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Vector Meson Exchange and Radiative Nonleptonic Kaon Decays in ChPT

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Contents

Chapter 1

Kaon Physics and ChPT

1.1 History of Kaon Physics

Kaon, also called K meson, was discovered in 1947 by G.D. Rochester and C.C. Butler [75], by observing two events with typic V pattern in a cloud chamber exposed to cosmic rays. Photographs showed one neutral particle decaying into two charged particles, and another charged particle decaying into one charged particle and something without electric charge. They called the neutral particle as V^0, now known as K_S, and the observed decay is $K_S \to \pi^- + \pi^+$.

In first years of 1950s, more events of V type particles were founded. At the same time, W. Chinowsky and J. Steinberger [13] proved that pion is a pseudoscalar particle and has negative parity, and as a consequence two different charged decays

$$\theta^+ \to \pi^0 + \pi^+$$

$$\tau^+ \to \pi^+ + \pi^+ + \pi^-$$

attracted more attention. Their final states have different parity and then they should be two different particles. On the other side, mass and lifetime were found to be roughly equal from experimental measurements, and it seemed to indicate they were the same particle. This was the famous $\tau - \theta$ *puzzle*. In 1956, T.D. Lee and C.N. Yang [62] put forward a very brave idea that parity is not conserved in weak interaction and proposed several experiments to test the idea. Lee explained it to his friend and colleague C.S. Wu and this prompted her to work with E. Ambler, R.W. Hayward, D.D. Hoppes and R.P. Hudson to study beta decay of cobalt-60, and they found clear parity violation in 1957 [84]. At the almost same time, another group of R.L.Garwin, L. Lederman and R. Weinrich [40] got the same conclusion by a different experiment. Therefore, τ and θ were proven to be the same particle - Kaon, which decays into three pions via weak interaction or decays into two pions through strong interaction.

In the mean time, A. Pais [72] and M. Gell-Mann [45] suggested that Kaons have a new quantum number, and thus it could be produced only together with a particle with opposite quantum number. Experiments confirmed their idea and that lead to the study of underlying symmetries of strong interaction in the early 1960s. Study of flavor $SU(3)$, in which Kaons, Pions and η belong to octet of this approximate symmetry, leads to *Gell-Mann - Okubo formula* [44, 71] which relate the Kaon mass with that of the pion and the η. One important extension of $SU(3)$ is chiral symmetry $SU(3)_L \times SU(3)_R$.

On the other hand, Kaon physics plays an important role in the flavor structure of weak interaction. In 1963, N. Cabibbo [8] introduced the *Cabibbo Angle* θ_C to explain the weaker strength of Kaon weak decays relative to that of the pions and proposed the idea of flavor mixing of charged quark current of weak interactions. Later, the very suppression of neutral current decay $K_L \to \mu^+\mu^-$ compared with charged current decay $K^+ \to \mu^+\nu_\mu$ was explained by the GIM mechanism [48] and lead to the prediction of the fourth flavor quark - c(*charm*). Experimental discovery of the decay $K_L \to \pi\pi$, by

J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay [14], gave the first clear evidence that CP symmetry could be broken. The theoretical study by Kobayashi and Maskawa [59] on CP violation showed that there is the third generation of quarks which known as t(*top*) quark and b(*bottom*) quark now. In 1990s, direct CP violation evidence was found in $K^0 - \bar{K}^0$ oscillation. Although study on CP violations changed to B mesons because of running of new generation experiments include BaBar at SLAC in USA and Belle at KEK in Japan after 2000. Kaon physics is still playing an important role in frontier research at present. Some topics, such as test of *Flavor violation* and CPT, can still potentially improve our understanding of fundamental principles of Universe.

1.2 Properties of Kaons and pions

We can find all properties of Kaons and pions from new version of particle listing from Particle Data Group [67], and here we list their basic properties in Table 1.1 and Table 1.2.

Particle	Content	Rest Mass	$I(J^{PC})$
π^+	$u\bar{d}$	139.570 MeV	$1(0^-)$
π^-	$\bar{u}d$	139.570 MeV	$1(0^-)$
π^0	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	134.976 MeV	$1(0^{-+})$

Table 1.1: Properties of pions

For pions, the main decay modes are:

$$\pi^+ \to \mu^+\nu_\mu$$

$$\pi^0 \to \gamma\gamma$$

$$\pi^0 \to e^+e^-\gamma$$

As for Kaons, condition is much complicate. There are two neutral particles K^0 and $\bar{K}^0$, however, they can not be their own antiparticle like the neutral pion because they contain s quark.

Particle	Content	Rest Mass	$I(J^P)$
K^+	$u\bar{s}$	493.677 MeV	$\frac{1}{2}(0^-)$
K^-	$\bar{u}s$	493.677 MeV	$\frac{1}{2}(0^-)$
K^0	$d\bar{s}$	497.614 MeV	$\frac{1}{2}(0^-)$
$\bar{K}^0$	$\bar{d}s$	497.614 MeV	$\frac{1}{2}(0^-)$

Table 1.2: Properties of Kaons

For neutral Kaons K^0 and $\bar{K}^0$, because of neutral Kaon oscillations by which these two kinds of Kaons can turn from one into another via weak interaction, they can be thought as composites of two weak eigenstates K_S (for short lived) and K_L (for long lived), which are eigenstates of CP and with different eigenvalues:

$$CP|K_S >= |K_S >, \qquad CP|K_L >= -|K_L >$$

Because of CP symmetry, they have different main decay modes: K_S decays primarily into two pions while K_L decays into three pions. However this is not exactly true if we consider that CP is not a perfect symmetry in weak interactions. Historically, CP violation in weak interaction was proved by observing the decay $K_L \to \pi\pi$, which shows that although K_S and K_L are weak eigenstates, they are not exactly pure CP eigenstate K_1 and K_2 and they can be expressed as

$$K_S = K_1 + \epsilon K_2, \qquad K_L = K_2 + \epsilon K_1$$

where the small parameters ϵ measures CP violation and $|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$ is listed in updated version of PDG [67].

After all basic informations of Kaons, we will introduce the theoretical framework to study Kaon physics, which is called *Chiral Perturbation Theory* which is an effective field theory constructed with a Lagrangian, which contains a few relevant particles, consistent with chiral symmetry of quantum chromodynamics (QCD), and those discrete symmetries of Charge conjugation and Parity.

1.3 Chiral Symmetry and QCD

Let ψ_i, i=1 to N, be free, massless and spin $\frac{1}{2}$ fermion fields which satisfy Dirac equation $i\gamma^\mu\partial_\mu\psi_i = 0$, and Lagrangian for this free field is therefore

$$\mathcal{L}(\psi) = i\overline{\psi}\partial\!\!\!/\psi, \quad \text{where}\psi = \begin{pmatrix} \psi_1 \\ \dots \\ \psi_N \end{pmatrix} \tag{1.1}$$

Consider a unitary $N \times N$ matrix $U = e^{i\epsilon_\alpha T^\alpha}$ acting on ψ,

$$\psi \to U\psi \tag{1.2}$$

where T^α are N^2 Hermition $N \times N$ matrices and ϵ_α are infinitesimal parameters independent on spacetime. Then it is obvious that Lagrangian (1.1) is invariant under the global $U(N) = SU(N) \times U(1)$ transformation (1.2).

Moreover, this symmetry can be extended to a bigger symmetry. At first, let us define chirality projection operators as

$$P_\pm \equiv \frac{1 \pm \gamma_5}{2} \qquad \text{with} \quad P_+^2 = P_-^2 = 1 \tag{1.3}$$

and define left-handed field and right-handed field

$$\psi_L \equiv P_+\psi, \qquad \psi_R \equiv P_-\psi. \tag{1.4}$$

Then by virtue of $P_+\gamma^\mu = \gamma^\mu P_-$,

$$\overline{\psi_L} = (P_+\psi)^\dagger\gamma^0 = \psi^\dagger\gamma^0 P_- = \overline{\psi}P_-, \qquad \overline{\psi_R} = \overline{\psi}P_+. \tag{1.5}$$

therefore Lagrangian (1.1) can be rewritten to a new form

$$\begin{aligned}
\mathcal{L}(\psi) &= i\overline{\psi}\partial\!\!\!/\psi \\
&= i\overline{\psi}\partial\!\!\!/P_+^2\psi + i\overline{\psi}\partial\!\!\!/P_-^2\psi \\
&= i\overline{\psi}P_-\partial\!\!\!/P_+\psi + i\overline{\psi}P_+\partial\!\!\!/P_-\psi \\
&= i\overline{\psi_L}\partial\!\!\!/\psi_L + i\overline{\psi_R}\partial\!\!\!/\psi_R
\end{aligned} \tag{1.6}$$

Evidently, we can make independent $SU(N)$ transformation on ψ_L and ψ_R respectively,

$$\psi_L \to U_L\psi_L, \qquad \psi_R \to U_R\psi_R \tag{1.7}$$

and clearly the new form Lagrangian (1.6) is invariant under transformation (1.7), so it has a $SU(N)_L \times SU(N)_R$ symmetry which is called *Chiral Symmetry*.

