

Dissipated energy and Entropy Production for an Unconventional Heat Engine: The Stepwise “Circular Cycle”

Francesco di Liberto¹, Raffaele Pastore² and Fulvio Peruggi²

¹ Dipartimento Scienze Fisiche - Università di Napoli “Federico II” - SPIN-CNR, CNISM and INFN, Sezioni di Napoli

² Dipartimento Scienze Fisiche - Università di Napoli “Federico II”

diliberto@na.infn.it

Abstract

When some entropy is transferred, by means of a reversible engine, from a hot heat source to a colder one, we have the maximum of efficiency, i.e. we obtain the maximum available work. Similarly the reversible heat pumps transfer entropy from a cold heat source to a hotter one with the minimum expense of energy. On the contrary if we are faced with non reversible devices there is some Lost Work for heat engines, and some Extra Work for heat pumps. These quantities are both related to the Entropy production. The Lost Work, i.e. $W_{\text{Lost}} = W_{\text{Rev}} - W_{\text{Irrev}}$, is also called ‘degraded energy’ or ‘Energy unavailable to do work’. The Extra Work, i.e. $W_{\text{Extra}} = W_{\text{Irrev}} - W_{\text{Rev}}$, is the excess of work performed on the system in the irreversible process with respect to the reversible one (or the excess of heat given to the hotter source in the irreversible process). In this paper, which follows two previous ones on the Lost Work [*Phil. Mag.* **87**, 569 (2007), *Phil. Mag.* **88** 4177-4187 (2008)] both quantities are analyzed in deep and are evaluated for a process with complexity, i.e. the stepwise Circular Cycle which is similar to the stepwise Carnot cycle [*Physica A* **314**, 331 (2002)]. The stepwise Circular Cycle is a cycle performed by means of N small weights dw which are first added and then removed from the piston of the vessel containing the gas or *viceversa*. The work performed by the gas can be found as increase of the potential energy of the dw 's. We identify each single dw and thus evaluate its rising i.e. its increase in potential energy. In such a way we find how the energy output of the cycle is distributed among the dw 's. The size of the dw 's affects the Entropy production and therefore the Lost and Extra work. The rising distribution depends on the removing process we choose.

1 - Introduction

As pointed out in a previous paper [1], entropy production and its relation to the available energy are fascinating subjects which in last years have attracted many physics researches [5-11]. It is well known [1-10] that for some elementary irreversible process, like the irreversible isothermal expansion of a gas in contact with a heat source at temperature T , the work done by the gas $W_{\text{Irrev}} \equiv W_{\text{out}}$ is related to the reversible isothermal work W_{Rev} (i.e. the work performed by the gas in the corresponding reversible process) by the relation

$$W_{\text{out}} = W_{\text{Rev}} - T\Delta S_U \quad (1)$$

where ΔS_U is the total entropy change of the universe (system + environment). The degraded energy $T\Delta S_U$ is usually called ‘the Lost work’ W_{Lost}

$$W_{\text{Lost}} = W_{\text{Rev}} - W_{\text{out}} \quad (2)$$

The latter can be interpreted as the missing work: i.e. the additional work that could have been done in the related reversible process (here the reversible isothermal expansion); it is also called ‘energy unavailable to do work’.

On another hand in the irreversible isothermal compression $T\Delta S_U$ is called W_{Extra} i.e. the excess of work performed on the system in the irreversible process with respect to the reversible one.

$$W_{\text{Extra}} = W_{\text{in}} - W_{\text{Rev}} \quad (3)$$

where now $W_{\text{Irrev}} \equiv W_{\text{in}}$. Due to the energy balance, the same relation holds for the amounts of heat given to the source T , i.e. we have

$$Q_{\text{out}} = Q_{\text{Rev}} + T\Delta S_U \quad (4)$$

Therefore $T\Delta S_U$ is also called the ‘Excess of heat’ (Q_{Extra}), i.e. the additional heat that has been given to the source [8,9].

The total variation of Entropy, ΔS_U , is usually called ‘Entropy production’; we shall call the latter π_U . The second Law claims that $\pi_U \geq 0$ and the entropy is an extensive quantity which in the transfers between systems can only increase or stay unchanged.

2.1 - Entropy production and Lost Work and Extra Work in isothermal irreversible processes.

