4.8 Conclusions

In this chapter, we presented a novel cooperative covering model, namely HNCL, aiming at maximizing the covered demand by locating facilities arranged in a two-level nested hierarchy. Facilities provide a decaying coverage to demand nodes according to defined functions. First, the two cooperations among facilities at the same hierarchical level are treated separately and modelled as joint coverage functions. Then, a further joint coverage function is introduced to aggregate the coverage provided by all facilities according to their hierarchical level. The three introduced joint coverage functions present three key parameters –  $\theta^U, \theta^L, \theta$  – regulating the cooperation degree among facilities at the same and different levels. A given threshold defines the minimum coverage level that must be provided to a demand node to be covered. Two distinct available budgets are considered to locate facilities of each level.

A first non-linear HNCL formulation has been presented. Then, we applied a linearizing approach to the HNCL developed by extending the linearization by [Karatas \(2017\)](#).

Both non-linear and linear formulations have been tested on instances randomly generated as well as extracted from real-world networks of a retail company. Several results have been obtained by varying the budget for locating facilities of the lower level and the parameters  $\theta^U, \theta^L, \theta$ . We first compared the two formulations from a computational point of view. The results have shown that our linearizing approach outperformed the non-linear formulation both in computing times and quality of solutions. Moreover, the

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results demonstrate the capability of the model to provide meaningful solutions and how decision-makers may leverage, by implementing the HNCL, the key parameters  $\theta^U, \theta^L, \theta$  in order to regulate the cooperation degree depending on the specific real scenario at hand, where some sort of dependency among the potential facilities of their network may occur.

# General conclusions

The delivery of services by modern businesses and organizations to final consumers has become much more dynamic to cater to the customers' rapidly changing needs due to digital transformation. We focused, in particular, on the retail sector. The trend to customize services, either individually or collectively, implies that firms need to serve their customers in an adaptable, flexible and, at the same time, cost-efficient manner. In order to adapt their business model to the evolving competitive landscape, retail businesses are integrating their existing physical channels with digital ones to deliver their services to the final consumers.

Moreover, the networks of physical retail facilities should be reorganised to be still effective. Therefore, firms develop their network of facilities by establishing facilities that offer multiple and different services according to customers' needs.

In this emerging context, the present thesis provided new tailored mathematical programming models for the spatial reorganization of retail services, implementing (some) location-based strategies (e.g., closing, specialization or outsourcing of services) able to provide decision support and valuable insights for decision-makers depending on the reference context and the specific services.

To this end, we firstly provided in [Chapter 1](#) a framework of the digital transformation phenomenon and its impact on the changing business models to deliver retail services. Then, the location analysis literature, the field we refer to model our problems, has been presented in [Chapter 2](#). Its main classes of problems was described, focusing in particular on covering, cooperative and hierarchical location problems. Two new models have been introduced and analysed in [Chapters 3](#) and [4](#).

Various contributions have been highlighted based on the extant literature analysis. In [Chapter 3](#), we presented the *branch network restructuring problem*, whose main

contribution is modelling reorganization strategies in the banking sector not previously considered in the literature. The addressed problem is more complex due to the multi-faceted and interrelated characteristics of banking services. In particular, the model aims at covering all the demand while: (i) shrinking the existing network of bank branches to cut costs; (ii) diversifying remaining branches in terms of provided services to satisfy different customers' segments; (iii) outsourcing basic services to staffed external facilities to preserve the proximity to customers reluctant to adopt digital channels and guarantee high accessibility to such services. Four types of facilities have been considered arranged in a nested hierarchy. The model has been tested on a real-world case study of an Italian banking institution operating in a city. Several restructuring scenarios have been produced by varying key parameters of the model, showing its capability to offer meaningful managerial insights. Specifically, it has shown that banks interested in restructuring their networks have to find a trade-off between consolidating the internal network to reduce costs and internalising banking services to guarantee proximity to clients. The number of more complex and costly branches should remain high if the bank intends to preserve customer loyalty by internally handling clients. On the other hand, if the interest is to push toward a more significant shrinking of the branch network, the bank has to outsource a higher percentage of basic services.