However, chiral symmetry is broken by a mass term. The general mass term of spin $\frac{1}{2}$ fermion field in terms of left-handed field and right-hander field is

$$\overline{\psi_L}M\psi_R + \overline{\psi_R}M^\dagger\psi_L \tag{1.8}$$

The mass term (1.8) breaks $SU(N)_L \times SU(N)_R$ symmetry directly, because it mix ψ_L and ψ_R together.

Quantum Chromodynamics (QCD) is the theory of strong interaction, and its basic matter fields are six types quark with $\frac{1}{2}$ spin, and every type quark has 3 quantum numbers, generally called as red, green and blue. Moreover, quarks interact with each other via gauge bosons which are called gluons. The Lagrangian of QCD is invariant under transformation of gauge group $SU(3)_C$ and then can be written as

$$\mathcal{L}_S = \overline{q_f}(iD\!\!\!\!/ - m_f)q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}_a^{\mu\nu}, a = 1, ..., 8 \tag{1.9}$$

where f∈{u,s,d,c,b,t} to represent flavor of quark, and every flavor quark is a color triplet and has form

$$q_f = \begin{pmatrix} q_{f,R} \\ q_{f,G} \\ q_{f,B} \end{pmatrix}$$

$D^\mu = \partial^\mu - ig\frac{\lambda_a}{2}\mathcal{A}_a^\mu$ is the covariant derivative in which g is coupling constant, $\mathcal{A}_a^\mu$ is gauge field, and $\mathcal{G}_a^{\mu\nu} = \partial^\mu \mathcal{A}_a^\nu - \partial^\nu \mathcal{A}_a^\mu + g f_{abc}\mathcal{A}_b^\mu \mathcal{A}_c^\nu$ are field strength of gauge field $\mathcal{A}_a^\mu$, where f_{abc} are structure constant of $SU(3)$. Under $SU(3)_C$ gauge transformation,

$$q_f \to U q_f$$
$$D_\mu q_f \to U D_\mu q_f$$
$$\mathcal{A}_a^\mu \to U \mathcal{A}_a^\mu U^\dagger - \frac{i}{g}\partial^\mu U U^\dagger$$

From experimental data, we know that $m_u \sim 5MeV$, $m_d \sim 10MeV$, $m_s \sim 175MeV$ are much smaller than $m_c \sim 1.15GeV$, $m_b \sim 4GeV$ and $m_t \sim 174GeV$, so following the basic idea of effective field theory that dynamics of low energy does not depend on dynamics of high energy, when we study the low energy hadron physics which are made up of light quarks include u,d and s, it is reasonable to focus only on u, d and s quarks and to discard c, b and t quarks, since the energy of light quarks well below the mass of c quark.

When we only consider light quarks u, d and s, and set them massless, the free Lagrangian of these quarks has global $SU(3)_L \times SU(3)_R$ symmetry. However, there is no this global symmetry in low energy hadron particle spectrum which are made up of these light quarks. On the other hand, eight members, consisting of proton, neutron, Σs, Ξs and Λ, form a baryon octet of approximate $SU(3)_V$ symmetry. This problem is solved by introducing the conception of spontaneous symmetry breaking, which is very important in standard model of particle physics. Because fundamental particles and gauge particles are massless under unified $SU(2)_L \times U(1)_Y$ group, and their masses are generated by *Higgs Mechanism* which is one type of spontaneous symmetry breaking. This conception is the basic stone of ChPT which study mesons physics and baryons physics in low energy region, and we will introduce the main idea of spontaneous symmetry breaking in next section.

1.4 Spontaneous Symmetry Breaking and Goldston Theorem

In 1960s, studies [50, 51, 68–70] by Nambu, Jona-Lasinio, Goldstone, Salam and Weinberg, showed that there must exist massless and spinless particles in physical spectrum after spontaneous breaking of continuous symmetry, which are in correspondence with the generators of broken symmetry. These particles are called as *Nambu-Goldstone Bosons* or simply as *Goldstone Bosons*. If continuous symmetries are not exact symmetries because there are small symmetry-breaking terms, for example as small masses of light quarks, in the Lagrangian, their spontaneous breaking generates small mass spinless particles instead of massless Goldstone bosons, generally called *Pseudo-Goldstone Bosons*.

Coming back to low energy hadron physics, on the one hand, there are baryon like proton, neutron, form octet of $SU(3)_V$, on the other hand, there are also some spinless particles like πs, Ks and η with masses which are small compare with those of baryons, can be treated as octet of $SU(3)_A$, therefore it seems reasonable to treat these particle as *Pseudo-Goldstone Bosons*.

1.5 Chiral Perturbation Theory

Below the energy scale of resonance m_ρ, hadronic spectrum contains only an octet of light psedoscalar particles(π^+, π^-, π^0, K^+, K^-, K^0, $\bar{K^0}$, η), and all relevant constituents are light quarks(u, d, s). However, it is not possible to use perturbation method in terms of these light quarks because the coupling constant g in QCD has a special property called *Asymptotic Freedom* which means interaction between particles become weak at high energy scale while become strong in low energy domain. The interaction of these light pseudoscalar particles can be described by global symmetry, based on chiral symmetry of QCD and idea of effective field theory, ChPT was developed to a powerful theoretical framework to analyze low energy hadronic physics.

Assume that light quarks u, d, s are massless, the Lagrangian of QCD (1.9), contains only u, d and s, will be invariant under global chiral symmetry $SU(3)_L \times SU(3)_R$, and then it can be rewrited as

$$\mathcal{L}_{uds} = i\overline{q_L}\not{D}q_L + i\overline{q_R}\not{D}q_R - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a, \qquad a = 1, ..., 8 \tag{1.10}$$

According to *Neother's Theorem*, there are conserved current

$$J^{a\mu}_X = \bar{q}_X\gamma^\mu\frac{\lambda_a}{2}q_X \qquad (X = L, R)$$

and corresponding conserved charge $Q^a_X = \int d^3x J^{a0}_X$ which satisfy commutation relations: $[Q^a_X, Q^b_Y] = i\delta_{XY}f_{abc}Q^c_X$. Moreover, linear combinations $Q^a_V = Q^a_R + Q^a_L$ and $Q^a_A = Q^a_R - Q^a_L$ commute with free Hamiltonian $\mathcal{H}^0$ which is correspond with free Lagrangian (1.10), and have opposite parity. Thus, one expect to find parity doubling states in physical spectrum. However, low energy hadron spectrum contains only an approximate degenerate baryon octet with positive parity, and octet of pseudoscalar mesons whose masses are much smaller than those other hadronic particles, they are candidates for Goldstone bosons of spontaneous symmetry breaking of

$SU(3)_L \times SU(3_R) \to SU(3)_V$ as one application of *Goldstone Theorem*. The interaction of Goldstone bosons become very weak for decreasing energy even though the underlying interaction of their ingredient quarks become extremely strong. It is the basis to construct an effective chiral Lagrangian which is a systematic low energy expansion of Goldstone bosons fields. Therefore, we should make clear the transformation properties of Goldstone bosons fields at first.

Thanks to those leading works by S.Weinberg [80], by S.R. Coleman, J. Wess and B. Zumino [17], by C.G. Callan, S.R. Coleman, J. Wess and B. Zumino [9], Weinberg proposal and then others proved that there is a standard procedure to implement symmetry transformation on Goldstone boson fields. Goldstone fields ϕ (Pions, Kaons and η_8) are coordinates of the coset space $\frac{SU(3)_L \times SU(3)_R}{SU(3)_V}$, and can be brought together in field $\xi(\phi) \equiv (\xi_L(\phi), \xi_R(\phi))$. In general, an element g of $SU(3)_L \times SU(3)_R$ induce a transformation on $\xi(\phi)$

$$\xi(\phi) \xrightarrow{g \in SU(3)_L \times SU(3)_R} g\xi(\phi) = \xi(\phi')h(g, \phi) \tag{1.11}$$

where $h(g, \phi)$ is an element of conserved subgroup $SU(3)_V$, if we set g corresponding to a spontaneously broken symmetry and it means $g \notin SU(3)_V$, we can have a nonlinear realization of Goldstone field ϕ.

$$\xi_L(\phi') = g_L \xi_L(\phi) h(g, \phi)^{-1}, \qquad \xi_R(\phi') = g_R \xi_R(\phi) h(g, \phi)^{-1}, \tag{1.12}$$

We have no interest in the explicit form of $h(g, \phi)$, and we can get ride of it by defining a new field $U(\phi) = \xi_R(\phi)\xi_L^\dagger(\phi)$ since $U(\phi)$ has a simple transformation property $U(\phi) \xrightarrow{SU(3)_L \times SU(3)_R} g_R U(\phi) g_L^\dagger$. We can choose a canonical representation $\xi_R(\phi) = \xi_L^\dagger(\phi) = u(\phi)$, and thus

$$U(\phi) = u^2(\phi) = exp\left\{\frac{i\sqrt{2}\phi}{f}\right\}, \tag{1.13}$$

where $\phi = \frac{\lambda_a \phi_a}{\sqrt{2}} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}$.