Let us first consider the isothermal irreversible expansion (A→B) of an ideal gas in contact with a heat source at temperature T where $V_B = V_A + \Delta V$ and $P_B = P_A - \Delta P$ with $\Delta P > 0$, $\Delta V > 0$. In such a simple process some heat¹ $Q_{\text{in}} \equiv W_{\text{out}} = P_B \Delta V = (P_A - \Delta P) \Delta V$ goes from the heat source T to the

ideal gas. There is an increase of entropy of the ideal gas, $\Delta S_{\text{gas}} = \int_A^B \frac{\delta Q_{\text{Rev}}}{T} = \frac{Q_{\text{Rev}}}{T}$ and a decrease of

the entropy of the heat source $\left(-\frac{Q_{\text{in}}}{T}\right)$, where

$$Q_{\text{Rev}} = \int_A^B \delta Q_{\text{Rev}} = W_{\text{Rev}} = \int_A^B P dV = RT \ln \frac{V_B}{V_A} = RT \ln \left(1 + \frac{\Delta V}{V_A}\right) = RT \left[\frac{\Delta V}{V_A} - \frac{1}{2} \left(\frac{\Delta V}{V_A}\right)^2 + \dots \right]$$

therefore the entropy production is

$$\pi_U \equiv \Delta S_U = \frac{Q_{\text{Rev}}}{T} - \frac{Q_{\text{in}}}{T} > 0 \quad (5)$$

Since $Q_{\text{Rev}} > Q_{\text{in}}$ we find in the ideal gas an amount of entropy greater than that taken from the heat source T . If, for example, $\Delta V = 3V_A$ we have $\pi_U = R \ln 4 - \frac{3}{4}R = 0.636R$. On the other hand for the

isothermal irreversible compression of the ideal gas (B→A) some heat Q_{out} goes from the gas to the source at temperature T . We have a decrease of the gas entropy $\Delta S_{\text{gas}} = \int_A^B \frac{\delta Q_{\text{Rev}}}{T} = -\frac{Q_{\text{Rev}}}{T}$ and an

increase of the source entropy, $\left(\frac{Q_{\text{out}}}{T}\right)$, where $Q_{\text{out}} = W_{\text{in}} = P_A(V_B - V_A) = P_A \Delta V$. Therefore the entropy production in the compression is

$$\pi_U \equiv \Delta S_U = \frac{Q_{\text{out}}}{T} - \frac{Q_{\text{Rev}}}{T} > 0 \quad (\text{since } Q_{\text{out}} > Q_{\text{Rev}}), \quad (6)$$

which, for $\Delta V = 3V_A$ gives $\pi_U = \frac{P_A \Delta V}{T} - R \ln \left(1 + \frac{\Delta V}{V_A}\right) = 3R - R \ln 4 = 1.614R$.

Observe that in the compression the entropy production is much bigger than in the expansion; here we will show how this is related to the wasting of energy. In order to find how the previous entropy productions affects the dissipation of energy, we have to remark that the irreversibility of a generic process (A→B) is due, in general, to internal and external irreversibility, therefore, as shown in [1,2,5] the related entropy production π_U can be expressed as a sum of two terms: the internal entropy production, $\pi_{\text{int}} \geq 0$ and the external entropy production, $\pi_{\text{ext}} \geq 0$ i.e.

$$\pi_U = \pi_{\text{int}} + \pi_{\text{ext}} \quad (7)$$

¹ The quantities Q_{in} , Q_{out} , Q_{Rev} are positive.

This result is not trivial since $\pi_{\text{int}} \neq \Delta S_{\text{sys}}$; there are in fact many processes for which $\Delta S_{\text{sys}} < 0$ and $\pi_{\text{int}} \geq 0$. The system entropy production π_{int} is defined [1,2,5] by the relation

$$\Delta S_{\text{sys}} = S_{\text{in}} - S_{\text{out}} + \pi_{\text{int}} \quad (8)$$

where S_{in} and S_{out} are respectively the quantity of entropy which respectively comes into and comes out of the system in the irreversible process; $\Delta S_{\text{sys}} = \int_A^B \frac{\delta Q_{\text{Rev}}}{T_{\text{sys}}}$ is the entropy variation of the system from A to B and does not depend on the particular process. Similarly the external Entropy production, π_{ext} is given by the relation

$$\Delta S_{\text{ext}} = S_{\text{in}}^{\text{ext}} - S_{\text{out}}^{\text{ext}} + \pi_{\text{ext}} \quad (9)$$

or by relation (7). It is easy to verify that for both previous irreversible isothermal processes $\pi_{\text{ext}} = 0$ and therefore that for both the expansion and the compression $\pi_{\text{int}} = \pi_U$. In the Appendix we give the relations for the Lost Work and Extra Work for isothermal processes with internal and external irreversibility ($\pi_{\text{ext}} \neq 0$). From relations (A4) and (A6) it follows that the Lost Work for an isothermal expansion at temperature $T = T_0$ and without external irreversibility ($\pi_{\text{ext}} = 0$) is

$$W_{\text{Lost}} = W_{\text{Rev}} - W_{\text{out}} = T_0 \pi_{\text{int}} \quad (10)$$

and that the Extra Work for an isothermal compression at $T = T_B$ (with $\pi_{\text{ext}} = 0$) is

$$W_{\text{Extra}} = W_{\text{in}} - W_{\text{Rev}} = T_B \pi_{\text{int}} \quad (11)$$

Therefore we understand that in the irreversible compression much more energy is wasted than in the irreversible expansion.

