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Risk Management and Supervision for Pension Funds: critical implementation of ALM Models

PhD Dissertation

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Contents

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Preface

Risk Management and Supervision for pension Funds: critical implementation of ALM models was initiated in response to the increasing interest in the development of new approaches in pension fund management and new trends in pension supervision, more oriented toward a full evaluation of the system of risks and the imposition of risk-based solvency constraints, similarly to what was previously experienced in the banking and insurance industry (Basle II and Solvency II projects). The interest with respect to these topics grow out also after the several attempts to stimulate the development of Italian pension market through the introduction of regulatory innovations, in terms of coverage ratio and managed assets. Our market is however still characterized by many delays and structural gaps with respect to the pension fund markets developed in other European countries in terms of management practise, development of risk management architecture and supervision activities. Given these considerations, this dissertation takes an international perspective and focuses on the *best practices* developed in more sophisticated markets at risk management and supervision level.

The thesis is composed by two parts. The first part takes a theoretical perspective, while the second presents empirical asset-and liability management (ALM) applications.

The first chapter has an introductory function and describe the typical decision problem faced by pension funds, which takes into account the dynamics of assets, e liabilities and also the interaction between different policies which the board of pension fund can apply, given the conflicting interests of the different stakeholders involved and the risks faced by the pension funds in its activities. The ALM represents a risk management approach which supports these decisions of the Board and leads to the definition of ALM strategies and policies. The second chapter deals with the recent trend in pension supervision towards risk-based standards. The experience of the four early adopter is presented and the main components of this new regulatory framework are discussed. They sign a significant distance from previous supervisory systems based on investment limits, which have been strongly criticized because they affected the ability of the fund to reach efficient portfolio position. Indeed, in most of these countries the risk-based supervision has represented a quid pro quod for a less strict supervision on investment decision. These new frameworks include also risk-based solvency constraints which must be included in traditional ALM models. The chapter three analysis the most common ALM models in the Netherlands, the most sophisticated pension markets in Europe. The Dutch pension funds adopt a simulation/optimization model based on scenario analysis and evaluate different ALM strategies by means of ALM-scores.

The second part applies the mentioned model together with the supervision solvency standards described in the first part to three different aspects of the pension fund management. The first application deals with a typical risk management decision with reference to the hedging decision of the strategic currency risk by usage of the forward market. The second application presents an optimization model aimed at maximize the conditional indexation of a defined benefit pension fund' benefit payments by introducing alternative assets in the portfolio, under risk-based solvency constraints. Finally the third application aims at the evaluation of the conditional indexation policy as embedded option to assess the potential impact on the market value of the liability.

Prefazione

La tesi di Dottorato "*Risk Management and Supervision for Pension Funds: critical implementation of ALM Models*" si focalizza in primo luogo sull'analisi del processo decisionale che coinvolge in numerosi *stakeholders* presenti all'interno dei fondi pensione in una prospettiva di Asset and Liability Management (ALM) al fine di definire gli obiettivi della loro *strategic asset allocation* (attraverso diverse strategie di ALM) e le diverse policies necessarie al conseguimento di questi obiettivi. Successivamente sono descritte le motivazioni che hanno condotto all'adozione dei nuovi sistemi di vigilanza per i fondi pensione basati sul rischio e i principali obiettivi da esso perseguiti sulla scia di quanto è avvenuto nel settore bancario con Basilea 2 e con Solvency II per le compagnie di assicurazione. In particolare, l'analisi si sofferma sui requisiti e le metodologie di *risk management* adottati dalle autorità di vigilanza che si distanziano considerevolmente dai precedenti modelli basati unicamente su limitazioni agli investimenti. Il terzo capitolo presenta la metodologia basata sull'analisi per scenario sviluppata presso il mercato dei fondi pensione olandesi, che si caratterizza per significatività, dimensioni e sofisticazione della gestione. La metodologia combina i tradizionali modelli di ottimizzazione di portafoglio utilizzando come vincoli gli *standard* imposti dalle nuove norme di vigilanza.

La seconda parte applica l'analisi per scenario al fine di analizzare diversi aspetti della gestione finanziaria dei fondi pensione dalla gestione del rischio di cambio con strumenti derivati in un contesto ALM, alla massimizzazione dell'indicizzazione all'inflazione delle loro passività con strumenti alternativi quali investimenti in *commodities* e *real estate,* e infine, alla valutazione delle politiche di indicizzazione che dipendono dal raggiungimento di specifici livelli del valore delle attività sulle passività e che si configurano come opzioni implicite vendute dal fondo ai propri membri.

PART I

DECISION MAKING PROCESS AND RISK-BASED SUPERVISION

1
Asset and Liability Management for Pension Funds

1.1 ALM decision process

Asset and liability management (ALM) is important when it is essential to deliver the liability cash flows (i.e. pensions payments) with high degree of probability (Fabozzi and Konishi, 1996; Campbell and Viceira 2002; Blake, 2003). For pension funds ALM consists of a risk management approach, which takes into account the assets, the liabilities and also the interaction between different policies which the board of pension fund can apply. ALM decision problem is the risk budget of all stakeholders and all the available policy instruments are taken in to account in order to accomplish adequate pensions at acceptable cost and risk (Boender, Dert, Heemskerr and Hoek 2007). ALM should help the board of a pension fund to find acceptable policies that guarantee, with large probabilities, that the solvency of the fund is sufficient during the planning horizon and, at the same, time, all the promised payments will be made. The solvency is the ability of the pension fund to fulfil all promised payments in the long run. The solvency at a certain time moment is measured as the funding ratio that is the ratio of assets and liabilities.

The ALM decision process consist in the definition of a planning horizon which specifies the total number of years which are considered in the decision making process. The pension funds is typically considered a long-term investor due to the long duration of its liabilities, but actual decision plan usually refers to shorted horizons, also due to the short term constraints imposed by supervisor. Once the planning horizon is defined, it is split in to sub periods of one year. In every year, benefit payments are made, premiums are received and changes in the status of participants are recorded appropriately. At the end of the year, the board also knows the return of the asset portfolio. The value of the asset is determined using market price at that moment and the liabilities as well. Once the value of assets and liabilities is known, the level of the funding ratio is determined. This value is compared to the values of the previous years to evaluate the effects of the actual strategy.

When all last year's information is revealed, the board looks forward to define the expectations with respect to the future and given the financial position of the fund at the end of the year, should make certain decision aiming at a sufficiently high future funding ratio. These decisions consist on policies re-definition or adjustments: for example changes in the composition of the asset portfolio, but under the restriction which may be imposed by the regulator; or changes in the contribution rate, under the consideration that a too volatile contribution rate strongly affect the soundness of the sponsor's business; or decision about the indexation policy, which affect the purchasing power of the payments received by the pensioners. All these policies must be accurately evaluated by the board in term of risk-return consequence. But the most important question is that in this decision process, policy definition and evaluation, the board is influenced by the interests of different parties involved in the decision making process, whose interest are often conflicting.

In terms of problem formulation, the ALM can be analyzed by its main components (see Ziemba and Mulvey 1998):

- *The Stakeholder objectives and constraints.*

The interests of several parties are involved in the decision process of a pension fund. Their interest and the definition of the targets for each of them differ depending on the scheme, Defined Benefit or Defined Contribution, but also on the type of supervisory standards to comply. For instance, in a Defined Contribution scheme, the only stakeholder is an individual whose objective could be a high expected pension at retirement but with a sufficiently small risk of falling below some minimum pension level. But for a Defined Benefit scheme, the number of stakeholder increases because the risk is contractually shared between employee and employers and then, the necessity to define and distribute the risk that each part is willing/obliged to bare emerges. Also the supervisor can be considered a stakeholder and influences more or less severely pension fund decision making process. For instance, in some countries the supervisory action on pension funds is limited to the definition of investment regulation, which essentially requires high level of diversification and reduced level of default risk. More recently however, especially after the several financial crises of the last 10 years which cause the failures of many pension funds, a new risk-based approach has been adopted by few countries like in Netherlands, which is considered to have the most sophisticated pension funds market. This new approach, which implies a new set of "interests" and objectives of the stakeholder "supervisor", will be extensively explained in the Chapter 2. They represent the new constraints to be considered in the definition of ALM models.

-*Policy instruments* are the tools which the decision maker (i.e. the board of trustee for a pension fund) can use to meet the stakeholders objective and constraints as best as possible. For instance, the contribution policy, the indexation policy, the investment policy and so on.

-The third component of a typical ALM problem consists on the identification of the *risk and return factors* that represents one of the key roles for ALM. The consequence of each possible policy with respect to the stakeholder's objective and constraints must be evaluated according to several ALM measures of risk and return. A policy is called more efficient than others if it results in a higher expected return at the same level of risk or, stated otherwise, lower risk at the same level of expected return (as in Markowitz 1968). A policy is called more effective if it makes optimal use of some defined risk budget. Although a policy with risk above this level yields a higher expected return, the associated risk is obviously too much given the risk-appetite of the decision makers. A policy with risk below this level on the other hand produces a lower expected return that is strictly possible given the risk appetite of the decision makers.

The determinants of risk and return are all the factors that can cause a policy to turn out good or bad depending on the future development of such a factor. These factors consist to a large extend to general macroeconomics variables such as interest rate, inflation, asset returns and also demographic trends. Once ALM provides all the relevant information to support decision maker by providing insight in the relevant risk-return relationship, the next step consist on identifying and communication optimal ALM strategies, which yield each risk-provider in the pension deal to a maximal benefit in return.

Next paragraph describes the actors involved in the risk-budgeting of the pension funds, then the instruments and policy under their control and the identification of the main sources of risk rising from them. Then, types of ALM strategies are presented: immunisation, cash flow matching horizon and liability driven investing (LDI). Finally the section concludes presenting the main critics invoked against ALM.

1.2 Stakeholders Objectives and Constraints

As mentioned before, the role of ALM in practise is to sustain the specification of the risk profile of the stakeholders, and the identification of the basic integral pension-contribution and investment policy. A Pension deal consists on the composition of all these different and conflicting interests. The very nature of the pension funds rely on a negotiation between employers and employees for the definition of a pension scheme in which the employees either earn pension rights (as in the Defined Benefit scheme) and/or obtain pension contributions (as in the Defined Contribution scheme). Increasingly, employers and employees agree on a hybrid pension schemes with both DB- and DC- components (see Ambachtsheer and Ezra, 1998).

The agreed pension scheme is carried out in the pension plan under the responsibility of the board of trustee. A board of trustee has to take into account the interest and requirements of many pension stakeholders.

At least five parties are involved in the decision making process or have interests in its results.

First of all, the members of the fund are involved, distinguished in the employees or active participants and beneficiaries. The formers are especially concerned about the level of the contribution rate. In particular older active participants are also interested in the degree of indexation: they would like to be compensating for inflation in all years. Active participants make contribution on regular basis to the fund to build up their pension rights. If the contribution rate increases the active participants have to make a large contribution to the pension fund, which results in a lower disposable income.

A second interested group consist of retired persons and surviving relatives often. For this group especially the indexation policy is important. Of course they would like to have full compensation for increases in prices or wages.

The sponsor of the fund is also involved. Not only does the sponsor pay a part of the regular contributions, but also in case of financial distress the sponsor plays an important role. If the funding ratio drops below a certain threshold, the sponsor of the fund may be contractually forced to restore the funding ratio. On the other hand, in case of financial prosperity, the sponsor may also benefit from contribution holidays. Intuitively, the sponsor wish to profit by low pension contributions to the plan deriving from a more aggressive (risky) investment strategy and then leading to higher expected return on the pension assets. But on the other hand, sponsors need to set constraints on the extent that pension investment risk and other pension risk drivers are allowed to affect the balance sheet of the sponsor and their Profit & Loss account. This limit defined as pension risk should be constrained to responsible value. This is of particularly important if we consider that deficits in pension plan are likely to coincide with also a bad profitability of the sponsor in the case of financial crisis.

Last party is the supervisor of the fund. Pension funds have to justify and report their activities to the supervisor. The role of supervisor differs from country to country. The supervisor has "interests" about the investment policy, because investment directly influences the risk of underfunding affecting the solvency of the pension funds. The supervisory framework specifies the maximum risk that the plan encounters a deficit and it is mainly related to the investment risk which can not directly be transferred to the sponsors and the members. It indirectly imposes the pension fund to make choice as risk-averse investor, especially when it imposes short-term risk-based constraints. This means that even if the board of trustee has the input from the sponsor to increase the pension risk, it has to comply with the solvency regulations. These are the means by which the supervisor preserves its interests:

-Investment regulation

Assets should be invested in a solid way. In addition, for pension funds related to a single company, rules exist with respect to the fraction of the assets which may be invested in their own company. In some case the restriction also relates the use of derivatives. As it will be discussed later, there is a general trend to reduce the restriction on the investment policy especially where a risk-based supervision approach is adopted.

-*Valuation of liabilities*

Liabilities are values using a market to market criterion. The methodology used to discount the cash flows must be validated by the supervisory authority and considers, among others, the use of interest yield curve, swap curves but also AAA-rated bonds.

-Indexing

In most of the pension funds, the indexation is voluntary. However, it is possible that a fund is compelled to index benefit payments, because such provision is part of the pension fund's statutes. The only prescription by the regulators with respect to the indexing of the pension rights usually refers to a commitment of equal treatment: if retired people get compensation, a corresponding compensation has to be given to deferred members.

All parties discussed here can be easily satisfied in case of financial prosperity. Otherwise tensions between these groups are expected when the financial position of the fund is weak or close to insolvency. Pensioners would like to receive index-linked pensions. Sponsors may find difficult to pay extra-contribution. In the attempt to recover the previous financial conditions the fund can only increase the investment risk, if it cannot cut indexation, but in this case supervisor will put even more attention on the activities of the fund.

1.3 Policies and Agreements

The board of pension fund has many instruments to its disposal to manage all these interests and control the funding ratio. The board should take into account the interests of all parties involved in the decision making process, to find the best “policy” mix. Main policies and rules by which the fund can control the funding ratio are:

-Pension policy

The pension policy deals with decisions with respect to the different types of the pensions that the fund includes in the pension internal regulation (Retirement pension, widow’s pension, partner pension, orphan pension, pension in case of disability). Active participants, deferred members and retired persons are interested in the pension policy, because they are the one who will receive money from the pension fund. Because this policy is part of the statute of the fund, the changes or modifications of this policy are very difficult and then considered as given in the ALM models.

-Pension system

The rules with respect to the pension benefits are registered in the pension rules. In these rules the pension system is described. Main pension systems are the final pay system, the average earned salaries system (also defined as defined benefit system) and the defined contribution system. In the first system, every wage increase not only affects the rights which will be building up in the remaining years of service, but also in the previous build up rights. In the second system, every wage increase influences the pension that will be build up in the remaining years of service, while the pension over previous years of service remains unaltered. Finally, in the defined contribution system the employer yearly transfers money (usually a percentage of the pensionable salary) to purchase a part of the employees' pension. The level of the pension depends on the number of years the pension contributions have been paid, the realized return in the years the pension has been build up, and the interest rate at the moment of retirement.

-Indexation policy

When benefit payments are only expressed in nominal terms, and are not corrected for increase in prices and wages, the purchasing power of the retired people is harmed considerably. To prevent this, nominal benefit payments are often increased in line with inflation. This is called indexing benefit payments.

The rule of indexing the benefit payments is defined as the indexation policy and it is also important in valuing the liabilities and the future benefit payments. The board of a fund has to decide which base to use, consumer price index or wage index. Generally, the indexation policy is defined in such a way that every year the pension fund has to decide whether the financial position of the fund suffices to give (full) compensation, or only partial and also when there are enough resources to recover past lost indexation. Retired people, deferred members and active participants all would like to be compensated for increases in prices or wages. These are the parties who benefit from indexing pension rights. Up to few years ago, the full indexation of the liabilities was granted. Since the equity market collapse of 2001-2003 many pension funds opted for conditional indexation policy (mainly in Netherlands) or limited indexation (as in UK). The definition of the indexation policy usually lead to the so called COLA (Cost of Living Allowance) agreement which defines at which low levels of the funding ratio the members abstain from full indexation, and which additional indexation they get in return at high levels of funding ratio.

-*Reinsurance policy*

Pension fund can sublet certain risks, like the risk of decease or disability, partially or entirely to an insurance company. The supervisor judges the reinsurance policy of the pension fund and tries to avoid that the pension funds are exposed to too much risk.

-*Contribution policy*

The board of a pension fund can not only manage its liabilities, also the assets can be managed. One of the instruments to manage the assets is by means of the contribution policy. In the contribution policy, the system is chosen on which the level of the contribution rate is determined. Most pension funds use a dynamic contribution rate. In this system, the level of the contribution rate can be modified over time. However, it is also possible that the different parties involved in the decision process agree about a fixed contribution rate. The active participants and the sponsor are the parties who are mainly interested in the level of the contribution rate, because they have to finance the system. The definition of this policy usually leads to the definition of the contribution agreement. It defines at which low level of the funding ratio the sponsor donates additional contributions, thereby specifying its *pension risk* budget, and which rebates it gets in return at high levels of the funding ratio.

-Investment policy

The value of the assets is also influenced by the investment policy. In this policy, the board of the pension fund decides in which asset classes the fund invests its assets, the levels of the lower and upper bounds on the fraction of the total assets invested in each asset class and rules concerning the rebalancing frequency. The investment policy can merely replicate an index (passive management) or the assets can be managed actively. Also investments to reduce risks, like currency hedging, are considered. In the definition of this policy, the board has to respect the supervisory standards aimed at control the underfunding risk.

1.4 Risk factors within Pension Funds

In order to take sound decisions, pension funds managers need first to identify and then to quantify the financial risks facing by the fund. The first step is to systematically structure these risks. Once this has been done and the pension fund's management knows what the financial risks are, they then need to define the fund's risk attitude.

Pension funds are exposed to many sources of risks. As explained before, the funding ratio is very important in determining the financial soundness of a fund. As a result, one of the greatest concerns of the board of a pension fund, and also of the sponsor and the supervision, is the *risk of underfunding*. This risk deals with the risk that the market value of the assets is not able to compensate the market value of the liabilities, which represent the present value of all the future obligations towards the members of the fund. To categorize the risk drivers of the funding ratio, we can refer to the risks involved in the specification of every policy discussed before.

-Risks regarding the asset portfolio (investment risk)

The investment risk is a measure of the extent to which a pension fund's financial position is sensitive to investment portfolio choices.

This category comprises all the risk factors affecting the asset portfolio, usually referred to as “market risk” with reference to banks and financial institutions portfolio: currency risk, interest rate risk, default risk and volatility risk. Currency risk is created by investments which are made in other currencies than the one in which the liabilities of a pension fund are expressed. It is a typical risk for large pension funds who usually invest their assets in international diversified portfolios. Pension funds usually also invest a consistent fraction of their assets in bonds with the aim of matching liability interest rate risk. With reference to bonds, there is always the risk that the issuer of the bond is not able to make the promised payments, which recall the risk of default. On the other hand, the fraction of the portfolio invested in equity is exposed to the risk of adverse fluctuations in the prices, which can be different from the estimated equity risk premium. Moreover, the volatility risk is present to if the returns on the asset classes fluctuate more than expected and it is more valuable when derivatives instruments are included in the portfolio.

-Risks regarding the liabilities

The liabilities are affected by interest rate risk and demographic risks. For long time, the interest rate risk was not really an issue for pension funds, because in most of the case a conventional 4% interest rate was assumed. Since the market-to-market valuation of the liabilities has been imposed, fluctuation in the long term interest rate yield curve can severely affect the present value of the liabilities. Regarding the demographic risk, it also has assumed greater important due to the ageing of the population. The so called “ longevity risk” exists due to the increasing life expectancy trends among policy holders and pensioners, and can result in payout levels that are higher than what the pension fund originally accounts for. Another source of risk arise when all outgoing future cash flows constituting the plan's liabilities are not exactly and with complete certainty matched by future incoming cash flows generated by its assets. If such a match cannot be realized, then a shortfall may occur in the future. Apart from the consequences of any initial surplus or deficit, this is called mismatch risk.

-Risks with respect to contributions

The sponsor of the fund may not be able to make its part of the contributions, or to make a remedial contribution if the financial position of the sponsor is bad. Actually, there is a high joint probability that a weak financial position of the fund corresponds to a weak financial position of the sponsor, due to fact that both are affected by macroeconomic shocks. Therefore, *the risk of default of the sponsor* is a source of risk from the perspective of the pension fund. On the other hand, from the sponsor perspective, there is a sort of *contribution risk* that deals with the volatility of contributions to be paid to the scheme by the sponsor. It is strongly linked to the underfunding risk. The only way to have lower average contributions is to invest more of the fund in equities and to accept greater underfunding risk (i.e. a large fall in the value of the equities held in the pension fund that is not matched by a corresponding fall in the value of liabilities) as a consequence. The trustees face this complex trade off between underfunding risk-contribution risk and the expected level of contributions into the fund. In general terms, this trade off can be summarized as follows: when equity market is booming, there is likely to be an employer contribution holiday, but when the equity market slumps, there will be a scheme deficit that needs to be removed over a recovery period agreed by trustees. Moreover, if there is an equity market downturn, this is likely to affect also employer's own share price and his ability to raise funds on the market to put into pension fund.

-Risk regarding reinsurance

Risk of default is also present in reinsurance contract. This is the case if the insurance company is not able to make its promised payments.

-Risks with respect to indexing (wage growth/inflation risk)

The wage growth/inflation risk is the result of pension scheme indexing clauses. An indexing clause determines the extent to which general salary measures or inflation lead or leads to changes in pension levels. The extent to which a pension fund's financial situation is sensitive to changes in the general salary level is called the wage growth risk. The extent to which a pension fund's financial situation is sensitive to inflation is called the inflation risk.

For example, high inflation rates may lead to higher than expected benefit payments, and therefore also to a higher value of liabilities. Active participants, deferred members and retired people are concerned about the risks with respect to indexing, because they benefit from indexation pension rights. Most pensions in payments in UK are subject to limited price indexation or LPI (i.e. retail price inflation up to a maximum of 2.5% p.a.), so the nearest available matching asset for this liability in an index-linked bond. In this way the inflation risk is partially minimized. However these markets have still limited size and liquidity. In Netherlands, as a consequence of the financial crisis of 2001-2003, most pension funds opted for a conditional indexation which depends on nominal funding ratio. In this case, the risks with respect to indexing result from a combination of inflation risk and underfunding risk.

-Assumption risks

These risks are relative to the modelling of the ALM and take the form of model risk for the estimation underlying the generation of scenarios and the evaluation methods of the ALM strategies.

1.5 ALM strategies

Many types of ALM strategy have been developed from the classical “immunisation” (or duration matching) to the last innovative ‘Liability-Driven-Investment’ (LDI) strategies with the purpose to manage, in different ways, the risks of the pension fund.

Immunisation is the process of constructing a portfolio that is not exposed to interest rate risk (Luenberger 1998), with the present value and duration of the future obligation of the pension fund matching those of the portfolio, mainly fixed-income assets, e.g. bonds. The construction of such a bond portfolio has to assure a return over a given investment horizon (equal to that of the payout on the fund’s liabilities) regardless of changes in the level of interest rate. In short, the bond portfolio is immunised against interest-rate changes in such a way that the present value of the bond portfolio equals the present value of liabilities. This construction is done by means of the durations. The duration of a fixed-income instrument (and also of the liabilities that can be regarded as a bond for their cash flows structure) is the weighted average of the times that payments (cash flow) are made, and it always lies between the first and last cash flows (Macaulay 1938). It is a typical measure of interest rate risk used by bond analysts and assumes a linear relationship between the price of the bond and the interest rate. It can be derived as first-order measure of the well-known Taylor expansion of the present-value profile. It measures the slope of the present-value profile at any given interest rate and represents how (linearly) the bond's price changes with respect to interest rates changes. However, given that the present value of a bond is computed as sum of discounted cash flow using an appropriate yield curve, as interest rates change, the price does not change linearly, but it rather is a convex function of interest rates. Convexity is a measure of the curvature of how the price of a bond changes as the interest rate changes. Specifically, duration can be formulated as the first derivative of the price function of the bond with respect to the interest rate in question, and the convexity as the second derivative. It is derived as the second order measure of Taylor expansion of the present-value profile. The lower the duration and the greater the convexity, the less sensitive the bonds and liabilities are to interest rate changes and then the lower the degree of interest rate risk they contains. To immunize the interest rate risk arising from liabilities implies that the bond portfolio is constructed to have the same duration and (at least) the same convexity as the liabilities. To match the duration of the liabilities, it is possible to construct portfolios with a specified duration from a whole range of bonds with different durations. For example, the portfolio could be constructed from bonds with durations close to that of the liabilities (so called a focused portfolio strategy), or it could be constructed from bonds with durations distant from that of the liabilities (barbell portfolio strategy). The latter strategy has the advantage of a much wider range of portfolios with different durations that can be constructed than with the focused portfolio. However, it leads to a greater immunisation risk that arises whenever there are nonparallel shifts in the yield curve. Non parallel shifts in the yield curve will lead to the income components of the value of the portfolio changing either too much or too little compared to the change in the capital component. This risk is reduced if the durations of the individual bonds in the immunising portfolio are close to that of the liabilities (as a focused portfolio), because the bond portfolio and the liabilities are affected in a similar way. In a barbell portfolio the effects of such a shift will be different according to duration of each bond in the portfolio, even if the portfolio its self has the same duration as that of the liabilities. The same issues are valid also when indexing is granted, whereas the immunization should imply a bond portfolio composed by inflation-linked bond.

Bodie (2004) is a strong proponent of ALM strategies based on only-bonds investment. He does not believe equities offer higher returns and inflation protection in the long run. He suggests default-free and inflation protected government bonds are the most sensible investment instrument for retirement provision. As noted by Davis (2005) and Hu (2005a), however, such long-term bonds with price protection are not available in many countries. Even in those countries, like the UK and the US where financial markets are deepest, total outstanding inflation-indexed government bonds were much less than the aggregated pension fund assets. For example, in the UK, as of 2003, inflation-indexed government bonds were at the order of $139bln, while pension assets were equivalent to $954bln. Moreover, even if fixed income and inflation protected bonds perfectly matching the liabilities are available, the implementation of this strategy is not simple. Once the bond portfolio is determined, it is necessary to periodically rebalance this portfolio to take into account the changes in the interest rates but also the passage of time, which reduce automatically the duration of the portfolio. In practice, the pension fund has to meet its liabilities on an annual basis and this implies that there is a schedule of liabilities over time, and not a single future date. When there are multiple liabilities, it is no longer sufficient simply to match the duration of the portfolio to the average duration of the liabilities as in the classical immunisation. Instead, it is necessary for each liability payment to be immunised (duration-matched) individually by multi period immunisation.

One of the main techniques of multi-period immunisation is cash-flow matching. It consists on the construction of a lowest-cost portfolio able to generate a pattern of cash flows that exactly matches the patterns of liability payments. This (the lowest-cost) bond should have the same maturity and value as the last liability payment. The coupon payments on the bond help to finance the earlier liabilities. Taking these coupon payments into account, another bond (again the lowest cost) is purchased with the same maturity as the penultimate liability payment. Working backwards in this way, all the liabilities can be matched by payments on the bonds in the portfolio.

There are two main advantages of this matching technique with respect to immunisation strategy. First, there is no need for duration matching. Second, there is no need to rebalance the portfolio as interest rate changes or with the passage of time, as there is with immunisation. Cash flow matching is a very simple passive buy-and-hold strategy, but its implementation is not simple. This because in the real world it is unlikely to find bonds with appropriate maturity dates and coupon payments. To guarantee that the liabilities are paid when due in the absence of perfect matching, the cash flow strategy would have to be constantly monitored and rebalanced, implying extra costs. In the comparison with a simple immunisation strategy, this latter would result more efficient. However, in some countries as UK, the introduction of a strips markets has allowed the principal and income components on bonds to be negotiated separately, helping in reducing the cost of cash flow matching strategy. Slightly different is the horizon matching strategy which combines the cash flow matching and immunisation. In other countries like Netherlands, it has became quite common to use derivatives instruments, especially long-dated interest rate swap to hedge the interest rate risk (in nominal terms, while for inflation hedging it has became available the inflation swap) as a way of mitigating their cash flow risks. Interest rate derivatives are an alternative which can help immunizing portfolios. Compared to long term bonds, long-duration interest rate derivatives may have more liquid market and do not require large changes in existing asset portfolios. Some interest rate derivatives, for instance, Roller Coaster Swaps, are designed to have different underlying notional in order that for each tenor, the interest rate sensitivity is zero. However, no strategy exists which can fully hedge interest rate risk since liability structure changes over time, and there are always credit risks from counterparties. Engel, Kat, Kocken (2005) investigated the adoption of derivatives to handle interest rate risk problems for pension funds. They draw the conclusions that the decision whether to choose swaps or swaptions is highly interest rate environment dependent: swaps can hedge most of the interest rate risks except when interest rates are lower than historical means.

To overcome most of the disadvantages of the previous approaches and also the poor performance on asset returns which derived from them, a number of so-called liability-driven investment (LDI) techniques have been promoted over the past few years by several investment banks and asset management firms. LDI is the latest and most sophisticated form of ALM (Scherer, 2005). The coherent implementation of risk-immunising portfolios lies at the heart of the new 'Liability-Driven-Investment' (LDI) strategies, the understanding of which, however, varies across countries. In its general meaning, which is mostly applied in the Netherlands, for example, LDI refers to an investment strategy that is aligned with the liabilities of an investor and explicitly considers their stochastic nature. The impact of relative differences between liabilities and assets on the goals and constraints set by the decision maker make it crucial to look at both sides simultaneously. In the UK context, LDI concepts aim to immunize the sponsor from certain risk factors, whereas duration and cash flow matching strategies aim at eliminating interest rate risks.

Generally speaking, the aim of LDI is to secure the expected liability cash flow at the lowest cost. It advocates the design of a customised liability-hedging portfolio (LHP) also called as matching portfolio, the sole purpose of which is to hedge away as effectively as possible the impact of unexpected changes in risk factors affecting liability values. LDI usually begins with a forecast of the liability cash flows followed by an analysis of all the sources of risks attached to the liabilities. Among these, the most notably interest rate and inflation risks, but also longevity risks and in the UK also the contribution risks of the sponsors (if the sponsor became insolvent). In hedging these risks, LDI recognises that the key traditional assets are either poor short term hedges for liabilities (as equity) or highly illiquid (as property), with the result that the asset portfolio can be volatile and unpredictable in comparison with the liabilities dynamics. It is mainly composed by fixed income instruments and inflation linked bonds.