It is clear that $U(\phi)$ transforms linearly under $SU(3)_L \times SU(3)_R$, while Goldstone boson fields do not, this is so-called nonlinear realization of chiral symmetry.

To write the most general Lagrangian in terms of $U(\phi)$, we must consider all known symmetries of relevant field. Moreover, $UU^\dagger = 1$ because of unitarity of U field. The Lagrangian can be organized in terms of increasing number of derivatives, in addition, parity conservation of strong interaction requires that the number of derivative must be even.

$$\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \ldots . \tag{1.14}$$

$U(\phi)$ is not the only field which is consistent with chiral symmetry, we can incorporate more classical field by using external field technique. Next, we list transformations of possible fields which could be used to construct the most general effective Lagrangian.

$$\begin{aligned} &U \to g_R U g_L^\dagger, && U^\dagger \to g_L U^\dagger g_R^\dagger \\ &\partial_\mu U \to g_R \partial_\mu U g_L^\dagger, && \partial_\mu U^\dagger \to g_L \partial_\mu U^\dagger g_R^\dagger \\ &s + ip \to g_R (s + ip) g_L^\dagger && \end{aligned} \tag{1.15}$$

For gauge fields v_μ and a_μ, they follow local $SU(3)_L \times SU(3)_R$:

$$\begin{aligned} l_\mu &\equiv v_\mu - a_\mu \to g_L l_\mu g_L^\dagger + i g_L \partial_\mu g_L^\dagger \\ r_\mu &\equiv v_\mu + a_\mu \to g_R r_\mu g_R^\dagger + i g_R \partial_\mu g_R^\dagger \end{aligned} \tag{1.16}$$

They involve in effective Lagrangian through the covariant derivatives

$$D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu, \quad D_\mu U^\dagger = \partial_\mu U^\dagger + i U^\dagger r_\mu - i l_\mu U^\dagger \tag{1.17}$$

and through the invariant field strength.

$$F_L^{\mu\nu} = \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], \quad F_R^{\mu\nu} = \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] \tag{1.18}$$

1.5.1 The lowest effective Lagrangian

In Chiral perturbation theory, all known fundamental elements are counted as:

$$
\begin{aligned}
& U, U^\dagger \sim \mathcal{O}(p^0), \\
& \partial_\mu U \sim \mathcal{O}(p), \qquad l_\mu, r_\mu \sim \mathcal{O}(p) \\
& F^{\mu\nu}_{L,R} \sim \mathcal{O}(p^2), \qquad \chi = 2\mathcal{B}_0(s+ip) \sim \mathcal{O}(p^2)
\end{aligned}
\tag{1.19}
$$

Invariants can be taken as the trace of products of types $AB^\dagger$ when one has object A, B, $\ldots$, which transform as $A \to g_R A g_L^\dagger, B \to g_R B g_L^\dagger$. Consequently, those possible invariants, under the transformation (1.15) and (1.16), are given at order $\mathcal{O}(p^2)$ by:

$$
\begin{aligned}
& \left\langle D_\mu D_\nu U U^\dagger \right\rangle = \left\langle U D_\mu D_\nu U^\dagger \right\rangle = - \left\langle D_\mu U D_\nu U^\dagger \right\rangle \\
& \left\langle U \chi^\dagger \right\rangle, \qquad \left\langle \chi U^\dagger \right\rangle \\
& \left\langle U F^{\mu\nu}_L U^\dagger \right\rangle = \langle F^{\mu\nu}_L \rangle = 0, \qquad \langle F^{\mu\nu}_R \rangle = 0
\end{aligned}
\tag{1.20}
$$

With the requirement of Lorentz invariance and Parity conservation, the most general effective chiral Lagrangian at $\mathcal{O}(p^2)$ [41–43, 81] is

$$
\mathcal{L}_2 = \frac{F^2}{4} \left\langle D_\mu U D^\mu U^\dagger + U\chi^\dagger + \chi U^\dagger \right\rangle \tag{1.21}
$$

where F and $\mathcal{B}_0$ are phenomenological coefficients and it can be shown that F is related to pion decay constant $f_\pi = 92.4 MeV$, while $\mathcal{B}_0$ is related to the quark condensate.

Setting

$$
s = \mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}
$$

and moreover taking $m_u = m_d$ and $q = 0$, the χ term in (1.21) gives

$$\begin{aligned}\frac{F^2\mathcal{B}_0}{2}\left\langle \mathcal{M}(U+U^\dagger)\right\rangle =& \mathcal{B}_0\left[-\left\langle \mathcal{M}\phi^2\right\rangle + \mathcal{O}(\phi^4)\right] \\ =& 4m_u\pi^+\pi^- + 2(m_u+m_s)K^+K^- + 2(m_u+m_s)K^0\bar{K^0} \\ &+ 2m_u\pi^0\pi^0 + \frac{2}{3}(m_u+2m_s)\eta^2\end{aligned} \tag{1.22}$$

From this equation, we can get relations between masses of mesons and masses of light quarks:

$$\begin{aligned}m_\pi^2 &= 2\mathcal{B}_0 m_u \\ m_K^2 &= \mathcal{B}_0(m_u+m_s) \\ m_\eta^2 &= \frac{2}{3}\mathcal{B}_0(m_u+2m_s)\end{aligned} \tag{1.23}$$

And we can deduce the well-known *Gell-Mann-Okubo relation* immediately from these relations,

$$4m_K^2 = 3m_\eta^2 + m_\pi^2 \tag{1.24}$$

After fixing these two coefficients, we can make predictions for some specific interaction process. One simple example is scattering of $\pi^+\pi^0 \to \pi^+\pi^0$. Through a easy expansion of $\mathcal{L}_2$, the relevant ϕ^4 terms are:

$$\frac{1}{12F^2}\left\langle 2\phi^2\partial_\mu\phi\partial^\nu\phi - \partial_\mu\phi\phi^2\partial^\mu\phi - \phi\partial_\mu\phi\phi\partial^\mu\phi\right\rangle + \frac{\mathcal{B}_0}{6F^2}\left\langle \mathcal{M}\phi^4\right\rangle \tag{1.25}$$

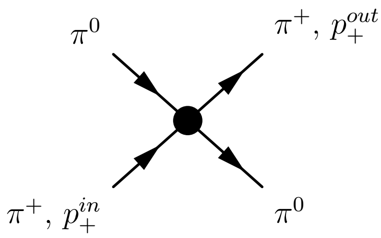

Figure 1.1: $\pi^+\pi^0 \to \pi^+\pi^0$

The scattering amplitude of $\pi^+\pi^0 \to \pi^+\pi^0$ can be read directly from this expression.

$$\mathcal{A}(\pi^+\pi^0 \to \pi^+\pi^0) = \frac{(p_+^{out} - p_+^{in})^2 - m_\pi^2}{F^2}$$

and it is perfect in agreement with the previous result from *Current Algebra* [79].

Though the lowest effective Lagrangian can give prediction to a few processes in a simple and elegant way, next to leading order are needed. The first reason is that we want to get precise correction to leading order, and another reason is that many processes can't be generated or have vanishing contributions at leading order, and their leading contribution come from next order $\mathcal{O}(p^4)$ or even higher order.

1.5.2 ChPT at $\mathcal{O}(p^4)$

At the next to leading order $\mathcal{O}(p^4)$, there are three types of contributions from different ingredients:

i) Tree diagrams deduced from the most general effective Lagrangian $\mathcal{L}_4$.

ii) One loop diagrams in which all vertices are obtained from $\mathcal{L}_2$.

iii) Tree Feynman diagrams in which one vertex given from *Wess-Zumino-Witten* anomaly functional.