2.2 - Entropy production, Lost Work and Extra Work in isobaric irreversible processes

Let us consider the irreversible isobaric expansion at pressure P_A (heating) of one mole of monatomic ideal gas from the state A to the state B for which, for example, $T_B = 2T_A$. The ideal gas, initially at temperature T_A , is brought in thermal contact with the source $T_B = 2T_A$, then an

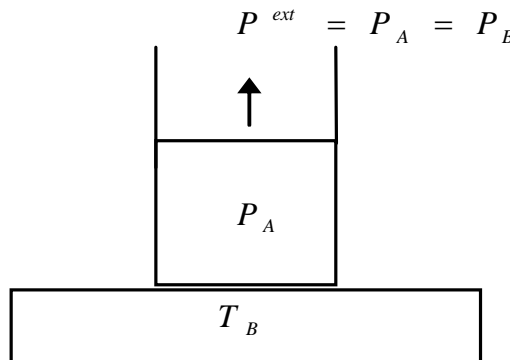


Figure 1 - ...

irreversible isobaric expansion at pressure $P_A = P_B = P^{\text{ext}}$ takes place and the ideal gas reaches the final state B. Let V_A be the initial volume and V_B the final volume. In the expansion the gas has performed the work ($= E_{\text{out}}$)

$$W_{\text{irrev}} \equiv W_{\text{out}} = P_A (V_B - V_A) = R(T_B - T_A) \quad (12)$$

and has extracted from the source T_B the heat ($=E_{in}$) $|Q_{irrev}| = C_p(T_B - T_A)$. From the energy balance we understand that there has been an increase of internal energy

$$\Delta U_{AB} = E_{in} - E_{out} = C_V(T_B - T_A)$$

where C_p and C_V are the molar specific heats respectively at constant pressure and at constant volume. For this process

$$\pi_U = \Delta S_{sys} + \Delta S^{ext} = C_p \ln \frac{T_B}{T_A} - \frac{|Q_{irrev}|}{T_B} \quad (13)$$

As in the previous case we want to find π_{int} and π_{ext} , i.e. the Entropy production due to the internal irreversibility and the Entropy production due to the external irreversibility. The path we follow is to analyse the related externally reversible process (Eso-reversible process); for this we evaluate the Entropy production, which is therefore due only to the internal irreversibility. This will be π_{int} . From this we can have π_{ext} (the Entropy production due to the external irreversibility) by subtracting π_{int} from π_U , i.e.

$$\pi_{ext} = \pi_U - \pi_{int} \quad (14)$$

To perform the Eso-reversible process we need a sequence of heat sources ranging from T_A to T_B , from which the gas takes, at each infinitesimal step, the heat δQ to perform the irreversible isobaric expansion, and an auxiliary reversible heat engine which takes the heat $\delta Q^{Eso} = \frac{T_B}{T} \delta Q$ from the source at temperature T_B and gives the heat $\delta Q = C_p dT$ to the source at temperature T of the sequence. Such an engine performs the work $\delta W_{Rev} = \delta Q^{Eso} \left(1 - \frac{T}{T_B}\right)$ at each step. Obviously

$$Q^{Eso} = \int_A^B \delta Q^{Eso} = C_p T_B \int_A^B \frac{dT}{T} = T_B C_p \ln \frac{T_B}{T_A}.$$

In this Externally reversible process (Eso-reversible) the Entropy production due to the Internal irreversibility $\delta\pi_{int}$ at each step is due to the infinitesimal variation of Entropy of the gas (i.e. $dS_{sys}^{Eso} = C_p \frac{dT}{T}$) and to the infinitesimal variation of Entropy of the heat source of the sequence

which is active in the step (i.e. $dS_{ext}^{Eso} = -C_p \frac{dT}{T}$), hence

$$\delta\pi_{int} = dS_{sys}^{Eso} + dS_{ext}^{Eso} = C_p \frac{dT}{T} - C_p \frac{dT}{T} = 0$$

We find therefore that $\pi_{int} = 0$, which means that there is no internal Entropy production in this Eso-reversible process; therefore

$$\pi_{ext} = \pi_U = C_p \ln \frac{T_B}{T_A} - \frac{|Q_{irrev}|}{T_B} \quad (15)$$

Remark that the global Entropy change is related the local Entropy productions by means of the following relation

$$\pi_U \equiv \Delta S_U = \Delta S_{sys} + \Delta S^{ext} = \pi_{int} + \pi_{ext}$$

For the irreversible isobaric expansion at pressure P_A (heating) of one mole of monatomic ideal gas from the state A to the state B for which, for example, $T_B = 2T_A$ and $C_p = \frac{5}{2}R$ the irreversible work done by the gas and the heat taken from the source T_B are $W_{irrev} = P_A(V_B - V_A) = R(T_B - T_A) = RT_A$ and $Q_{irrev} = C_p(T_B - T_A) = C_p T_A$; therefore

$$\pi_{\text{ext}} = \pi_U = C_p \ln \frac{T_B}{T_A} - \frac{|Q_{\text{irrev}}|}{T_B} = C_p \left(\ln 2 - \frac{1}{2} \right) = 0.193 C_p. \quad (16)$$