This LHP complements the traditional performance seeking portfolio (PSP) (or return-seeking portfolio), the composition of which is not impacted by the presence of liabilities. In this way, the LDI also allows the objective of the two portfolios to be separated, so that if the investment risk budget is large enough, it allows the trustee to take advantage of additional investment opportunity (alpha). Within the aforementioned LDI paradigm, a variety of cash instruments (treasury inflation-protected securities, or TIPS) as well as dedicated OTC derivatives (such as inflation swaps) are typically used to tailor customised inflation exposures that are suited to particular institutional investor's liability profile.

In setting the investment objectives the trustees try to outperform their liabilities by a certain level each year. However, given the limited information on the nature of liabilities available historically, naturally the focus fell more on the assets resulting in a significant and often unappreciated mismatch with the liabilities.

Today the tools are available for a very close translation of expectations allowing trustees to define the investment objectives explicitly in terms of the liabilities. For example, a liability driven investment objective might be of the form: *match the change in liabilities plus outperformance of x % p.a.*

The linkage is transparent and explicit – the assets should outperform the liabilities by *x%* each year. The liability investment objective focuses on the liabilities first (matching portfolio) and then addresses the desired level of outperformance over the liabilities (return-seeking portfolio), subject to various risk constraints. The implementation of LDI solutions depends also on the attitude towards risk. It is typically understood that high risk aversion leads to a predominant investment in the LHP, which in turn implies low extreme funding risk (zero risk in the complete market case), as well as low expected performance and therefore high contributions. On the other hand, low risk aversion leads to a predominant investment in the PSP, which implies high funding risk as well as higher expected performance, and hence lower contributions

Liability driven investment solutions offer trustees the opportunity to structure their investments so that performance relative to liabilities is the primary measure of investment success. Investments can take advantage of evolutions in the financial markets, whereby many pension fund risks can now be efficiently hedged and investment manager skill can be accessed in a variety of ways. The foundation for any liability driven strategy is the cash flow forecasts. This will estimate year-by-year cash flows as well as the proportion of these cash flows that is sensitive to inflation including the LPI caps and floors.

This forecast facilitates the identification of the liability matching portfolio, or least risk portfolio, which is a combination of assets exhibiting similar sensitivities to interest rates, inflation and other variables as the liabilities. Using gilts and index-linked gilts, a pension scheme can construct a low risk cash flow matching portfolio with the objective of producing the required cash flow at the time it is needed. In other words, the liability cash flows are approximately equal and synchronised with the asset cash flows.

The problem with this simplistic approach is the limited range of government securities available. Any match using conventional fixed income and index-linked securities will inevitably be 'lumpy' and will not extend far enough into the future to cover all the liabilities. Matching can be made more accurate by including other assets, such as supra-national and corporate bonds.

Swaps might also be used to fine tune the exposure, or alternatively, it is possible to construct a predominantly swaps based solution using interest rate swaps, inflation swaps and credit default swaps to achieve the same bond exposures as a conventional portfolio. This 'synthetic' solution usually offers greater flexibility than the physical approach. Typically a liability matching portfolio will include the asset classes.

One outstanding problem with LDI approach, however, is that such solution generates very modest performance. In fact, real returns on inflation-protected securities, negatively impacted by the presence of a significant inflation risk premium, are usually very low. In other words, while these solutions offer substantial risk management benefits, the lack of performance makes them costly options for pension funds and their sponsors. In addition, the inflation linked securities market does not have the capacity to meet the collective demand of institutional and private investors, while the OTC inflation derivatives market suffers from a perceived increase in counterparty risk. Moreover, other risk dimensions as mortality risks however cannot be properly addressed for lack of adequate financial products. Nevertheless this approach has been widely adopted in many pension funds.

2
Risk-based Supervision and Risk Management

2.1 Introduction

Over the past several decade privately managed pensions have evolved to became an important, and in some cases crucial, element of social insurance systems. Private pensions funds accumulated asset levels exceeding those of more traditional financial institutions in a number of countries, in some cases more than 100 percent of gross domestic product (GDP), leading to a commensurate increase in attention to their systemic importance. Hence, their supervision has operated a similar transition to meet the requirements of this new role. It has evolved from merely ensuring compliance with tax laws and labour contracts and relatively simple methods to limit investment risk, toward a much more comprehensive approach ensuring proper management of all the risks associated with complex institutions relied on to provide secure sources of retirement income. This new approach is defined as risk-based supervision (RBS).

The traditional supervision regimes are based on simple portfolio limits with very proactive compliance enforcement, and the primary concern was limiting downside risk over short periods through investment controls. The new supervision regimes focus more on the risk-return efficiency and effective capital allocation. In general terms, this transition shifted the nexus of supervision from controlling agency risk to managing systemic financial and operational risks (Brunner, Hinz and Rocha, 2008). The reasons behind these evolutions of the supervision regimes are cleared analyzing the general economic conditions of the beginning of the new millennium and the several factors, which accelerated these changes in supervision methods. The so called “perfect storm” of rapidly declining interest rates coincident with collapsing equity prices exposed the fragility of the loose funding requirements especially for DB pension scheme. Many of them, across Europe and United States, failed or succeed to survive switching to DC systems. At the same time, the capacity of the new DC plans to produce adequate levels of retirement’s income also focused the attention on the efficacy of their design and operation. Hence a number of countries began to adopt supervision systems based on various risk-based approaches that established new standards for the operation of the pension funds and guided the conduct of their oversight activities. These systems have only recently been introduced or are in development phase, but their origins can be found on the consolidated applications of risk-based methods in the supervisions of the banks. Indeed, the trend toward risk-based supervision of pensions reflects an increasing focus on risk management in both banking and insurance, which is based on three key elements: capital requirements, supervisory review and market discipline.

The earliest of these systems was developed for the banking industry in the 1998 by the Basle Committee on Banking Supervision that implemented the Capital Adequacy Accord (Basle I). It provided a risk-based framework for assessing the capital adequacy of banks to cover credit risks. The development of this framework was an important step toward the adoption of RBS. It aimed to ensure an adequate level of capital in the banking system, by applying weighting to credit exposures based on broad risk classifications. In 1999 the Basel Committee began the process of replacing the Basle I Accord with a more sophisticated framework, which requires banks to improve risk management and corporate governance in conjunction with improved supervision and transparency. The new framework, known as Basle II, is designed to encourage good risk management by tying regulatory capital requirements to the results of internal systems and processes assessment, thereby creating incentives for improvement in risk management. It defines calculation of regulatory capital in a more risk sensitive way and is extended to two new area, defined as pillar two and three. The former concerns the supervisory review, the latter the market discipline. More specifically, the Pillar one requires a more extended implementation of an effective risk management system that includes credit, market and operational risk.

Pillar two allows supervisors to evaluate a bank's assessment of its own risks and assures themselves that the bank's processes are robust, in the sense that the bank understands its risk profile and is sufficiently capitalized against its risks. This should encourage adoption of risk-focused internal audits and the development of risk management units.

Pillar three deals with market discipline and ensure that the market is provided with sufficient information to allow it to undertake its own assessment of a bank's risk. It is intended to strengthen incentives for improved risk management through greater transparency. This should allow market participants to better understand the risk inherent in each bank and ultimately support banks that are well managed at the expense of those that are poorly managed.

The movement toward greater risk focus is also reflected in the insurance industry. The International Association of Insurance Supervisors (IAIS) is working to develop a common international framework for assessing the solvency of the insurers. This project in Europe is called Solvency II and aims to adopt a risk-based approach to capital requirements for insurance companies and introduce qualitative requirements for senior management, risk management, model validation, and internal controls.

Solvency II will involve a three-pillar approach similar to that of Basel II, introducing a supervisory review process and enhanced transparency. The current solvency framework defines capital requirements for insurers in terms of solvency margins typically based on simple rules applied to technical provisions or premiums. Under Solvency II, the Pillar one will define the resources that a company needs to be considered solvent. It will define two thresholds for capital: the solvency capital requirements will set a threshold for supervisory action and the minimum capital requirements will provide a basis for stronger action or even withdrawal of the company's license to write new business. As with Basle II, the capital requirements can be computed using either a simple standardized model or an internal model approved by supervisor. Pillar two will take into account qualitative measures of risks focusing on risk management processes, individual risk capital assessment, and aspects of operational risk, including stress test. Pillar three will address disclosure requirements incorporating more consistent international accounting standards.

From banking and insurance industry, the trends toward risk-based supervision have gradually interested the pension industry. In the following chapter we will describe the experience of the four countries that are the earliest adopter of risk-based supervision. They implement risk-based principles and standards which are different from those implemented in banking and insurance industry, because they must take into account the specific characteristics of their own pension funds market, for instance, the dominance of DB pension plans on DC pension plans. Moreover, different approaches across countries can be also referred to the absence of a super-national authority defining the general principles for the whole industry, as it has happened for banks and insurance companies with the Basle Committee. The chapter continues describing the common aim and the drivers that led to the adoption of a risk-based supervision and the main components of the regulatory framework. Then, the sophisticated system developed in Netherlands is analyzed and conclusions are drawn.

2.2 The early experiences in Pension Funds Industry

The countries that, to different extents, have adopted RBS standards so far are the Netherlands, Denmark, Australia and Mexico. They followed different approaches to RBS and that reflects the different types of pension markets they represent. All of these countries have mandatory or quasi-mandatory private pension systems. In Australia and Mexico, contributions to private pension plans are imposed by legislation. In Denmark and the Netherlands, contributions take place in the context of collective labour agreements that are classified as quasi-mandatory because most workers are covered by these agreements. The mandatory or quasi-mandatory nature of contributions results in high coverage rates except in Mexico. The pension systems in these countries are very large, with assets exceeding 100 percent of GDP in all cases except Mexico. The relatively small size of assets relative of GDP in the Mexican case is due to the lower coverage ratio and the fact that the Mexican system is much younger, having started operations only in 1998. Three countries have a large number of funds, ranging from 111 in Denmark to 1000 in Australia; these funds may operate more than one pension plan. Many of them are occupational funds structured as non-profit trust or foundations originally created on a voluntary basis and operating for several decades. They include single funds and larger multiemployer or industry-wide funds. Australia and Denmark also have several for-profit commercial institutions managing pension funds, including life insurance companies in the Danish case. Mexico has only 18 funds currently licensed. The difference in the number of funds is a result of the different origins and characteristics of the Mexican system. The Australian, Danish and Dutch systems have their roots in voluntary arrangements with employers. Most funds were initially established with liberal licensing and authorization rules designed to encourage participation and coverage. By contrast, the Mexican system was established as a mandatory system of open funds subject to a strict regulatory framework, including much stricter licensing rules. Dutch pension funds manage primarily DB plans. The Netherlands has been one of the few countries that has successfully resisted the move toward DC plan. The Danish system is a DC system that offers benefit guarantees and operates on risk-sharing or profit-sharing basis. The guarantees introduce a core liability and the risk insolvency of the provider. Therefore, the Danish system exhibits some of the characteristics of a DB system, although it operates with more flexible rules than pure DB systems and seems to be moving in the direction of DC plans with fewer guarantees. Australian pension funds manage primarily traditional Dc plan with no formal guarantees. There still some DB plans, but these are mostly restricted to public sector and account for a small share of total assets. Australia best represents a pure DC system.

Given their starting points, these countries have adopted models of risk-based supervision developed with different degree of sophistication. The Australian case provides a model of risk-based supervision that applies to both DC and DB pension funds and covers a wide range of institutions in terms of size and complexity, and applies to both open "public offer" funds and closed occupational funds. They have developed a structured methodology for ranking pension funds according to relative threat of failure, weights this in accordance with the impact of such a failure, and map this to a supervisory response framework. The Australian method demonstrates how DC pension funds can be subjected to risk-based assessment (Thompson and Graeme, 2006). The model makes a distinction between larger funds that are subject to detailed assessment and smaller funds that are subject to a streamlines and more automated assessment.

The Danish case provides a model of risk-based supervision applied in a voluntary occupational system that has achieved a high degree of coverage through collective agreements. Danish funds operate on a DC basis but offer guarantees that result in DB type of arrangements. The model demonstrates how the move toward a risk-based supervision can be a gradual process and doesn't involve the development of a holistic risk-rating model (Andersen, Brink and van Dam, 2006). The "traffic light" approach utilizes a stress test that can feed into a broader and more subjective assessment of pension funds. Nevertheless the results are still used to guide the intensity and scope of supervision.

In the Netherlands, the supervisory authority corresponding to the Dutch National Bank (DNB), applies a sophisticated risk-based system in a defined-benefit pension environment. The Dutch have integrated a risk-scoring system with sophisticated solvency standards designed to ensure adequate buffers to absorb investment and other risks. They provide with a comprehensive set of tools to evaluate all the key risks faced by pension funds and establish a capital rule which defines buffers and funding level according to the risk profile of the institution, in a way similar to banks and insurance companies (Hinz and van Dam 2006). In this way, the single supervision authority can more easily integrate its activity.

The Mexican case utilizes an alternative model in a DC setting that is in the early stage of implementation, and which includes a Value at Risk (VaR) approach to control market risk, as well as a detailed regulation on internal risk management. In particular, it includes a limit on downside risk, defined by a ceiling on the daily absolute value at risk (Berstein and Chumacero, 2006).

The experience of these early adopters has shown so far high potential for the application across the full range of pension system designs. More generally, the application to DC systems such as Australia and Mexico represent the greatest challenge. Transferring investment risk to members requires the formulation of alternative financial risk concepts. Mexico has been innovative in applying the concept of VaR as an attempt to contain downside losses. However, this remains controversial due to the limited linkage between short-term measure and the longer horizon of pensions. These techniques may involve tradeoffs between security and optimizing long-term returns. Australia has sidestepped this challenge by simple incorporating process-based investment standards into its risk-scoring techniques, even if this is a viable options only in systems grounded on well-established and supervised financial service provider.

Evidence of the impact of risk-based methods is still preliminary, but we can suppose that this trend toward RBS will probably have a set back due to the recent financial crisis. This latter has shown indeed several shortcomings of the risk-based systems in the banking industry, in particular the inconvenience to induce banks to behave in a highly pro-cyclical manner. The same critic should also be discussed in pension industry, even if RBS are likely to continue to gain acceptance because they offer the prospect of advantages relative to other approaches.

2.3 Objectives and drivers of RBS approaches

One of the main objective of RBS in banking and insurance is to ensure that institutions adopt sound risk management procedures and hold appropriate levels of capital. Banks and insurance companies have already recognized that sound risk management practices are also in the interest of stakeholders and are rewarded by the market, as indicated by the growing consideration of the quality of internal risk management by rating companies. Pension supervisors face challenges that are in many aspects similar to those faced by banks and insurance supervisors. They recognize the need to evolve to an approach that emphasizes sound risk management by the supervised institution in order to strengthen financial stability and ensure more efficient outcomes for pensioners.

Apart from this more general convenience for RBS, different motivating factors lie behind the introduction of RBS in pension industry: some are partially common to all the countries, but some are country-specific. Preventing underfunding of DB plans was a strong factor motivating the adoption of RBS in the Netherlands, especially after the equity collapse in 2001-2003. Dutch funds enjoying the equity boom in 1990's and started taking contribution holidays when funding ratio reached levels considered high. However, these funding ratios proved insufficient to absorb the adverse price movements in the early 2000s. Regulators interpreted the outcome as indicating a weakness of the supervisory approach that was perceived as lacking sufficient foresight and concern for the risks facing the institutions.

The introduction of a more risk-oriented supervision in Denmark was also motivated by concerns with the solvency providers, but the surrounding conditions were quite different from Netherlands. In particular, the new system was introduced as a quid pro quo for a more liberal investment regime in which the ceiling on equity investments was raised to 70 percent. Concerns with adverse price movements was also one of the motivating factors in Mexico, although the Mexican system is a DC system, where the investment risk is shifted to the individual and there is little risk of provider insolvency. The Mexican policy concern is more related to the exposure of retired workers to extreme downside losses and the extreme volatility of benefits across cohorts. It is interesting to underline that, as in the Danish case, the adoption of VaR ceiling in Mexico and the introduction of strict risk management rules were a quid pro quo for the introduction of a more liberal investment regimes, that allowed pension funds managers to make riskier investment and use derivatives. The search for efficiency gains was also one of the main motivating factors in Denmark and Mexico. In both cases, the investment regime was liberalized and pension funds were allowed to invest more in equity and other assets perceived as risky. In Mexico, pension funds were allowed to use derivatives, subject to authorization by supervisor.

The relaxation of the investment regimes was motivated by perceptions that pension funds were constrained below the efficient investment frontier and that there was scope for longer-term improvement in the risk-return trade-off; in other words, the possibility to exploit the long duration of the liabilities to gain from long term benefits (as mean reversion effects in equity markets). However, to control the assumption of excessive risks, the relaxation of investment rules was accompanied by other rules designed to strengthen risk management. The general need to establish a risk management regulation was essential to enable pension funds to take advantage, as other institutions, of the increasing sophistication and complexity of financial instruments and markets. From the institutional point of view, the integration of financial supervisory functions in one entity for banking, insurance and financial markets seems to have been a motivating factor in Australia, Denmark and the Netherlands, also driven by the need to allocate scarce supervisory resources efficiently. In these cases, all financial institutions could have adopted the same basic supervision approach and that has also accelerated transfer of supervisory expertise from banking and/or insurance supervision to pension supervision. Only Mexico represents an exception, as the supervisory agency (CONSAR) was a single entity when the new approach was adopted and has remained a single entity.

Even if different drivers has lead to the adoption of RBS, the target that pension supervisors across all these countries aim to reach is to ensure that all licensed institutions comply with minimum standards of risk management and hold appropriate level of capital in the systems where this is relevant. The ability of the institution to identify, measure, and manage all the relevant risks would be, for instance, reflected in the presence of a sound internal architecture of risk management that includes the definition of risk management strategies. However, this is not the only components of a RBS: every country has developed different tools according to different environments.

2.4 Requirements for Risk Management Architecture

Similarly to the Pillars in Basle II and Solvency II, the pension supervision is built on four main components. These are (*i*) the requirements for the internal risk management architecture, which can be more or less defined and structured across the different approaches, and assigns different involvement of the Board of the fund; (*ii*) the risk-based solvency rules, similar to the Pillar one in the Basle II/Solvency II framework, which is relevant in DB systems or DC systems that offer benefit guarantees; (*iii*) risk-scoring models, aimed to understand the risk profile of pension funds through their normal activities and can be assimilate to second pillar in Basle II, becoming an essential tool around which pension supervisors organize their offsite and onsite supervisory actions; (*iv*) the role of market discipline as in Pillar three, whose relevance more closely depends on the particular type of system.

The construction of risk management capacity in pension funds is a supervisory purpose in Australia, Denmark and the Netherlands. However supervisors try to achieve this aim through different means. They all impose some requirements on risk management as part of licensing or initial registration procedures. These include the elaboration of a risk management plan or guidelines. These requirements are not very detailed, with the supervisor allowing for differences depending on the size of the institution. These countries do not seem to impose specific regulatory requirements on the internal risk management architecture, although Dutch funds must have an internal body reviewing long-term risk management, as well as independent risk management functions. The institutions must comply with corporate governance rules that emphasize the role and responsibilities of the Board. By contrast, Mexican pension funds have to adopt a very specific and detailed risk management architecture laid out in a specific regulation issued by regulators. All the Mexican funds must have two board committees dedicated to risk management. Each committee must have at least five members, three of whom are board members. At least one of the members must be independent, while the other members are the chief executive officer and the chide risk officer. The regulation specifies in detail the duties and obligations of each unit, including the interactions with other key executives. The regulation also requires the presence of a compliance officer to ensure observance of all the regulations.

However the Mexican approach can only be implemented in systems with fewer and larger pension funds. In Australia, Denmark and the Netherlands the adoption of risk management practise, even if not regulated in detail, is induced by their risk-scoring model presented in the next section. These models measure the exposure of the institution to risk and their capacity to manage these risks. This capacity is assessed in some detail, entailing the assessment of the quality of very specific elements of risk management, procedures, and control.

2.5 Risk-Based Solvency Standards

Risk-based solvency rules are relevant in Denmark and the Netherlands because of the nature of their system (DB). Dutch supervisors have recently implemented a detailed and formal risk-based solvency rule that addresses longevity, market, credit, currency and interest rate risk and that penalises asset-liability mismatches. This system originated with a set of solvency standards first developed in 1997 and subsequently refined and introduced with the new pension act that became effective on January 1st, 2007. It includes a minimum solvency margin and solvency buffers designed to minimize the risk of underfunding due to longevity improvements or fluctuations in interest rates and asset prices. Liability (technical provision) is measured with a mortality table that reflects predicted longevity improvements and a buffer to deal with unforeseen improvements. The discount rate used is the market yield curve measure by euro swap curve or, in the case of indexation to inflation, the interest rate yield curve. All pension funds must comply with a minimum solvency requirement equivalent to 5 percent of technical provision. However, funds must also build additional solvency buffers whose magnitude depends on the degree of asset-and-liability mismatches and that are designed to reduce the probability of underfunding to only 2.5 percent within a one-year horizon. In line with the approach followed in Basle II, pension funds may opt to comply with a standardized model or build their own internal model to compute their solvency requirements, although these models need to be approved by the supervisor. In the standardized model, the solvency buffers are calculated through a stress test based on six broad risk factors and a formula for aggregate risk that takes partially into account correlation across asset classes.

The methodology implies that the typical Dutch fund will need to maintain a sizeable buffer amounting to 30 percent of technical provisions. This clearly incentives pension funds to build their own internal model, because a more refined methodology and more accurate parameters will probably reduce the size of the required solvency buffer. However, if pension funds decide to build their own model, this may prove challenging to supervisor, who will have to assess each of these models, as in Basle II. Next to the solvency constraints, Dutch supervisor has allowed for some flexibility by allowing a relative long period of 15 years for compliance, because the Dutch solvency rules had been criticized for being too costly and not taking into account that long run risks are lower due to lower correlations of asset classes or mean reversion of equity returns (Barberis 2000).

Denmark has adopted a model that can be classified as hybrid. The formal solvency rule is not risk-based, but it is complemented by a standard stress test called the “traffic light system” that entails a test of the resilience of the institution in response to fluctuations in interest rates and asset prices. Although the traffic light system is similar to the new Dutch solvency rule, it also has some important differences, mainly due to the particular risk-sharing features of that system (DC with guarantees). As with the Netherlands, there is a minimum solvency margin based on the current valuation of liabilities that is supplemented by a stress test based on the composition of assets. The stress test places each fund into one of three traffic-light zones that indicate the current solvency position. It is distinguished from the Dutch approach because it does not explicitly link remedial measures to the status of the fund, but it just signals devices and market pressure. A solvency status is calculated for every institution twice a year and places each institution in one of the three categories: a green light for those within acceptable solvency status, a yellow light for those in danger in facing solvency problems, and a red light for those that face severe and immediate problems. Rather than impose a single potential scenario of adverse market conditions, the Danish approach establishes two sets of parameters for each risk factor, which effectively imply a mild and a strong stress test. Factors are stipulated for decline in equity, in real estate, for variation in the duration of fixed income instrument, credit risk and others. Funds that remain theoretically solvent after the strong test are put in the green zone. If a fund is insolvent by the mild test, it is deemed to be in the red zone. Those that remain solvent under the mild test but not the strong test are placed in the yellow zone. Failure to meet the yellow scenario is treated as an early warning indicator. An institution that receives a yellow light is placed under intensified supervision, which consists with requirements for increased risk awareness of the management of the pension institution. When a fund is in the red zone, the supervisor may order the institution concerned to take the measures necessary within a specified time limit if its financial position has deteriorated to such a degree that it puts the interest of policyholders and other affected parties at risk. A red light does not necessary imply that the institution will immediately be subject to crisis management, but the supervisor could require monthly reporting as well as a commitment that it will not increase its overall risk exposure. The Danish Financial Supervisory Authority decides the maximum period for the restoration of the financial position, depending on the size of the shortfall and anticipated market developments.

Risk-based solvency rules are not so relevant in DC system as Australia and Mexico. Australia, which has an almost complete Dc system, does not incorporate explicit solvency requirements on the risks of the DC fund portfolios. However the exposure to financial risks is captured in the risk-scoring model and the supervisor will check if the institution has the capacity to manage these risks. If the institution proves to be unable to manage the risks associated with a more aggressive or complex portfolio, it becomes subject to more intensive supervision. On the other hand, Mexico has taken a completely different approach to volatility risk. Within a DC system, the relevant characteristic is the volatility of the value of member's account rather than the asset-liability balance. Even if it does not represent a solvency measure in the traditional meaning, the parameters that Mexico requires the pension funds to remain within, aim to a similar purpose: to ensure the adequacy of the asset base and retain its fluctuation with its pre-specified level. It consists on a limitation that takes the form of a maximum permissible VaR that the funds are permitted to have. Mexico now permits two types of portfolios within each of the pension companies. The standard portfolio established at the outset of the system design is limited to a composition that is estimated through the VaR methodology associated with a maximum loss in a day of less than 0.6 percent of its value. The higher risk/return portfolio that was recently introduced into the system must maintain a VaR of less than 1.0 percent. The VaR is calculated by the supervisor on a daily basis, based on a rolling 500-day sample of the prices of all of the permissible assets. The price vector is provided by two independent price vendors to ensure a common valuation methodology and comparability.

The VaR is historic and calculated with a 5 percent level of significance for each portfolio. If any of the funds drifts outside of the permissible limits, the supervisor is able to intervene and provide specific instructions regarding the reallocation required to move back within the prescribed standard. Even if innovative in its methodology, toward this approach the traditional critics to VaR methodology has been addressed as relying on historical data and only manage financial instruments with linear payoffs.

2.6 Supervisory Risk-Scoring Systems

Any supervision framework implies the collection of data from pension funds obtained by offsite and onsite supervision. The analysis of the collected data gives a picture of the financial status of the funds and is combined for the computation of the overall risk scores for each institution. Australia was the first of the four countries to introduce a fully developed scoring system with the introduction in 2002 of a structured framework for risk assessment in pension funds, called “probability and impact Rating System” (PAIRS). It results in a ranking of the pension funds computed according to the relative threat of failure that is mapped into a supervisory response. The model makes a distinction between larger funds subject to detailed assessment and smaller funds subject to a streamlined and automated assessment. It considers the significance of the risks, the mitigation factors and the extent to which each contributes to or reduces the overall risk of the fund. Weighted numerical assessments are combined into an overall score. This score is converted to a risk rating using a nonlinear function to ensure that higher risk funds are given greater attention. After taking into account an impact rating based on fund size, the scores are converted into a supervisory attention index that maps into a supervisory stance and action plan. In this way, the rating directly defines how the supervisor will manage the relationship with the pension fund. Funds in the “normal” category are subject to regular supervision activities. Those in the “oversight” category receive more intense monitoring and more frequent contacts. Funds rated for mandated improvement are expected to develop and implement plans for improvement, while those rated “restructure” require strong enforcement actions. One of the advantages of this system is that it allows allocating more supervisory resources toward institution whose failure would have a greater impact on financial system.

In the Netherlands, the DNB introduced an integrated method for analyzing risk for all financial institution called FIRM (Financial Institution Risk Analysis Method). The FIRM model adopts a four-stage approach to build the risk assessment. The first step is the delineation of a detailed profile of the pension fund. The second step identifies relevant management units and functions and assigns weights to these. Using this functional breakdown, the third step evaluates gross risk and assigns a score to this assessment. In contrast to Australia, this system combines probability and impact into a single score within the system rather than assessing these separately, based on the view that the two elements are so closely related that they should not be independently considered. The various types of risks identified in Australia and the Netherlands are similar. The additional focus in the Netherlands on technical insurance and mismatch risk reflects the primarily DB nature of the pension system. Finally, the fourth step in the Dutch scoring model seeks to obtain an insight into the quality of risk control for each risk category to derive a final value that represents the net risk of the pension fund. The aggregation of risks is based on mathematical algorithm that puts emphasis on high risk and poor controls. On the base of the final score, the supervisory activities are planned.

In Mexico and Denmark only recently there has been the introduction of elements of a risk-scoring model in the form of early warning indicators for assessing operational and financial risks. In Mexico, the current methodology entails three risks factors – low, medium, high – and gives emphasis to irregularities detected during supervision activity. Denmark has developed a risk-scoring model in the form of an internal system with three internal quality scores covering organization, procedures, and internal control, as well as ratings on insurance risks that mainly cover longevity risk exposure of different institutions. The results deriving from this system are combined with the traffic-light results to guide the intensity and scope of supervisory activity.

2.7 Role of Market-Based Discipline

The importance of market discipline in RBS depends fundamentally on the type of pension system and the extent to which supervisor ensures disclosure and enhances the roles of third parties, such as external auditor. Generally speaking, the market discipline pillar is more relevant in those systems that allow selection of the provider. In any case, supervisor must ensure proper accounting, auditing and disclosure rules ensuring the access of fund members and market analysis to relevant and accurate information. It is a consolidate practise in DC system as well as in Australia and Mexico, but it has only been introduced recently in the Netherlands and Denmark. In all these countries, external auditors need to verify the accuracy of financial statements, but in Australia and Mexico their role is expanded to include an assessment of the quality of risk management systems. Mexico imposes extensive disclosure requirements, including monthly disclosure of individual portfolios, returns, fees and VaRs. Denmark discloses annually a large number of performances and solvency indicators of individual providers, allowing for direct comparison of performance. Australia has detailed product disclosure requirements for funds that allow members to direct their investment strategies, but not on fund performance. The less-demanding disclosure requirements in the Dutch regulatory framework reflects the closed nature of the Dutch system. Overall, the market discipline Pillar seems to play a more important role in Mexico and Denmark, followed by Australia and the Netherlands. With reference to the risk scoring models presented above, we can state that none of these countries disclose ratings to the markets. Denmark only provides summary solvency indicators, while the Netherlands limits disclosure of risk management scores to pension funds. Australia does not even disclose to a fund its rating. It is clear there may be scope for being more open in disclosing rating for pension funds in order to strengthen market discipline and promote sound risk management.