The general Lagrangian $\mathcal{L}_4$ [41–43] is given in terms of those elements (1.19).

$$
\begin{aligned}
\mathcal{L}_4 = & L_1 \left\langle D_\mu U D^\mu U^\dagger \right\rangle^2 + L_2 \left\langle D_\mu U D_\nu U^\dagger \right\rangle \left\langle D^\mu U D^\nu U^\dagger \right\rangle \\
& + L_3 \left\langle D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger \right\rangle + L_4 \left\langle D_\mu U D^\mu U^\dagger \right\rangle \left\langle \chi U^\dagger + U \chi^\dagger \right\rangle \\
& + L_5 \left\langle D_\mu U D^\mu U^\dagger (\chi U^\dagger + U\chi^\dagger) \right\rangle + L_6 \left\langle \chi U^\dagger + U \chi^\dagger \right\rangle^2 \\
& + L_7 \left\langle \chi U^\dagger - U \chi^\dagger \right\rangle^2 + L_8 \left\langle U \chi^\dagger U \chi^\dagger + \chi U^\dagger \chi U^\dagger \right\rangle \\
& - i L_9 \left\langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \right\rangle + L_{10} \left\langle F_R^{\mu\nu} U F_{L\mu\nu} U^\dagger \right\rangle \\
& + H_1 \left\langle F_{L\mu\nu} F_L^{\mu\nu} + F_{R\mu\nu} F_R^{\mu\nu} \right\rangle + H_2 \left\langle \chi \chi^\dagger \right\rangle
\end{aligned}
\tag{1.26}
$$

All coupling coefficients L_i can not be fixed by chiral symmetry, because they contain information on the underlying dynamics of QCD. In principle, they can be given in terms of the remaining parameters of QCD like heavy quark masses and QCD scale Λ_{QCD}. Up to now, these coefficients have been fixed by fitting those experimental data.

On the other side, ChPT is a quantum field theory, so we must consider quantum loops with pseudoscalar boson internal propagators. One loop diagram given by $\mathcal{L}_2$ generate infinites which are counted as $\mathcal{O}(p^4)$ following Weinberg's power counting [81]. Although effective field theories are non renormalizable, they define a renormalizable theory in the momentum expansion order by order. Coefficients H_i and L_i, except for L_3 and L_7, are required by the renormalization of one loop diagrams. Using dimensional regularization, coupling coefficients can be written as

$$L_i = L_i^r(\mu) + \Gamma_i \frac{\mu^{2\epsilon}}{32\pi^2}\left(\frac{1}{\hat{\epsilon}} - 1\right); \quad H_i = H_i^r(\mu) + \tilde{\Gamma}_i \frac{\mu^{2\epsilon}}{32\pi^2}\left(\frac{1}{\hat{\epsilon}} - 1\right) \tag{1.27}$$

where $L_i^r(\mu)$ is renormalized coupling which depend on the dimensional regularization scale μ, and Γ_i and $\tilde{\Gamma}_i$ can be given by one loop generating functional Z_4 [41–43]:

$$\begin{aligned}
&\Gamma_1 = \frac{3}{32}, \quad \Gamma_2 = \frac{3}{16}, \quad \Gamma_3 = 0, \quad \Gamma_4 = \frac{1}{8} \\
&\Gamma_5 = \frac{3}{8}, \quad \Gamma_6 = \frac{11}{144}, \quad \Gamma_7 = 0, \quad \Gamma_8 = \frac{5}{48} \\
&\Gamma_9 = \frac{1}{4}, \quad \Gamma_{10} = \frac{-1}{4}, \quad \tilde{\Gamma}_1 = \frac{-1}{8}, \quad \tilde{\Gamma}_2 = \frac{5}{24}
\end{aligned} \tag{1.28}$$

factor $\frac{1}{\hat{\epsilon}} - 1$ is related with divergent pieces in $\overline{MS}$ renormalization scheme.

Chiral symmetry is broken at the quantum level by anomalies of the fermionic determinant [2–4]. In ChPT, anomaly vertices can be generated by a functional constructed by Wess and Zumino [82]. Witten reformulated the functional in a nice geometrical way [83]. Its explicit form is

$$\begin{aligned}
S[U,l,r]_{WZW} = &-\frac{iN_C}{240\pi^2}\int d\sigma^{ijklm}\left\langle \Sigma_i^L \Sigma_j^L \Sigma_k^L \Sigma_l^L \Sigma_m^L \right\rangle \\
&-\frac{iN_C}{48\pi^2}\int d^4x \epsilon_{\mu\nu\alpha\beta}\left(W(U,l,r)^{\mu\nu\alpha\beta} - W(1,l,r)^{\mu\nu\alpha\beta}\right);
\end{aligned} \tag{1.29}$$

with

$$W(U,l,r)_{\mu\nu\alpha\beta} = \langle U l_\mu l_\nu l_\alpha U^\dagger r_\beta + \frac{1}{4} U l_\mu U^\dagger r_\nu U l_\alpha U^\dagger r_\beta + iU\partial_\mu l_\nu l_\alpha U^\dagger r_\beta$$
$$+ i\partial_\mu r_\nu U l_\alpha U^\dagger r_\beta - i\Sigma^L_\mu l_\nu U^\dagger r_\alpha U l_\beta + \Sigma^L_\mu U^\dagger \partial_\nu r_\alpha U l_\beta$$
$$- \Sigma^L_\mu \Sigma^L_\nu U^\dagger r_\alpha U l_\beta + \Sigma^L_\mu l_\nu \partial_\alpha l_\beta + \Sigma^L_\mu \partial_\nu l_\alpha l_\beta$$
$$- i\Sigma^L_\mu l_\nu l_\alpha l_\beta + \frac{1}{2}\Sigma^L_\mu l_\nu \Sigma^L_\alpha l_\beta - i\Sigma^L_\mu \Sigma^L_\nu \Sigma^L_\alpha l_\beta \rangle - \langle L \leftrightarrow R \rangle \tag{1.30}$$

where $\Sigma^L_\mu = U^\dagger \partial_\mu U, \Sigma^R_\mu = U\partial_\mu U^\dagger$ and $(L \leftrightarrow R)$ stands for the interchanges $U \leftrightarrow U^\dagger$, $l_\mu \leftrightarrow r_\mu$ and $\Sigma^L_\mu \leftrightarrow \Sigma^R_\mu$. The anomaly functional is responsible for interactions which break the intrinsic parity, and gives rise to $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$, ...

For special case $\pi^0 \to \gamma\gamma$, the anomaly WZW functional gives one relevant term

$$\mathcal{L}_{WZW} = \frac{e^2}{32\pi^2 f}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}\left(\pi^0 + \frac{\eta}{\sqrt{3}}\right) \tag{1.31}$$

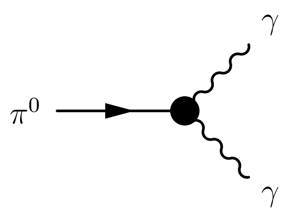

Figure 1.2: $\pi^0 \to \gamma\gamma$

And then, the corresponding amplitude can be easily read as:

$$\mathcal{A} = \frac{ie^2}{4\pi^2 f}\epsilon_{\mu\nu\alpha\beta}q_1^\mu \epsilon_1^\nu q_2^\alpha \epsilon_2^\beta \tag{1.32}$$

Summing over the final photon polarizations and integrating over phase space, the decay rate is $\Gamma_{\pi^0\to\gamma\gamma} = 7.6eV$, which fits the experimental value $(7.7 \pm 0.6)eV$ very well.

1.5.3 ChPT at $\mathcal{O}(p^6)$

ChPT at $\mathcal{O}(p^4)$ has led to lots of successful applications in low energy hadron physics, thus, naive thinking is to extend it to next order $\mathcal{O}(p^6)$. On the one hand, we can get more precise theoretical prediction, on the other side, it is possible to reveal the low energy dynamics of heavy particles by including these heavy resonances. However, condition at $\mathcal{O}(p^6)$ is much complicated. First, there are a very large number of invariant terms at $\mathcal{O}(p^6)$ which satisfy chiral symmetry, Lorentz invariance and discrete C, P and CP symmetries. Moreover, we have no general formula to determine how many independent invariant terms at this order. Furthermore, contributions are coming from not only one loop diagrams but also two loop diagrams, as a result, more technical methods are required to evaluate these two loop diagrams. Finally, more heavy mesons are involved at this order, and their contribution can not be decided by chiral symmetry, so we have to construct theoretical models beyond chiral symmetry to evaluate contributions of heavy mesons.