To find the Lost Work we need the available total Reversible Work. The total Reversible work is the reversible work made by the gas (which is identical to the irreversible work) and the work made by the auxiliary reversible engine working between T_B and the variable temperature T of the sequence of sources which we use to perform the reversible isobaric expansion.

$$W_{\text{Rev}}^{\text{Total}} = W_{\text{Rev}}^{\text{gas}} + W_{\text{Rev}}^{\text{engine}} = P_B (V_B - V_A) + \int_A^B \delta Q^{\text{Eso}} \left(1 - \frac{T}{T_B} \right)$$

where

$$\delta Q^{\text{Eso}} = \frac{T_B}{T} C_p dT$$

Therefore

$$W_{\text{Lost}} = W_{\text{Rev}}^{\text{Total}} - W_{\text{irrev}} = C_p T_B \ln \frac{T_B}{T_A} - C_p (T_B - T_A)$$

On the other hand by relation (A4)

$$W_{\text{Lost}} = T_B \pi_U = T_B \pi_{\text{ext}} = C_p T_B \ln \frac{T_B}{T_A} - C_p (T_B - T_A) \quad (17)$$

i.e. $W_{\text{Lost}} = T_B \pi_U = 2T_A \pi_{\text{ext}} = 0.386 C_p T_A$.

For the irreversible isobaric compression at pressure P_A (cooling) of one mole of monatomic ideal gas from the state B to the state A for which as before, $T_B = 2T_A$, the irreversible work is

$$W_{\text{irrev}} = -P_A (V_B - V_A) = -R(T_B - T_A).$$

Following the same steps as for the expansion we find

$$\pi_{\text{ext}} = \pi_U = -C_p \ln \frac{T_B}{T_A} + \frac{|Q_{\text{irrev}}|}{T_A}$$

and

$$W_{\text{Extra}} = T_A \pi_u = |Q_{\text{irrev}}| - C_p T_A \ln \frac{T_B}{T_A} \quad (18)$$

i.e. for $T_B = 2T_A$

$$\pi_{\text{ext}} = C_p (1 - \ln 2) = 0.307 C_p \quad \text{and} \quad W_{\text{Extra}} = T_A \pi_u = 0.307 C_p T_A$$

Observe that here, as opposed to the previous isothermal process, we have

$$W_{\text{Lost}} \geq W_{\text{Extra}}$$

In the next section by means of relations (10,11) and (17,18) we study the Lost Work and the Extra Work for the Stepwise Circular Cycle.

3 - The step-wise ideal gas Circular cycle and dissipated energy

In order to perform an ideal gas stepwise cycle we need a lot of heat sources (N) and heat sinks (N), a vessel with a free piston and a large number (N) of small “driving weights” to increase or decrease slowly, step by step, the external pressure P . If the steps are infinitesimally small the cycle is “reversible”. In order to evaluate the work performed by the ideal gas during the cycle, the displacements of the small driving weights (dw) must be done carefully. We let them move on and off the piston only horizontally. To this end we assume that the handle of the piston is endowed with

so many shelves that we can move each dw horizontally (and without friction) from (or to) the corresponding fixed shelf which belongs to the dw 's Reservoir. (The dw 's Reservoir is a vertical sequence of horizontal shelves on which the dw 's are initially located). Such an ideal device is shown schematically in Figure 2.

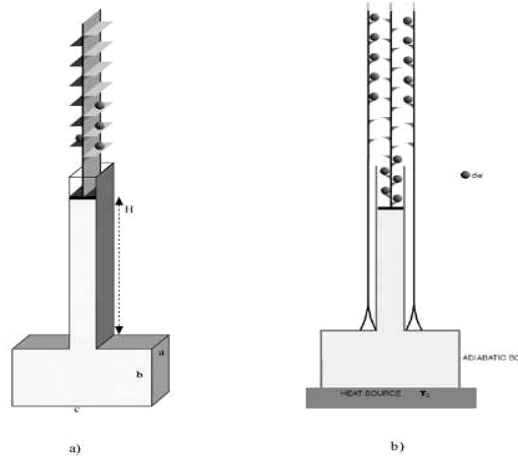


Figure 2

a) The adiabatic vessel with some dw 's on the piston.

b) Cross section view of the vessel showing two supports for the dw 's (the dw 's Reservoir).