In [Chapter 4](#), we extended the above model by allowing cooperation among facilities to cover the demand and introduced the *hierarchical nested cooperative location* model. We purposely removed the focus on the banking sector and opened up the model to be applied to various contexts. In this setting, the coverage provided by a facility has been modelled as a decreasing function over the distance, namely the coverage decay function, and interpreted as a probability of coverage. The proposed (non-linear) model extended the available literature on the class of cooperative location problems by considering nested hierarchical facilities in which cooperation is not only among facilities of the same level but also at different levels. The inclusion of this last type of cooperation increased the non-linearity in the model. Then, we developed an original approach to obtain an equivalent linear formulation of the problem. A further innovative contribution is given by the framework provided to evaluate the actual value of cooperation in covering the demand. The proposed model has been tested on random instances as well as (small) real instances extracted by the branch network case study in [Chapter 3](#). The results have shown how decision-makers may leverage identified key parameters of the HNCL to regulate the cooperation degree depending on the specific real scenario at hand, where some sort of dependency among the potential facilities of their network may occur.

The present thesis opens to different research directions.

As regards the branch network restructuring problem presented in [Chapter 3](#), an interesting perspective may involve a multi-period version of the problem to allow the bank to gradually consolidate the internal network while outsourcing services according to the proximity constraints that can progressively be relaxed according to users' digitalization.

Although the model has been developed for the banking sector, the proposed approach can be generalised and adapted to other business sectors. The strategies WE considered may be applied in contexts where service networks need to be restructured due to digitalization to match the conflicting goals of cost-efficiency and satisfying the changing demand (e.g., courier services, retail shops, healthcare). Therefore, the managerial implications could be more significant.

Finally, as regards the case study under investigation, more accurate results could be obtained by considering real facilities' costs. Therefore, due to the unavailability of data, a project should be seeded to evaluate such costs (e.g., fixed costs, employees, and the type of agreement between the bank and external facilities).

In line with this last point, the hierarchical nested cooperative location problem, presented in [Chapter 4](#), may be applied to larger real instances (e.g., the full-size branch network) and several contexts. Depending on the specific real context at hand, the decay coverage functions should be carefully modelled to be able to shape the customers' perceptions concerning the coverage of services based on real customer data obtained, for instance, employing surveys.

However, the problem may be challenging from a computational point of view as the dimension of the instance increases. Therefore, tailored solution approaches (exact or heuristics) could be developed. For instance, given the structure of the problem, an exact solution approach could be applied based on the Benders' decomposition and Combinatorial Benders' cuts (e.g., [Cordeau et al., 2019](#)).

Finally, the hierarchical nested cooperative location problem may be easily extended by formulating a gradual version of the model involving the possibility of partial coverage of the demand.

# Appendix A

## Exploding $\Phi_i^U \Phi_i^L$

In order to handle the prod-part  $\Phi_i^U \Phi_i^L$  included in the inter-level aggregate coverage  $\Phi_i$ , let  $\Phi_i^U$  and  $\Phi_i^L$  be defined as in (4.1):

$$\Phi_i^U \Phi_i^L = \left[ \theta^U \max_{j \in J^U} \{ \phi_{ij}^U x_j^U \} + (1 - \theta^U) (1 - Q_i^U) \right] \left[ \theta^L \max_{j \in J^L} \{ \phi_{ij}^L x_j^L \} + (1 - \theta^L) (1 - Q_i^L) \right] \quad (\text{A.1})$$