1.6 ChPT in weak sector

The framework discussed so far focus only on QCD, and it is not complete since standard model include weak interaction as well, and weak interaction are responsible for flavor-changing transition, as well as most of Kaon decay modes. For nonleptonic $\Delta S = 1$ decays, Kaon mass is much smaller than mass of W which is the mediator between two charged weak currents. Thus, according to effective field theory, W can be integrated out. Using operator product expansion and renormalization group techniques, nonleptonic $\Delta S = 1$ decays can be described by an effective Hamiltonian [47]

$$\mathcal{H}_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum c_i(\mu) Q_i + h.c. \tag{1.33}$$

where G_F is Fermi constant, V_{ud} and V_{us}^* are elements of Cabbibo-Kabayashi-Maskawa matrix, $c_i(\mu)$ are Wilson coefficients which are function of heavy particles masses and Q_i are local four-fermion operators.

For nonleptonic Kaon decays, there is another simple effective theory. The $\Delta S = 1$ nonleptonic weak interaction is incorporated in chiral theory as a perturbation to strong effective Lagrangian. The general leading order effective weak Lagrangian contains two types of contributions:

$$\mathcal{L}_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \left\langle \lambda L_\mu L^\mu \right\rangle + g_{27} \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + h.c \right\}. \tag{1.34}$$

where $\lambda = \frac{1}{2}(\lambda_6 - i\lambda_7)$, $L_\mu = if^2 U^\dagger D_\mu U$, couplings g_8 and g_{27} represent the strength of two parts in effective Hamiltonian (1.33) transforming as $(8_L, 1_R)$ and $(27_L, 1_R)$ under chiral transformation. Their value, $|g_8| \simeq 5.1$ and $\frac{g_{27}}{g_8} \simeq \frac{1}{18}$, can be fixed from $K \to \pi\pi$ decays [74], and the big difference between them can explain the enhancement of the octet $|\Delta I| = \frac{1}{2}$ transitions.

After fixing the two coupling, the Kaon hadronic weak decays can be calculated in a straightforward way from the terms in (1.34). However, the experimental data require the next to leading order corrections to the leading order theoretical predictions, and in particular, for radiative nonleptonic Kaon decays, many decay modes do not occur at $\mathcal{O}(p^2)$.

The same problem of the strong sector appear again in the weak sector, and even worse. At $\mathcal{O}(p^4)$, there is a huge number of possible terms which satisfy $(8_L, 1_R)$ and $(27_L, 1_R)$ transformation properties; even after using equation of motion of U deduced at $\mathcal{O}(p^2)$ to reduce the number of terms, there are still 35 independent structures left in octet sector [28, 37, 57], in which 22 relevant terms [31] contribute to nonleptonic Kaon decays where external fields are photons. Due to the very large number terms at $\mathcal{O}(p^4)$, their coupling constants contain more information than strong sector. In addition to long distance and short distance contributions, CP violating imaginary part are involved in some coefficients. From the theoretical side, to estimate these coupling coefficients, there are several different approach such as factorization model, weak deformation model, resonance exchange saturation, or effective action approach. Up to now, more work in this direction is still needed.

As for $\mathcal{O}(p^6)$, there is no complete independent term list. Fortunately, one simple way is to consider which terms are relevant to peculiar decay modes, and this method was used first by Cohen, Ecker and Pich [16] and after was developed to fix several coupling coefficients at $\mathcal{O}(p^6)$ [7].

Chapter 2

Radiative Non-leptonic Kaon Decays in ChPT

2.1 Classification of radiative nonleptonic Kaon decays

There are a lot of different modes of Kaon decays, among of them, radiative nonleptonic Kaon decays play an important role in ChPT framework, since there involve chiral loops, chiral anomalies, long-distance contribution, short-distance contribution, CP violation and other possible new physics concepts beyond the standard model. From a theoretical viewpoint, we can understand low energy structure of nonleptonic weak transitions, which can not be deduced from standard model directly up to now, in terms of some undetermined low-energy constants in ChPT. Moreover, specific models which based on assumptions beyond pure ChPT and satisfy those low-energy theorems will give more details to those low-energy constants. On the other hand, experiments of Kaon decays provide precise data to evaluate these models by comparing with their predictions of higher order contribution.

In general, we can classify radiative nonleptonic Kaon decays into three groups:

- One photons in final states, such as $K \to \pi\pi\gamma$, ...
- Two photons in final states, such as $K \to \gamma\gamma$, $K \to \pi\gamma\gamma$, ...
- More than two photons in final states, such as $K \to \pi\gamma\gamma\gamma$, ...

In this thesis, we will focus on those modes with two photons in final states and without consideration of CP violation. CP violation in ChPT can be found in reviews by D' Ambrosio and Isidori [25]. In the following sections, we will give introduction to application of ChPT in radiative nonleptonic Kaon decays at first, and then browse previous works on decay modes with two photons. Finally, I will give a overview of other types of radiative nonleptonic Kaon decay.

2.2 ChPT for radiative nonleptonic Kaon decays

The nonleptonic Kaon decays are not well understood in standard model, since the hamiltonian for nonleptonic $\Delta S = 1$ Kaon decay can not be determined completely by $SU(3)_L \times SU(3)_R$ symmetry. At the low energy, after integrating out those heavy boson $W^{\pm}$, Z and those heavy quarks c, b and t, the nonleptonic $\Delta S = 1$ weak transition can be described by the effective Hamiltonian [47] (1.33). Under chiral symmetry, part of the effective Hamiltonian transform as $(8_L, 1_R)$ and generate $\Delta I = \frac{1}{2}$ transition, and another part transform as $(27_L, 1_R)$ and induce $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ transition.

The most general chiral Lagrangian at $\mathcal{O}(p^2)$ is (1.34) with the same transformation properties as effective Hamiltonian. After coefficients G_8 and G_{27}

are fixed phenomenologically, chiral lagrangian (1.34) can give prediction to some other decays like $K \to \pi\pi$ or $K \to \pi\pi\gamma$ at this order. However, many radiative nonleptonic decay modes can not be generated at the first order, because powers of momenta provided by $\mathcal{O}(p^2)$ chiral Lagrangian is still smaller than minimum number of powers of momenta required by gauge invariance. Works by Ecker, Pich and da Rafael [33, 34] proved that amplitudes of radiative nonleptonic Kaon decays with at most one pion in final states vanish at $\mathcal{O}(p^2)$ chiral Lagrangian. Thus, it is necessary to consider contributions from the next order $\mathcal{O}(p^4)$.

At the $\mathcal{O}(p^4)$ order, the chiral Lagrangian has a lot of possible operator terms which satisfy $(8_L, 1_R)$ and $(27_L, 1_R)$ transformation, and it is proved that there are 35 independent terms left. The general $\mathcal{O}(p^4)$ nonleptonic weak effective lagrangian in terms of octet part only are given:

$$\mathcal{L}_4^{\Delta S=1} = G_8 F^2 \sum N_i W_i + h.c. \tag{2.1}$$

where N_i are dimensionless coupling coefficients and W_i are octet operators given in [47]. Here, we only consider 22 relevant terms which give contributions to radiative nonleptonic Kaon decays. However, this is not complete contribution to $\mathcal{O}(p^4)$ order. In fact, at the $\mathcal{O}(p^4)$ order, there are three types of contributions to radiative nonleptonic Kaon decays:

- The direct tree diagram given by $\mathcal{O}(p^4)$ chiral Lagrangian $\mathcal{L}_4^{\Delta S=1}$.
- One loop diagram with a $\mathcal{L}_2^{\Delta S=1}$ vertex in the loop.
- Tree diagram with a $\mathcal{L}_2^{\Delta S=1}$ vertex and one vertex from strong effective $\mathcal{L}_4$ chiral lagrangian or from anomaly Wess-Zumino-Witten functional [82, 83].

The nonleptonic weak loop diagram are generally divergent, so all coefficients N_i should absorb the remaining divergences after renormalization. Using dimensional regularization, N_i can be wrote at arbitrary scale μ:

$$N_i = N_i^r(\mu) + Z_i \frac{\mu^{d-4}}{16\pi^2}\left\{\frac{1}{d-4} - \frac{1}{2}[ln(4\pi) + \Gamma'(1) + 1]\right\} \tag{2.2}$$

Z_i are chosen to absorb one loop divergences in the amplitudes, the final amplitudes of $\mathcal{O}(p^4)$ are finite and scale independent.