The Circular cycle can be performed through $Z=2N$ steps. In each of the first N steps one dw is *added* on the piston (and removed from the Reservoir at its initial height h_0); in each of the following N steps one dw is *removed* from the piston (and brought back to the Reservoir at its final height, say h_f). The k -th dw is the dw which has been added on the piston at the k -th step in the compression. When the dw is added on the piston of the vessel in thermal contact with the sink T_i , the gas performs an isothermal compression; when it is removed the process is the isothermal expansion. Each isothermal process is followed by an isobaric process: this is a compression (volume reduction) in the first $N/2$ steps and an expansion in the following $N/2$ steps. The reverse happens in the N removing steps. Therefore at the end of the cycle the overall raising, on the dw 's Reservoir, of the k -th dw from its initial height (h_{k0}) to the final one (h_{kf}) is

$$h_k = h_{kf} - h_{k0} \quad (16)$$

Since a friction-less process is assumed, the vertical motion of the dw 's is only due to the gas and the total work (W) performed by the ideal gas can be found as increase of potential energy of the dw 's on the Reservoir, i.e.

$$W = \sum_{i=1}^{2N} P_i \Delta V_i = mg \sum_{k=1}^N h_k \quad (17)$$

where P_i is the external pressure at step i (after the addition or removal of the i -th dw) $\Delta V_i = V_i - V_{i-1}$, is the volume variation from step $(i-1)$ to step i , and mg is the weight of the generic dw . Relation (17) has been proved elsewhere [3]. In the next section the raisings of the single dw on the reservoir are evaluated.

3.1 - The raisings of the dw 's for a step-wise Circular cycle

The cycle we consider is reported in Figure 3. The chosen values of P and V are easily available in ordinary conditions. In the first N steps the dw 's are *added* on the piston to perform first the process (A \rightarrow B). In the remaining N steps the dw 's are *removed* from the piston in order to return to the initial state (B \rightarrow A). The working fluid is the ideal gas and we assume the free piston has no mass.

The vertical vessel's walls are heat insulating and the vessel's diathermal floor is made adiabatic when needed. The chosen circular cycle is described in the PV plane by the relation

$$\left(\frac{P-P_0}{r_p}\right)^2 + \left(\frac{V-V_0}{r_v}\right)^2 = 1 \quad (18)$$

where $P_0 = 15 \times 10^{-1}$ atm, $V_0 = 17.4$ l, $r_v = 5$ l and $r_p = 5 \times 10^{-1}$ atm. Since T_{\min} is at $\theta_{\min} = -(3/4)\pi$ and T_{\max} is at $\theta_{\max} = -(7/4)\pi$, it follows that the steps from 1 to $N/4$ are cooling steps. These are followed by N heating steps and $(3/4)N$ cooling steps.

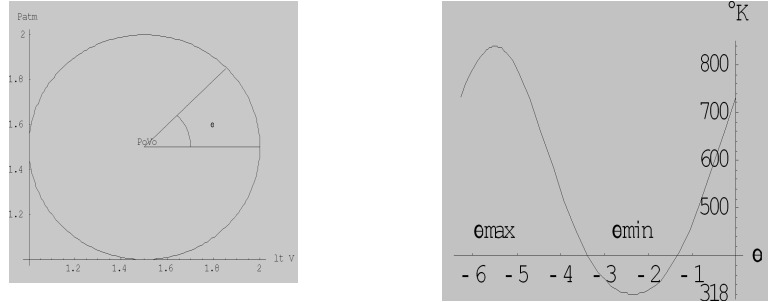


Figure 3 - The step-wise Circular cycle with very small steps and $T(\theta)$ the temperature along the clock-wise cycle.

Let us call $P_A = 1$ atm and $P_B = 2$ atm, respectively, the values of the pressure at bottom and at the top of the cycle. We have considered here $N = 1033$ dw's and therefore $2N = 2066$ steps. The mass of each dw is $m = 0.1$ Kg. The surface of the piston is $S = 100$ cm², so that at each step in the compression the pressure increase is $\Delta P = P_A/1033$, i.e.

$$P_i = P_A + i\Delta P \quad \text{for } i \in [1, N] \quad \text{and} \quad P_B = P_A + N\Delta P \quad (19)$$

And for each step in the expansion the pressure decreases by ΔP i.e.

$$P_{N+l} = P_A + (N-l)\Delta P = P_B - l\Delta P \quad \text{for } l \in [1, N] \quad (20)$$

Notice moreover that $V_{2N} = V_A$ and $V_N = V_B$, i.e. the volume at step $2N$ is the initial volume and the volume at step N is the volume at the top of the cycle. Of course for each P_i , $i \in [1, N]$, by means of relation (18), there are $V(P_i)_{\text{exp}}$ and $V(P_i)_{\text{comp}}$ which are the volume at the pressure P_i taken respectively in the "expansion" ($B \rightarrow A$) and in the "compression" ($A \rightarrow B$), and also two temperatures $T(P_i)_{\text{exp}}$ and $T(P_i)_{\text{comp}}$. All that can also be written in the following way: for each P_i , $i \in [1, 2N]$ there is a volume $V_i = V(P_i)$ and a heat (source or sink) at temperature $T_i = T(P_i)$. Keeping in mind how we perform the Circular cycle, let us take a closer look at the last dw: when this small weight leaves the Reservoir and is added on the piston, it (together with the piston and the previous dw's) moves downward, performing the isothermal compression step ($P_{N-1}V_{N-1} \rightarrow P_N V_{N-1}^*$) at temperature T_{N-1} ; then the gas is heated by the heat source T_N at pressure $P_N = P_B$ performing the expansion ($V_{N-1}^* \rightarrow V_B$); afterwards, at step $N+1$, the small weight is removed and goes to rest on the fixed shelf of the Reservoir in front of it. It will stay on the piston for one step only! i.e.