2.5 Conclusions

Overall, the models of risk-based supervision have demonstrated the benefits of moving away from approaches based in strict compliance, specific rules and quantitative controls with respect to an approach that puts more emphasis on the identification and management of relevant risks, encouraging supervised entities to place greater focus on risk management in their daily operations, which promotes a stronger pension system and more effective outcomes for the members of the system. Evidence of the impact of risk-based methods is preliminary at best, and it remains far too early to draw any decisive conclusions. But several challenges must be faced for their improvements. Among other critics, some point out the absence of a direct linkage between risk-based concept and the "nature" of the pension funds. For instance, in the definition of a VaR measure of 1% daily in the Mexican case or the 97.5 percent probability of underfunding of the Dutch pension funds, there is not a direct foundation in the capacity of pension funds to remain solvent over the long term. Similarly, to the extent to which these RBS models are based on a perceived "average" member of the fund, they may be poorly aligned with the diverse requirements of members with widely varying time horizons or differing risk appetite.

A second critic underlies that the solvency constraints are potentially procyclical in nature. For the same critics also Basle II has been criticized and a reviewing process has started. Funds holding more volatile assets will have incentives to sell these when faced with market fluctuations. If pension funds are sufficiently large, these can became potentially self-reinforcing cycles that exacerbate instability.

Despite these challenges, RBS methods are likely to continue to gain acceptance because they provide a forward-looking paradigm around which to organize supervision that offers the promise of reduced risk of insolvency of DB and potential efficiency gains in DC systems that impose investment restrictions, even if it requires new technical requirements and a higher level of sophistication from all parties.

3 Asset and Liability Management Modelling: a scenario-based approach

3.1 Introduction

Asset-and-liability modelling is a key method in strategic risk management. It is a financial risk assessment and asset planning tool used by pension funds to help them choose the strategic pension policy under uncertainty in a coherent and consistent balance sheet approach. It involves developing mathematical scenarios of the future evolution of pension fund assets and liabilities, given certain assumptions about the statistical properties of economic, financial and biometric variables that affect the evolution of assets and liabilities. There are many ways to generate economic, actuarial and financial market scenarios. The traditional method was to create a central scenario and to carry out some stress testing around it. Successively the models have become more sophisticated, moving from the 'one-period static' type to 'multi-period dynamic' models, involving the consistent stochastic simulation of assets and liabilities. Modern studies in this field rely on stochastic models that generate thousands of scenarios with different probabilities attached to each. The traditional AL modelling studies focused on asset-optimisation with a deterministic view on liabilities, but today the context is increasingly used to simulate the consequences of pension policies on different stakeholders while complying with the requirements of the regulating authorities. In this sense, ALM systems are used as integrated planning systems to simultaneously determine investment, funding and – if applicable – indexation policies thereby balancing the goals of the different stakeholders.

ALM models are common in many countries, however there are differences in how they are carried out, and the stringency with which the resulting strategic asset allocation is implemented. In all countries, ALM studies are carried out by outside actuaries or consultants; only the very large Dutch and US funds run ALM studies internally, often in parallel to an externally conducted study. Dutch pension funds can be regarded as most sophisticated in terms of ALM models. In the Netherlands, ALM is a widely accepted risk management tool. The new regulatory framework introduced the use of ALM studies, with stochastic analysis prescribed as of 2010. In Austria, the financial supervisory authority (FMA) has developed a scenario analysis model in order to simulate the consequences for members and beneficiaries, pension funds (Pensionskassen) and employers of different investment returns on asset classes. Germany also requires 'Pensionskassen' to regularly perform an ALM study, although the German market still lacks the Dutch sophistication. In the United Kingdom, on the other hand, there are still reservations against ALM. Though ALM models have proven a better fit for the real world scenarios encountered by pension funds, they do have their drawbacks, partly due to their complexity, making it harder for fund trustees or directors to understand and interpret. Arguably, in some countries investment oversight and trustee training have not always been able to keep pace with improvements in the sophistication of mathematical modelling techniques. Furthermore, it has been proclaimed that many ALM studies generated high-risk, high-return portfolios, rather than strictly liability- matching portfolios, as it is proposed by a school in financial economics that proclaims pension funds should avoid exposing sponsoring employers to risks that can be taken directly by shareholders of the sponsoring company.

However, more and more pension fund regulators are also starting recently to consider the use of ALM techniques to assess the resilience of the pension fund sector to different shocks.

In this chapter we will focus on the typical sophisticated internal ALM models used in Dutch pension funds, which adopts scenario based analysis combined with optimization model. They can offer better insight on the potentiality of the risk management approach. This general model will be applied in the second part of this dissertation. Next Paragraph provide an review of the literature on ALM Model, Paragraph three extensively describe the scenario-based approach, Paragraph four and five presents the optimization techniques and the evaluation of ALM strategies by means of ALM scores. Finally some advantages and disadvantages of the model are discussed and conclusions are drawn.

3.2 Literature on ALM Models

The existing contributions in the academic literature fall within two different and somewhat competing approaches to ALM. On the one hand, several authors have attempted to cast the ALM problem in a continuous-time framework, and extent Merton's intertemporal selection analysis (see Merton (1969, 1971)) to account for the presence of liability constraints in the asset allocation policy. A first step in the application of optimal portfolio selection theory to the problem of pension funds has been taken by Merton (1990) himself, who studies the allocation decision of a University that manages an endowment fund. In a similar spirit, Boulier et al. (1995) have formulated a continuous-time dynamic programming model of pension fund management. It contains all of the basic elements for modelling dynamic pension fund behaviour, and can be solved by means of analytical methods. Rudolf and Ziemba (1994) extend these results to the case of a time-varying opportunity set, where state variables are interpreted as currency rates that affect the value of the pension's asset portfolio. Also related is a paper by Sundaresan and Zapatero (1997), which is specifically aimed at asset allocation and retirement decisions in the case of a pension fund. This continuous-time stochastic control approach to ALM is appealing because it enjoys the desirable property of tractability and simplicity, allowing one to fully and explicitly understand the various mechanisms affecting the optimal allocation strategy. On the other hand, because of the simplicity of the modelling approach, such continuous-time models do not allow for a full and realistic account of uncertainty facing institutions in the context of asset-liability management. A second strand of the literature has therefore focused on developing more comprehensive models of uncertainty in an ALM context. This has led to the development of a stochastic programming approach to ALM, including Kallberg *et al*. (1982), Kusy and Ziemba (1986), or Mulvey and Vladimirou (1992). This strand of the literature is relatively close to industry practice, with one of the first successful commercial multistage stochastic programming applications appearing in the Russell-Yasuda Kasai Model (Cariño *et al*. (1994, 1998), Cariño and Ziemba (1998).

Other successful commercial applications include the Towers Perrin-Tillinghast ALM system of Mulvey *et al*. (2000), the fixed-income portfolio management models of Zenios (1995) and Beltratti *et al*. (1999), and the InnoALM system of Geyer *et al*. (2001). A good number of applications in asset-liability management are provided in Ziemba and Mulvey (1998) and Ziemba (2003). In most cases, stochastic programming models require the uncertainties be approximated by a scenario tree with a finite number of states of the world at each time.

Important practical issues such as transaction costs, multiple state variables, market incompleteness due to uncertainty in liability streams that is not spanned by existing securities, taxes and trading limits, regulatory restrictions and corporate policy requirements can be handled within the stochastic programming framework. On the other hand, this comes at the cost of tractability. Analytical solutions are not possible, and stochastic programming models need to be solved via numerical optimization. One solution is represented by the hybrid simulation / optimization scenario model developed by Boender (1996). It represents the starting point of the ALM studies in the Netherlands, characterized by opting for a practitioners/consultant perspective. This ALM model uses scenarios in an iterative learning process of evaluating and improving asset/liability strategies, sustained by simulation and optimisation. It is described in details in the next paragraph.

3.3 Methodology

The concepts of scenario analysis (see Kingsland L., 1982), also called Monte Carlo simulation or stochastic simulation, are often applied in ALM to model the economic risk and return factors. Instead of focus on a single future development, a large number of scenarios of economic variables are generated. Together with the strategic policy under consideration there are fed into a model which states all relations between policy instruments, scenario variables and relevant output measures with respect to the objectives of the stakeholders. Using these relations the model simulates what would happen to the objective of the stakeholders if the policy under consideration would be applied during the simulation period. It is important to underline that the scenarios should be neutral with respect to the objective and constraints of the various stakeholders. The scenario should represent one and the same, independent, macroeconomic world in which the pension fund as financial institution and its stakeholders need to operate.

The simulation/ optimization scenario model can be distinguished in two phase. In the first, the diagnostic phase, the asset/liability playing field is explored to reveal how potential ALM strategies behave in various economic environments with respect to costs and risks. . In particular, based on scenarios, different risk and returns can be calculated (as the expected return or probability of the solvency ratio falling below a given threshold) for a selected policy/strategy.

In the second, the phase of judgement and decision making, this process of successively testing and improving strategies is repeatedly carried out until a strategy emerges which agreed upon by all who carry responsibility for the pension fund and its sponsors and trustee. The decision makers evaluates the risk and returns measures (ALM-score) of the policy and decide whether it is satisfactory and in line with their objectives. Otherwise, alternative policies are analyzed and the process goes on until a satisfactory policy is obtained which meets the stakeholder's objectives and constraints as best as possible, given the assumptions made with respect to the simulated scenarios.

There are many reasons why scenario analysis is often preferred over alternative approaches. The first reason is the flexibility it offers to model complex interactions and relations within and between the components of an ALM problem. The second reason for the popularity of the scenarios, nowadays extended to the supervision authority, is that it offers great possibilities for learning about the problem under investigation besides just obtaining some "optimal" solution. The third reason, which applies more to the practical than to the academic applications of ALM, is that the model and the solutions obtained can be easily read by the decision maker and the more easily accepted by the stakeholders. Acceptation of these models by decision makers is crucial for the recommendations coming from these models to be actually implemented instead of remaining some interesting theoretical experiment.

3.3.1 Scenario generation

The quintessence of scenario analysis is that external uncertainties which ALM-decision makers have to take into account, i.e. inflation, interest rates, risk premiums of equity as well as actuarial dynamics, are modelled by a set of possible plausible future developments, referred to as scenarios. A definition of scenario is described in Brauers and Weber (1988): “a scenario is a description of a possible state of an organization’s future environment, considering possible developments of relevant interdependent factors of the environment”. The generation of good scenarios that well represent the future evolution of the key parameters is crucial to the success of the modelling effort. In a scenario analysis the uncertainties are modelled as a fan of scenarios, and not as a tree, which of course is only responsible since we restrain the use of the scenarios to simulation and not-anticipating optimization. These scenarios are generated in two steps. The scenarios of the economic environment are generated starting from historical data considering macroeconomics factors such as inflation and short and long interest rates. After that, financial market factors such as yield curve, credit spreads, dividend yields and their growth, earning forecasts and currency exchange values are considered. Typically these information are integrated with help of expert opinion of investment advisory committee, especially with regards to expected development of inflation, interest rate and equity risk premium. In our model these are the inputs for the construction of scenarios by means of a Vector Autoregressive Model (VAR). In the second step, the relevant actuarial quantities are developed using Push

Pull Markov probability model to determine the status (i.e. active/ non active, age, salary group) of the current and future member in each node of the scenarios, whereas the pension scheme of the plan is used to determine the corresponding actuarial quantities of the members in each node.

For being used in ALM models, the scenarios must satisfy some requirements. The first is the "comprehensiveness": the scenario must be generated having complete awareness of the model and the assumptions adopted and the scope behind their generation. In this way they provide a common framework for discussion and contribute to a better understanding between stakeholders and managers. The second is "coherence" of the scenarios with the financial and economic theories. The third is the scenarios must be "consistent with statistical expectation", that's to say, that they show the same expected values, standards deviation and correlations as observed in the historical data. The scenario generation process is further described in this chapter. Usually for an ALM application a pension funds used to work with a scenario set of 2500 scenarios with a horizon of 20 years.

3.3.2 Vector Autoregressive Model

The Vector Autoregressive Model (VAR) model was used in Russell-Yasuda Kasai Model, as in Carino and Ziemba (1998).VAR models were introduced by Sims (1980) as a forecasting method using historical data. In particular to construct year-frequency scenarios of the future development of the economic time series, we apply (log-)Normal Vector Autoregressive VAR models, where the values of the economic quantities in any year follow a multidimensional (log-)normal probability distribution, whose expected values are linear combinations of the realizations of the economic quantities in the previous years:

$$y_t \approx N(\mu + \Omega * [y_{t-1} - \mu], \Theta) \tag{3.1}$$

The model assumes stationarity, such that it may be necessary to transform the raw historic data, or that it may be necessary to include dummy variables for periods, such as the oil crisis, which violate stationarity. The estimation of the model proceeds in two steps. First the sample estimators are determined of the variance and covariance matrices denoted as V and W (to preserve stationarity, the denominators of these estimators is the number of sample points, and not the number of sample points minus the number series).

In the second steps, applying Yule Walker estimation method Ω and Θ are, respectively:

$$\Omega = V * W^{-1}; \Theta = V - W * V^{-1} * W^T \tag{3.2}$$

An important characteristics of the VAR model, which is crucial for the quality of the ALM analysis which is sustained by the model, is that if the parameters are estimated using Yule Walker method, then also with limited historic data the scenarios which are generated by the model will asymptotically display the same expected values, standard deviations and (auto)-correlations as observed in the applied historical dataset (see Steehouwer, 2005).

The next step, given parameters estimates of the VAR, which can be rewritten as:

$$y_{t+1} = a + By_t + \varepsilon_{t+1} \tag{3.3}$$

is to simulate recursively from the VAR model. For this, the estimated covariance matrix of the residuals is decomposed by means of the Cholesky (Gentle, J. E. 1998) matrix (C), such that $CC' = \sum_{\varepsilon}$. The decomposition is used to estimate values of ε_t. This is done by sampling a vector u from a standard normal distribution *N (0,1)* so that: $u \sim N(0,1)$ of which $Cu \sim N(0, CC')$ is derived. By multiplying the Cholesky decomposition with a vector of random numbers from a standard normal distribution, new shocks to the system are generated which gives simulations of $\varepsilon = Cu$. These values are used to recursively solve equation (3.3) in order to generate thousands of scenarios.

Starting from historical data of each asset class to be included in the portfolio, the VAR model is applied to generate scenarios of assets returns. Moreover, due to the need to analyze duration strategies, and due to the new regulations which impose the market-to-market valuation of the liabilities, the model generates yield curves. In ALM this implies that a yield curve has to be generated in each year of each scenario, in such a way that the relevant dynamics and correlations are in accordance with statistical expectation.

This is accomplished by using the Nelson Siegel model (Nelson & Siegel 1987), which is characterized by four parameters: β_0 is the long-term interest rate, β_1 is the difference between the interest rate with short maturity and the interest rate with longer maturity, β_2 is the curvature of the curve and affects the shape of the curve and τ is a scaling parameter that determines the rate of convergence to the long-term interest rate. The term structure of interest rate in each scenario and each year will be determined by combining the values of these four parameters using the following formula:

$$r_k = \beta_0 + (\beta_1 + \beta_2)\frac{(1 - e^{-k/\tau})}{k/\tau} - \beta_2 e^{-k/\tau} \tag{3.4}$$

$$\begin{aligned} \lim_{k\to\infty} r_k &= \beta_0 \\ \lim_{k\to 0} r_k &= \beta_0 + \beta_1 \end{aligned} \tag{3.5}$$

Where k is the maturity for each cash flow and τ is set by 1.8.

Despite the drawback that this model lacks a theoretical underpinning, it is the most widely applied model by the major central banks in the world as well as by the European Central Bank and by practitioners. The advantage of the Nelson & Siegel model is the ability to capture many of the typically observed shapes that the yield curve assumes over time. The three Nelson-Siegel components have a clear interpretation as short, medium and long-term components. These labels are the result of the contribution of each element to the yield curve. The long-term component is β_0, because it is constant at 1 and therefore the same for all maturities. The component β_1 is designated as the short-term component. It starts at 1 but then decays to zero at an exponential rate. The medium-term component is $\frac{(1 - e^{-k/\tau})}{k/\tau} - e^{-k/\tau}$, which starts at 0, increases for medium maturities and then decays to zero again, thereby creating a hump-shape. The decay parameter τ determines at which maturity this component reaches its maximum. The interest rate yield curve is generated by the four parameters and formula (6) for each (s,t) and it is used to discount all the future cash flows by changing the value of k

3.3.3 Liability estimation

The simulation of the liabilities in each node is accomplished in three phases. In the first phase a so-called Push Markov model is applied to generate the status of each current active and non-active plan member in each node. That is, given characteristics of the members, especially gender, age, salary group and years of service, matrices of transition probabilities are used to simulate future developments of the members with respect to survival, disability, resignation and career. This part of the model is called a Push Markov model since the stochastic behaviours of the members are independent. The survival probabilities are based on public actuarial tables. The expected future development of the size and structure of the employee force is input of the ALM model. Given the results of the Push Markov model, in the second phase a so called Pull Markov model is applied. This model successively fills vacancies by hiring new employees until the number of employees in each category in each node is as much as possible in accordance with specified numbers. The result of the first two phases of the generation process of the liabilities is that we know the status of each current and future active and non active member in each node of each scenario. Then, the pension scheme is applied to compute all the relevant actuarial quantities in each node, especially concern the actuarial cost, the pension payments and the value of the pension liabilities. Of special importance is the determination of the pension liabilities in each node. These are determined by discounting the future payments of the members in each node by the Nelson Siegel interest rate structure generated for the corresponding node.

3.4 Measure of evaluation of ALM strategies

The scenarios are used to evaluate the risk-return consequences of ALM policies by means of so called ALM-scores. In every nod, different risk and return measures can be computed depending on the definition of the ALM decision problem. With respect to the contribution policy, an indicator can be the "Expected contribution rate", which is defined as the average value of the observed contribution rates across scenarios. It can also define as a measure of the costs associated with a certain policy. The "Expected funding ratio", analogously to the previous definition, is defined as the average of the observed funded ratios over all the combinations time-scenario. Of course this is important information concerning which is the financial status of the fund deriving from the adoption of a certain policy. However, as the funding ratio expresses the ability of the fund to be solvent, it is also necessary to consider the volatility of this measure. In particular, the pension fund is not interested in symmetrical measures of risk as the variance, but only in the downside deviations from the expected funding ratio which can actually affects the ability of the fund remain solvent. Following the definition of portfolio return (Sortino and van der Meer, 1991), we can compute the "Downside deviation of the funding ratio" in year t as the standard deviation of the funded ratios which are smaller than 100% in year t. Another measure of risk related to the funding ratio is the probability of underfunding, which has been also adopted by the Dutch supervision authority for the definition of the solvency constraint. This risk measure is defined as the percentage of scenarios in which the pension fund is ever over a certain horizon confronted with underfunding. More sophisticated measures of risk are also based on Value-at-Risk methodology with the advantage to provide information also about the amount of underfunding. For instance, “the 1% 1year Surplus at Risk (SaR)” is defined as the amount of underfunding which occurs with 1% probability: i.e. if the 1%1-year SaR is equal to 10, then with a probability of 1% the funded ratio in any year will be smaller than 90%.

A similar VaR measure can be also computed with reference to the contribution policy as the 5% 3-year Contribution at Risk (CaR): it is defined as the minimal amount of contributions (expressed as the percentage of salaries in any year) which the sponsor has to pay with 5% over a period of 3 years.

For the indexation policy two main indicators are computed: the probability of missing indexation and the 5% 3-year Pension at Risk (PaR). The first is easily defined as the percentage of scenario-time combinations where the pension rights will not be fully compensate for the inflation prices. The latter gives more precise information also about the amount of missed indexation and is defined as the minimal indexation cuts which will occur with 5% probability over a 3 years horizon.

Clearly many different ALM scores can be developed to evaluate ALM policies/strategies/products and select those which constitute the efficient frontier with respect the applied ALM score according to the purpose of the scenario analysis.

3.5 Optimization

The model optimization deals with the definition of the initial asset mix, also defined as the strategic asset allocation. A strategic asset allocation (SAA) represents a set of portfolio weights showing how a particular investor, a pension fund in our case, wishes to spread his/her wealth between different generic asset classes over a long-term horizon. In pension fund literature, the traditional asset classes are bonds, equities and cash. In SAA some exogenous decision parameter must be defined: the length of the investment horizon, the revision frequency; the composition of the investment universe; the specification of an objective function; the risk appetite of the funds, that is to say, the amount of overall risk is willing to bear.

The investment horizon specifies the period over which expectations to risks and returns are formed, and thus the period over which the portfolio optimization is optimal *ex ante.* A precise quantification of the length of the investment horizon is also important for the explicit generation of risk and return measures (ALM score in our case). The actually chosen time-span should naturally follow from the institution's definition of long term. A pension funds should refer to the long-term maturity of its liabilities, so its investment horizon should be around 40 years. However in practice, 20 years are considered a long-term horizon and for those pension funds that choose to not rebalance the portfolio composition the horizon is reduced to 3-5 years. In the last years the financial world has been characterized by several crises that impose a higher frequency of an optimization analysis.

By revision frequency is meant the regular time intervals between dates when it is investigated whether the current strategic asset allocation is still in accordance with the overall

Once all this variables are defined, ALM decision problem must be considered.

Ideally in an ALM setting, the optimization models should also take into account all available policy instruments. That is, the decision variables of these models should not only concern the asset allocation, but also the contribution and indexation policies at least. Moreover, the ALM optimization should ideally take into account that a current decision can optimally be adapted in future circumstances. Important examples of ALM-models who optimally adapt current decision to future circumstances are the dynamic recourse optimization model as in Dert (1995); Geyer et al. (2005); Rudolf and Ziemba (2004), Siegmann (2003).

Due to the complex decision of the integral ALM-problem, in practise the Boender (1997) hybrid simulation/ optimization method is still the most used in practise. This model randomly generated and evaluates tens thousand random ALM policies and select ALM policies which constitute the efficient frontiers with respect to the applied ALM-scores (see Chapter 4). In principle, any parameter of an ALM policy can be a decision variable in this process. In this way the complete consistency between optimization and simulation is guaranteed.

As decision variable can be also used the traditional mean variance objective function based on ALM criteria as the maximization of the expected funding ratio or as in Chapter 5, the maximization of the indexation decision. In this case a different process can be followed. That's to say, in spite of evaluate thousand of different asset mix, it is possible to optimize the objective function by using an optimizer as the Solver in Excel. In this way the solver produces the best asset mix for each risk aversion parameter across all the scenarios as in the traditional portfolio optimization and provide with the efficient frontiers. Moreover, when the LDI paradigm is applied (see Chapter 1), the optimization can be focused also on one side of the portfolio, usually the return-seeking. That's to say, under the assumptions that the matching portfolio is able to perfectly match the liabilities in terms of cash flows, the return seeking portfolio is optimized in such a way to reach a return able to compensate for risk as inflation risks, convexity risk, and longevity risks and also improve the financial situation of the fund.

Of course, also these solutions can offer computational problems due to excessive complexity. In particular, since an objective function cannot be linked to too many decision variables, the others decisions related to the other policies (as contribution rate), must be included as constraints (as it also happens for the solvency constraints) or included in some assumption of the model. In these cases the analysis can result somewhat partial, but still useful to give insights in the dynamics of each policy through comparative analysis.

3.6 Conclusions

The scenario analysis presented here is widely adopted by several pension funds in the Netherlands and will be soon part of the risk assessment put in place by the supervisory authority. The several advantages of this methodology can be found in the simplicity in the interpretation of his results, a clear definition of the assumption underlying it and also the possibility to gain an insight in the future development of the economics environment.

However, many critics arise from practitioners against the VAR methodology relative to other model classes. The VAR model assumes stationarity while recently theories have revived that the economic environment is not stationary, but moves in compositions of longer term and shorter term business cycles. The effort in this sense must be to identify adequately these cycles and replicating them in the scenarios. VARs model are a-theoretical, since they use little theoretical information about the relationship between the variables to guide the specification of the model. Modelling returns with a vector auto-regressive (VAR) model on log-returns omits any information on price dependencies and long-term equilibrium to purely focus on short-term effects in return series. In order to address this shortcoming of the VAR model, the cointegration relationships should be taken into account and consider the sensitivities of model-implied dynamics with respect to these additional factors that capture price dependencies in addition to return dependencies. Only recently, the widely used macroeconomic error correction form of the vector-autoregressive model (VECM), or cointegrated VAR model has been suggested by the literature to replace the traditional VAR. It has the striking advantage, as compared to the standard VAR representation, that it explicitly distinguishes between short-term and long-term dynamics in the joint distribution of asset returns and inflation.

PART II

ALM MODELS IN PRACTICE

4
Hedging strategic currency risk

4.1 Introduction

Pension funds has shown an increasing interest in globally invested portfolios. The decrease in fixed income returns and the equity market turmoil justify a strong interest in the protection of the return on the assets. The first point raises the debate about how to manage currency risk, which can be defined as the additional risk by having exposures to exchange rate movements in the portfolio, that is to say that the currency risk causes the local currency value of the foreign receivables or investments to fluctuate. The second point underlines the question if an investor, in particular pension funds, should be exposed to the currency exposures of the investments or use the forward or future market to hedge the currency risk. Therefore the hedging decision of a pension fund is a relevant and complex area of investigation.

This chapter focuses on a risk management application based upon the ALM model analyzed in Chapter 3, to investigate about the decision concerning the hedging of the currency strategic risk. We take a Swiss-based pension fund perspective, answering the main question: *should a Swiss-based globally invested pension fund hedge currency risk?* Large Swiss investors like pension funds have to invest in foreign assets because of the limited investment opportunity set in the home market. Therefore such an investor will be faced with currency risk. Seen from a Swiss point of view, the Euro and US dollar are the main investment markets.

Therefore, we will consider a portfolio composed of stocks and bonds denominated in these two currencies. However, the analysis could be easily applied also to differently based pension funds, for instance to pension plans whose sponsor has multinational business.

The analysis is divided in two sub-questions: (*i*) Given different horizons, does hedging the currency risk affect the downside-risk of the Swiss-based pension? (*ii*) Does hedging the currency risk affect the expected return on the portfolio of the Swiss-based pension fund?

In order to give a recommendation about how the Swiss investor should approach the management of the strategic currency risk, we combined different methodologies on strategic asset allocation and asset and liability management (ALM). We compare hedged and unhedged returns, and evaluate them with respect to funding ratio return (FRR) and downside-risk measures as Value-at-Risk. Our results show that for a short-term investor different portfolios could be preferable, depending on the risk tolerance of the investor. For the conservative investor an unhedged portfolio seems to be the better choice, whereas for the more aggressive investor a US fully hedged and a partially (50%) EUR-hedged portfolio is preferable. In the long-run, the better portfolio is the EUR unhedged and US 50% hedged position, no matters what the risk tolerance of the investor is.

The structure of this chapter is as follows. Next paragraph describes the main findings in the literature on strategic asset allocation and globally invested portfolios. Paragraph 3 presents the VAR methodology and the scenario generation used in our research. Paragraph 4 describes the dataset. Paragraph 5 discusses our results on optimal hedging strategies for different horizons and risk tolerance. Paragraph 5 gives the conclusions and suggests recommendation for further research.

4.2 Main finding in the literature

Underlying the decision to hedge strategic currency risk, different intuitions and theoretical arguments can be found, and different academic results and empirical evidence have been discussed. Nevertheless the debate is still ongoing. The most important intuition in favor of a decision not to hedge a global portfolio is, of course, the geographical diversification benefits that can be obtained. A second consideration is that the expected currency return in the long-term is zero. This is also known as the covered interest parity (CIP) theorem. The exchange rate, even if traded daily, is not an asset generating cash flow or dividend. Thanks to mean-reversion effects, currency movements will cancel out each other in the long-term. In this perspective, hedging currency risk is not necessary in the long-term, therewith avoiding the cost of hedging.

On the other hand, the foreign exchange market shows a very high volatility in the short-term that could strongly affect the risks of international investments. It has been argued that even though the pension funds are by definition long-term investors, they also have to consider the short-term impact of their strategy (Boender et al., 2007). It occurs because the effects of the short-term volatility on the funding ratio (the ratio of assets to liabilities) affects the ability of the fund to comply to supervision one-year solvency standards (Boender and Vos, 2004), and increase the volatility of contribution rate paid by the sponsor.

The academic researchers are still debating on the existence of an optimal hedge ratio, which would be able to give the correct trade-off between the diversification benefits and the additional volatility risk. Solnik (1974) develops an equilibrium model of international CAPM of an investor who can choose to include domestic and foreign bonds and equities (100% hedged) in his portfolio. Under the assumption that local-currency values are uncorrelated with exchange rate movements, he finds the need for internationally diversified mutual funds: the optimal portfolio is diversified internationally in equities which are 100% hedged, but currency home-biased. Foreign currency is considered as a speculative asset and inclusion in the portfolio is justified only by the attempts to exploit short-term deviation from uncovered interest parity (UIP).

In their influential paper, Perold and Schulman (1988) argue that currency hedging reduces risk without a negative effect on returns, concluding that the optimal hedge ratio should be 100%.

Black (1989) has identified a universal formula for the optimal hedge ratio under perfect market conditions, defined as a fraction of total investment abroad to be covered. In his economic model, he finds that for equities this ratio should lie in a range between 30% and 75%, rejecting the 100% results of previous studies. His result is universal, meaning that it does not depend on which currency to hedge. Regarding the inclusion of foreign bonds in the portfolio, the Black's universal formula suggests a hedge ratio of 100% as the most appropriate.

More recently, Campbell, Viceira, and White (2003) argue that domestic currency is almost riskless in real terms in the short-term, while in the long-term, domestic currency is risky because the real interest rate varies over time. This implies that conservative long-term investors, as pension funds can be considered, should show interest in alternative assets that hedge real interest rate fluctuations. One of the possibilities they mention is to include foreign currency in the portfolio. This is due to the fact that investors to hedge against domestic real interest rate fluctuations can use foreign T-bills. The authors use the long-term portfolio choice theory of Campbell, Chan, and Viceira (2003), which allows for inter-temporal hedging demands as a framework for the analysis. Their main contribution to the debate is the fact that currency hedging on currencies with stable real interest rates, which are not correlated with their exchange rates (such as the US-dollar and the Euro), are the most attractive currencies to foreign investors. However, they excluded equities from their analysis, so nothing can be concluded about the optimal currency hedge ratio for a foreign equity position.