Chiral anomaly give contributions to nonleptonic weak decay from $\mathcal{O}(p^4)$ order, and there are two different contributions from anomalous. The first one produces the so-called reducible anomalous amplitudes, which arise from the contraction of meson lines between a $\mathcal{L}_2^{\Delta S=1}$ vertex and another vertex by Wess-Zumino-Witten terms. Let us consider a simultaneous diagonalization of the kinetic parts of $\mathcal{L}_2$ and $\mathcal{L}_2^{\Delta S=1}$, this type anomaly can be described by local Lagrangian.

$$\begin{aligned} \mathcal{L}_{an}^{\Delta S=1} = & -\frac{\imath e G_8}{8\pi^2 F}\tilde{F}^{\mu\nu}\partial^\mu \pi^0 K^+ D_\nu \pi^- + \\ & + \frac{\alpha G_8}{6\pi F}\tilde{F}^{\mu\nu}F_{\mu\nu}\left(K^+\pi^-\pi^0 - \frac{1}{\sqrt{2}}K^0\pi^+\pi^-\right) + h.c. \end{aligned} \tag{2.3}$$

Another anomalous in nonleptonic weak decay arises from contraction of W boson between a vertex from strong Green function and another vertex from Wess-Zumino-Witten functional. As always, integrating out all heavy particles, operators due to the presence of the anomaly must be realized at the bosonic level. Bosonic operators contain factorizable parts and non-factorizable parts; factorizable contributions produce all relevant octet operators contain ϵ tensor, while the non-factorizable are also with the same form as $W_{28}, \ldots, W_{31}$. In summary, the $\Delta S = 1$ effective Lagrangian in the anomaly parity part can be characterized by coefficients

$$\begin{aligned} N_{28}^{an} &= \frac{a_1}{8\pi^2} \qquad N_{29}^{an} = \frac{a_2}{32\pi^2} \\ N_{30}^{an} &= \frac{3a_3}{16\pi^2} \qquad N_{31}^{an} = \frac{a_4}{16\pi^2} \end{aligned} \tag{2.4}$$

where a_i are dimensionless constant expected to be positive and of order one.

Though most radiative nonleptonic Kaon decay modes have been studied in the $\mathcal{O}(p^4)$ order chiral Lagrangian framework, and many predictions from ChPT fit experiments data very well, more precise experiments are being perfermanced and higher-order corrections beyond $\mathcal{O}(p^4)$ may be sizeable to be measured. However there is no complete investigation on higher order effects in strong sector, and the understanding of nonleptonic weak sector is even worse. Some works have been done to consider *Vector Meson Dominance* and *Unitarity correction* in $\mathcal{O}(p^6)$ to specific decay modes. We postpone to discuss *Vector Meson Dominance* in next chapter since it is the topic of this thesis.

2.3 Two photons in final state

Two photons in final state is very special topic, since they can have either $CP = +1$ or $CP = -1$; thus the amplitude for these decay modes will be proportional to $F^{\mu\nu}F_{\mu\nu}$ which $CP = +1$ or to $\epsilon_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda}$ which $CP = -1$. According to how many pion appear in final state, we classify these decay modes into three groups.

- No pion in final state. $K_S \to \gamma\gamma$, $K_L \to \gamma\gamma$.
- One pion in final state. $K_S \to \pi^0\gamma\gamma$, $K_L \to \pi^0\gamma\gamma$, $K^+ \to \pi^+\gamma\gamma$.
- Two pions in final state. $K_S \to \pi^0\pi^0\gamma\gamma$, $K_L \to \pi^0\pi^0\gamma\gamma$, $K^+ \to \pi^+\pi^0\gamma\gamma$, $K_S \to \pi^+\pi^-\gamma\gamma$, $K_L \to \pi^+\pi^-\gamma\gamma$

Next, we will follow the classification to browse these decay modes one by one.

2.3.1 No pion in final state

The amplitude for $K \to \gamma\gamma$ has the general form compatible with gauge invariance, and can be decomposed to

$$M^{\mu\nu} = (q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu})\lambda_a(q_1, q_2) + (i\epsilon_{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma})\lambda_b(q_1, q_2) \tag{2.5}$$

where $\lambda_a(q_1, q_2)$ and $\lambda_b(q_1, q_2)$ are invariant coefficents and which are function of photons four momentum q_1 and q_2, and satisfy photon exchange symmetry.

$K_S \to \gamma\gamma$

In the framework of ChPT, there is no tree diagram for this decay at $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$, because K_S is neutral and then it can not couple to photons directly. It implies that the chiral meson loops are the only contribution at $\mathcal{O}(p^4)$ and they must be finite. This decay has parallel polarizations, so there is no anomaly contributions $\lambda_b(q_1, q_2)$ to amplitude. The main contribution come from pion loops [19,49], and its amplitude is proportional to $M_K^2 - M_\pi^2$; Kaon loop amplitude is proportional to $M_{K_0}^2 - M_{K_+}^2$, therefore neglecting its contribution is reasonable. We list all one loop diagrams in Figure 2.1.

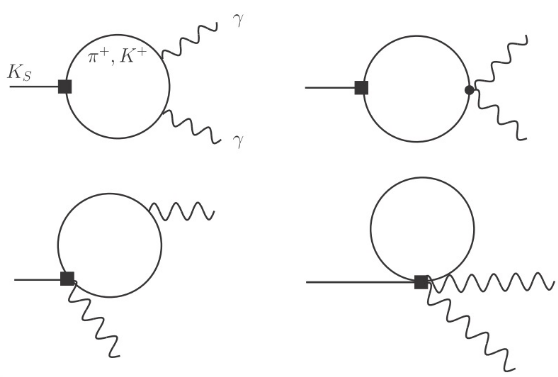

Figure 2.1: All one loop diagrams for $K_S \to \gamma\gamma$

The invariant coefficient is:

$$\lambda_a = \frac{2\alpha F}{\pi}\left(G_8 + \frac{2}{3}G_{27}\right)(1 - r_\pi^2)\mathcal{F}\left(\frac{1}{r_\pi^2}\right) \tag{2.6}$$

Where $r_\pi = \frac{M_\pi}{M_K}$ and the function $\mathcal{F}$ is the meson loop function, and is given by

$$\mathcal{F}(z) = \begin{cases} 1 - \frac{4}{z}arcsin^2(\frac{\sqrt{z}}{2}) & z \leq 4 \\ 1 + \frac{1}{z}\left(ln\frac{1-\sqrt{1-\frac{4}{z}}}{1+\sqrt{1-\frac{4}{z}}} + \imath\pi\right)^2 & z > 4 \end{cases} \tag{2.7}$$

The decay rate is given by

$$\Gamma(K_S \to \gamma\gamma) = \frac{M_K^3}{64\pi}|\lambda_a|^2 \tag{2.8}$$

and taking into account the values of chiral coupling G_8 and G_{27}, the result is $1.5 \times 10^{-11}eV$, with a branch ratio $BR(K_S \to \gamma\gamma) = 2.1 \times 10^{-6}$, which compares well with the new experimental data [67] $BR(K_S \to \gamma\gamma)_{ex} = (2.63 \pm 0.17) \times 10^{-6}$.

At $\mathcal{O}(p^6)$, there is no vector meson dominance contribution, while the unitary correction [56] from $K_S \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$ gives:

$$\frac{2\alpha F}{M_K^2}G_8\left(\frac{1}{\pi}(m_\pi^2 - s)\mathcal{F}(s, m_\pi^2) - \frac{1}{\pi}(M_K^2 - s)\mathcal{F}(s, M_K^2)\right) \tag{2.9}$$

Another contribution comes from local terms at $\mathcal{O}(p^6)$ with an unknown coupling constant [7]

$$\frac{4\alpha G_8}{\pi F}M_K^2(1 - r_\pi^2)a_1 \tag{2.10}$$

Putting known contributions from $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ together, and comparing with experimental data, we can achieve the conclusion that unitary corrections are enough to produce the measured branch ratio and $a_1 = (-1.2 \pm 1.3) \times 10^{-3}$.

$K_L \to \gamma\gamma$

This decay has two photons with perpendicular polarizations ($\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$). Historically, it is related with the GIM mechanism and it has a short-distance contribution [63] from one loop diagram in quark level, which is function of $\frac{m_i^2}{M_W^2}$, where m_i are masses of intermediate quarks. Obviously, the contribution is negligible.