$$h_N = (V_N - V_{N-1})/S.$$

For this last step let us call $\Delta T_N = T_N - T_{N-1}$, $\Delta V_{N-1}^* = V_{N-1}^* - V_{N-1}$, $\Delta V_N^p = V_N - V_{N-1}^*$; from which we have $\Delta V_N = V_N - V_{N-1} = \Delta V_N^p + \Delta V_{N-1}^*$. In this last step the sink at temperature T_{N-1} takes the

entropy $\frac{Q_{N-1}^{\text{out}}}{T_{N-1}}$, and the source at temperature T_N gives the entropy $\frac{Q_N^{\text{in}}}{T_N}$ to the system, where

$$Q_{N-1}^{\text{out}} = P_N \Delta V_{N-1}^* \quad \text{and} \quad Q_N^{\text{in}} = P_N \Delta V_N^p.$$

Since $P_{N-1}V_{N-1} = P_N V_{N-1}^* = RT_{N-1}$, it follows that $\frac{Q_{N-1}^{\text{out}}}{T_{N-1}} = \frac{V_{N-1}\Delta P}{T_{N-1}} = \frac{R\Delta P}{P_{N-1}}$ and

$$\frac{Q_N^{\text{in}}}{T_N} = C_P \frac{\Delta T_N}{T_N}.$$

Similarly for the last but one dw :

$$h_{N-1} = \frac{1}{S}(V_{N+1} - V_{N-2}) = \frac{1}{S}(V_{2N-(N-1)} - V_{(N-1)-1}) \quad (21)$$

Therefore for the k -th dw

$$h_k = \frac{1}{S}(V_{2N-k} - V_{k-1}),$$

and

$$h_1 = \frac{1}{S}(V_{2N-1} - V_0).$$

For a very large number N we can write

$$h(P) = \frac{1}{S}[V(P)_{\text{exp}} - V(P)_{\text{comp}}]$$

Where $V(P)_{\text{exp}}$ and $V(P)_{\text{comp}}$ are the volume at the pressure P in the “expansion” and in the “compression”, and $h(P)$ is the raising of the dw which, added on the piston, gives rise to the pressure P . From this:

$$V(P)_{\text{exp}} = V_0 + \sqrt{r_v^2 - (P - P_0)^2 r_v^2 / r_p^2} \quad \text{and} \quad V(P)_{\text{comp}} = V_0 - \sqrt{r_v^2 - (P - P_0)^2 r_v^2 / r_p^2}$$

For $r_v \neq r_p$ one has elliptic cycles.

For a reversible Circular Cycle that starts from $V_0 = 17.4$ l and $P_A = 10 \times 10^{-1} = 1$ atm, the raisings $h(P)$ are easily obtained from relations (18) and (21):

$$h(P) = \frac{2}{S} \sqrt{r_v^2 - (P - P_0)^2 r_v^2 / r_p^2}, \quad (22)$$

whose values are shown in Figure 4.

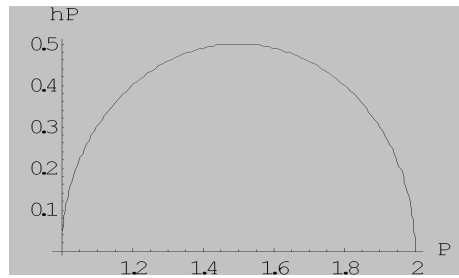


Figure 4 - Overall raising on the reservoir of each dw .

3.2 - Lost work and extra work step by step and the total dissipated energy

One may observe that in the cycle there has been an Entropy production: in fact when the last dw is added on the piston (the N -th step) we had an isothermal compression at temperature T_{N-1} and an

isobaric expansion from T_{N-1} to T_N . By means of relation (6) and (19), for the isothermal compression ($\Delta V_{N-1}^* < 0$, $\frac{\Delta P}{P_{N-1}} = -\frac{\Delta V_{N-1}^*}{V_{N-1}}$) we have

$$\pi_{N-\text{int}} = \frac{Q_{N-1}^{\text{out}}}{T_{N-1}} + R \ln \left(1 + \frac{\Delta V_{N-1}^*}{V_{N-1}} \right) \cong \frac{R}{2} \left(\frac{\Delta P}{P_{N-1}} \right)^2.$$