On the other extreme, Froot (1993) argues that hedging over short-horizons reduces risk substantially, while over long-horizons, equity returns are correlated with exchange rate movements and hedging often does not reduce risk at all. Moreover, Froot (1993) shows that complete hedging at horizons of several years (from one-year to eight years) actually increases the return variance of many portfolios. This is due to the fact that hedge returns at different horizons are driven by different risk factors. Short-term risk in foreign currency is mainly due to exchange rate movements. However, in the long-run purchasing power parity (PPP) holds, meaning that exchange rate movements are only temporarily affecting the hedging returns and real exchange rates over time remain roughly constant. At long-horizons, fluctuations in cross-country differences are due to unexpected inflation and the hedge returns are dominated by real interest differentials. This is why Froot (1993) decomposes hedge returns into real exchange rate movements and inflation/real interest rate surprises.

In one of the most recent papers, Chincarini (2007) investigates global currency hedging over 19 countries during the period 1999-2006. He finds that currency hedging does substantially lower portfolio risk, but concludes that hedging is not necessarily an optimal investment strategy over any given time period.

Campbell, Medeiros, and Viceira (2007) find that risk-minimizing investors should short (hold long positions in) those currencies that are more positively (negatively) correlated with equity returns. They also find that optimal currency positions tend to be long on USD, CHF, and EUR, and short on AUD, CAD, Yen, and GBP.

More closely to our investigation is the paper by Hoevenaars et al. (2007). They study the strategic asset allocation for a pension fund in an asset-and-liability context subject to inflation and interest rate risk and explore the inter-temporal covariance structure of assets and liabilities. Although their analysis is not focused on currency hedging with ALM model, they investigate the alternative asset classes that add value for long-term investors using VAR methodology that follows previous work by Campbell and Viceira (2002).

4.3 Methodology

The research questions are investigated through a comparison of hedged with unhedged returns of a globally invested portfolio of a Swiss-based pension. The analysis will refer to different horizons (one-year, 5, 10 and 20-years), for a pension fund can be considered as a long-term investor, but also has to take into account the short-term implications of its investment strategy. We generated scenarios from a VAR model estimation of asset and liability returns of a portfolio composed of EUR and US denominated stocks and bonds, liabilities and a Swiss risk-free rate with constant weights. Our hedging strategy consists in a one-year rolling forward strategy of which the returns are also generated from the VAR.

We will first describe the construction of foreign excess stock returns and forwards, continued by the generation of liabilities and bonds; then the VAR model is estimated; finally, we will present a description of the scenario generations and the risk measures used to evaluate the scenarios in terms of the FRR. All returns are expressed in logarithmic.

4.3.1 Foreign stock returns and forwards

Our starting point is the construction of foreign stock and forward returns, both denominated in Swiss-franc. Following Froot (1993), we construct these as follows:

$$r_{S,t}^{EUR} = s_{EUR,t} + e_{CHF/EUR,t} \tag{4.1}$$

and

$$r_{S,t}^{US} = s_{US,t} + e_{CHF/USD,t} \tag{4.2}$$

where $r_{S,t}^{EUR}$ and $r_{S,t}^{US}$ are respectively the EUR and US-stock returns denominated in Swiss-franc, $s_{EUR,t}$ and $s_{US,t}$ are respectively the EUR and US-stock returns denominated in their local currency, and $e_{CHF/EUR,t}$ and $e_{CHF/USD,t}$ are respectively the EUR and US exchange rate returns.

The forward returns are constructed as follows (Froot, 1993):

$$f_t^{EUR} = e_{CHF/EUR,t} + i_{EUR,t} - i_{CHF,t} \tag{4.3}$$

and

$$f_t^{US} = e_{CHF/USD,t} + i_{US,t} - i_{CHF,t} \tag{4.4}$$

where f_t^{EUR} and f_t^{US} are respectively the EUR and US-forward returns, $i_{EUR,t}$ and $i_{US,t}$ are respectively the EUR and US one-year interest rate returns and $i_{CHF,t}$ is the one-year Swiss interest rate return. Forward returns are thus calculated as exchange rate returns denominated in Swiss-franc plus the interest differential between the foreign and domestic one-year interest rate. If the forward returns show an unconditional mean different from zero, it means that the uncovered interest parity does not hold and that the hedging policy will affect the average returns of the assets and vice versa. These forward returns are often referred to as the exchange risk premium (Froot and Thaler, 1989).

4.3.2 Generation of liabilities and bonds

Pension funds are by definition characterized by long-run liabilities, which gives them the nature of long-term investors. For the Swiss investor we consider Swiss-franc denominated fully-indexed liabilities with only the interest rate as a risk driver[1].

The returns of liabilities are constructed by using the log-linear approximation described in Campbell and Viceira (2002) and assuming duration of 17 as in Hoevenaars et al. (2007):

$$r_{L,t+1} = \frac{1}{4} rr_{CHF,n-1,t+1} - D_{n,t}(rr_{CHF,n-1,t+1} - rr_{CHF,n,t}) \quad (4.5)$$

where $r_{L,t}$ is the return on liabilities, $rr_{CHF,n,t}$ is the 10-year Swiss nominal interest rate and $D_{n,t}$ denotes the duration. $rr_{CHF,n-1,t}$ is approximated by $rr_{CHF,n,t}$ (Hoevenaars et al., 2007).

[1] We exclude from our analysis the actuarial risk (as mortality risk and demographic risk) as in Hoevenaars *et al.* (2007).

For the construction of EUR and US-denominated bonds, the same log-linear approximation is used:

$$b_{n,t+1}^{EUR} = \frac{1}{4} y_{EUR,n-1,t+1} - D_{EUR,n,t}(y_{EUR,n-1,t+1} - y_{EUR,n,t}) \tag{4.6}$$

and

$$b_{n,t+1}^{US} = \frac{1}{4} y_{US,n-1,t+1} - D_{US,n,t}(y_{US,n-1,t+1} - y_{US,n,t}) \tag{4.7}$$

where $b_{n,t}^{EUR}$ and $b_{n,t}^{US}$ are respectively the return on EUR and US-bonds, n is the bond maturity, $y_{EUR,n,t}$ and $y_{US,n,t}$ are respectively the EUR and US-bond yields on the n-period maturity bond at time t, and $D_{EUR,n,t}$ and $D_{US,n,t}$ are respectively the duration of EUR and US-bonds. $y_{EUR,n-1,t}$ is approximately equal to $y_{EUR,n,t}$ and $y_{US,n-1,t}$ is approximated by $y_{US,n,t}$.

The duration $D_{n,t}$ of the bonds can be approximated by:

$$D_{n,t} = \frac{1-(1+Y_{n,t})^{-n}}{1-(1+Y_{n,t})^{-1}} \tag{4.8}$$

where $Y_{n,t}$ is the bond return. The foreign bond returns denominated in Swiss-franc are generated in a similar way as the foreign stock returns denominated in Swiss-franc using the following formulas:

$$r_{B,t}^{EUR} = b_{n,t}^{EUR} + e_{CHF/EUR,t}$$ (4.9)

$$r_{B,t}^{US} = b_{n,t}^{US} + e_{CHF/USD,t}$$ (4.10)

where $r_{B,t}^{EUR}$ and $r_{B,t}^{US}$ are respectively the returns on EUR and US-bonds denominated in Swiss-franc and $b_{n,t}^{EUR}$ and $b_{n,t}^{US}$ are respectively the returns on EUR and US-bonds, denominated in their local currency.

4.3.3 Vector Autoregressive model and scenario generation

The generation of the scenarios is based on an unrestricted vector autoregressive (VAR) first-order model, which is estimated using a historical dataset from 1988-Q1 to 2007-Q3. A VAR model of first order is preferable, given the number of variables included; this will give a more parsimonious model. Next to this, Campbell and Viceira (2002) show that every VAR model can be rewritten to a VAR (1) model.
The model can be defined as follows:

$$z_{t+1} = a + Bz_t + \varepsilon_{t+1} \tag{4.11}$$

where a denotes a vector of the intercepts, B denotes the matrix of coefficients, z_t is the state vector and ε_t is the vector of shocks to the system which is assumed to be normally distributed with zero mean and variance-covariance matrix Σ_ε: $\varepsilon_t \sim N(0, \Sigma_\varepsilon)$. The state vector is composed of eight variables, namely the return on US and EUR-stocks, on US and EUR-bonds, the return on liabilities, EUR and US-forward returns and the Swiss risk-free rate return. All variables are denominated in Swiss-franc.

The whole VAR system can be written as follows:

$$z_{t+1} = \begin{pmatrix} r_{t+1} \\ f_{t+1} \\ r_{f,t+1} \end{pmatrix} = \begin{pmatrix} a_{10} \\ a_{20} \\ a_{30} \end{pmatrix} + \begin{pmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{pmatrix} \begin{pmatrix} r_t \\ f_t \\ r_{f,t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \\ \varepsilon_{3,t+1} \end{pmatrix} \tag{4.12}$$

where r_t is a vector of returns on foreign stocks, foreign bonds and liabilities denominated in Swiss-franc, f_t a vector of forward returns, and $r_{f,t}$ is the Swiss risk-free rate return. As initial values of the state vector we use the unconditional historical mean of the variables as calculated from the dataset. In order to model the economic risk and risk factors of asset and liabilities, scenario analysis is often applied in ALM.

As described in the previous sub-section, the first part of generating the scenarios is accomplished by estimating a VAR model. The next step, given parameters estimates, is to simulate recursively from the VAR model. For this, the estimated covariance matrix of the residuals is decomposed by means of the Cholesky matrix (C), such that $CC' = \sum_{\varepsilon}$. The decomposition is used to estimate values of ε_t. This is done by sampling a vector u from a standard normal distribution N (0, 1) so that: $u \sim N(0,1)$ of which $Cu \sim N(0, CC')$ is derived. By multiplying the Cholesky decomposition with a vector of random numbers from a standard normal distribution, new shocks to the system are generated in such a way that $\varepsilon = Cu$. These values are used to solve equation (11) in order to generate 2,500 scenarios, from which the returns on the portfolios and liabilities are calculated.

4.3.4 Funding ratio returns and Value-at-Risk measures

In the literature on the topic, the ability of a pension fund to meet the liabilities is usually approached from a FRR perspective. Leibowitz, Kogelman and Bader (1994) and Hoevenaars et al. (2007) propose to use the funding ratio (F) as defined by the ratio of assets (A) to liabilities (L). The general funding ratio log-return r_F is defined as:

$$r_F = r_{A,t} - r_{L,t} \tag{4.13}$$

where $r_{A,t}$ is the return on assets and $r_{L,t}$ the return on liabilities.

This approach is more appropriate to compare hedged and unhedged returns, because it has the property to be independent from the initial funding ratio level (Leibowitz, Kogelman and Bader, 1994). The initial funding ratio value is a relevant factor that influences the level of risk tolerance. However to simplify our analysis we do not take the initial funding ratio value into consideration.

Partly following the Froot's (1993) approach, the formula for the return on assets is constructed as follows:

$$\begin{aligned} r_{A,t} &= [w_{S,EUR}(r_{S,t}^{EUR} - \phi^{EUR} f_t^{EUR}) + w_{S,US}(r_{S,t}^{US} - \phi^{US} f_t^{US}) \\ &+ w_{B,EUR}(r_{B,t}^{EUR} - \phi^{EUR} f_t^{EUR}) + w_{B,US}(r_{B,t}^{US} - \phi^{US} f_t^{US}) + w_{rf,CHR}(r_{f,t})] - c \end{aligned} \tag{4.14}$$

where $w_{EUR,x}$, $w_{US,x}$, $w_{EUR,b}$, $w_{US,b}$ and $w_{rf,CHF}$ are the portfolio weights on respectively EUR and US-stocks, EUR and US-bonds and the Swiss risk-free rate. ϕ^{EUR} and ϕ^{US} are the hedge ratios and c is the cost of implementing the hedging strategy. A cost of hedging of 20 basis-points per annum is used as assumed in Boender et al. (2007), which is a proxy for the transaction costs on forward markets. We set the weights according to the current strategic asset allocation of the Swiss pension fund, which is composed of 60% bonds (5% CHF, 30% EUR and 25% US) and 40% equity (20% EUR and 20% US).

To obtain the FRR in each quarter and for every scenario the corresponding return on liabilities is subtracted from equation (14). Several measures of evaluation of the FRR are used in ALM to obtain insight in the FRR dynamics, and are used in our analysis as criteria of comparison between different hedging policies.

First, the expected funding ratio is defined as the average of the observed FRRs over a certain horizon and over all the scenarios generated. Second, the standard deviation of the expected funding ratio is considered as an absolute measure of risk. However, these measures appear to be inappropriate for pension funds, due to the fact that these institutions have typically to oblige to downside-risk constraints (Boender et al., 2007). For this reason a measure of downside-risk often applied in ALM studies is the probability of underfunding, defined as the percentage of scenarios in a certain horizon in which the pension fund is confronted with a FRR below zero, $prob.(FRR < 0)$. This measure gives an indication of the ability of the pension fund to meet its obligations over a certain horizon and it is usually used as the definition of the minimum required regulatory buffer. Another downside-risk measure often applied in ALM studies is the value-at-risk (VaR) measure of the FRR at a 95% and 99% confidence level. Assuming normality of the FRR, we can compute the parametric VaR respectively as the 1% and 5% percentile of the FRR distribution over a certain horizon expressed in monetary value.

First, we choose to analyze three basis cases: a full-hedged position of the portfolio, an unhedged position of portfolio and a portfolio with 50% as hedge ratio for both currencies. Each portfolio and each horizon is compared with respect to the measures mentioned above, in order to answer our sub-questions.

Then, a EUR-hedged of 0%, 25%, 50%, 75% and 100% (keeping the USD-hedge at 50%) and a USD-hedged of 0%, 25%, 50%, 75% and 100% (keeping the EUR-hedge at 50%) are analyzed in order not only to decide whether the Swiss-based pension fund should hedge or not, but also to see what could be the best hedge position.

4.4 Dataset

Our quarterly data series start in 1988-Q1 and end in 2007-Q3 covering a period of 20 years. Data are obtained from DataStream[2] and the Swiss National Bank[3] and transformed in logarithmic returns. The total return indices of the MSCI EMU[4] and the MSCI USA[5] are used to represent the opportunity investment set of a Swiss investor on the US and EUR equity stock market, while the exchange rates Swiss-franc per Euro and Swiss-franc to US dollar are used to construct the stock returns denominated in Swiss-francs.

2 Thomson DataStream, accessed at the Vrije Universiteit.

3 http://www.snb.ch/en/iabout/stat/statpub/zidea/id/zidea, 22-01-2007.

4 Stock market index of European Economic and Monetary Union, consisting of almost 299 stocks in 11 developed European countries and maintained by Morgan Stanley Capital International, *MSCI Euro Indices: Methodology & Constituents January 1999*, available at http://www.msci.com. The MSCI EMU captures approximately 85% of the market cap of developed European countries.

5 Stock market index of United States of America consist of approximately 5,000 stocks in the US equity universe and is maintained by Morgan Stanley Capital International, *MSCI US Equity Indices: Methodology Book November 2005*, available at http://www.msci.com. The MSCI USA covers more than 98%5 of the market cap of the US equity universe.

In order to construct the forward returns we use the one-year German Interbank, one-year US Interbank and one-year Swiss Interbank. Due to the introduction of the Euro in 1999, we have chosen Germany as a proxy for the EUR interest rates, as Germany was the leading economy for this region during this time frame. The return of the series of the one-year German Interbank interest rate was denominated in German mark and then converted into Euro by using the synthetic exchange rate German-mark to Euro.

The liabilities were constructed using the ten-year spot interest rate on Swiss Confederation bonds. The three-month Swiss Treasury bill (T-bill) is used as a proxy for the Swiss risk-free rate.

For the construction of EUR and US-denominated bonds, the ten-year German benchmark bond yield and the ten-year US treasury benchmark bond yield were used and were converted into Swiss-franc denominated returns.

In total, our dataset consists of eight series with 78 observations per series. Table 4.1 shows an overview of the summary statistics of the series. The values for the mean and the standard deviation seem to confirm the conventional wisdom that stocks are a better investment in the long run than bonds with respect to risk, in particular for the foreign investment in US assets. The highest risk is associated with the three-month Swiss T-bill that could be explained by the reinvestment risk that affects the short-term investment in the long run, as in Hoevenaars et al. (2007). The statistics show a positive mean for the US-forward returns and a negative mean for the EUR-forward returns. The values of the historical means were used as initial values for the VAR estimation.

Table 4. 1: Summary statistics

The table reports the summary statistics of the following series: US-stock and EUR-stock, Swiss-liabilities, US-bonds and EUR-bonds, US-forwards and EUR-forwards and the Swiss risk-free rate. All series are in log-returns in Swiss-franc and on quarterly basis during the period 198- Q1 to 2007-Q3.

	$r_{S,t}^{US}$	$r_{S,t}^{EUR}$	$r_{L,t}$	$r_{B,t}^{US}$	$r_{B,t}^{EUR}$	f_t^{US}	f_t^{EUR}	$r_{f,t}$
MEAN	0.019	0.014	0.012	0.023	0.024	0.003	-0.003	0.002
MEDIAN	0.022	0.012	0.006	0.041	0.017	0.009	-0.014	-0.010
MAX	0.222	0.129	0.176	0.255	0.243	0.618	0.506	0.756
MIN	-0.186	-0.115	-0.112	-0.314	-0.201	-0.800	-0.422	-0.779
STD. DEV.	0.081	0.042	0.059	0.104	0.079	0.174	0.195	0.249
Observations	78	78	78	78	78	78	78	78

Table 4.2 reports the parameter estimates of the VAR model as defined in equation (11). The reported coefficients and the residual covariance matrix are used to generate 2.500 scenarios as explained in the methodology section. We tested the stability of this model and the results show that the roots of the coefficients are outside the unit circle, implying that the model is stationary.

All variables included do not show strong predictive power for the stock returns denominated in Swiss-franc. They are not correlated with their own values and seem to be following almost unpredictable patterns ("random walk"), which is also indicated by the low R^2. The liabilities show a significant coefficient for its own lag (positive) and for the risk-free rate (negative). The first relation can be explained by the flat dynamics of the 10-year Swiss interest rate and the assumption of duration of 17 for the liabilities, while the second could be due to the impacts of short-term shocks to the long-term structure. Next to this, the lagged value of the EUR-forward returns show a negative and significant coefficient with respect to the liabilities, but this cannot be easily explained. It could be driven by the relationship between the Swiss long-term interest rate (which is part of the liabilities) and the one-year interest differential (which composes the forward returns).

Table 4. 2: VAR estimation

The table reports the parameter estimates of the VAR model $z_{t+1} = a + Bz_t + \varepsilon_{t+1}$ (1). The first row shows the variables of our model, whereas the first column shows the lagged values of these variables. Significant coefficients at the 90% level are in bold. The t-statistics are given in parenthesis.

	$r_{S,t}^{US}$	$r_{S,t}^{EUR}$	$r_{L,t}$	$r_{B,t}^{US}$	$r_{B,t}^{EUR}$	f_t^{US}	f_t^{EUR}	$r_{f,t}$
$r_{S,t-1}^{US}$	-0.133	0.073	-0.155	**-0.421**	**-0.202**	-0.723	-0.522	0.159
	(-0.558)	(0.235)	(-0.937)	(-1.760)	(-1.712)	(-1.502)	(-1.295)	(0.243)
$r_{S,t-1}^{EUR}$	0.006	-0.142	0.008	0.283	0.121	0.515	0.248	0.012
	(0.033)	(-0.559)	(0.060)	(1.455)	(1.254)	(1.314)	(0.756)	(0.022)
$r_{L,t-1}$	**-0.404**	0.180	**0.334**	**-0.442**	0.045	0.286	0.299	-0.066
	(-1.844)	(0.629)	(2.195)	(-2.013)	(0.416)	(0.647)	(0.808)	(-0.110)
$r_{B,t-1}^{US}$	0.028	-0.162	0.105	0.275	0.123	**0.968**	**0.761**	-0.260
	(0.114)	(-0.499)	(0.609)	(1.105)	(1.000)	(1.933)	(1.813)	(-0.381)
$r_{B,t-1}^{EUR}$	0.124	-0.037	-0.083	-0.193	0.110	-0.250	-0.776	0.129
	(0.418)	(-0.096)	(-0.403)	(-0.647)	(0.744)	(-0.416)	(-1.541)	(0.158)
f_{t-1}^{US}	0.077	0.026	0.015	0.088	0.023	**0.311**	-0.080	0.165
	(0.915)	(0.232)	(0.263)	(1.036)	(0.548)	(1.818)	(-0.557)	(0.707)
f_{t-1}^{EUR}	0.048	0.162	**-0.125**	-0.114	-0.065	**-0.873**	**-0.613**	0.213
	(0.473)	(1.230)	(-1.790)	(-1.129)	(-1.297)	(-4.295)	(-3.601)	(0.770)
$r_{f,t-1}$	0.010	**0.161**	**-0.087**	**-0.110**	**-0.071**	**-0.561**	**-0.593**	**0.656**
	(0.194)	(2.310)	(-2.342)	(-2.072)	(-2.684)	(-5.230)	(-6.605)	(4.486)
C	0.031	0.025	0.011	0.027	0.012	-0.013	0.003	-0.002
	(2.910)	(1.786)	(1.491)	(2.586)	(2.341)	(-0.625)	(0.171)	(-0.075)
R^2	0.093	0.103	0.209	0.126	0.208	0.392	0.459	0.307

Both the liabilities and the EUR-forward returns are affected by preceding shocks in the Swiss short interest rate in a negative manner. This can be reasonably explained by the previous argument. An interesting result is that the EUR-forward return is a quite strong predictor (coefficient of -0,8) for the subsequent US-forward value. This suggests that forward returns depend on the dynamics between the two exchange rates and that this relation is dominated by the EUR fluctuations.

The bond returns are mostly affected by previous values of the Swiss-franc denominated US-stock returns in a quite similar way.

The risk-free rate presents only a significant coefficient for its own lag, due to the fact that it is a short-term rate and therefore does not encounter many shocks that are outside its own dynamics.

4.5 Optimal hedging strategies

To answer the question whether or not a Swiss-based pension fund should hedge currency risk, we first need to take into consideration three basis hedge positions, namely a zero-hedge, a full-hedge and a 50% hedge for both currencies.

Table 4.3 shows an overview of the estimated expected FRR, the standard deviation of the FRR, the probability of underfunding and the value-at-risk at the 95% and 99% level with respect to the different horizons. Even though the standard deviation of the FRR is not that relevant for a pension fund, we show this measure for completeness, giving more importance to the downside-risk measures.

Table 4. 3: Overview of results

The table shows expected FRR, the standard deviation of the FRR, the probability of underfunding and the value-at-risk at the 95% and 99% levels at the different horizons. We consider three basis hedge positions, namely zero-hedge, a full-hedge and a 50% hedge for both currencies. The value-at-risk is computed with respect to an investment of 1.000.000 CHF.

Horizon	*T=1*	*T=5*	*T=10*	*T=20*
		Hedging strategy: φus = φeur = 0		
Mean (FRR)	2.293%	2.514%	2.579%	2.597%
St.dev. FRR	3.072%	1.372%	0.967%	0.688%
P(FRR < 0)	24.360%	3.200%	0.480%	0.000%
VaR (95%)	25,736.87	---	---	---
VaR (99%)	46,090.58	6,460.81	---	---
		Hedging strategy: φus = φeur = 0.5		
Mean (FRR)	2.409%	2.555%	2.620%	2.622%
St.dev. FRR	5.297%	2.415%	1.718%	1.192%
P(FRR < 0)	33.280%	15.000%	6.200%	1.360%
VaR (95%)	62,739.97	13,212.41	1,703.95	---
VaR (99%)	93,343.90	29,097.28	14,736.51	1,024.73
		Hedging strategy: φus = φeur = 1		
Mean (FRR)	2.526%	2.597%	2.661%	2.646%
St.dev. FRR	8.181%	3.737%	2.668%	1.841%
P(FRR < 0)	37.920%	24.440%	15.720%	7.800%
VaR (95%)	108,141.22	35,332.16	16,590.08	4,233.62
VaR (99%)	160,678.47	58,178.11	35,237.72	15,292.49

Figure 4.1 shows that full-hedged portfolio at the one-year horizon has a higher expected return than the unhedged portfolio, while the 50%-50% hedged portfolio lies in between. However, the differences decrease as time progresses and almost disappear at the 20-year horizon.

Hence, from an expected return point-of-view, the impact of a hedging strategy is only valuable in the short-run, but it does not affect the expected FRR in the long run.

As for the risk measures, the standard deviation of the FRR and the probability of underfunding show similar results. In both cases the full-hedged portfolio is the most risky, even though they follow different dynamics over time. Concerning the probability of underfunding it is evident that the higher risk is related to the shorter term, but also that the highest risk spreads between the three portfolios is at the 5-year horizon and then decreases progressively.

The value-at-risk shows that the highest maximum potential loss at the two confidence levels is related to the one-year horizon, and that it reduces over time. Both at the 95% and 99% levels the riskier is the full-hedged portfolio at each horizon. Comparing the unhedged and the 50%-50% portfolios, the latter presents higher maximum expected loss at one-year and 5-years. At 10-years horizon the 50%-50% portfolio is still risky but only at the 99% level, while both these portfolio present a VaR value of 0 at the 20-year horizon.

An interesting result is that at the 20-year horizon the spread in the probability of underfunding between the full-hedged and unhedged position has become smaller (but is still positive) while the spread between the unhedged and the 50%-50% hedge portfolio disappears. The full-hedged position clearly has a higher risk than the 50%-50% portfolio.

Looking at the results from the basis case, it seems that the 50%-50% hedge portfolio appears to be the most preferable portfolio in the long run. It ensures higher returns in each horizon and it has the same probability of underfunding as the zero-hedge position in the long run. However, for shorter horizons, the downside-risk of the 50%-50% portfolio is higher than the downside-risk of the zero-hedge-portfolios.

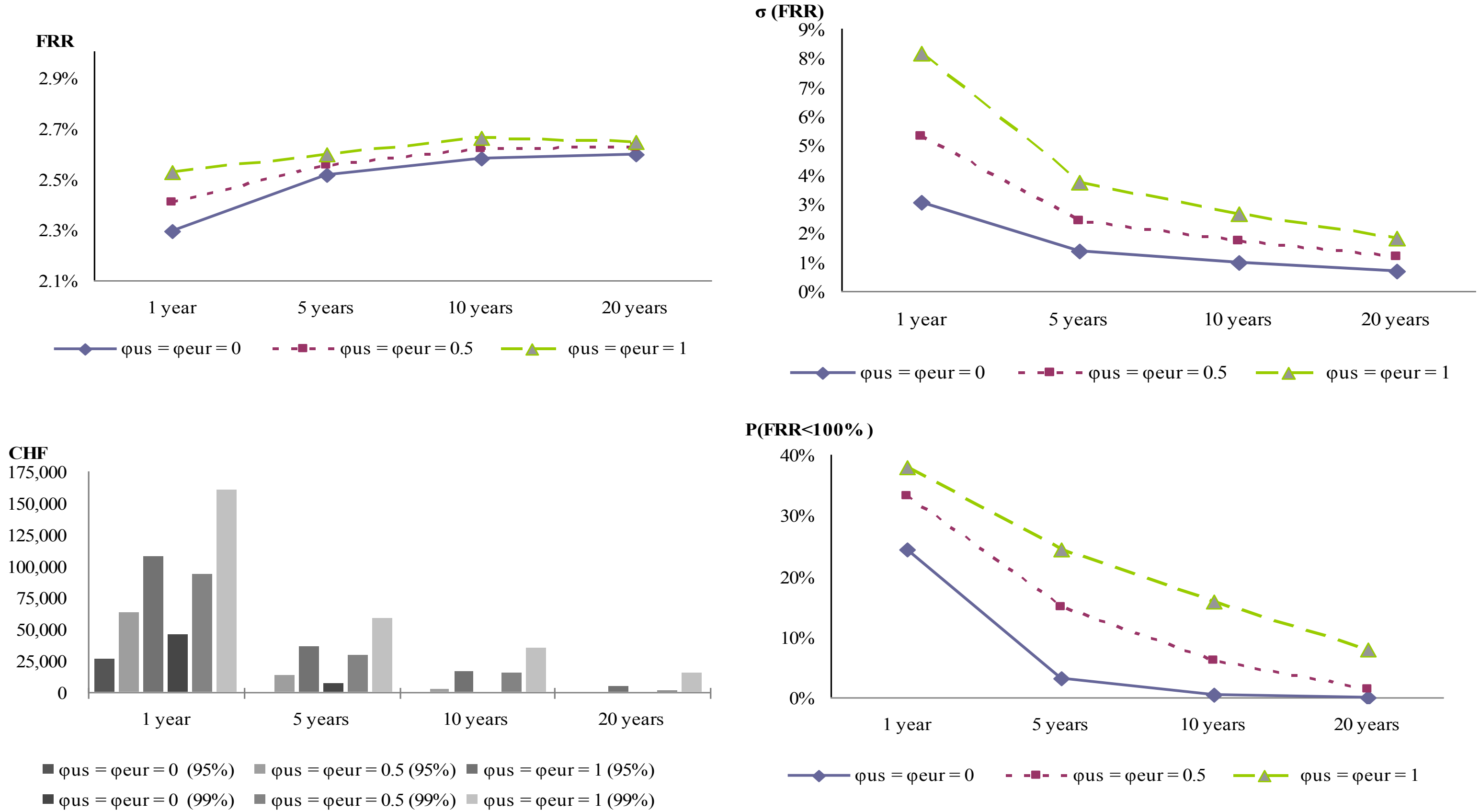

Figure 4. 1: Expected FRR, standard deviation of FRR, value-at-risk expressed in Swiss-franc and probability of underfunding at different horizons. We consider three basis hedge positions, namely zero-hedge, a full-hedge and a 50% hedge for both currencies. The value-at-risk is computed with respect to an investment of 1.000.000 CHF.

Figure 4.2 shows the risk-return trade-off at different horizons and for different hedge positions. The 50%-50% hedge portfolio is used as a reference point, because it seems to be the better solution among the three basis portfolios in the long run. To analyze other hedge positions, we let one currency hedge remain at 50%, whereas the other is allowed to change from 0% to 100%.