Let us recall the auxiliary decision variable  $z_{ij}^k$  that is equal 1 when the located facility  $j$  of level  $k$  provides the maximum coverage to  $i$  among all the located facilities of level  $k$  and 0 otherwise, as defined in Section 4.3.3. Therefore,  $\max_{j \in J^k} \{ \phi_{ij}^k x_j^k \}$  is equivalent to  $\sum_{j \in J^k} \phi_{ij}^k z_{ij}^k$  and can be replaced by the latter in (A.1).

$$\begin{aligned} \Phi_i^U \Phi_i^L &= \left[ \theta^U \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + (1 - \theta^U) (1 - Q_i^U) \right] \left[ \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + (1 - \theta^L) (1 - Q_i^L) \right] = \\ &= \left[ \theta^U \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + (1 - \theta^U) - (1 - \theta^U) Q_i^U \right] \left[ \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + (1 - \theta^L) - (1 - \theta^L) Q_i^L \right] \end{aligned} \quad (\text{A.2})$$

The expression (A.2) can be exploded as follows:

$$\begin{aligned} \Phi_i^U \Phi_i^L &= \theta^U \theta^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + \theta^U (1 - \theta^L) \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + \\ &\quad - \theta^U (1 - \theta^L) Q_i^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + (1 - \theta^U) \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + \\ &\quad + (1 - \theta^U) (1 - \theta^L) - (1 - \theta^U) (1 - \theta^L) Q_i^L - (1 - \theta^U) (1 - \theta^L) Q_i^U + \\ &\quad - (1 - \theta^U) \theta^L Q_i^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + (1 - \theta^U) (1 - \theta^L) Q_i^U Q_i^L \end{aligned}$$

By separating and aggregating all the linear contributions, we obtain:

$$\begin{aligned} \Phi_i^U \Phi_i^L &= \theta^U (1 - \theta^L) \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + (1 - \theta^U) \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + \\ &+ (1 - \theta^U) (1 - \theta^L) - (1 - \theta^U) (1 - \theta^L) Q_i^L - (1 - \theta^U) (1 - \theta^L) Q_i^U + \\ &+ \theta^U \theta^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L - \theta^U (1 - \theta^L) Q_i^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + \\ &- (1 - \theta^U) \theta^L Q_i^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + (1 - \theta^U) (1 - \theta^L) Q_i^U Q_i^L \end{aligned}$$

$$\Phi_i^U \Phi_i^L = \theta^U (1 - \theta^L) \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + (1 - \theta^U) \theta^L \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L \quad (\text{A.3})$$

$$+ (1 - \theta^U) (1 - \theta^L) (1 - Q_i^U - Q_i^L) +$$

$$+ \theta^U \theta^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L - \theta^U (1 - \theta^L) Q_i^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U + \quad (\text{A.4})$$

$$- (1 - \theta^U) \theta^L Q_i^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L + (1 - \theta^U) (1 - \theta^L) Q_i^U Q_i^L \quad (\text{A.5})$$

(A.3) includes the linear contributions, which we denote by the label  $\Lambda_i$ , and the non-linear contributions are (A.4) – (A.5). The latter are separately linearized by extending the linearizing process by Karatas (2017), as described in Section 4.3.3. Let  $\Delta_i^n$  denote the  $n$ -th non-linear contribution. Then, (A.3)-(A.5) is given by:

$$\Phi_i^U \Phi_i^L = \Lambda_i - \Delta_i^1 - \Delta_i^2 + \Delta_i^3 + \Delta_i^4$$

$$\text{where } \Delta_i^1 = \theta^U (1 - \theta^L) Q_i^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U,$$

$$\Delta_i^2 = \theta^L (1 - \theta^U) Q_i^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L,$$

$$\Delta_i^3 = (1 - \theta^U) (1 - \theta^L) Q_i^U Q_i^L,$$

$$\Delta_i^4 = \theta^U \theta^L \sum_{j \in J^U} \phi_{ij}^U z_{ij}^U \sum_{j \in J^L} \phi_{ij}^L z_{ij}^L.$$

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