For the expansion and isobaric heating we have

$$\pi_{N-\text{Ext}} = \frac{Q_N^{\text{in}}}{T_N} + C_P \ln \left(1 + \frac{\Delta T_N}{T_{N-1}} \right) \cong \frac{C_P}{2} \left(\frac{\Delta T_N}{T_N} \right)^2.$$

Observe that the same result holds for isobaric cooling. Therefore in the N -th step we have $W_{N-\text{extra}} = T_{N-1} \pi_{N-\text{int}}$ and $W_{N-\text{Lost}} = T_N \pi_{N-\text{Ext}}$. Finally we can conclude that the Dissipated energy i.e. W_D is

$$W_D = \sum_{i=1}^{2N} (T_{i-1} \pi_{i-\text{int}} + T_i \pi_{i-\text{Ext}}) \cong \sum_{i=1}^{2N} V_{i-1} \frac{1}{2} \left(\frac{\Delta P}{\sqrt{P_{i-1}}} \right)^2 + \sum_{i=1}^{2N} \frac{C_P}{2} \left(\frac{\Delta T_i}{\sqrt{T_i}} \right)^2$$

Now we give some upper bound to $W_D(N)$. Let

$n_1 = N/2$ be the first $N/2$ adding steps for which $P \geq P_A$, $V \leq V_0$, $T \geq T_{\min}$

$n_2 = N/2$ be the second $N/2$ adding steps for which $P \geq P_0$, $V \leq V_0$, $T \geq T_C$

$n_3 = N/2$ be the third $N/2$ removing steps for which $P \geq P_0$, $V \leq V_D$, $T \geq T_0$

$n_4 = N/2$ be the fourth $N/2$ removing steps for which $P \geq P_A$, $V \leq V_D$, $T \geq T_{\min}$

$$W_D = \sum_{i=1}^{2N} (T_{i-1} \pi_{i-\text{int}} + T_i \pi_{i-\text{Ext}}) \leq \frac{N}{2} \left[V_0 \frac{1}{2} \left(\frac{\Delta P}{\sqrt{P_A}} \right)^2 + \frac{C_P}{2} \left(\frac{\Delta T_i}{\sqrt{T_{\min}}} \right)^2 + V_0 \frac{1}{2} \left(\frac{\Delta P}{\sqrt{P_0}} \right)^2 + \frac{C_P}{2} \left(\frac{\Delta T_i}{\sqrt{T_C}} \right)^2 \right. \\ \left. + V_D \frac{1}{2} \left(\frac{\Delta P}{\sqrt{P_0}} \right)^2 + \frac{C_P}{2} \left(\frac{\Delta T_i}{\sqrt{T_0}} \right)^2 + V_D \frac{1}{2} \left(\frac{\Delta P}{\sqrt{P_A}} \right)^2 + \frac{C_P}{2} \left(\frac{\Delta T_i}{\sqrt{T_{\min}}} \right)^2 \right]$$

And, since $\Delta T \leq \frac{T}{N}$,

$$W_D = \sum_{i=1}^{2N} (T_{i-1} \pi_{i-\text{int}} + T_i \pi_{i-\text{Ext}}) \leq \frac{N}{4} (\Delta P)^2 \left[\frac{V_0}{P_A} + \frac{V_0}{P_0} + \frac{V_D}{P_0} + \frac{V_D}{P_A} \right] + C_P \Delta T$$

Since $V_0 \cong \frac{4}{5} V_D$, $P_0 \cong \frac{3}{2} P_A$, choosing $C_P \Delta T \cong \frac{5}{2} \frac{RT}{N} \cong \frac{5}{2} \frac{PV}{N}$, we have

$$W_D = \sum_{i=1}^{2N} (T_{i-1} \pi_{i-\text{int}} + T_i \pi_{i-\text{Ext}}) \leq \frac{N}{4} \left(\frac{P_A}{N} \right)^2 3 \frac{V_D}{P_A} + \frac{5}{2} \frac{PV}{N},$$

$$W_D = \sum_{i=1}^{2N} (T_{i-1} \pi_{i-\text{int}} + T_i \pi_{i-\text{Ext}}) \leq 3 \frac{P_A V_D}{N}.$$

From this upper bound we see that, for $N \rightarrow \infty$, $W_D(N) \rightarrow 0$.

4 - Summary

In this paper we have introduced the Extra Work which, together with the Lost Work, gives the Dissipated energy in the irreversible processes. The analysis is very accurate for irreversible isothermal process and for isobaric processes. The new and previous results are used to evaluate the Dissipated energy for a stepwise ideal gas Circular Cycle, a system with complexity.

Acknowledgements

We thankfully acknowledge G. Monroy, P. Ruggiero and M. Zannetti for helpful discussions.

This paper is dedicated to the memory of Giuliano Martinelli for his pioneering works on energy preservation.