At the one-year horizon, Figure 4.2 shows that among the three basis cases there is not a preferable portfolio and that the best way to hedge the portfolio will depend on the risk tolerance of the investor. Taking into account the other alternative hedging policies, the full-hedged position is dominated by the portfolios with the hedge ratios for EUR equal to 25% and 0% while the US dollar hedge ratio is set at 50%. However, these portfolios cannot be compared to the unhedged position, because it has the lowest return but also the lowest risk. Therefore, the latter portfolio should be preferred by the conservative investor. The highest expected return belongs to the portfolio with a EUR-hedge ratio equal to 50% and fully US-hedged. A more aggressive investor should choose this portfolio. Hence, among all the combinations the choice is restricted between these four portfolios.

At the 5-years horizon the results are quite similar even though the probability of underfunding ranges now between 3% and 28%, while at the one-year horizon it was between 24% and 39%. This shift is even more pronounced at the 10-year horizon, where the range of the probability of underfunding is now within 0% and 17%. At this latter horizon, the two curves representing all the alternative hedging policies get closer to the vertical axis, but still the four preferable portfolios are the same as one-year horizon.

At the 20-year horizon the preferable portfolio is US-hedged at 50%, and not EUR-hedged. This portfolio is preferable compared to the full-hedge and unhedged positions as well as in comparison with all the other combinations. At this horizon the probability of underfunding ranges between 0% and 7% and all the portfolios lie very close to each other. There are two portfolios with a downside-risk equal to zero, but this portfolio is the one that presents a higher expected FRR.

Our results show that in the long run a hedged portfolio should be preferred with respect to an unhedged portfolio for the Swiss investor. However, in the short-run an unhedged position could be an option for a conservative investor. It is not possible to point out which portfolio is the better one in the short-run. For a conservative investor, the unhedged position seems to be the better choice; as for the more aggressive investor the US fully hedged and the 50% EUR-hedge seems to provide a better risk-return trade-off.

The results confirm the previous findings, when the transaction costs of the forward strategy used for hedging the portfolios is taken into account (*see Appendix*).

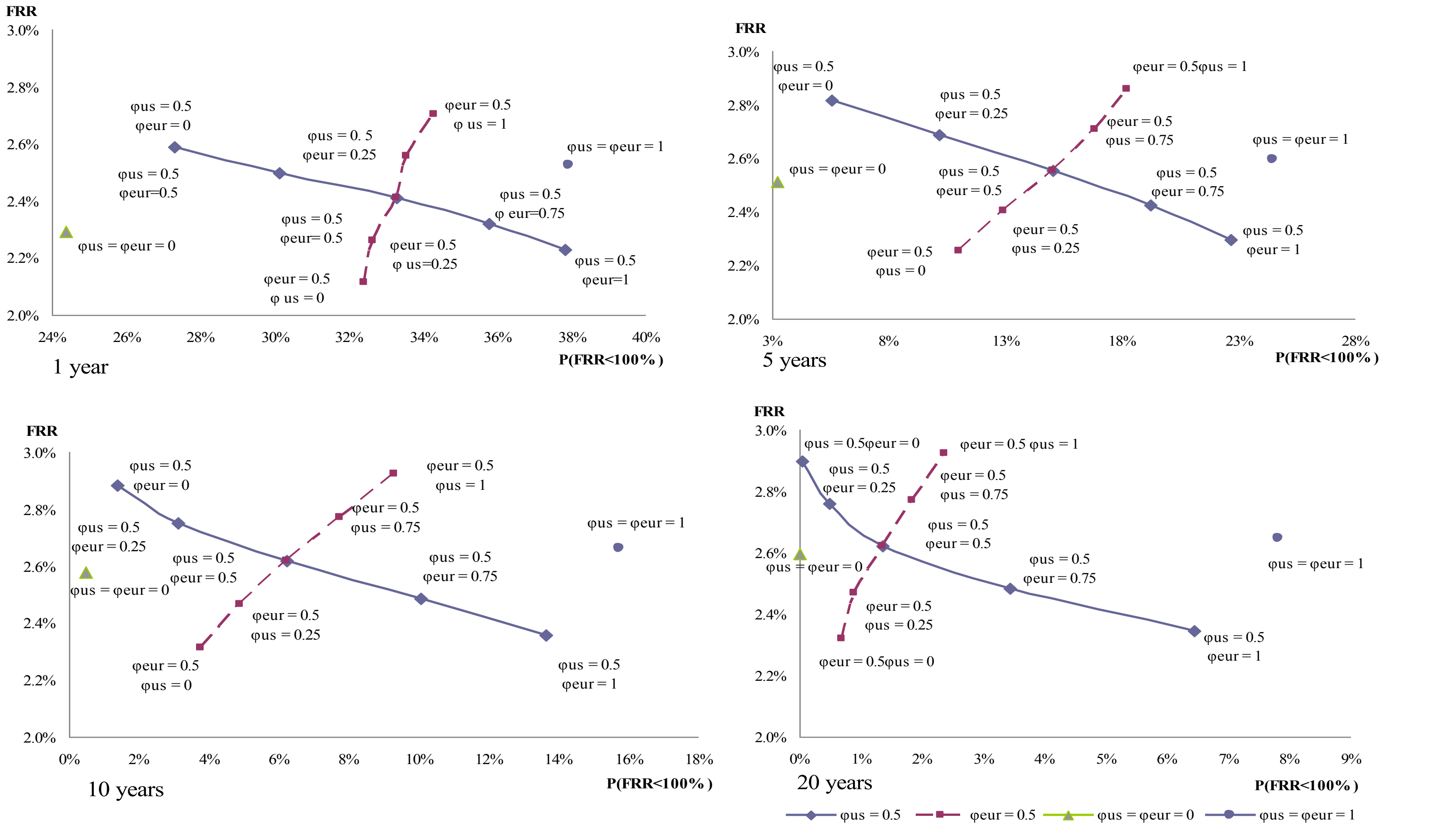

Figure 4. 2: Alternative hedge positions at different horizons. The x-axis denotes the probability of underfunding, whereas the y-axis shows the expected FRR. The reference point is the 50%-50% hedged portfolio, and from this point, while one currency hedge remains at 50%, the other one is allowed to change from 0% to 100%. The positions shown are 0%, 25%, 50%, 75% and 100%. For comparison, the zero hedge portfolios and the fully hedged portfolio are also shown.

4.6 Conclusion

This chapter has investigated if a Swiss based globally invested pension fund should hedge its current strategic asset allocation composed by assets denominated in EUR and US dollar. Because pension funds are long term investors but also have to take into consideration the short term implications of their long term strategy, we argue that this decision should be analyzed at different horizon of investment. We choose to evaluate this portfolio from a funding ratio return perspective. We considered three basis cases, namely a zero hedge, a full hedge and a 50%-50% hedge portfolio where the hedging policies are implemented through a one-year rolling forward strategy. Next we also considered alternative hedging policies.

The analysis of the basis cases reveals that the hedging strategy both affects the expected funding ratio return and the measure of downside risk (i.e. probability of underfunding and the value at risk). Partly in contrast with the results of the literature, the comparison of alternative hedging positions with the basis cases reveals that the unhedged position is a preferable portfolio in the short term for a conservative investor, while in the long run an optimal portfolio should have a US hedge ratio of 50% and EUR hedge ratio of 0%. The inclusion of the transaction costs in our analysis confirms our results.

As mentioned in Campbell and Viceira (2004), relying on the estimation procedure, to infer the parameters of the VAR(1) model might lead to biased estimates of the coefficients of the variables included in the model because some return forecasting variables are highly persistent. To correct these biases they suggest a bootstrapping procedure to check the robustness of the VAR coefficients. Other possible extensions of the methodology concern the variance-covariance structure of residuals and the definition of the intercepts and the slope coefficients. Regarding the first case, in our analysis we assume a constant variance-covariance matrix of the shocks to generate our scenarios. It could be argued that there is a need for a time-varying matrix. However, as argued in Campbell and Viceira (2004), this argument might not be too relevant for an investor with a long-term investment perspective. On the other hand, given the short-term implications of long-term strategies for a pension fund, this extension of the model could be valuable to investigate the robustness of the results of this application. To estimate the VAR, we relied on historical data. However, it is possible to introduce prior views about some of these parameters using Bayesian methods and possibly combining them with estimates from the data (Boender et al., 2007).

A more sophisticated methodology could be to implement a structural VAR model which takes into account the possible co-integration of the different time-series by imposing restrictions on the residual covariance matrix (Garratt et al., 1999). Another consideration regards relaxing the assumptions we put in place in order to simplify our analysis. In our analysis we assume a fixed current strategic asset allocation for the Swiss pension funds and hedging policies based only on forwards markets. Next to this, we impose that the pension fund does not rebalance its portfolio. Constant rebalancing of the portfolio over time is not common for a pension fund with long-term investments. However, as mentioned in Hoevenaars et al. (2007), the strategic investment plan of a pension fund is normally reviewed once every three to five years. Allowing for this kind of rebalancing could better fit the actual investment strategy of pension funds.

Another way of relaxing the static portfolio weights is by investigating the optimal asset allocation of this pension fund, for instance by minimizing the probability of underfunding. The optimization could also include the hedge ratios and the risk tolerance (i.e. dependence on the initial funding ratio). However, by doing so the focus of the research shifts to finding the optimal weights of the internationally diversified portfolio, and will therefore not investigate anymore what is the optimal hedge position for the current asset allocation of the Swiss investor. In our research we assumed that the asset allocation decision is taken before the hedging decisions are made, as usually happens in practice. Hedging policies can be also implemented through the currency option markets. Even though at the moment the forward markets are the reference market to hedge against currency risk, the option markets are rapidly growing in importance. Optimal hedging ratios could be investigated by using other hedging instruments. Next to this, other currencies could be included in the portfolio, to check the robustness of our results. An out-of-sample comparison could be useful as well.

4.7 Appendix A: Overview of literature on global invested portfolios

Table A. 1: Overview of literature on currency hedging

The table represents an overview of the main findings concerning currency hedging (published between 1988 and 2007). The list is not exhaustive.

Authors of study	***Main focus***	***Main variables and measures risk-evaluation***	***Evidence for currency hedging over long horizons***	***Investment horizons***	***Data used***
Campbell, Medeiros, and Viceira (2007)	Exploration of the particular demand for foreign currency that results from the desire to manage equity and bond risks.	Main variables: exchange rates, short-term interest rates and yields on long-term bonds. Evaluation: variance of portfolio.	No	From one month to a year.	Data of seven countries: Australia, Canada, Euroland, Japan, Switzerland, UK and US form 1975:7-2005:12 (monthly / quarterly)
Campbell, Viceira, and White (2003)	Empirical importance of foreign currency as a hedge against real interest rate risk for long-term investors. Violation of the presumption that short-term debt portfolios should always be fully domestic.	Main variables: ex post real short rate, real exchange rate and CPI inflation. Evaluation: annualized standard deviation.	Yes, foreign currency is an attractive asset class for long-term investors as a hedge against domestic interest rate risk.	From one quarter to 25 years.	Data of four countries: US, UK, Germany, and Japan from 1973:1-2001:4 (quarterly)

Chincarini (2007)	Analysis of hedging currency following three hedging techniques and compare them against the strategy of no hedging: (*i*) a hedge ratio of 1, (*ii*) a historically optimal hedge ratio, and (*iii*) a strategy of hedging in which only a part of the entire currencies in the global portfolio is exercised.	Main variables: summary statistics of equity, spot currency, and futures currency data Evaluation: annualised monthly mean returns, annualised standard deviation of monthly returns and Sharpe ratio	Findings for currency hedging in a single currency: use a hedge ratio of one. Findings for currency hedging for a global portfolio (analysed from 1999 to 2006): hedging currency risk was not beneficial; it did not significantly reduce the monthly volatility of the portfolio neither did it improve the risk-adjusted performance of the portfolio.	He does not consider different investment horizons, but the period from 1999 to 2006	Data of the countries: Australia, Brazil, Britain, Canada, Denmark, Europe, Hong Kong, India, Japan, Mexico, New Zealand, Norway, Singapore, South Africa, South Korea, Sweden, Switzerland, Taiwan, Thailand, and US from 1980:2-2006:8 (monthly)
Froot (1993)	Re-examination of the widely held wisdom that currency exposure of international investments should be entirely hedged.	Main variables: stock prices, interest rates, exchange-rates and CPI inflation Evaluation: mean and standard deviation	No	From one to eight years	Data of the countries: United States and United Kingdom from 1802-1990

Hoevenaars et al. (2007)	Strategic asset allocation for long-term investors (i.e. pension funds) and exploration of the intertemporal covariance structure of assets and liabilities.	Main variables: stocks, government bonds, corporate bonds, T-bills, listed real estate, commodities and hedge funds Evaluation: annualized volatility, correlations with stocks and bonds, inflation hedge properties and real interest rate properties	No	One, five, ten and twenty-five years	Data of the United States from 1952:2-2005:4 (quarterly)
Perold and Schulman (1988)	Analysis on currency hedging as a long run investment policy.	Main variables: stock- and bond markets Evaluation: annualized standard deviation of quarterly real USD returns of unhedged vs. hedged portfolios	Yes, they argue that it is better to plan long run investment strategy in terms of hedged portfolios than unhedged portfolios	They do not consider different investment horizons, but the period from 1978 to 1987	Data of the countries: US, Japan, UK, Germany and Non-US from 1978:1-1987:4 (quarterly)

4.8 Appendix B: Results including transaction costs

Table A. 2: Overview of the results including transaction costs.
The table shows an overview of the estimated expected funding ratio return, the standard deviation of the FRR, the probability of underfunding and the value at risk at the 95% and 99% levels with respect to the different horizons, all including transactions costs of the forward strategy. Transaction costs are set at 20 basis points per annum. We consider three basis hedge positions, namely zero hedge, a full hedge and a 50% hedge for both currencies. The value at risk is computed with respect to an investment of 1.000.000 CHF. The results in this table are comparable to the results in table 4.3.

Horizon	*T=1*	*T=5*	*T=10*	*T=20*
	Hedging strategy:	$\varphi us = \varphi eur = 0$		
Mean FRR	2.320%	2.531%	2.522%	2.53%
St.dev.FRR	3.122%	1.397%	0.991%	0.71%
P(FRR < 100%)	22.000%	3.960%	0.440%	0%
VaR (95%)	28,524.14	---	---	---
VaR (99%)	51,851.66	8,585.06	---	---
	Hedging strategy:	$\varphi us = \varphi eur = 0.5$		
Mean of FRR	2.471%	2.595%	2.555%	2.551%
St.dev. FRR	5.478%	2.412%	1.699%	1.203%
P(FRR < 100%)	31.320%	14.360%	6.960%	2.120%
VaR (95%)	68,025.89	14,999.95	2,765.91	---
VaR (99%)	107,878.32	31,431.35	14,171.22	3,515.88
	Hedging strategy:	$\varphi us = \varphi eur = 1$		
Mean of FRR	2.622%	2.659%	2.588%	2.574%
St.dev. FRR	8.446%	3.701%	2.613%	1.849%
P(FRR < 100%)	36.840%	23.640%	16.200%	8.480%
VaR (95%)	115,077.62	35,662.02	16,553.24	5,449.86
VaR (99%)	175,982.05	61,925.01	34,751.92	19,746.51

5

Conditional indexation policy and alternative assets:

A model for maximizing purchasing power of participants

5.1 Introduction

The recent turmoil in the financial market sets even more challenges in terms of performance for pension funds, among the major investors in the stock markets. These challenges must be added to the difficulties already faced by these investors in the last decades (in particular during the pension crisis of the 2000-2003), because of the strong reduction of the equity premium, the decline in long-term bond rates, the ageing of the population, the stricter supervision adopted by the regulators and the accounting innovation in terms of fair valuation of the liabilities (IFRS). As a consequence, nowadays interest rate risk, equity risk, longevity risk and the inflation risk have to be taken into account in the definition of the investment policies as crucial risk-drivers for the solvability. As for a defined-contribution pension plan, the impact of the financial crisis depends critically on pension fund asset allocation and the member's age, for the main concern of a defined-benefit pension fund is the reduction of the funding level. The retirement income provided by defined benefit pension plan is in principle unaffected by changes in investment return, but lower asset prices worsen their financial solvency. In the last years many of the DB pension funds in OECD countries reported lower funding levels and in some cases large funding gaps (OECD 2009). Whereas the impact of the financial crisis is not such to harm the solvability of DB pension plans, the reduction of the funding levels resulted mainly in a reduction in the indexation granted to pension fund's participants in countries where the indexation of benefits is conditional. These pension funds will most likely react to lower funding ratio by stopping the indexation of benefits to wage or price inflation until funding level recovers.

The indexation represents a correction of the pension rights aimed at compensating the loss in terms of purchasing power due to inflation rate increases and therefore offers a hedge against the purchasing power risk faced by pension participants. The full indexation of the liabilities has been for last decades an undisputed guarantee offered to the participants of a pension fund, but it has become less sustainable for many DB pension funds since the 2000-2003 stock market collapse. Most of them opted to voluntary and conditional/limited indexation policy, depending on the financial position of the fund. It means that the compensation can also be null or only partial when the funding ratio falls below required level. In the UK, indexation is typically restricted to the range of 0%-5% per year (limited indexation). In the Netherlands pension funds mostly opted for a solution consisting in a conditional indexation: the decision to grant indexation depends on the nominal funding ratio defined as the ratio of assets to liabilities. If the funding ratio falls below threshold level, indexation is limited or skipped altogether (assuming the features of an option) (de Jong 2008). However, even if not explicitly stated in the pension contracts, most of the Dutch pension funds states that the maximal price or wage indexation is aimed for (Bikker and Vlaar 2006). From a participant's perspective, the conditional indexation implies that the "indexation risk" (or purchasing power risk) partly translates from the pension fund to its participants. This solution has been strongly rejected by pension fund's participants, given the worldwide recognised assumption that pensioners aim to keep constant their standard of living after retirement (Modigliani 1986). Indeed, inflation risk can strongly affect the pension rights accrued during the working years resulting in a loss of the purchasing power of savings at the retirement. The Figure 5.1 shows the time series of inflation in Euro Area and in the Netherlands. The Dutch inflation shows increasing volatility in the years between 2000 and 2003, whereas the inflation relative to the Euro Area is stable around the 2%, which is the target set by European Central Bank to reach the main goal of the price stability. The figure supports the evidence that the inflation trend have shown over time wide fluctuations around the average of 2% from which the risk arises. To give an example of the significant impact of inflation on the purchasing power over a long horizon, the average inflation in the Netherlands in the last 10 years of 3.21% has corresponded to a loss of 271 Euro in the purchasing power of a pension right of 1000 Euro. Given that the horizon of investment of a pensions fund's participant is around 40 years, clearly the indexation policy is dramatically important for the participants of a pension fund. Moreover, more recently, the consumer price index (CPI) inflation has been revised up more than one percentage point, increasing the need for investors, and especially for pension funds that face pension payments that are indexed with respect to CPI or wage level index, to hedge against unexpected changes in price levels. This trend is likely to continue for the next future, despite the current crisis, given the current long-term increased demand pressure on food and energy resources. Inflation hedging should remain an important component of long run investment policy.

Source: Eurostat

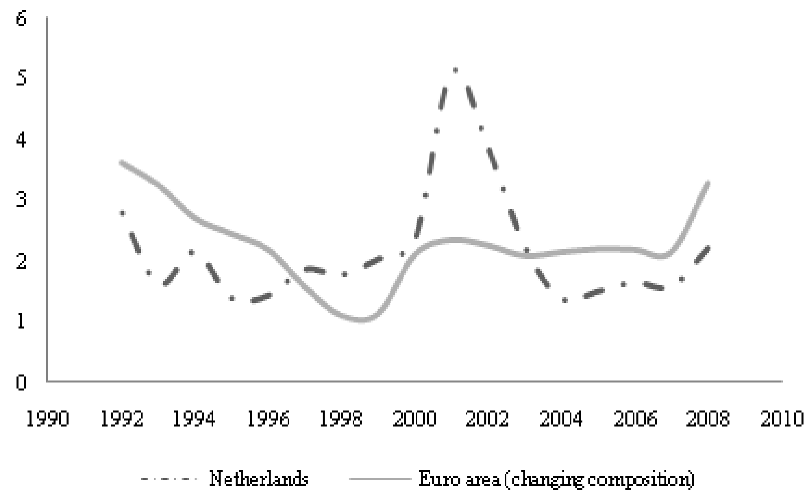

Figure 5. 1: Inflation in Euro Area and Netherlands as %.

The Figure shows the annual average rate of change of Harmonized Indices of Consumer Prices (HICPs) for the last 20 years in Euro Area and in the Netherlands, expressed as percentage. HICPs are also used by the ECB for inflation monitoring.

More precisely, as conditional indexation are often defined in terms of the nominal funding ratio, the indexation risk is not purely on inflation risk, but a combination of inflation, interest rate and equity risk. From the pension fund management perspective, the solution to offer only conditional indexation has been seen as a good compromise given the adverse financial market conditions. However, several criticisms have been raised on pension fund management because it strongly relies on positive equity premium and it does not take into the appropriate consideration the indexation target. The traditional asset allocation is typically composed by a 40-60% invested in equity and real estate and the remainder invested in nominal bonds. As underlined by de Jong (2006) this portfolio definition implies that “the expected return on the actual portfolio is higher than the expected return on index-linked bonds, if there is a positive equity premium. But the risk is also larger and especially the inflation hedge of the portfolio is rather weak. Given the importance of the purchasing power of pension rights and the recognised social and political role played by the pension funds, we assess that a new definition of the "pension deal" also implies a new definition of the criteria underlying the asset allocation of the funds. In the recent financial crisis, due to the current return-oriented pension fund management, we have seen the pension funds to be preoccupied in reducing their exposure to highly risky investment and be forced to sell part of their equity holding, even at a loss, to respect the regulatory standards. This type of investment strategy (combined with risk-based supervision) has also the characteristic to be highly pro-cyclical: during economic expansion the pension fund is willing to bear more risk to obtain higher return, but when there is a downturn it leads to severe losses and consequentially to the reduction of the indexation at the expense of the pension fund participants.

In this application we aim at a definition of a pension fund's portfolio having as target the maximal indexation of the liabilities, under the consideration that is not the maximisation of the return but the full indexation of the pension rights to have the priority. The ability to reach this target will be tested by introducing real asset as property and commodity in the portfolio that should offer inflation-hedging properties in the long run, and imposing annual regulatory constraints. By the definition of a simulation/optimization model in an Asset and Liability Management (ALM) context, we adopt a new objective function represented by the indexation decision, conditional on the nominal funding ratio. We use the traditional mean-variance framework (Markovitz 1952) combined with a simulation model as in Boender (1997). According to the "liability driven investment" (LDI) technique promoted by a number of investment banks (as Morgan Stanley) over the past few years, the model assumes that the pension fund divides its portfolio in two parts (*see* Chapter 1). The first part (the Matching Portfolio) must be able to meet the nominal liabilities over time adopting duration matching strategy. We set this portfolio to earn a return equal to the nominal growth of the liabilities. It can be considered as an ideal asset perfectly correlated with the liabilities. The second part (the risk-return Portfolio) is composed by return-seeking assets, which are represented by equities categories, property and commodity. We define an indexation-based objective function, which also allows for partial and recovery indexation, depending on the financial status of the fund. Maximising the objective function will give us the Optimal asset mix able to maximise the purchasing power of the pension rights of the participants and if possible, to give full indexation of the liabilities. Secondly, we will examine the contribution of the real asset to the definition of the composition of these Optimal portfolios. The analysis compares the Optimal portfolios for different investment horizons, risk-aversion levels and initial funding ratios.

The model is applied to the real case of the ABN AMRO BANK pension fund. ABN AMRO kindly provided us with the scenarios of the relevant economic time series, the nominal payments they have to face in the future and the conditional indexation rule. The pension fund only guarantees the nominal payments, but is willing to provide indexation of their future payments. We work under the "liquidation perspective": the pension participants and the invested assets are fixed at 2009 and will not be increased by new contributions. We start from assuming an initial funding ratio level of 110. Afterwards, wealthier positions are considered by setting the initial funding ratio at 120 and 130. We expect that the richer the fund is, the more the indexation policy is sustainable, without affecting negatively the capability of the fund to meet its solvency constraints. On the other hand, when the funding ratio is relatively low, the fund is forced to take even more risk to meet his nominal obligations (constraints) plus the provision of the indexation. The Optimal portfolio will also depend on the levels of the risk aversion parameter. In our model this parameter represents a penalty to the volatility of the indexation decision, implicitly corresponding to the possibility of indexation cuts. We expect that a higher risk-aversion parameter could lead to choose a safer asset mix when the funding ratio is relatively high, and vice versa. As in the traditional analysis, this parameter could be considered as a proxy of the fund flexibility to react to other variables such as extra-contribution by the participants or financial support of sponsors. We set the risk level at 5, 10 and 20. Finally, a third dimension is represented by the investment horizon. A pension fund is typically considered to be a long-term investor, due to the long maturity of its liabilities. However, the Dutch regulatory framework FTK (Pensioen-en Verzekeringskamer 2004) imposes solvency constraints on the one-year probability of underfunding. Both the short and the long horizon have to be taken into account simultaneously. We investigate the three years, 5 years and 10 years horizons. These horizons do not correspond to the long term horizon of the liabilities which is about 40 years, but in the practise the definition of the asset allocation tends to be much shorter. The investigation through this direction is important to provide insight in the inflation hedging property of the assets in the portfolio, the extend to which they can be exploited and the horizon able to offer the higher utility in the maximisation of our objective function. The structure of the chapter is as follows. Paragraph 2 presents the optimisation model for a DB pension fund. Paragraph 3 describes the dataset provided by the ABN AMRO Pension Fund. Paragraph 4 discusses the results and Paragraph 5 concludes.

5.2 Literature Review

The ALM literature initially focused on mean-variance single-period optimisation analysis, having as objective function either the optimisation of the surplus (difference between asset value and liabilities value) or the "universal" measure represented by the funding ratio return (Leibowitz et al. 1994). Successively, the analysis was extended to consider the long-term nature of the pension fund, with the imposition of adequate short-term risk constraints to the maximisation of the funding ratio. Recently, ALM studies mostly apply operations research model to optimise funding and investment policies under uncertainty (see Ziemba and J.M. Mulvey 1998). Using stochastic programming techniques, they assume as objective function the end-of-period wealth of the funds, or the minimisation of the risk of underfunding, and impose as constraints several requirements with respect to solvency, contribution rate and indexation policy. Several models have been developed using chance constraints to limit the probability of underfunding for the next years (see Dert 1995) or assuming measures of underfunding risk such as the conditional value at risk (Bogentoft et al. 2001). In this field, Drijven (2005) formalises indexation decision, though in a rough way, considering the conditional indexation policy in the objective function as a penalty associated with not giving full indexation. The main difference of this model relative to our analysis is that they assume unconditional indexation to be one of the constraints that the pension fund has to meet. In our model we consider the indexation "decision", conditional on the funding ratio, as objective function, setting a direct link between the definition of the Optimal portfolio and the purchasing power of the participants. It represents the novelty of our approach. Moreover, we adopt a simulation-optimisation model as in Boender (1997), avoiding the complexity of these previous works related to the need for analytical solutions.

Our analysis also differs with those mentioned above because we consider the inclusion of alternative assets in the pension fund portfolio. At the beginning of the millennium, the debate on investing was mainly focused between bonds and equities (see e.g. Benartzi and Thaler 1995). The main results suggested a preference towards equities in the long run. Because of the mean reversion effect, it seems to be a safer investment compared to the Bond (Campbell and Viceira 2005). However, the equity premiums are now dramatically reduced, while bonds are the important to hedge the interest rate risk arising from the new valuation of the liabilities market-to-market. Relatively to the inflation property of equity, the empirical evidence (see e.g. Fama and Schwert 1997) suggests a negative relationship between expected stock returns and expected inflation. This result seems to be consistent with the idea (e.g. Fama 1981) of a negative influence of higher inflation on economic activity and thus on stock returns. On the other hand, stock dividends are positively influenced by higher future inflation (Campbell and Shiller 1988) and this means they can offer inflation hedging protection in the long run. Controversial are the results of Dert (1995) for Dutch data, as he found a negative correlation between stock returns and Dutch price inflation. Regarding the bonds, we expect a positive long-term correlation between bonds returns and changes in inflation in the long run, while in the short run we can expect lower or negative correlation, due to deviations of actual realised inflation and expected inflation.

Next to bonds and equities, in the recent years the investment policies of the pension funds have been characterized by the introduction of a wider mix of alternative assets and also derivatives instruments such as interest rate swaps and the inflation swaps. Commodities are generally considered to be leading indicators of inflation and more recently to be one of the main drivers in the inflation increases, especially in the domain of agriculture, minerals and energy. As shown in Gorton and Rouwenhorst

(2006), commodities futures show good hedging-inflation properties in long and short run, having a positive correlation with inflation which increases with the holding period and that are larger when annual or 5-years frequency is considered. Real estate investments also allow for enhanced inflation protection as showed in Fama and Scwert (1997) and this effect is particularly significant over long horizon. They can be considered as a traditional asset in the pension fund asset allocation. Moreover, as argued in Froot (1995) and in Hoevenaars et al. (2008), real estate investments closely behave like stocks, showing good inflation properties even if they do not add benefit in terms of risk diversification to the portfolio.

As far as regard the investment in emerging economies, a large body of literature investigates the over-under performance of these markets in terms of equity premium, compared to the most developed countries. They are considered relatively risky because they carry additional political, economic and currency risks. This could also explain why they are not often included in the literature on pension funds. However, they can offer significant diversification benefits because their performance is generally less correlated with developed markets. In terms of inflation protection, their demand for food and energy in the next decades could strongly affect the global inflation trends.

Concerning the interest rate swaps and the inflation swaps, they are attracting more and more pension fund managers. The formers are a good alternative to investments in nominal bonds to manage the interest rate risk, due to the higher liquidity of the corresponding market. On the other hand, inflation swaps are considered a good alternative for inflation hedging strategies. They are viewed to be a better investment than inflation-linked bonds because they can offer a better return performance. However, there is still reluctance towards both these instruments. Indeed the capacity of the inflation linked security market is not sufficient to meet the collective demand of institutional and private investor. On the other hand, the over-the-counter markets suffer from a perceived increase in counterpart risk. For these reasons, our analysis will exclude the derivatives since these instruments are not really common in the pension funds' portfolios so far and the literature about their use is still at early stage.