In the ChPT framework, we can't have tree diagram at $\mathcal{O}(p^2)$ for the same reason for $K_S \to \gamma\gamma$, however, unlike $K_S \to \gamma\gamma$, there could be long-distance contribution [63] from tree diagram at $\mathcal{O}(p^4)$ which generated by Wess-Zumino-Witten term and the weak chiral Lagrangian $\mathcal{L}_2^{\Delta S=1}$.

$$\begin{aligned}\mathcal{A}(K_L \to \gamma\gamma) &= \mathcal{A}(K_L \to \pi^0 \to \gamma\gamma) + \mathcal{A}(K_L \to \eta_8 \to \gamma\gamma) \\ &= \mathcal{A}(K_L \to \pi^0)\mathcal{A}(\pi^0 \to \gamma\gamma)\left(\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3(M_K^2 - M_{\eta_8}^2)}\right)\end{aligned} \tag{2.11}$$

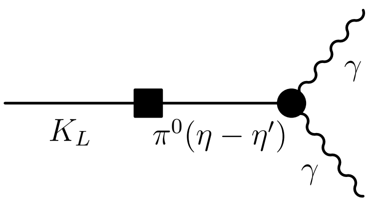

Figure 2.2: $K_L \to \gamma\gamma$

However, this contribution is exactly zero due to the Gell-Mann-Okubo mass formula. Moreover, there is no one loop diagram contribution to this decay, therefore, the leading contributions of $K_L \to \gamma\gamma$ start at $\mathcal{O}(p^6)$, and at this order, we need consider $\eta - \eta'$ mixing to include η_1 in the process. the amplitude is given by

$$\lambda_b = -\frac{2}{\pi}\alpha F(G_8 - G_{27})\mathcal{F}_2 \tag{2.12}$$

with

$$
\begin{aligned}
\mathcal{F}_2 =& \frac{1}{1-r_\pi^2} + \frac{1}{3(1-r_\eta^2)}\left[(1+\xi)\cos\theta_\eta + 2\sqrt{2}\hat{\rho}\sin\theta_\eta\right]\left(\frac{F_\pi}{F_{\eta 8}}\cos\theta_\eta - \frac{2\sqrt{2}F_\pi}{F_{\eta 0}}\sin\theta_\eta\right) \\
& - \frac{1}{3(1-r_{\eta'}^2)}\left[2\sqrt{2}\hat{\rho}\cos\theta_\eta - (1+\xi)\sin\theta_\eta\right]\left(\frac{F_\pi}{F_{\eta 8}}\sin\theta_\eta + \frac{2\sqrt{2}F_\pi}{F_{\eta 0}}\cos\theta_\eta\right)
\end{aligned}
\tag{2.13}
$$

where F_π, F_{η_1} and F_{η_8} are decay constants of π, η_1 and η_8 respectively, and they are fixed to $\frac{F_{\eta_8}}{F_\pi} = 1.34$ and $\frac{F_{\eta_1}}{F_\pi} = 1$. θ_η is the mixing angle of $\eta - \eta'$

$$
\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\theta_\eta & -\sin\theta_\eta \\ \sin\theta_\eta & \cos\theta_\eta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_1 \end{pmatrix}
$$

Large-N_C analyses give $\theta_\eta \simeq -20^\circ$ [54, 55]. ξ labels how exact the $SU(3)$ symmetry is and is given [26] by

$$
\xi = \sqrt{3}\frac{< \eta_8|\mathcal{L}^{\Delta S=1}|K_2^0 >}{< \pi^0|\mathcal{L}^{\Delta S=1}|K_2^0 >} - 1 \tag{2.14}
$$

Its value is not precisely known since it was estimated to be about 0.17 [26] at first and then another work [52] claim that it was canceled. $\hat{\rho}$ represents the information of the breaking of nonet symmetry.

$$
\hat{\rho} = -\sqrt{\frac{3}{8}\frac{< \eta_1|\mathcal{L}^{\Delta S=1}|K_2^0 >}{< \pi^0|\mathcal{L}^{\Delta S=1}|K_2^0 >}} \tag{2.15}
$$

It was indicated [12, 23] that $\hat{\rho} \simeq 0.8$.

The new experimental data [67] shows

$$
BR(K_L \to \gamma\gamma)_{ex} = (5.47 \pm 0.04) \times 10^{-4}.
$$

Using this result, the theoretical prediction at $\mathcal{O}(p^6)$ with η mixing angle $\theta_\eta = -20^\circ$ gives that $\xi = 0$ leads to $\hat{\rho} \simeq 0.7$ while $\xi = 0.2$ requires $\hat{\rho} \simeq 0.8$.

2.3.2 One pion in the final state

The general amplitude for $K(k) \to \pi(p)\gamma(q_1)\gamma(q_2)$ has form

$$\mathcal{A}(K \to \pi\gamma\gamma) = \epsilon_\mu(q_1)\epsilon_\nu(q_2)\mathcal{M}^{\mu\nu} \tag{2.16}$$

where $\mathcal{M}^{\mu\nu}$ can be decomposed to

$$\begin{aligned}\mathcal{M}^{\mu\nu} &= \lambda_a(z,y)(q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu} \\ &\quad + \lambda_b(z,y)(-k\cdot q_1 k \cdot q_2 g^{\mu\nu} - q_1 \cdot q_2\, k^\mu k^\nu + k\cdot q_1\, q_2^\mu k^\nu + k \cdot q_2\, k^\mu q_1^\nu) \\ &\quad + \lambda_c(z,y)(\varepsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma}) \\ &\quad + \lambda_d(z,y)[\varepsilon^{\mu\nu\rho\sigma}\,(k\cdot q_2\, q_{1\rho} + k \cdot q_1\, q_{2\rho})\, k_\sigma + (k^\mu \varepsilon^{\nu\alpha\beta\gamma} + k^\nu \varepsilon^{\mu\alpha\beta\gamma})k_\alpha q_{1\beta} q_{2\gamma}]\end{aligned} \tag{2.17}$$

where $z = (q_1+q_2)^2/M_K^2$ and $y = k\cdot(q_1 - q_2)/M_K^2$ are dimensionless parameters. λ_i are coefficients which satisfy photon exchanging symmetry. When CP is conserved, $K_L \to \pi^0\gamma\gamma$ has contribution from λ_a and λ_b, while $K_S \to \pi^0\gamma\gamma$ get contribution from λ_c and λ_d; as for $K^+ \to \pi^+\gamma\gamma$, it has all λ_is contribution.

The differential decay rate for unpolarized photons is given

$$\frac{d^2\Gamma}{dy\,dz} = \frac{M_K}{2^9\pi^3}\left\{z^2(|A+B|^2 + |C|^2) + \left[y^2 - \frac{1}{4}\lambda(1, r_\pi^2, z)\right]^2 (|B|^2 + |D|^2)\right\} \tag{2.18}$$

where $0 \le |y| \le \frac{1}{2}\lambda^{1/2}(1, r_\pi^2, z)$ and $0 \le z \le (1-r_\pi^2)^2$ are physical region.

This type of decays has no tree diagram at $\mathcal{O}(p^2)$, and only λ_a and λ_b get contribution at $\mathcal{O}(p^4)$. Moreover, λ_c and λ_d are higher order effect and arise only at $\mathcal{O}(p^6)$.

$K_S \to \pi^0\gamma\gamma$

The leading contribution to this decay appear at $\mathcal{O}(p^4)$ and proceeds through a reducible anomalous amplitude with one anomalous vertex $(\pi, \eta) \to \gamma\gamma$

which come from Wess-Zumino-Witten term, and at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$, if we do not consider $\eta - \eta'$ mixing, the coefficient is given [33] by

$$C(z) = \frac{G_8 M_K^2 \alpha}{\pi} \left[\frac{2 - z - r_\pi^2}{z - r_\pi^2 + i r_\pi \frac{\Gamma_{\pi^0}}{M_K}} - \frac{F_\pi (2 - 3z + r_\pi^2)}{3 F_\eta \left(z - r_\eta^2 \right)} \right] \tag{2.19}$$

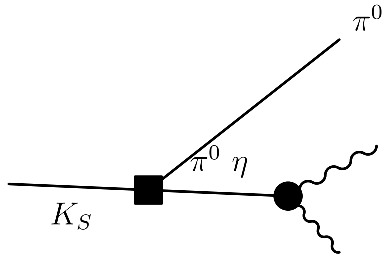

Figure 2.3: $K_S \to \pi^0 \gamma\gamma$

After including η' [46], the amplitude is dominated by the pion pole. In order to eliminate the overwhelming background from $K_S \to \pi^0\pi^0$, kinematic region should be chosen to $z > 0.2$, therefore the theoretical prediction gives

$$\mathrm{BR}(K_S \to \pi^0 \gamma\gamma)_{z>0.2} = 3.8 \times 10^{-8} \tag{2.20}$$

The updated experimental data [61] show that

$$\mathrm{BR}(K_S \to \pi^0 \gamma\gamma)_{z>0.2} = (4.9 \pm 1.8) \times 10^{-8}. \tag{2.21}$$

and it is a good sign to ChPT since the result fit totally in with theoretical prediction given by ChPT.