Appendix - Lost Work and Extra Work for isothermal processes with external irreversibility

Here we evaluate the Lost Work for the expansion and the Extra work for the compression when there is external irreversibility. In Sec. 2.2 and in Sec. 3 of paper [1] we have shown that, if the irreversible isothermal expansion is performed by means of a (shorter) contact with a heat source at $T_{\text{ext}} > T$, we have $\pi_{\text{ext}} \neq 0$, i.e.

$$\pi_{\text{ext}} = \frac{Q_{\text{in}}}{T} - \frac{Q_{\text{in}}}{T_{\text{ext}}} \quad (\text{A1})$$

and for the Endo-reversible process, i.e. the process in which the gas performs the reversible isothermal expansion $A \rightarrow B$,

$$\pi_{\text{ext}}^{\text{Endo}} = \frac{Q_{\text{Rev}}}{T} - \frac{Q_{\text{Rev}}}{T_{\text{ext}}} \quad (\text{A2})$$

Similarly, if the irreversible isothermal compression is performed by means of a (shorter) contact with an heat source at $T_{\text{ext}} < T$, we have $\pi_{\text{ext}} \neq 0$, i.e.

$$\pi_{\text{ext}} = \frac{Q_{\text{out}}}{T_{\text{ext}}} - \frac{Q_{\text{out}}}{T} \quad \text{and} \quad \pi_{\text{ext}}^{\text{Endo}} = \frac{Q_{\text{Rev}}}{T_{\text{ext}}} - \frac{Q_{\text{Rev}}}{T}. \quad (\text{A3})$$

To evaluate the Lost Work for the expansion with $T_{\text{ext}} > T$ we calculate the work available in the related Reversible process and subtract from it W_{out} , the effective work done in the irreversible process. This difference gives the Lost Work.

The Reversible Work is the Reversible work of the gas plus the work of an auxiliary reversible engine working between T_{ext} and T . For the gas $W_{\text{Rev}}^{\text{gas}} = Q_{\text{Rev}}$. The auxiliary reversible engine, which brings the heat Q_{Rev} to the system (the ideal gas at temperature T) and takes from the heat source

at temperature T_{ext} the heat $Q_{\text{Rev}} \frac{T_{\text{ext}}}{T}$, performs the work $W_{\text{Rev}}^{\text{engine}} = Q_{\text{Rev}} \frac{T_{\text{ext}}}{T} \left(1 - \frac{T}{T_{\text{ext}}}\right)$. Therefore the total reversible work is

$$W_{\text{Rev}}^{\text{Total}} = W_{\text{Rev}}^{\text{gas}} + W_{\text{Rev}}^{\text{engine}} = Q_{\text{Rev}} + Q_{\text{Rev}} \frac{T_{\text{ext}}}{T} \left(1 - \frac{T}{T_{\text{ext}}}\right)$$

The Work performed by the gas in the irreversible expansion is $W_{\text{out}} = Q_{\text{in}}$, therefore

$$W_{\text{Lost}} = W_{\text{Rev}}^{\text{Total}} - W_{\text{out}} = Q_{\text{Rev}} - Q_{\text{in}} + Q_{\text{Rev}} \frac{T_{\text{ext}}}{T} - Q_{\text{Rev}} = T\pi_{\text{in}} + T_{\text{ext}}\pi_{\text{ext}}^{\text{Endo}} \quad (\text{A4})$$

On the other hand for the compression with a heat source at $T_{\text{ext}} = T$

$$W_{\text{Extra}} = W_{\text{in}} - W_{\text{Rev}} = Q_{\text{out}} - Q_{\text{Rev}} = T\pi_{\text{int}} \quad (\text{A5})$$

but if one uses a heat source at $T_{\text{ext}} < T$, we have to subtract the work of the reversible engine from the Reversible work necessary to perform the isothermal compression at temperature T , which

subtracts Q_{out} from the heat source at temperature T (the gas) and gives the heat $Q^{\text{Min}} = \frac{T_{\text{ext}}}{T} Q_{\text{out}}$ to the source at temperature T_{ext} , i.e.

$$W_{\text{Rev}}^{\text{ext}} = Q_{\text{out}} - Q^{\text{min}} = T_{\text{ext}} \left(\frac{Q_{\text{out}}}{T_{\text{ext}}} - \frac{Q_{\text{out}}}{T} \right) = T_{\text{ext}} \pi_{\text{ext}} ;$$

therefore the Extra work is

$$W_{\text{Extra}} = W_{\text{in}} - (W_{\text{Rev}} - W_{\text{Rev}}^{\text{ext}}) = Q_{\text{out}} - Q_{\text{Rev}} + Q_{\text{out}} - T_{\text{ext}} \frac{Q_{\text{out}}}{T} = T \pi_{\text{int}} + T_{\text{ext}} \pi_{\text{ext}} . \quad (\text{A6})$$

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