Closer to our application is the paper by Hoevenaars et al. (2008) who analyses a diversified portfolio in an asset and liability context. They construct an optimal mean-variance portfolio with respect to inflation-driven liabilities based on model implying forward looking variance and expected returns. This paper suggests that alternative asset classes add value to the portfolio: commodities are good risk diversifier also at long horizons; stocks are inflation hedger in long run; hedge funds are interesting for return enhancement and listed real estate behave like stocks. Differently from this work, we use a scenarios approach and impose current risk-based regulatory constraints which can heavily affect the asset allocation decision in real assets and the capacity to exploit their long term properties.

Our analysis can be somehow considered partial. A real pension fund is characterized by multiple competing objectives defined as risk-budgeting (in Boender and Vos 2000), while our stylized pension fund solely aims for maximal indexation with respect to the short-term regulatory rules on the probability of underfunding. It does not take into account, for instance, the contribution policy. However, as in Siegmann (2007), we can invoke the 1-1 relation of the indexation policy (conditional on the financial position of the fund) with the funding ratio. If the funding is high, the constraints are satisfied and also the contribution level can be lowered, and vice versa. The results suggest that the sustainability of the indexation-based portfolios is easily affordable at short horizon, even if the full indexation can be reached only at higher level of the initial funding ratio (120 and 130). The initial funding ratio strongly affects the capability of the fund to set an investment strategy over longer horizon. This can be easily explained considering the cumulative effect of the indexation policy: once the indexation is granted, it is permanently part of the nominal liabilities which will be eventually indexed the next year and so on. Another main evidence is the limited impact of different risk aversion parameter in the definition of the compositions of the Optimal portfolio. Concerning the composition, there is a convergence in the results towards a portfolio composed by Matching Portfolio (around 88-90%), Property (8-9%) and a residual part in Equity (1-2%). There is no strong role for typical inflation hedger assets as Commodity and Equity, since Dutch inflation shows low correlation with all the other assets. Property represents a better investment opportunity than Equity at every horizon. This composition changes when riskier strategies are needed to reach higher level of indexation. In this case there is a significant shift of resources from the Matching Portfolio to Property. Commodity is included in the portfolio only at the longest horizon and when the fund has a solid initial financial position.

5.3 Methodology

The application analyzes the portfolio choice for a stylized DB fund aimed to maximize the decision about the indexation of the liabilities to the inflation rate, that is to say to maximize the purchasing power of the participants. In a defined benefit pension scheme, the employer pays every year a contribution to the pension fund, which frequently includes also a contribution by the employee. Each year, the employee gets an additional pension right in terms of a percentage of the pensionable salary. At the end of the working life, these rights will define the pension as a percentage of the salary. In our stylized fund the number of the participants is fixed and the invested collected assets will not be increased by contributions, but they will increase or decrease only depending on the portfolio returns.

At time 0, the pension funds have a certain current value of the assets $A_{t=0}$ and liabilities $L_{t=0}$. The initial funding ratio is defined as:

$$FR_{t=0} = \frac{A_{t=0}}{L_{t=0}} \tag{5.1}$$

where $A_{t=0}$ corresponds to the market value of the invested assets and $L_{t=0}$ to the present value of all the future obligation of the fund towards the participants as a whole. For each time t, according to the LDI paradigm, the asset portfolio A_t is divided in two parts: the Matching Portfolio $A_{M,t}$ and the risk-return Portfolio $A_{R-R,t}$. The Matching Portfolio is assumed to earn exactly the liability return to match nominal liabilities as a result of an immunization strategy. The risk-return Portfolio consists of equity and alternative assets. The nominal liabilities L_t are fair valued and grow at a rate defined as liability return. The risk-return Portfolio is meant to provide with enough resources to grant indexation. The amount of A_t invested in each portfolio is defined as ($w_m, w_{r-r,j}$) and the portfolio is rebalanced to these weights each year. The funding ratio expresses the financial status of the fund as the capability of the amount of the resources available to cover the related nominal liabilities. It is expressed in percentage terms. In the following notation the percentage will be omitted. A funding ratio equal to 105 stands for a 5% surplus of the assets over the liabilities. Conditional indexation depends on the financial status of the fund, summarized by the funding ratio. Only if the nominal liabilities are covered in terms of assets, the pension fund will proceed to consider an update of the nominal liabilities to the inflation rate, also known as the indexation decision. We develop an optimization model having this indexation decision as objective function.

5.3.1 Optimizing indexation conditional on Funding Ratio

The indexation decision at each time t is defined as delta (δ_t). It takes values between 1 (full indexation), if the funding ratio is greater than 115 (required funding ratio) and 0 (no indexation) if the funding ratio is smaller than 105 (minimum required funding ratio). Between the two thresholds, partial indexation is granted according to a pre-specified indexation rule. The model also allows for recovering indexation: if enough resources are available in year t, once the indexation relative to the year t is granted, the remaining resources are devoted recovering the missing indexation of the previous years.

We want to maximize the expected value of δ_T, where T is a time in the future which represents a relevant horizon of investigation. The decision to be taken is the amount of the Matching Portfolio and risk-return portfolio ($w_m, w_{r-r,j}$) to invest in each asset class *j* (no short selling). The model is static: over the horizon T, the asset allocation is kept constant, that is to say there are no policy changes between 0 and T. This means that the decision about ($w_m, w_{r-r,j}$) gives the Optimal starting mix for a buy and hold strategy over the whole planned period. To the maximization of the expected value of delta is associated with a penalty consisting in the variance of delta. Higher volatility of the delta penalises the utility associated with the indexation. As in Leibowitz et al. (1992), where the objective function is represented by the return on the portfolio or the funding ratio return, the mean-variance model does not consider that the pension funds are more sensitive to downside risk measures than to symmetric measure of risk. Also for our objective function, this consideration is valuable. The pension fund is sensitive only to risk of not being able to grant the indexation (indexation cuts). However, since the complexity of the mean-shortfall model and to let our model to be numerically tractable in a simple way, we use this symmetric measure of risk. The formulation of the optimization problem is given by:

$$Max_{w_m, w_{r-r,j}} E(\delta_T) - \gamma\sigma^2(\delta_T) \tag{5.2}$$

s. t.

$$P(FR_{t+1} > 105 \mid FR_t) \geq 0.975 \tag{5.3}$$

where γ (gamma) is the risk-aversion parameter of the pension funds.

As constraint of our analysis, we consider the conditions on the solvency as promoted by FTK. However, even though we refer to the Dutch regulatory framework, there is no loss of generality in our model since recently more and more countries worldwide are valuating the opportunity to implement risk-based supervision for pension funds (Brunner, Hinz, and Rocha 2008). As mentioned in Chapter 2, FTK sets a first condition defined as a minimum required solvency with respect to the short-term. It imposes that the funding ratio should be greater than or equal to 5% of the liabilities for every year. This constraint is implemented as a condition for the indexation rule. The second condition imposes that the solvency should be such that the probability of underfunding in the next year is smaller than or equal to 2.5%. The probability of underfunding is defined as the percentage of scenarios in which the pension fund is confronted with underfunding.

We use a scenario-based ALM model as described in Chapter 3, to implement the optimization described above. It is the basic version of the well-known model used in pension funds industry to support their actual decision-making. In our application, the optimization will be based on a range of possible future developments of the deltas (scenario), depending on the range of possible future developments of all the other economic variables. The expected value of delta and variance of delta in our objective function are computed for each time across the scenarios. In the application developed in this chapter our analysis starts from a dataset already provided by the ABN AMRO Pension fund. It is composed of 2500 scenarios for all the economic series. The methodology that is applied behind their generation follows the model described in Chapter 3, but it will also include the specific view of the ABN AMRO Pension fund about the development of the economic environment. They provide us asset returns for several asset classes, the estimation of all the future benefit payments they are expected to pay and the parameters needed to develop interest rate curve and inflation scenario for each year. Next paragraph will describe the computation of different value of the assets and the liability which take into account the dynamics of the cash flow and their eventual indexation.

5.3.2 Market values of Assets and Liabilities

The fair value of the liabilities is computed under the hypothesis of the run-off of the pension fund (liquidation perspective). We set the time $t = 0$ as the moment from which the pension fund is formally closed to new participants and the old ones do not pay any contribution. The pension fund only has annual nominal payments (cash flows) to be paid to the participants at the end of each year until the definitive closing date (n)

. The present value of all these future nominal obligations is computed market-to-market as:

$$L_t^U = \sum_{k=0}^{n} \frac{CF_k}{(1+r_k)^k} \quad k = 0,1,2...n \tag{5.4}$$

where k is the maturity, in terms of the number of periods, corresponding to each cash flows (CF_k) and r_k is the interest rate associated to each cash flow maturity on the interest rate yield curve. The cash flows are computed under assumptions about the life expectation of the participants, the expected retirement date and other variables according to a defined actuarial model that takes into account actuarial and longevity risk. We will not investigate these aspects, but concerning the nominal liabilities we only consider the interest rate risk arising from their fair valuation.
The interest rate yield curve is generated by the Nelson & Siegel Model (Nelson & Siegel 1987). As described in Chapter 3,the term structure of interest rate in each scenario and each year will be determined by combining the values of these four parameters using the following formula:

$$r_k = \beta_0 + (\beta_1 + \beta_2)\frac{(1-e^{-k/\tau})}{k/\tau} - \beta_2 e^{-k/\tau} \tag{5.5}$$

$$\begin{aligned} &\lim_{k\to\infty} r_k = \beta_0 \\ &\lim_{k\to 0} r_k = \beta_0 + \beta_1 \end{aligned} \tag{5.6}$$

Where k is the maturity for each cash flow and τ is set equal to 1.8. The interest rate yield curve is generated by the four parameters and formula

(6) for each (s,t) and it is used to discount all the future cash flows by changing the value of k.

On the liabilities side, we define three different values for the liabilities to account for cash flow and indexation decision dynamics. The ultimo value of the liabilities at time t is computed as the present value of all the future nominal obligations including the cash flow to be paid at the end of year t, discounted at the interest rate yield curve according to the formula (6). This value only takes into account the nominal obligation as defined at time t, excluding the eventual increase of the nominal liabilities due to the indexation decision.

From the ultimo value, we derive the primo value of the liabilities at time t, by subtracting the nominal cash flow to be paid at time t:

$$L_t^P = L_t^U - CF_t \tag{5.7}$$

The primo value of the liabilities at time t represents the end of the year value of time t, and hereafter the initial value of the liabilities at the beginning of the next year.

Given these definitions, the "nominal" rate of growth of liabilities is given by:

$$r_{L,t+1} = \frac{L_{t+1}^U}{L_t^P} - 1 \tag{5.8}$$

This value gives the increase in the value of the nominal liabilities from their initial value (primo) at the beginning of the year to the end of the same year, only due to the dynamics of cash flow and changes in the interest yield curve from one year to another.

Once the nominal growth of liabilities is computes, every year the primo value of the liabilities at time t, that is to say the initial value of the liabilities at time $t+1$, is updated by the nominal rate of growth as in formula (8), to obtain the nominal ultimo value at time $t+1$ as below:

$$L_{t+1}^{U} = L_{t}^{P}(1 + r_{L,t+1}) \tag{5.9}$$

Secondly, depending on the value of the funding ratio at time $t+1$, the indexation decision is taken and applied to the ultimo value in formula (9), to obtain the ultimo indexed value of the liabilities, as following:

$$L_{t+1}^{Uindex} = L_{t+1}^{U} \cdot \widetilde{\delta}_{t+1} \cdot (1 + \pi_{t+1}) \tag{5.10}$$

where π_{t+1} is the inflation rate and depending on the indexation rule (see next section), $\widetilde{\delta}_{t+1}$ will assume values between 0 and 1, if there is no previous missing indexation, and values greater than 1 if in the passed periods partial or null indexation was granted. After the eventual update to indexation, and the subtraction of the cash flow for the corresponding year, we obtain the third value of the liabilities, called "primo value post indexation", which will be used to define the solvency constraints. In particular, if the indexation is granted at time $t+1$, the indexed ultimo value of the liabilities increases and becomes the starting point for the definition of the new nominal primo value of the liabilities for next year. Otherwise, if there is no indexation, the two values would simply coincide.

After the step shown in formula (10), by subtracting the corresponding cash flow to be paid at the end of the year (also updated by indexation decision), we compute a new primo value for the liabilities which also takes into account the eventual indexation decision:

$$L_{t+1}^{Pindex} = L_{t+1}^{Uindex} - (CF_{t+1} \cdot \widetilde{\delta}_{t+1} \cdot (1 + \pi_{t+1})) \tag{5.11}$$

This value represents the initial value of the liabilities for the next year that will be accordingly updated by the nominal growth estimated in formula (8) and eventually by the indexation decision (9). It is denominated "index" to be distinguished by the previously defined primo value, which does not include indexation. However, once the indexation is granted it is permanently part of the future nominal obligation and then the primo index value is actually the nominal obligation of the funds towards the participants as a whole. This means that formula (9) becomes:

$$L_{t+2}^{U} = L_{t+1}^{Pindex}(1 + r_{L,t+2}) \tag{5.12}$$

The initial amount of assets at time 0 is invested every year, and therefore A_t, represents the market value of portfolio of the pension fund. It is divided in two parts with two different targets. The value of the portfolio is the sum of the values of the two parts:

$$A_t = A_{M,t} + A_{R-R,t} \tag{5.13}$$

The first part is defined as Matching Portfolio $A_{M,t}$, because it is aimed to match the liabilities in term of duration.

We assume that this portfolio is composed by fixed-income assets with duration equal to the duration of the liabilities and that it earns every year a return equal to the nominal rate of growth of the nominal liabilities as defined in formula (8).

$$r_{L,t} = r_{M,t} \tag{5.14}$$

where $r_{M,t}$ is the rate of return of the Matching Portfolio at time t. In this way, the interest rate risk is partially offset. Due to the fact that the immunization is only in term of duration, it only hedges from a parallel shift of the interest rate yield curve. The remaining interest rate risk (convexity risk) and the inflation risk should be hedged by the dynamics of the returns of the other portfolio, called risk-return Portfolio $A_{R-R,t}$. This portfolio is composed by: Property, Commodity, Equity Value, Equity Passive, Equity Emerging Market and Equity Growth. It should earn enough to complete the hedging of the nominal liabilities and also provide with extra-return to allow for indexation. The return on the risk-return portfolio of the pension fund is given by:

$$r_{r-r,t} = \sum_{j=1}^{z} r_{j,t} \cdot w_{r-r,j} \tag{5.15}$$

The decision to be taken in our optimization model is about the definition of the percentages (w_m , $w_{r-r,j}$).

As for the liabilities, we can define two different values for the assets. The first one, defined as Ultimo asset value (A_{t+1}^{U}) is the reference value for the computation of nominal funding ratio on which the indexation decision will depend on. It is computed as:

$$A_{t+1}^{U} = A_{M,t}^{P}(1 + r_{L,t}) + A_{R-R,t}^{P}(1 + r_{r-r,t}) \tag{5.16}$$

It expresses the value of the invested assets before the indexation and the payment of the cash flow for the corresponding year, where the A_t^P is the primo value for each portfolio. Similarly to the Primo value of the liabilities, it is computed as:

$$A_{t+1}^P = A_{t+1}^U - (CF_{t+1} \cdot \tilde{\delta}_{t+1} \cdot (1+\pi_{t+1})) \tag{5.17}$$

The primo value of the assets is used for defining the constraints at each year relative to the next year and it is obtained excluding the cash flow (eventually updated to indexation), that has to be paid in the corresponding year.

5.3.3 Implementing indexation rule and solvency constraints

As mentioned above, the objective function is the indexation decision δ_T. It depends on the financial status of the fund (ultimo funding ratio), expressed by the funding ratio computed using the ultimo value for both assets and liabilities:

$$FR_t^U = \frac{A_t^U}{L_t^U} \tag{5.18}$$

We want to model the indexation decision in such a way that it allows for recovery and partial indexation. As in most of the DB pension fund, it should be defined as follows:

-- if the funding ratio is greater than the required funding ratio, full indexation is granted and previous missed indexation is recovered. The required funding ratio is defined by the Pension Law and depends on the Strategic Asset Allocation (SAA) of the fund and on duration mismatch between pension assets and liabilities. For simplicity we assume the required funding ratio to be equal to 115.

-- if the funding ratio is smaller than 105 (minimum required funding ratio) the nominal liabilities at time $t+1$ corresponds to the nominal liabilities at time t (no indexation).

-- if the funding ratio is between 105 and 115, partial indexation is granted.

To model this indexation rules in such a way that the optimization model is easier to be solved (using an optimizer as the Solver provided by Excel), we first define δ_T as the indexation decision which will assume value equal to 1 for full indexation, 0 for no indexation and a value between 0 and 1 if partial indexation is granted.

$$\delta_t \begin{cases} \mathbf{1} & \textit{Full indexation} \\ \mathbf{0} & \textit{No indexation} \\ \mathbf{0} < \delta_t < \mathbf{1} & \textit{Partial indexation} \end{cases}$$

To take in to account the recovery and partial indexation to be included, we define δ_t as:

$$\delta_t = \delta_{t-1} \cdot \tilde{\delta}_t \tag{5.19}$$

where delta tilde corresponds to:

$$\tilde{\delta}_t = \frac{1}{(1+\pi_t)} + F(x)\left(\frac{1}{\delta_{t-1}} - \frac{1}{(1+\pi_t)}\right) \tag{5.20}$$

where $F(x)$ is modelled as a logistic function and introduces the conditionality of the indexation decision on the ultimo funding ratio:

$$F(x) = \frac{1}{1+e^{(-c)(x)}} \tag{5.21}$$

$$c = 1$$
$$x = FR_t^U - 110$$

This definition of the logistic function has the property of a fast growth (exponential) in the beginning, but it reaches its maximum at 1, which is the limiting value of the function. In this way the indexation decision this year accounts for conditionality, but also for the previous indexation decision through δ_{t-1}, which allows when possible to recover the missing indexation of the previous years.

Setting the reference ultimo funding ratio at 110, we have a value of the logistic function equal to zero if the ultimo funding ratio is below 105. In this case, $\widetilde{\delta}_t$ becomes equal to $\frac{1}{(1+\pi_t)}$, which will affect the indexed value of the liabilities in the formula (10) in such a way that it will coincide with the indexed value of liabilities of the previous year (no indexation). Then, δ_t will assume a value equal to $\frac{1}{(1+\pi_t)}$ if full indexation was granted in the previous year ($\delta_{t-1} = 1$), otherwise it assumes value equal to $\frac{\delta_{t-1}}{(1+\pi_t)}$ including the information about the missing indexation in current year and in the previous year. Similarly, if the ultimo funding ratio is above 115, the value of the logistic function is

1, the $\widetilde{\delta}_t$ becomes equal to $\dfrac{1}{\delta_{t-1}}$. If δ_{t-1} is equal to 1 (full indexation in all the previous periods), the full indexation is granted because $\widetilde{\delta}_t$ assumes value 1 in formula (10). However if δ_{t-1} is below 1, as in the cases above (funding ratio in the previous year is below 105), $\widetilde{\delta}_t$ assumes value equal to $(1+\pi_{t-1})$ if the only missing indexation is related to the previous year, otherwise it is equal to the product of all the missing indexations $\Pi_{i=o}^{n}(1+\pi_{t-i})$, where i represents all the periods of previous missing indexation. In this way the liabilities are corrected for the full indexation to the current indexation, but they also recover the previous missing indexation. In both cases δ_t will assume value equal to 1 and the next year definition of $\widetilde{\delta}_{t+1}$ will contain the information that there is no indexation to be recovered. For values of the ultimo funding ratio between the two thresholds, the value of the logistic function will be between 0 and 1. In this case we have partial indexation granted and hence, partial missing indexation to be recovered. For example, when the ultimo funding ratio is equal to 110, we have $x = 0$, the logistic function assumes a value of 0.5 and only half indexation will be granted and in the scheme above, delta and delta tilde will take into account that only partial indexation has to be recovered. Figure 5.2 below shows the shape of the logistic function, which translates the conditionality of the indexation decision on the nominal funding ratio.

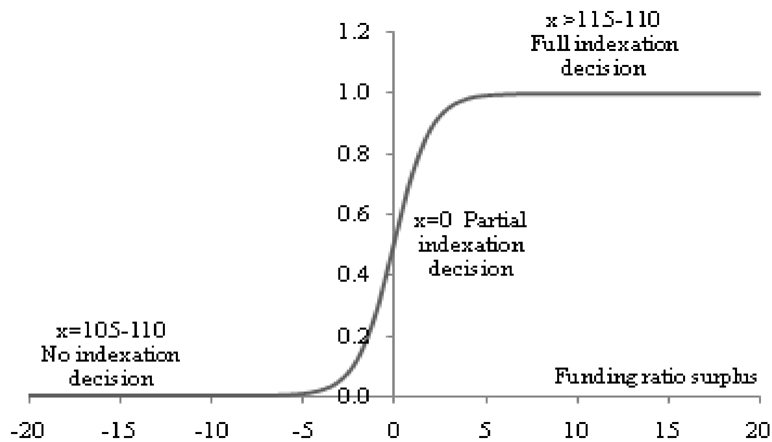

Figure 5. 1: The indexation rule

The Figure shows the logistic function $F(x)=\dfrac{1}{1+e^{(-c)(x)}}$ used to model the indexation rule, where c =1 and $x = FR_t^U - 110$. It assumes value 1 when the ultimo funding ratio is greater than 115, value 0 when it is smaller than 105 and values between 0 and 1 for ultimo funding ratio value comprises between the two thresholds. For instance, the value 0.5 is assumed when funding ratio is equal to 110.

In our optimization model we want also to include the new solvency constraints that have been introduced since January 2007 by the Dutch National Bank which supervises the pension funds. The risk-based supervision approach in the Netherlands has been analyzed in Chapter 2 and represents at the moment the most sophisticated approach to pension supervision. For this reason, in our model the Optimal mix that maximizes the objective function delta at time T should also be able to ensure for each $t+1$ from 0 until $T+1$, that the probability of the funding ratio for the next year being greater than 105, is greater than 0.975 across all the scenarios (3). By the direct implementation of these constraints arise (at least) two questions. The first one is about the use of chance constraints, which are, from a computational point of view, very hard. To overcome this problem we approximate the solvency condition by a logistic function, similarly to what we did for the indexation rule.

$$f(y) = \frac{1}{1+e^{(-c)(y)}} \tag{5.22}$$

$$c = 10$$
$$y = FR_t^U - 105$$

where y is defined as a difference between the primo funding ratio (computed using the primo value of the asset and liabilities) and 105 and $c = 10$. We define c and y in such a way that to each primo funding ratio equals or is greater than 105, corresponding respectively to the value of 0.975 or greater. Computing the expected value of this function across the scenarios, we have a proxy of the probability expressed in the formula (3).

A second question concerns the fact that for each combination (s,t) the computation of probability of underfunding for the next year needs the generation of 2500 scenarios for each of the 2500 funding ratios at time t. Given properly normalised returns, we can assume that the value at time t of the funding ratio is the expected value at time $t+1$ under a proper probability space (Markov chain). Therefore, we assumed that every funding ratio at time t is the result of the initial *n*-path generated on the basis of the initial of the value the funding ratio at time $t = 0$. As a consequence, the expected value of the funding ratio at time t can be regarded as the basis of the simulated funding ratio at time $t+1$, since we are only interested in the terminal values and not in the intertemporal value of delta.

5.4 Dataset

The model is applied to the ABN AMRO Bank Pension fund portfolio. It provide us with 2500 scenarios for the relevant economic time series and the asset classes generated by a VAR model, as described before, for the period 2009-2022 on an annual basis. As far as interest rate time series are concerned, they provide us with the parameters of the Nelson & Siegel model from which we generate the interest yield curves used in the liabilities valuation. The ABN AMRO indexation is based on the Dutch inflation and it also provides with the Dutch price inflation rate also generated by the application of the Nelson & Siegel model. The indexation the participants care more about is the Dutch price inflation rate (and not the wage inflation), as it is defined in the formal agreement between ABN AMRO Bank and ABN AMRO Pension Fund. The Nelson & Siegel model also generates negative values for inflation, but since negative indexation is not reasonable we substitute these values by 0.0005. In particular, we focus on annual realized Dutch inflation. The Figure 3 shows the annual realized inflation across the 2500 scenarios over the whole horizon under investigation. The long-term mean is slightly above the BCE target of 2%, while the standard deviation is about 0.01.

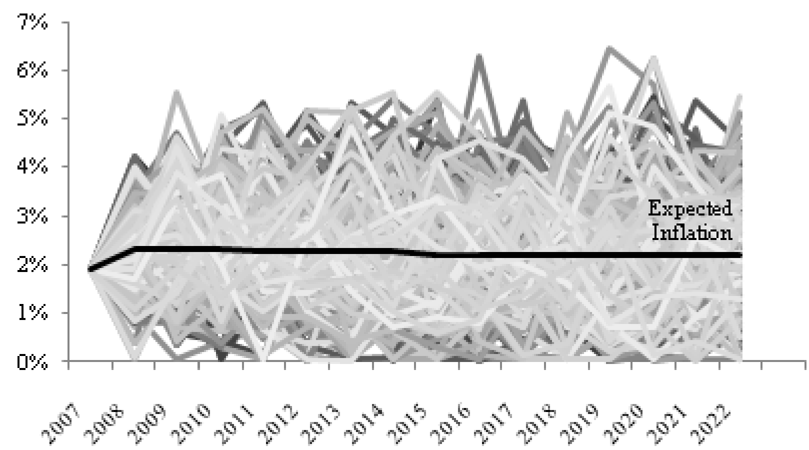

Figure 5. 2: Annual realized inflation rate over the period 2007-2022

The Figure shows the scenarios generated by the VAR model over the horizon 2007-2022 of the Dutch annual realized inflation rate used to index the liabilities. The black line represents the expected realized inflation for every year. The long-term inflation expectation is about 2%, in line with ECB targets and the standard deviation is very low, equal to 0.01.

On the liabilities side, the ABN AMRO provides us with all the future nominal payment under the assumption of the run off of the fund from 2009 until 2126, closing date of the fund. The Figure 5.4 shows the decreasing amount of the nominal payments over time. It is important to underline that these cash flows are estimated by actuarial simulation that are properly linked to the other simulated economic times series. Our analysis will be focused on the period 2009-2022. The present value of liabilities generated by the interest rate yield curve has an expected long-term annual growth of 5.71% while the standard deviation is around 12%. This annual growth is defined as liabilities return. To reach the full indexation of the liabilities in this ALM context, the invested asset available at beginning of 2009 must be ideally allocated in such a way to earn on average the annual nominal liabilities return plus an average inflation rate of around 2%, without incurring in risk of underfunding too high.

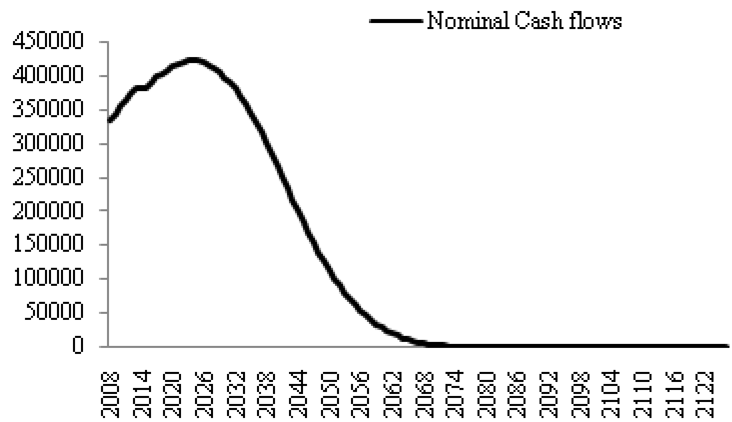

Figure 5. 3:Dynamics of the Cash flows in liquidation perspective

The Figure shows the nominal cash flow of the ABN AMRO Pension Fund under the hypothesis of run-off of the fund. In our model these payments will be updated to the Dutch inflation according to a specified indexation rule depending on the nominal funding ratio. They are estimated by actuarial simulations about number of participants, age, mortality, surrender, transfer, marriage, resignations, etc.

The asset side is composed by Property, Commodity, Equity Value, Equity Passive, Equity Emerging Market and Equity Growth. The alternative assets are represented by Commodity and Property. Commodity dataset is represented by Goldman Sachs Commodity Index (GSCI), a composite index of Commodity sector returns which represents a broadly diversified, unleveraged, long-only position in Commodity futures. Property data is represented by ROZ/IPD Dutch Property Index. This index measures the total returns on directly held real estate investments belonging to institutional investors and real estate funds in Netherlands. Concerning the investment in equities, Equity Growth is represented by worldwide used Morgan Stanley Capital International World Index (MSCIWI). It comprises more than 1.700 companies listed on the exchanges of 22 of the world's major developed economies and it is composed by 52.8% equity North America, 22.8% in equity Europe, 11.6% in equity UK, 12.8% in equity Asia. For the second equity category, Equity Passive, we use a slightly adjusted benchmark composed of 30% equity North-America, 50% equity Europe, and 20% equity Asia. Equity Value category is represented by MSCISWI hedged, which gives the performance of an index of securities where currency exposures affecting index principal are hedged against a specified currency. Finally Emerging Markets Equity category is represented by MSCI Emerging Markets Index, which is a float-adjusted market capitalization index investing in 26 emerging economies: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Israel, Jordan, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey and Venezuela. For each of this asset class, 2500 scenarios are available for the period under investigation.

The Table 5.1 below offers the descriptive statistics of the assets which are candidate to be included in our portfolio together with the Matching Portfolio, considered as an "asset" earning by definition exactly the liabilities return. Long term mean and the standard deviation are computed for each asset. In terms of risk-return trade-off, Property is the dominant asset, which presents an expected return close to the other assets, but associated with a very low standard deviation compared to the other assets. Illiquid assets as Property are often characterized by high return and low volatility. The less efficient asset is Commodity. Equity categories only dominate the Commodity. They offer a higher return but also a higher risk, in particular Equity Emerging Markets, compared to Property. These assets should be preferable for less risk-averse investor.

	MEAN	ST.DEV.
Matching PF	5.7%	12.0%
Inflation	2.2%	1.0%
Property	8.0%	5.8%
Commodities	6.5%	23.0%
Equity value	8.1%	19.8%
Equity passive	8.1%	19.5%
Emerging Mkts	9.0%	31.2%
Equity growth	8.1%	19.7%

Table 5. 1 Descriptive statistics of the assets in the portfolio

The table shows the long term mean and standard deviation of the assets, composing the portfolio. Property dominates all the other assets in terms of risk-return with exception of Equity in Emerging markets, which should be the preferable asset for riskiest investors. Commodity offers the less efficient trade-off and is dominated by Equity categories.

This partial description has to be extended to the analysis of the correlations of these assets with Matching Portfolio and the inflation. From the first value we obtain the liabilities hedging qualities of the assets when positive. Since the Matching Portfolio grows exactly as nominal liabilities, when the liabilities return increase, also the assets earn more. Similarly, positive correlation value with inflation gives the characteristic of inflation hedger to an asset, since when the inflation rate rises the asset also earns more return to support the nominal funding ratio and then the indexation decision get closer to 1. Moreover, the cross correlations between assets show the possibility of risk-diversification qualities of these assets. The Table 5.2 presents the correlations with Matching Portfolio, with inflation rate and the cross correlations between assets.

T=3								
Inflation	1.00							
Matching PF	-0.09	1.00						
Property	0.02	0.21	1.00					
Commodities	-0.05	0.09	0.10	1.00				
Equity value	-0.03	-0.07	-0.10	-0.18	1.00			
Equity passive	-0.05	-0.08	-0.12	-0.21	0.97	1.00		
Emerging Mkts	-0.07	-0.04	-0.29	-0.09	0.68	0.69	1.00	
Equity growth	-0.03	-0.07	-0.10	-0.18	1.00	0.97	0.68	1.00
T=5								
Inflation	1.00							
Matching PF	-0.15	1.00						
Property	-0.03	0.25	1.00					
Commodities	-0.08	0.07	0.07	1.00				
Equity value	-0.03	-0.06	-0.07	-0.25	1.00			
Equity passive	-0.07	-0.06	-0.09	-0.27	0.98	1.00		
Emerging Mkts	-0.09	-0.02	-0.30	-0.11	0.71	0.72	1.00	
Equity growth	-0.03	-0.06	-0.07	-0.25	1.00	0.98	0.71	1.00
T=10								
Inflation	1.00							
Matching PF	-0.17	1.00						
Property	-0.08	0.27	1.00					
Commodities	-0.13	0.05	0.07	1.00				
Equity value	0.00	-0.07	-0.07	-0.30	1.00			
Equity passive	-0.04	-0.06	-0.07	-0.31	0.98	1.00		
Emerging Mkts	-0.09	-0.04	-0.29	-0.11	0.71	0.72	1.00	
Equity growth	0.00	-0.07	-0.07	-0.30	1.00	0.98	0.71	1.00

Table 5.2: Cross correlation Matrix

The Table shows correlations of the assets of the portfolio with Dutch inflation rate, Matching Portfolio and between assets. The assets show limited inflation hedging properties, in particular the Equity categories. The Property and the Commodities are the best risk-diversification assets.

The best inflation hedging qualities belongs to Property, which presents a positive correlation value with inflation, but very low. All the other assets show a negative correlation showing a very low value. The contribution of these assets in terms of inflation hedging can not be considered significant. This could be explained by the evidence that the Dutch inflation rate is considered while the datasets (with exception of Property Index) refer to worldwide indices. The higher negative correlation with inflation belongs, as expected, to t Matching Portfolio, because its return corresponds to the liability return which is negatively affected by positive changes in the nominal interest rate curve, including the inflation rate. At longer horizon, the correlations with inflation are all negative and the values for the alternative assets become smaller. In particular Commodity shows little inflation risk hedging qualities. Moreover, the statistics confirm the negative correlation of the Dutch inflation with the Equities categories, even if they are all close to 0.

The best liabilities hedging qualities (correlation with Matching Portfolio) belongs again to Property, that presents a positive value of the cross correlation, followed by the Commodity. However, correlation values of Property increases over time, while that of Commodity declines. These qualities of Property asset could be explained by the link between the real estate sector and the interest rate risk underlying the liabilities valuation, due to the fact that both the dataset are refers to Dutch economy.

The cross correlations analysis reveals relevant diversification properties of Commodity and Property with respect to Equity Categories, which increase at longer horizon. These categories also offer a good diversification for the risk affecting liabilities, typically the interest rate risk.

5.5 Empirical Results

This paragraph presents the main results from the application of our model to the ABN AMRO Pension Fund dataset. The analysis is developed along three dimensions: the risk aversion level, the initial funding ratio and the investment horizon. The Appendix in Tables 5.4, 5.5 and 5.6 shows the Optimal Portfolios (Optimal PFs) obtained by optimizing the objective function for each combination of these three dimensions. The composition of the portfolio shows how resources are allocated between Matching Portfolio and the risk-return Portfolio, which alternative assets are included in the risk return portfolio and how their weights change at different horizon and risk aversion level.

The optimization does not allow to invest in short positions and is subject to the satisfaction of the solvency constraints for all the years included in the investment horizon.

For each portfolio utility, expected delta, standard deviation of delta, indexation loss and the composition of the portfolio are reported for 3, 5 and 10 years horizons. The first value allows investigating which portfolios offers higher utility in the mean-variance setting. The expected delta gives the average value of delta across scenarios. If the expected delta is equal to 1 at the end of the specified investment horizon (3, 5, 10 years), the full indexation in all the previous years has been granted, otherwise if delta is smaller than 1, the portfolio is able to ensure only partial indexation of the pension rights, in other words, there has been loss of indexation. The distance from the full indexation can be defined as the indexation loss associated with each portfolio at the end of the investment period. Given the formulation of our model, it can be defined as: $\frac{1-\delta_T}{\delta_T}$.

For instance, a value of expected delta equal to 0.98 approximately represents a loss of 2% in terms of missed indexation over the whole investment period. Standard deviation of delta gives a measure of the risk associated with the expected delta, and consequentially it is a measure of the implied risk of the investment strategy. Delta is depending on the nominal funding ratio, whose volatility changes according to liability volatility and portfolio volatility. Since liability volatility is the same for all the portfolios, higher standard deviation of delta are due to higher volatility of the optimized portfolio. The general overview of Optimal portfolios shows that the sustainability of the indexation-based optimisation is easily affordable at short horizon, even if the full indexation can be reached only at higher level of the initial funding ratio (120 and 130). At longer horizon, when the initial funding ratio is 110, which corresponds to a weak (but still solvent) financial position of the fund, the optimisation is not able to find feasible solutions which satisfy all the solvency constraints. Therefore, the initial funding ratio strongly affects the capability of the fund to set an investment strategy over longer horizon. These results can be explained considering the cumulative effect of the indexation policy. Once the indexation is granted, it is permanently part of the nominal liabilities which will be eventually indexed the next year and so on. It means that if indexation is granted, in the following years a greater amount of resources is needed to match the (new) nominal liabilities even if the solvency constraints prevent from assuming excessive risk. A solid initial financial position better sustain indexation over longer horizon.

An important evidence is the limited impact of different risk aversion parameter in the definition of the compositions of Optimal Portfolio. In most of the cases, changes for different value of gamma are around 0.5%. For this reason, this section will focus on the analysis of the results for gamma equal to 10. The complete overview of the results for different gammas is in the Appendix. Appendix also reports the statistics and the compositions of the portfolios investing 100% of the resources in the Matching Portfolio (100% MP). These portfolios represent bond-only portfolios aimed to match the nominal liabilities but also to provide with indexation. These strategies, earning liabilities return, are always dominated by the Optimal Portfolio. These results intuitively confirm that there is a positive contribution of the alternative assets and equity in the maximisation of the purchasing power of the participants' pension rights.

Once the Optimal Portfolio is defined, when needed, we impose two new different constraints to the optimisation relative to the weight of the Matching Portfolio and Property. The first constrain regards the market impossibility to invest a high percentage of the available resources in Matching Portfolio, due to the imperfections of the long term bond markets. This constraint, conventionally defined by ABN AMRO, limits the weight of Matching Portfolio to be equal or smaller than 63%. These portfolios defined as "MP-restricted PF" are available only for richer pension funds (funding ratio greater than 120) and, due to the limited investment opportunity set, are less efficient of the Optimal Portfolios. However these portfolios can easier be replicated in the financial market. The second constraint regards the investment in Property. As mentioned before, this asset is a valuable asset because of its comparative low volatility and high return. However, it is by definition, an illiquid asset. Most of the pension funds set a limit into this type of investments characterized by the fact that they can not be easily transformed in cash. Indeed, the pension funds is characterized by the annual liquidity pressure of the cash flows payment and should prefer liquidity assets. For this reason, we optimise imposing the weight of Property to be equal or smaller than 15%. When both the constraints are added to the solvency constraints, feasible solutions are available only when the initial funding ratio is set equal to 130.

We start discussing the Optimal Portfolios when the initial funding ratio is 120 and gamma is 10 (Table 5.5 in the Appendix). The highest utility is associated with the shortest horizon and decreases for longer horizons due to the cumulative effect of the indexation. Figure 5.5 shows how the composition of these three portfolios changes over time. The portfolios are composed by the Matching Portfolio and in the risk-return Portfolio by Property and a small contribution of Equity categories (in particular Equity growth).

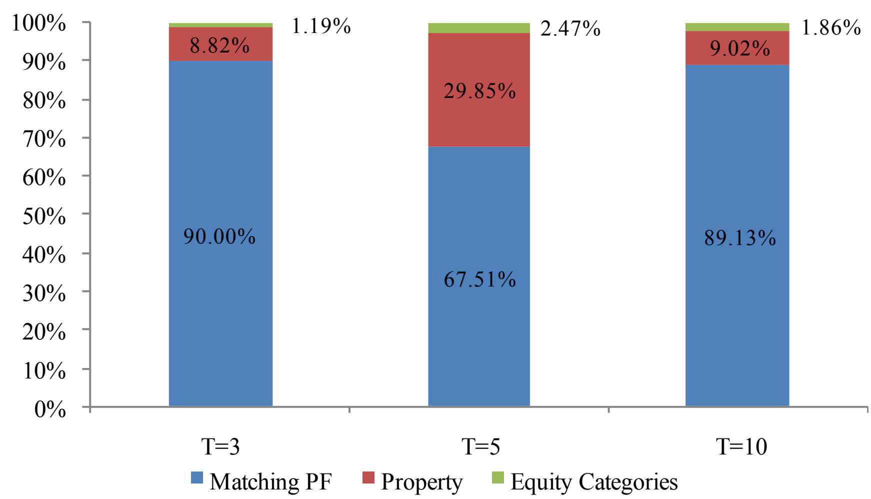

Figure 5. 4: Composition of Optimal Portfolios for initial funding ratio 120 and gamma 10

The Figure shows the asset allocation between Matching Portfolio and risk-return Portfolio. The Optimal Portfolios only include Property as alternative assets. At short and long horizon the compositions of the portfolios are quite similar, while at medium term there is a significant shift of resources from Matching Portfolio to Property. This medium term portfolio is characterized by higher return, but also higher volatility.

From these results emerge that Property, differently from literature results, is able to give a considerable contribution to the definition of the Optimal portfolio and it is a preferable asset compared to Equity in the short, medium and long term. Another result in contrast with the literature is the absence of the Commodities even at short horizon. Despite the good risk diversification properties of these assets supported by the cross correlations statistics, its high volatility could represent a threat to the satisfaction of the solvency constraints and lead to the exclusion of these assets. An interesting result regards the distribution of the weights at the medium term. At 3 years and 10 years the portfolio invests a high percentage in Matching Portfolio about 89-90%, 8-9% in Property and a residual 1-2% in Equity. At 5 years horizon the composition is quite different. There is substantial shift of resources from the Matching Portfolio (-22.5%) to Property and Equity, which increase respectively of 20.8 and 1.7%. Since the mean, the standard deviation and the cross correlation do not show significant changes from the short to the medium term, a better insight could derive from observing the distributions of the nominal funding ratio and delta. The Table 5.3 shows the descriptive statistics and distributions of delta and nominal funding ratio for each horizon.

Delta	
T=3	
Mean	1.000
St.deviation	0.001
Kurtosis	121.10
Asimmetry	-9.14
Min value	0.98
Max value	1
Pr(delta>0.98)	1
Pr(delta=1)	0.78
Obs	2500

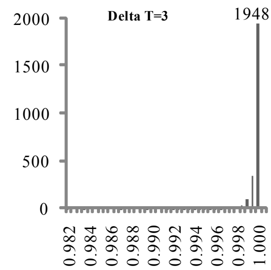

Figure a

Funding ratio	
T=3	
Mean	116.90
St.deviation	2.1040
Kurtosis	-0.11
Asimmetry	-0.001
Min value	109.46
Max value	123.83
Prob(FR<105)	0.00
Prob(FR>115)	0.78
Obs	2500

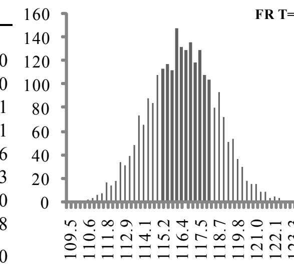

Figure b

Delta	
T=5	
Mean	0.997
St.deviation	0.008
Kurtosis	12.283
Asimmetry	-3.282
Min value	0.94
Max value	1
Pr(delta>0.98)	0.95
Pr(delta=1)	0.63
Obs	2500

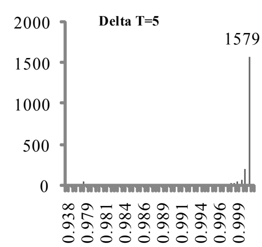

Figure c

Funding ratio	
T=5	
Mean	118.64
St.deviation	4.97
Kurtosis	0.15
Asimmetry	0.06
Min value	101.83
Max value	139.00
Prob(FR<105)	0.01
Prob(FR>115)	0.63
Obs	2500

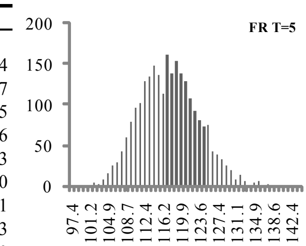

Figure d

Delta	
T=10	
Mean	0.959
St.deviation	0.029
Kurtosis	0.004
Asimmetry	-0.640
Min value	0.85
Max value	1
Pr(delta>0.98)	0.02
Pr(delta=1)	0.02
Obs	2500

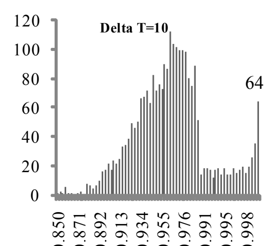

Figure e

Funding ratio	
T=10	
Mean	108.58
St.deviation	2.29
Kurtosis	2.29
Asimmetry	0.94
Min value	97.11
Max value	119.32
Prob(FR<105)	0.02
Prob(FR>115)	0.02
Obs	2500

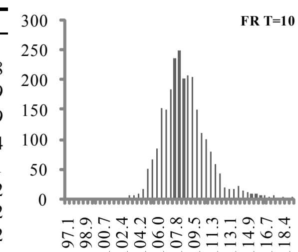

Figure f

Table 5. 3: Delta and Funding ratio distributions for Optimal Portfolio (FR=120, gamma=10)

The Figure shows probability distributions for delta and nominal funding ratio and the relative statistics. We observe that at longer horizon the mean of the distributions decrease, while the volatility of both increases. In particular we observe a higher volatility of the funding ratio at T=5 with respect to T=10, which can be explained by a riskier investment strategy.

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At 3 years horizon, the expected delta is 1 (on average the portfolio ensures the full indexation) associated with a standard deviation extremely low. The sub-figure a) shows that the distribution is within a range of high delta values, between 0.98 and 1. It has a negative asymmetry, meaning that the mass of the distribution is concentrated on the right of the figure.

It has relatively few lower values. The kurtosis value is extremely high. A high positive value of kurtosis suggests a concentration of the observations on the highest value of delta (1) and that more of the dispersions are due to infrequent extreme deviations. We also consider the probability of delta greater than 0.98 and 1 at 3 years horizon. These measures of probability can be considered as downside risk measure. The first indicator gives the probability not to lose more than 2% of indexation, while the second to have full indexation. These values for a 3 years horizon are equal to 100% and 78%. In the latter case, it means that in 1948 out of 2500 scenarios the full indexation is granted. These measures of downside risk can be helpful in the valuation of the portfolios whereas the expected delta only gives an averaged value of the indexation. The sub-figure b) presents the nominal funding ratio distribution at 3 years horizon. The distribution is close to the normal distribution shape as suggested by the low values of the asymmetry and kurtosis. The mean is higher than the minimum required level for the full indexation as defined by the indexation rule, ensuring full indexation. Downside risk measures are computed also for the nominal funding ratio. The probability of the funding ratio below 105 gives the probability of underfunding of the pension funds, which indirectly corresponds to number of scenarios where no indexation is granted. The probability of funding ratio greater than 115 gives the same information as the probability of delta greater than 1, as defined by the indexation rule. At 3 years horizon these probabilities are respectively 0 and 78%. These statistics reveal the sustainability of this investment strategy aimed at the maximisation of the purchasing power in the short term. At medium term the Optimal portfolio implies an indexation loss about 0.03% and also a higher dispersion of delta and in particular of the nominal funding ratio (sub-Figure d)). Also the downside risk measure suggests that the Optimal Portfolio at 5 years horizon is riskier. The indexation maximisation is obtained adopting a riskier strategy which could explain the shift to Property and risky equity. Property is less risky than the Matching portfolio, but since the latter is by definition perfectly correlated with the liabilities return, it offers less support to hedging liabilities. Figure 5.6 shows the evolution over time of the nominal funding ratio for the three Optimal Portfolios and the relative solvency constraints, assuming to keep constant their compositions for 10 years. For the first 3 years, all the three portfolios are able to grant full indexation and to oblige to the solvency requirements. However the Optimal Portfolio at T=3, is not able to offer full indexation over five years. Higher portfolio returns are needed to reach sustain the indexation, obtained by the introduction of Equity and Property. At longest horizon the Optimal Portfolio T=5 cannot be implemented because due to the risk of its strategy it does not satisfy all the solvency constraints, and the best investment strategy is similar to the Optimal Portfolio at T=3.

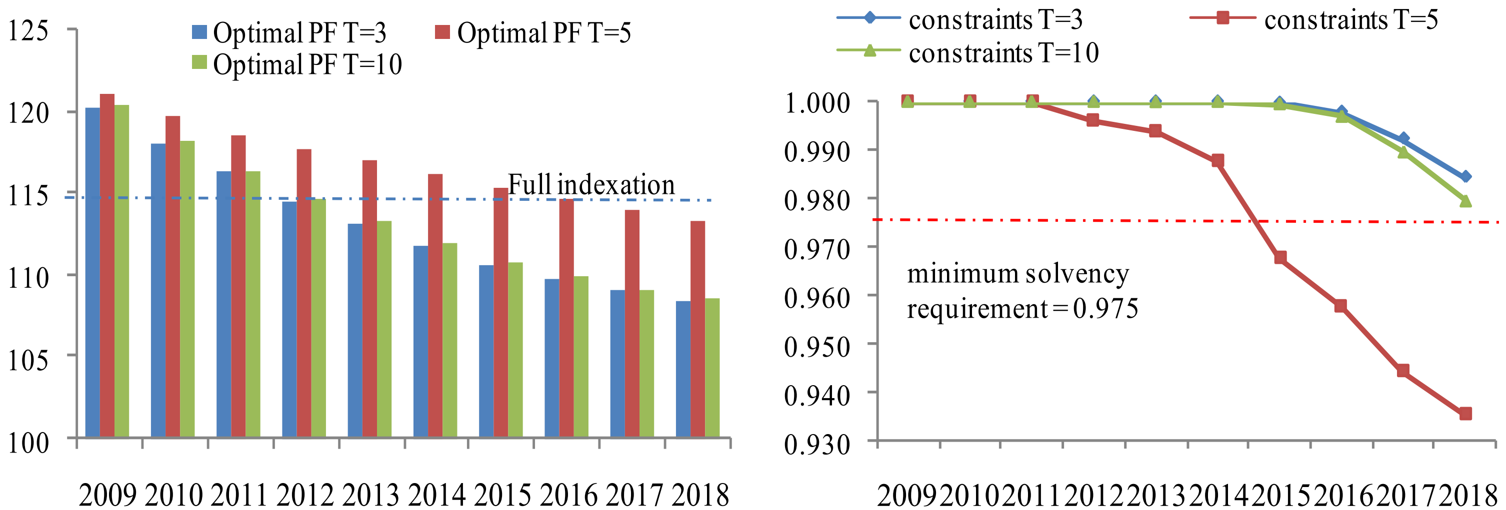

Figure 5. 5: Nominal funding ratio and solvency constraints over time (FR=120, gamma =10)

The Figure shows the evolution over time of the nominal funding ratio for the Optimal Portfolios and of the relative solvency constraints, assuming to keep constant their compositions for 10 years. For the first 3 years, the portfolios are all able to grant full indexation and to meet the solvency requirements. However the Optimal Portfolio at T=3, is not able to offer full indexation over five years. Higher portfolio returns are needed to reach higher indexation, obtained by the introduction of Equity and Property. At longest horizon the Optimal Portfolio T=5 cannot be implemented because it does not satisfy all the solvency constraints, and the best investment strategy is similar to the Optimal Portfolio at T=3.

At 10 years horizon the cumulative impact of the indexation strongly affect the liabilities which needs to be matched by a stronger investment in the Matching Portfolio. However, the indexation is fully granted only in the first three years and is halved at the end of the period (T=10). This implies an indexation loss equal to 4.3% (delta equal to 0.95), approximately corresponding to two years of missing indexation when the inflation rate is constant and equal to 2%. The probability of delta grater than 0.98 is only 26%, while the probability of underfunding rises to 2%, close to the regulatory constraint of 2.5%. These statistics suggest the riskiness relative to the implementation oft an indexation-based optimisation over longer horizon (with a static asset allocation) and a convenience for a 3 years investment horizon.

In this perspective it is important to examine how the composition of the portfolio changes at 3 years horizon, given different initial funding ratio. This kind of comparison is only available at this short horizon, because the Solver is not able to give feasible solutions at longer horizon. It is obvious to recognize that the higher utility is associated with the scenario with the higher funding ratio. The Figure 5.7 shows the compositions of the Optimal Portfolios for different initial funding ratio at 3 years horizon. All the portfolios are composed by the same type of assets: Matching Portfolio, Property and Equity. At initial funding ratio equal to 120 and 130 the weighting of the portfolio is very similar and the portfolios reach the full indexation. When the initial funding ratio is equal to 110, the delta is 0.976 and the volatility is significantly higher. As seen before, we can notice that when the pension fund needs to invest more aggressively to reach higher level of indexation in longer period, or when it starts from a weak financial position, the weight invested in Property increases with respect to the other Optimal Portfolios. The riskier strategy is confirmed by the standard deviation of delta (*see Appendix*).

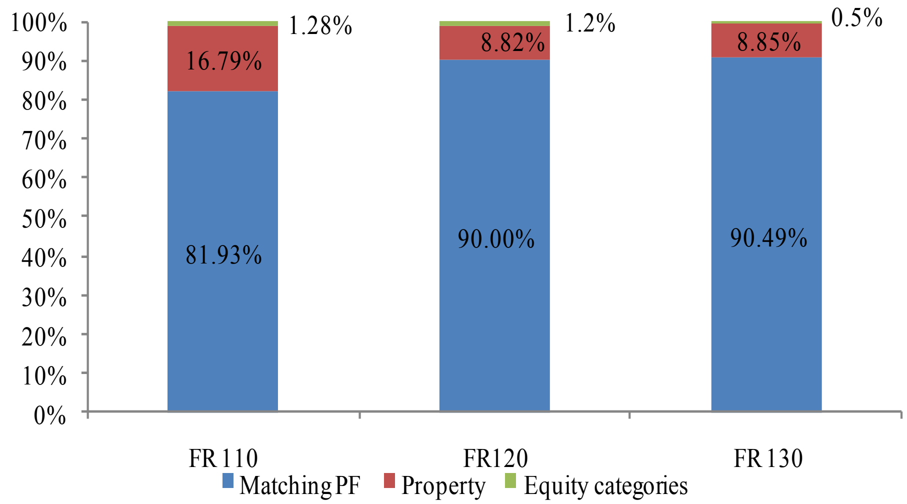

Figure 5. 6: Optimal Portfolios for different initial funding ratio (T=3, gamma =10)

The Figure shows the compositions of the Optimal Portfolio optimized over 3 years horizon assuming different initial funding ratio. At higher levels of funding ratio, the composition is quite similar. At funding ratio equal to 110 we observe a shift of resources from the Matching PF to Property. Commodity has no role in the short term.

As noticed before, Commodity does not play any role at short horizon. The inclusion of this asset is reported only at 10 years horizon, when the pension fund is in a solid financial position (FR=130). The Figure 5.8 shows the compositions of the Optimal portfolios for the three investment horizon under investigation, when gamma is set equal to 10 and the initial funding ratio is 130. An interesting result is that the composition of the Optimal portfolios at 3 and 5 years horizon is similar to the composition seen before, composed by Matching Portfolio, Property and Equity. In this case, given the stronger initial position, the full indexation is reached, also at 5 years horizon, associated with very low standard deviation (*see* Table 6 in the Appendix). At 10 year horizon, the composition is different. Once again there is a shift of resources from Matching Portfolio to Property, but also to Commodity (3.1%) and to Equity. This could be explained by a riskier investment strategy. These results confirm the crucial role played by Property, the secondary role played by Equity and that Commodity have risk diversification property exploitable only in the long-term due to the solvency constaints.

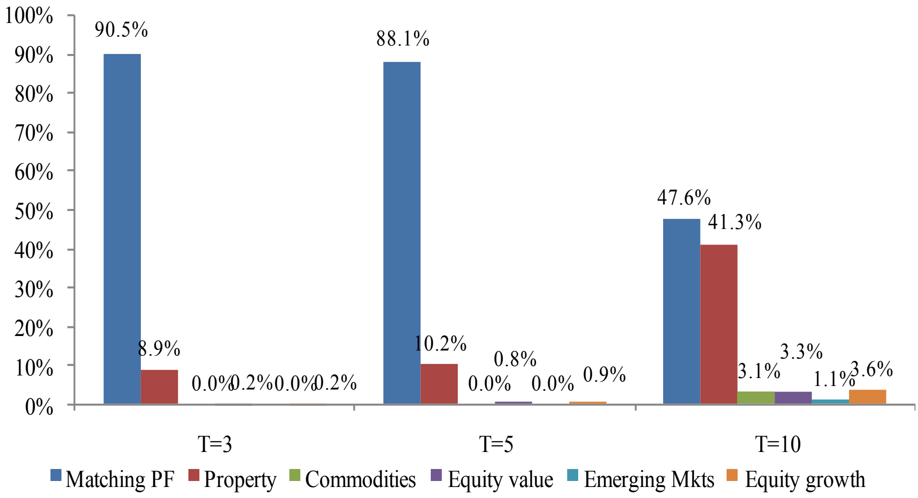

Figure 5. 7: Composition of Optimal Portfolios for initial funding ratio 130 and gamma 10

The Figure shows the compositions of the Optimal Portfolio for the horizons under investigation. At 3 and 5 years horizon, the compositions closely converge to the composition seen before. It is composed by Matching Portfolio, Property and Equity. At 10 year horizon, the composition is different. Once again there is a shift of resources from Matching Portfolio to Property, but also to Commodity (3.1%) and to Equity.

From the analysis developed so far, it emerges a convergence in the compositions of the Optimal Portfolios able to ensure the full indexation at different horizon or initial funding ratio. This composition invests around 88-90% in Matching Portfolio, 8.8-10% in Property and residual resources in equity categories. This composition changes only when riskier investment strategy are needed to rich higher level of indexation. However, this composition can not be easily replicated in the financial market. For this reason we restrict the Matching Portfolio to be equal or smaller than 63%. Feasible solutions are available only at higher initial funding ratio and at short and medium term. As reported in the Appendix, these portfolios reach slightly lower or equal level of utility than the Optimal Portfolios, but associated to a higher standard deviation. The compositions of these restricted Portfolios are shown in Figure 5.9. What can not be invested in Matching Portfolio goes, almost completely, to the investment in Property. The investment in Equity stays almost unchanged, with exception of a slightly higher investment in Emerging Market Equity at short horizons.

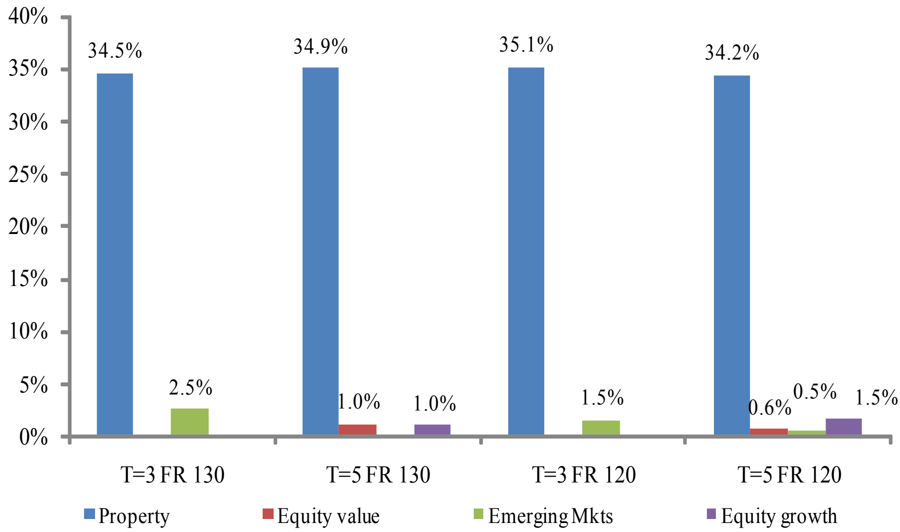

Figure 5. 8 Composition of restricted Optimal Portfolio (Matching Portfolio =63%, gamma =10)

The Figure shows the compositions of the restricted Optimal portfolio at short and medium horizon. The Matching Portfolio weight is equal to 63%. The remaining resources are mainly allocated in Property. A residual amount of resources is invested in Equity.

Once again, these restricted Optimal portfolios present a shortcoming relative to their composition, given the illiquid nature of the Property asset. A pension fund hardly invests such a large amount of its resources in an illiquid investment. Figure 5.10 shows what happen to the composition of the portfolio if we add a new constraint and restrict the weight of property to be equal or smaller than 15%. Feasible solutions are only available at medium and short term when the funding ratio is 130. We observe a relevant investment in Equity passive and a stronger investment in Commodity. The contributions of these assets in the short and medium term exist and are valuable when Property is not available.

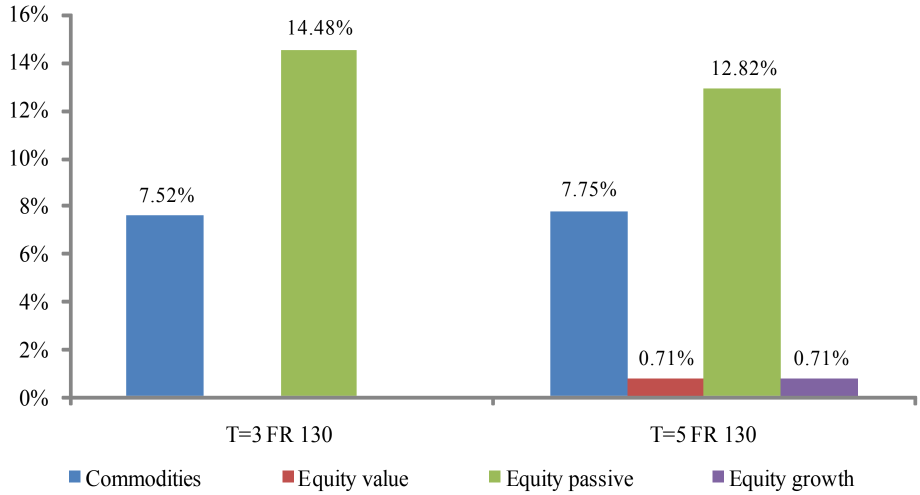

Figure 10: Composition of restricted Optimal Portfolio with restriction on Property and Matching Portfolio (Property=15%, MP=63%, gamma =10.

The Figure shows the composition of restricted Optimal portfolios when the investment in Property and in the Matching Portfolio are restricted. The role played by Equity and Commodity in short and medium term is valuable when Property is not available.

5.6 Conclusions

This chapter has developed an indexation-based optimisation model aimed to maximise the purchasing power of the participants of pension funds, characterized by conditional indexation policy. Given this new pension deal offered by pension fund, the indexation has to be considered as the objective function of the optimisation of the portfolio. Since the compensation of the liabilities for losses in the purchasing power is not guaranteed anymore by the pension fund, a specific model is needed to aim for its maximisation. The model has been applied to the real case of ABN AMRO Pension fund and considers also the inclusion of alternatives assets in the portfolio of the fund. The composition of Optimal Portfolios has been examined for different initial funding ratios, risk aversion levels and investment horizons. The influence of different risk aversion levels in the definition of the compositions of the Optimal portfolio is limited. The sustainability of the indexation-based portfolios is easily affordable at short horizon, even if the full indexation can be reached only at higher level of the initial funding ratio. The initial funding ratio strongly affects the capability of the fund to set an investment strategy over longer horizon. This can be easily explained considering the cumulative effect of the indexation policy. Concerning the composition, there is a convergence in the results towards a portfolio composed by Matching Portfolio (around 88-90%), Property (8-9%) and a residual part in Equity (1-2%). There is no strong role for typical inflation hedger assets as Commodity and Equity. Property represents a better investment opportunity than Equity at every horizon. These compositions change when riskier strategies are needed to reach higher level of indexation. In this case there is a significant shift of resources from the Matching Portfolio to Property. Commodity is included in the portfolio only at the longest horizon and when the fund has a solid initial financial position to overcome the risk-based regulatory constraint. These results partially contrast with the main finding in the literature, that is to say that Equity are a preferable asset to Property and that Commodity are good risk-diversifier at each horizon. However, when we restrict the optimisation imposing constraints to the investment in Matching Portfolio (due to the imperfections of the long-term bond market) and in Property (due to its illiquid nature), Commodity and in particular Equity Passive play a crucial role in the short and medium-term.

The analysis also reveals that for a Dutch Pension fund, linked to the Dutch price inflation, there is limited possibility to exploit inflation hedging properties of the assets in the portfolios. The correlations with the inflation are most of the same very close to zero. An indexation policy linked to the Euro Area inflation would probably be helped by higher correlations with the assets. In the same way, the sustainability if the indexation-based optimised portfolio would increase at longer horizon by considering dynamic asset allocations. Next to these, there are some obvious paths for developing this research. In order to have a tractable model we made several simplifying assumption along the way. A first step would be to remove part of these assumptions, for instance regarding the contribution policy. From a methodological perspective, the use of a mean-shortfall model can be considered. On the asset side, the analysis can be extended to consider a wider asset mix also including inflation linked bond and derivatives. On the liabilities side, the analysis can be extended to consider the conditional indexation policy in our model as an embedded option that should be adequately evaluated in the fair valuation of the liabilities.

5.7 Appendix C

Table 5. 4 Optimal Portfolio for initial funding ratio equal to 110

T= 3	g=5		g=10		g=20	
FR 110	Optimal PF	100% MP	Optimal PF	100% MP	Optimal PF	100% MP
U	0.9749	0.9591	0.9736	0.9577	0.9709	0.9549
E(δ)	0.976	0.960	0.976	0.960	0.976	0.960
σ(δ)	0.0163	0.0163	0.0163	0.0163	0.0163	0.0163
Indexation Loss	0.024	0.041	0.024	0.041	0.024	0.041
Matching PF	0.82	1.00	0.82	1.00	0.82	1.00
Property	0.17	0.00	0.17	0.00	0.17	0.00
Commodities	0.00	0.00	0.00	0.00	0.00	0.00
Equity value	0.00	0.00	0.01	0.00	0.01	0.00
Equity passive	0.00	0.00	0.00	0.00	0.00	0.00
Emerging Mkts Eq.	0.00	0.00	0.00	0.00	0.00	0.00
Equity growth	0.00	0.00	0.00	0.00	0.01	0.00

Table 5. 5: Optimal Portfolio for initial funding ratio equal to 120

g=5		T=3			T=5			T=10
FR=120	Optimal PF	100% MP	MP-restricted PF	Optimal PF	100% MP	MP-restricted PF	Optimal PF	100% MP
U	0.9997	0.9993	0.9987	0.9965	0.9893	0.9965	0.9547	0.9240
E(δ)	1.000	0.999	0.999	0.997	0.990	0.997	0.959	0.930
□ (δ)	0.0009	0.0021	0.0044	0.0077	0.0129	0.0079	0.0294	0.0336
Indexation Loss	0.000	0.001	0.001	0.003	0.010	0.003	0.043	0.076
Matching PF	0.90	1.00	0.63	0.66	1.00	0.63	0.89	1.00
Property	0.09	0.00	0.35	0.31	0.00	0.34	0.09	0.00
Commodities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity value	0.01	0.00	0.00	0.00	0.00	0.02	0.00	0.00
Equity passive	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Emerging Mkts Eq.	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00
Equity growth	0.01	0.00	0.00	0.02	0.00	0.00	0.00	0.00

g=10		T=3			T=5			T=10
FR=120	Optimal PF	100% MP	MP-restricted PF	Optimal PF	100% MP	MP-restricted PF	Optimal PF	100% MP
U	0.9997	0.9993	0.9986	0.9962	0.9885	0.9962	0.9504	0.9183
E(δ)	1.000	0.999	0.999	0.997	0.990	0.997	0.959	0.930
σ(δ)	0.0009	0.0021	0.0044	0.0077	0.0129	0.0079	0.0290	0.0336
Indexation Loss	0.000	0.001	0.001	0.003	0.010	0.003	0.043	0.076
Matching PF	0.90	1.00	0.63	0.68	1.00	0.63	0.89	1.00
Property	0.09	0.00	0.35	0.30	0.00	0.34	0.09	0.00
Commodities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Equity value	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00
Equity passive	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Emerging Mkts Eq.	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Equity growth	0.01	0.00	0.00	0.02	0.00	0.01	0.01	0.00

g=20		T=3			T=5			T=10
FR=120	Optimal PF	100% MP	MP-restricted PF	Optimal PF	100% MP	MP-restricted PF	Optimal PF	100% MP
U	0.9997	0.9993	0.9985	0.9956	0.9868	0.9956	0.9421	0.9070
E(δ)	1.000	0.999	0.999	0.997	0.990	0.997	0.959	0.930
σ(δ)	0.0009	0.0021	0.0044	0.0076	0.0129	0.0079	0.0287	0.0336
Indexation Loss	0.000	0.001	0.001	0.003	0.010	0.003	0.043	0.076
Matching PF	0.90	1.00	0.63	0.69	1.00	0.63	0.89	1.00
Property	0.09	0.00	0.35	0.28	0.00	0.34	0.09	0.00
Commodities	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Equity value	0.01	0.00	0.00	0.00	0.00	0.01	0.01	0.00
Equity passive	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Emerging Mkts Eq.	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
Equity growth	0.01	0.00	0.00	0.02	0.00	0.01	0.01	0.00

Table 5. 6: Optimal Portfolio for initial funding ratio equal to 130

g=5	T=3				T=5				T=10	
FR 130	Optimal PF	100% MP	MP-restricted PF	MP-Prop-restricted	Optimal PF	100% MP	MP-restricted PF	MP-Prop-restricted PF	Optimal PF	100% MP
U	1.0000	1.0000	1.0000	0.9998	1.0000	0.9999	0.9999	0.9988	0.9985	0.9796
E(δ)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.982
σ(δ)	0.0000	0.0000	0.0007	0.0019	0.0001	0.0006	0.0011	0.0051	0.0061	0.0221
Indexation Loss	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.018
Matching PF	0.91	1.00	0.63	0.63	0.88	1.00	0.63	0.63	0.46	1.00
Property	0.09	0.00	0.34	0.15	0.10	0.00	0.35	0.15	0.43	0.00
Commodities	0.00	0.00	0.00	0.07	0.00	0.00	0.00	0.08	0.03	0.00
Equity value	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.03	0.00
Equity passive	0.00	0.00	0.00	0.15	0.00	0.00	0.00	0.14	0.00	0.00
Emerging Mkts Eq.	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Equity growth	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.00	0.04	0.00

g=10	T=3				T=5				T=10	
FR 130	Optimal PF	100% MP	MP-restricted PF	MP-Prop-restricted	Optimal PF	100% MP	MP-restricted PF	MP-Prop-restricted PF	Optimal PF	100% MP
U	1.0000	1.0000	1.0000	0.9997	1.0000	0.9999	0.9999	0.9987	0.9983	0.9771
E(δ)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.982
σ(δ)	0.0000	0.0000	0.0007	0.0019	0.0001	0.0006	0.0011	0.0051	0.0060	0.0221
Indexation Loss	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.018
Matching PF	0.90	1.00	0.63	0.63	0.88	1.00	0.63	0.63	0.48	1.00
Property	0.09	0.00	0.35	0.15	0.10	0.00	0.35	0.15	0.41	0.00
Commodities	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.08	0.03	0.00
Equity value	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.03	0.00
Equity passive	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.13	0.00	0.00
Emerging Mkts Eq.	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Equity growth	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.04	0.00

g=20	T=3				T=5				T=10	
FR 130	Optimal PF	100% MP	MP-restricted PF	MP-Prop-restricted	Optimal PF	100% MP	MP-restricted PF	MP-Prop-restricted PF	Optimal PF	100% MP
U	1.0000	1.0000	1.0000	0.9997	1.0000	0.9999	0.9998	0.9984	0.9980	0.9723
E(δ)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.999	0.999	0.982
σ(δ)	0.0000	0.0000	0.0007	0.0019	0.0001	0.0006	0.0011	0.0051	0.0060	0.0221
Indexation Loss	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.018
Matching PF	0.91	1.00	0.63	0.63	0.88	1.00	0.63	0.63	0.50	1.00
Property	0.09	0.00	0.35	0.15	0.10	0.00	0.35	0.15	0.39	0.00
Commodities	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.08	0.03	0.00
Equity value	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.02	0.00
Equity passive	0.00	0.00	0.00	0.14	0.00	0.00	0.00	0.12	0.00	0.00
Emerging Mkts Eq.	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.01	0.00
Equity growth	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01	0.05	0.00

6
Evaluating conditional indexation as Embedded Option

6.1 Introduction

Pension funds recognizing inflation indexation targets are obliged to pay an additional payoff that is linked to the inflation rate through some specific rule as those defined in the previous chapter. The additional payoff normally takes the form of a contingent claim conditional to a "measure" of sustainability of the payoff itself (the funding ratio); able to capture and guarantee the solvability of the fund itself. Therefore, a full valuation of the obligation towards fund's participants cannot exclude the proper appraisal of this additional option. The option payoff is conditional to a measurement asset that is different from the reference underlying asset. This structure recalls a barrier option with different measurement and payoff asset.

The chapter investigates the opportunity to apply barrier option scheme to the case of a pension fund, whose indexation target is conditional to a specific value of the funding ratio, in order to provide a full valuation of the obligation towards participants. The main objective is to provide a value for the inflation indexation as embedded option. Results derive from a simulation procedure applied to an exemplar case by means of scenario-based analysis (see Chapter 3). The dataset and the indexation rule correspond to the previous implementations in the Dutch-based pension funds. Numerical results give the opportunity to state the absolute value of the “inflation option” and the relative value with respect to the fund’s liability.

6.2 Methodology

The conditional indexation agreement, depending on the funding ratio, configures as a structured product, in particular a barrier option, which is implicitly embedded in the pension contracts that the pension fund sells to its participants as suggested in de Jong (2008). Among different types of barrier option, we evaluate this indexation option (IO) as an outside barrier option call down-and-out. Next section describes the general functioning of the barrier options and in particular the payoffs of the outside barrier options chosen to describe the indexation option. The following paragraph evaluates this option by means of scenario analysis.

6.2.1 Outside Barrier Options

Barrier options are option that either are born (in barrier or knock in) or expire (out barrier or knock out) when the asset price reaches a specified value H defined as “barrier”. Given the presence of the barrier, these options are typically evaluate at a lower value (with higher expected return) with respect to plain vanilla options. Several types of barrier option (put and call) can be formulized which are divided in four categorises:

--*up-and-in option*, where the price of the underlying asset (S) have to growing up to the value H before the expiration date. Only in this case the holder will be entitled to exercise the option.

--*down-and-in option*, where the option born only if the asset price decreases and reach the barrier value H.
--*up-and-out option*, where the holder loses the right to exercise the option if the underlying asset S increases until it reaches the barrier H during the life of the option.
--*down-and-out option*, where the contract expired if the asset price reaches or falls below the value barrier H before the expiration date.

To configure the scheme of the conditional indexation policy we will refer to a particular type of the barrier down-and-out option, called as outside barrier option characterized by the presence of two underlying assets. The first asset represents the “measurement asset”: the possibility of knocking out depends solely on the fact that the measurement reaches the barrier level during the life of the option. The second asset represents the “payoff asset”, which ultimately defines the payoff of the option. In an outside barrier option call down-and-out an increase in the price of the measurement asset, will increase the value of the option only if also the payment assets will have an increase, that’s to say if there is a positive correlation between the two underlying assets.

To evaluate an outside barrier option closed form analytical formula has been developed (see Zhang 1995). The evaluation of the outside barrier option requires that the density function contains the lognormal distribution of the asset price payoff which is conditional upon the achievement or failure to achieve (depending of if it is knock in or knock out) of the barrier level by the price of the measurement asset during the life of the option.

The density function is:

$$\xi(x) = f(x)N\left\{\vartheta\varsigma\,\frac{\left[d_{bs}(M,H,\sigma_2)+\rho u\right]}{\sqrt{1-\rho^2}}\right\} - \varsigma e^{\frac{2av_2}{\sigma_2{}^2}} f\left(u - \frac{2\rho a}{\sigma_2\sqrt{\tau}}\right)$$
$$\times N\left\{\vartheta\left(\frac{\left[d_{bs}(M,H,\sigma_2)+\rho u\right]}{\sqrt{1-\rho^2}} + \frac{2a}{\sigma_2\sqrt{\tau}}\sqrt{1-\rho^2}\right)\right\}$$

(6.1)

where :

$$v_2 = r - g_2 - \frac{\sigma_2{}^2}{2} \text{ and } a = \ln\left(\frac{H}{S}\right); \tag{6.2}$$

and x represents the log returns of the payment asset;

$$u = \frac{(x - v_\tau)}{\left(\sigma_2\sqrt{\tau}\right)} \tag{6.3}$$

is a normal standard variable of x;

$$d_{bs}(M,H,\sigma_2) = \frac{\left[\ln(M/H) + v_2\tau\right]}{\sigma_2\sqrt{\tau}} \tag{6.4}$$

Has the same definition as in the traditional Black and Scholes (1973) formulation. It represents the probability of the option to be exercised.

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Moreover, ϑ e ζ are two binary operators which indicate the direction of the option (ϑ = 1 down, ϑ = -1 up), and if the option is a Knock in o Knock out (ζ = 1 out, ζ = -1 in).

Given this density function above, the payoff function is defined as follows:

$$OTDS(\omega,\vartheta,\varsigma,\rho)=e^{-r\tau}\int \max\left[\omega Se^{x}-\omega K,0\right]\xi(x)dx \tag{6.5}$$

As ϑ and ζ, ω is a binary operator which indicates the type of option (1 for a call and -1 for a put).

Finally the pricing of the formula is given by:

$$\begin{aligned} priceOTDS(\omega,\vartheta,\varsigma,\rho) &= \omega Se^{-g\tau}N_2\left[\omega d_{1bs}(S,K,\sigma),\vartheta\varsigma d_{12},\omega\vartheta\varsigma\rho\right] \\ &-\omega Ke^{-r\tau}N_2\left[\omega d_{bs}(S,K,\sigma),\vartheta\varsigma d_{bs}(M,H,\sigma_2),\omega\vartheta\varsigma\rho\right] \\ &-\varsigma\left(\frac{H}{M}\right)^{\frac{2v_2}{\sigma_2{}^2}}\left\{\begin{array}{l}\omega S\left(\frac{H}{M}\right)^{\frac{2\rho\sigma}{\sigma_2}}e^{-g\tau}N_2\left[\omega d_{21},\vartheta d_{22},\omega\vartheta\rho\right] \\ -\omega Ke^{-r\tau}N_2\left[\omega\left(d_{21}-\sigma\sqrt{\tau}\right),\vartheta\left(d_{22}-\rho\sigma\sqrt{\tau}\right),\omega\vartheta\rho\right]\end{array}\right\} \end{aligned} \tag{6.6}$$

where :

N_2 is a binary normal distribution function;

$$d_{12}=d_{bs}(M,H,\sigma_2)+\rho\sigma\sqrt{\tau}; \tag{6.7}$$

$$d_{21}=d_{1bs}(S,K,\sigma_1)+\frac{2a\rho}{\sigma_2\sqrt{\tau}}; \tag{6.8}$$

$$d_{22}=d_{12}+\frac{2a}{\sigma_2\sqrt{\tau}}; \tag{6.9}$$

Where M is the price of the asset measurement; H is the barrier layer and S is the payment asset.

More in general, the payoff of an outside barrier option call is given by the difference between the payment asset and the strike price conditional on the event that the measurement asst assumes a value greater than the barrier level. Otherwise the option is knocked out.

6.2.2 Evaluating indexation option

For the application of the outside barrier option to the indexation option case, the formulas defined above can not be appropriately used. This is due to the fact that they assume a continue barrier over the life of the option. In the pension fund case, the barrier represented by a specified level of the funding ratio, in not observed continuously, but in a discrete time and the observation period is set equal to the last day of each year, when the market value of the assets and liabilities is computed and the inflation rate is observed.

Therefore, we will define the indexation option (IO) as an outside barrier option (down-and-out) having a discrete barrier. For this reason we will proceed to a simulated evolution of the embedded option.
We define:

$$\begin{array}{lll} L_t(\pi_t) = S_t & \rightarrow & payoff \quad asset \\ K = 0 & \rightarrow & strike \quad price \\ FR_t = M_t & \rightarrow & measurement \quad asset \\ H & \rightarrow & barrier \end{array}$$

where:

$L_t(\pi_t)$ is the indexation value in terms of amount at time t

FR_t is the funding ratio value at time t

The IO payoff is given by:

$$IO = \begin{cases} \max\left[L_t(\pi_t), 0\right] & \text{IF} \quad (FR_t) > barrier \quad level \\ 0 \quad \text{IF} \quad (FR_t) \leq barrier \quad level \end{cases} \tag{6.10}$$

$$IO = \begin{cases} \max\left[S_t - K, 0\right] & \text{IF} \quad (M_t) > H \\ 0 \quad \text{IF} \quad (M_t) \leq H \end{cases} \tag{6.11}$$

The indexation rule can be expressed as a function of :

--nominal value of the liabilities at the end of year t, L_t

--inflation rate of the year t, π

--nominal funding ratio at the end of the year t , FR_t

Therefore the full updated value $\widetilde{L}_t$ (post indexation) is given by:

$$\widetilde{L}_T = \begin{cases} \max\left[L_t(1+\pi_t), L_t\right] & \text{IF} \quad (FR_t) > barrier \quad level \\ L_t \quad \text{IF} \quad (FR_t) \leq barrier \quad level \end{cases} \tag{6.12}$$

If we isolate L_t :

$$\widetilde{L}_T = L_t + \underbrace{\begin{cases} \max\left[L_t(\pi_t), 0\right] & \text{IF} \quad (FR_t) > barrier \quad level \\ 0 \quad \text{IF} \quad (FR_t) \leq barrier \quad level \end{cases}}_{indexation \quad option \quad payoff}$$

(6.13)

From this formula we derive as the second component of the left side the payoff structure of the indexation option (OIP).

Successively, the IO is evaluated by numerical methods, based on scenario analysis as far as the asset and liability values are concerned.

Let us denote:

M is the total number of scenario (i.e. $m = 2500$)

j correspond to each single scenario (with $j = 1, 2, \ldots, m$)

n is the total number of relevant (residual) time nodes (i.e. $n = 14$)

t corresponds to each single time node (with $t = 1, 2, \ldots, n$)

OIP (t, j) is the payoff of IO at time t and scenario j

r(0,t,j) is the spot rate referred to time t and scenario j

We evaluate the option value at time 0 as:

$$Option\,value(time\ 0) = \frac{1}{m}\sum_{j=1}^{m}\sum_{t=1}^{n}\frac{OIP(t,j)}{(1+r(0,t,j))} \qquad (6.14)$$

The option value at time 0 gives the value of the option sell by the pension fund to the participants in the indexation agreement. The valuation of the IO is applied to the dataset assuming that:

--The assets are invested assuming static asset allocation equal to 37% for the risk return portfolio and 63% for the matching portfolio (as often in the practice of the Dutch pension funds)

--The liabilities are conditionally (only) fully indexed to inflation rate

--The barrier is set equal to 105 (as minimum solvency requirement).

6.3 Results

The methodology is applied to the dataset by means of MATLAB. The figure below shows the option payoff (OIP) for each scenario at time equal to 2009 at varying of the payoff asset and the funding ratio. The option payoff has value equal to zero when the option expires because the option in that scenario is knocked out or the payoff asset is not positive (negative inflation). On the left side we also present a histogram showing the distribution of the frequencies associated with each payoff. The x-axis the histogram represents the distribution frequency of the payoff assets in 2009 across scenarios.

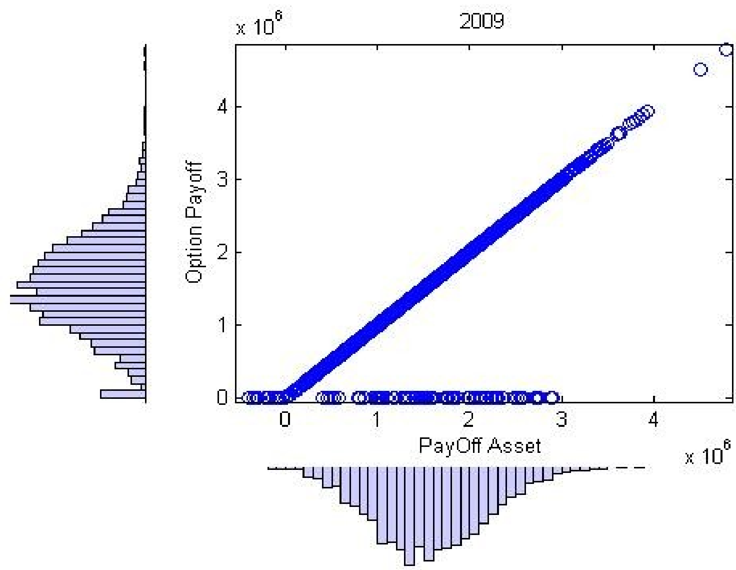

Figure 6. 1: Option Payoff and Payoff asset in 2009.

The figure shows the option payoff (OIP) for each scenario at time equal to 2009 at varying of the payoff asset and the funding ratio. In particulate the two histograms show the frequency distribution of the option payoff and the payoff assets.

The graph below gives relates the option payoff (and the relative frequency distribution) to the funding ratio dynamics in 2009.

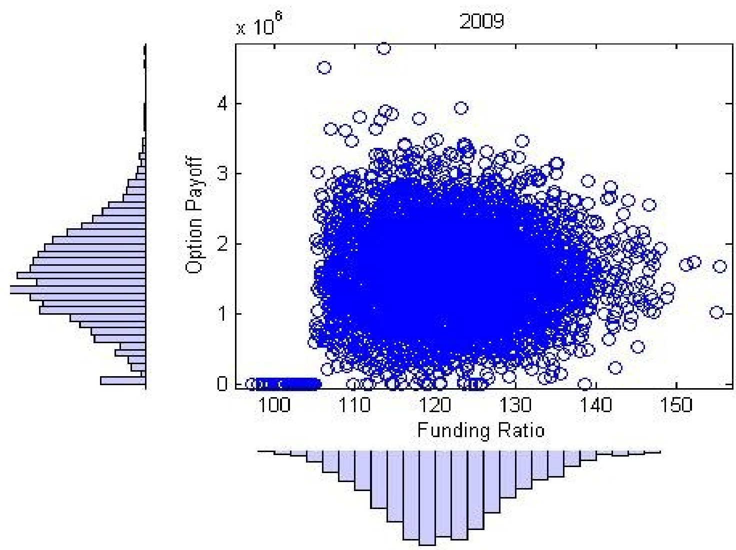

Figure 6. 2: Option payoff and Funding ratio in 2009.

The figure shows the option payoff (OIP) for each scenario at time equal to 2009 at varying of the funding ratio. This value is equal to zero when the option is knocked out.

The option price at time 0 (2009) for the indexation policy over 14 years is approximately equal to 22.38% of the nominal liabilities. It is not an irrelevant percentage of the value of the liabilities and should be probably taken in to account in their valuation.

The graph below shows the distribution of the option payoff for each year under consideration as a stochastic process. We can notice that the means and the standard deviations of the payoff increase over time. This is due to the increasing volatility of the underlying scenario over time. We can also notice that due to the higher volatility of the funding ratio the frequency associated with the case where the option is knocked out increase over time. This is also due to the fact that we assume static asset allocation. Changes in the weights of assets with inflation hedging properties can improve this effect.

We also develop the same analysis setting the barrier level at 115. As we expected, the option value increase and reaches the value of 27% of the liabilities in 2009.

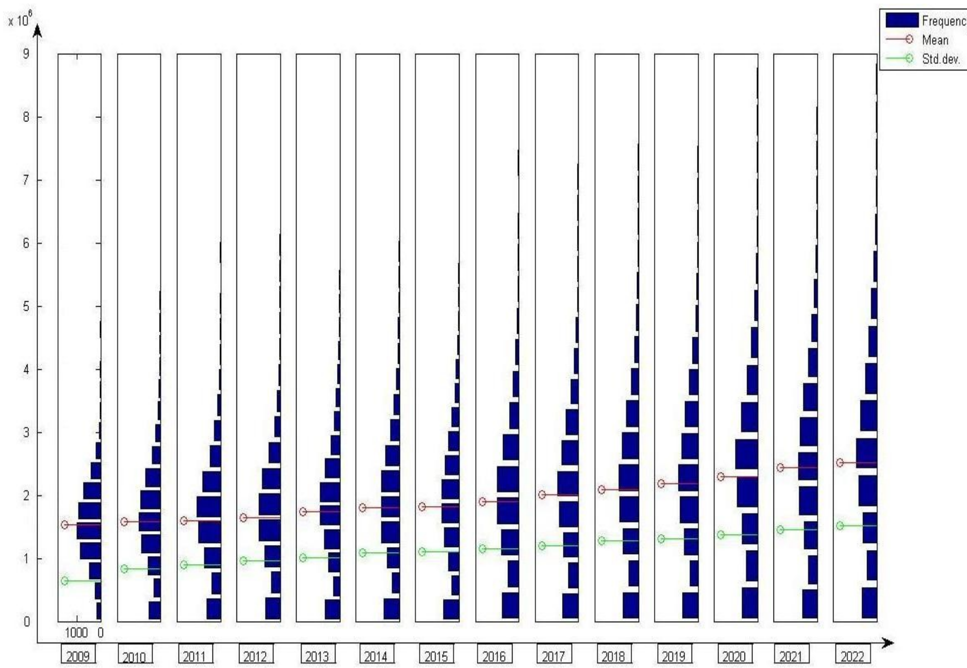

Figure 6. 3: The distribution frequency of the OIP over time.

The figure shows the option payoff (OIP) for each scenario at time equal to 2009 at varying of the payoff asset and the funding ratio. In particulate the two histograms show the frequency distributions of the option payoff and the payoff assets.

6.4 Conclusions

Conditional indexation is an important issue to be taken into account in the valuation the liabilities. It is an embedded option sells by the fund to the participants in the indexation agreements.

We model the indexation rule adopted by the Dutch pension funds to investigate which is the impact of this option on the present value of the liabilities. We show that a knock-out call barrier option (with two reference assets) provide with a good framework for this valuation. The option value in 2009 for the following 14 years amounts to 22% of the liability value when the barrier is 105 and 27% when the barrier is 115. Further investigations should try to remove several assumptions we impose as the static asset allocation or also allow for partial and recovering indexation. Also the definition of an optimal level for the barrier can be considered.

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