$K_L \to \pi^0 \gamma\gamma$

This decay has no $\mathcal{O}(p^4)$ counterterms and thus the one loop contributions [10, 33, 78] are finite.

$$\lambda_a(z) = \frac{G_8 M_K^2 \alpha}{\pi\, z} \left[(z - r_\pi^2) \mathcal{F}\left(z / r_\pi^2 \right) - (z - 1 - r_\pi^2) \mathcal{F}(z) \right] \tag{2.22}$$

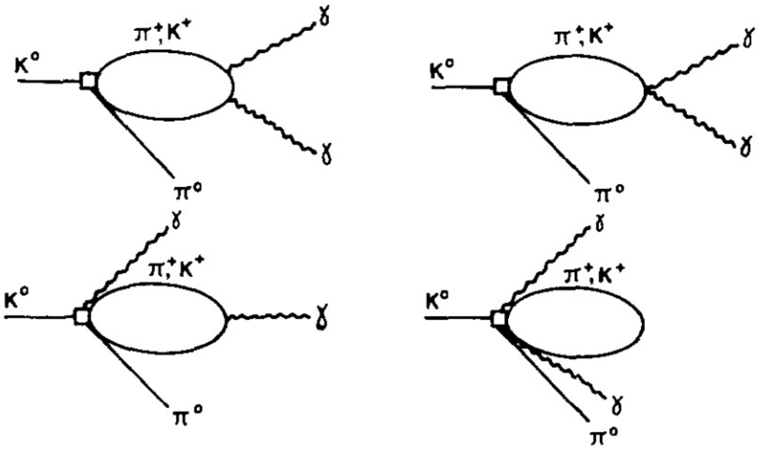

Figure 2.4: All one loop diagrams?? of $K_L \to \pi^0\gamma\gamma$

The amplitude gives branching ratio $BR(K_L \to \pi^0\gamma\gamma) = 6.8 \times 10^{-7}$, however this result is much smaller than the current experimental data from PDG [67]

$$BR(K_L \to \pi^0\gamma\gamma) \ = \ (1.27 \pm 0.03) \times 10^{-6} \tag{2.23}$$

so we must consider higher order chiral corrections. The unitarity correction from $K_L \to \pi^0\pi^+\pi^-$ have been studied [11, 16] and give contribution to both λ_a and λ_b. Moreover, the contribution from 27-plet also was been considered in [11]. Another important higher order contribution is from local counterterms which comes from vector meson exchange and non-resonant parts. There are several models which consider the weak vector meson dominance, like FM, WMD, FMD. There is one parameter which express vector meson's contribution. Theoretical estimation gives $a_V \ = \ -0.7 \pm 0.3$ [24]. Though experimental measurements give different values: NA48 [60] gives $a_V = -0.46 \pm 0.05$, while KTeV [1] obtains $a_V = -0.31 \pm 0.09$, both of them support results where vector meson dominance is assumed.

$K^+ \to \pi^+\gamma\gamma$

The leading contributions come from the $\mathcal{O}(p^4)$ counterterms, one loop diagrams and tree diagrams with one anomalous vertex. The counterterms and one loop diagrams give contributions to λ_a while anomalous tree diagram give contributions to λ_c as follows.

$$A^{(4)}(z) = \frac{G_8 M_K^2 \alpha}{2\pi z}\left[(z+1-r_\pi^2)\mathcal{F}\left(\frac{z}{r_\pi^2}\right) + (z-1+r_\pi^2)\mathcal{F}(z) - \hat{c}z\right] \quad (2.24)$$

$$C(z) = \frac{G_8 M_K^2 \alpha}{\pi}\left[\frac{z-r_\pi^2}{z-r_\pi^2+ir_\pi\frac{\Gamma_{\pi^0}}{M_K}} - \frac{3z-2-r_\pi^2}{3(z-r_\eta^2)}\right] \quad (2.25)$$

where $\hat{c} = 128\pi^2\left[3(L_9+L_{10})+N_{14}-N_{15}-2N_{18}\right]/3$ is unknown coupling constant from combination of local terms. Calculations indicate that the loop contributions are bigger than the contribution from anomalous diagram.

The known contribution at $\mathcal{O}(p^6)$ are unitarity correction and vector meson exchange. The unitarity correction [16, 22] from $K^+ \to \pi^+\pi^+\pi^-$ gives contribution to λ_a and λ_b and it will increase theoretical prediction from $\mathcal{O}(p^4)$ about 30%. As for vector meson exchange, their contributions [16, 24, 35] are related to parameter

$$a_V^+ = -\frac{\pi}{2G_8 M_K^2 \alpha}\lim_{z\to 0} B_V(z). \quad (2.26)$$

The strong vector meson exchange with a weak transition on the external legs and the model dependent weak vector meson contribution combine together to be very small and then can be discard.

The branching ratio were measured [58] to be

$$BR(K^+ \to \pi^+\gamma\gamma) = (1.1 \pm 0.3) \times 10^{-6} \quad (2.27)$$

and therefore it is possible to determine the value of $\hat{c}$, consider to $\mathcal{O}(p^6)$ order, it yield $\hat{c} = 1.8 \pm 0.6$

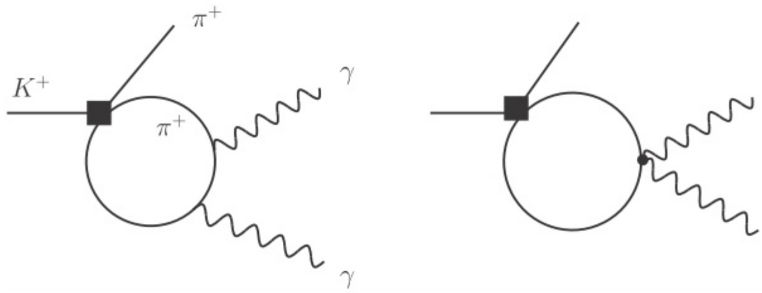

Figure 2.5: unitarity correction to $K^+ \to \pi^+\gamma\gamma$ at $\mathcal{O}(p^6)$

2.3.3 Two pions in the final state

For $K(k) \to \pi(p_1)\pi(p_2)\gamma(q_1)\gamma(q_2)$ decays, only $K_S \to \pi^0\pi^0\gamma\gamma$ and $K_L \to \pi^0\pi^0\gamma\gamma$ were studied in details. The general amplitude for these neutral final states can be decomposed as

$$\begin{aligned}\mathcal{M}^{\mu\nu} &= \lambda_a(q_2^\mu q_1^\nu - zg^{\mu\nu}) \\ &\quad + \lambda_b\left[z^2 p^\mu p^\nu + (p\cdot q_1)(p\cdot q_2)q_2^\mu q_1^\nu - z(p\cdot q_1)q_2^\mu p^\nu - z(p\cdot q_2)p^\mu q_1^\nu\right]) \\ &\quad + \epsilon^{\mu\nu\rho\sigma}q_{1\rho}q_{2\sigma}\end{aligned} \tag{2.28}$$

where $z = q_1 \cdot q_2$ and $p = p_1 + p_2$. Because of Bose symmetry, λ_i should be symmetric under the exchange of $q_1 \leftrightarrow q_2$ and $p_1 \leftrightarrow p_2$.

$K_L \to \pi^0\pi^0\gamma\gamma$

$K_L \to \pi^0\pi^0\gamma\gamma$ has been studied [27] at leading order at $\mathcal{O}(p^4)$ in ChPT, and the only contribution comes from tree diagram with Wess-Zumino-Witten vertex. and then it give contribution to λ_c

$$\lambda_c = i\frac{e^2 g}{48\pi^2 F^3}M_K^2\left(\frac{3}{m_\pi^2 - 2z - im_\pi\Gamma_\pi} + \frac{1}{m_\eta^2 - 2z}\right) \tag{2.29}$$

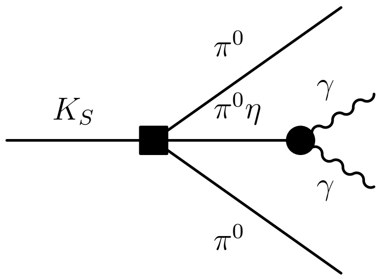

Figure 2.6: $K_L \to \pi^0\pi^0\gamma\gamma$

This amplitude is dominated by the pion pole contribution and is strongly dependent on the cuts on the invariant photon mass.

$K_S \to \pi^0\pi^0\gamma\gamma$

This decay has no counterterms and tree diagram at $\mathcal{O}(p^4)$, therefore the one loop contribution must be finite. Funch and Kambor [39] evaluated all one loop diagrams and related amplitudes. And because this decay has strong background from purely mesonic modes, phenomenological cuts for invariant mass of photons should be considered. For $\delta m_{\gamma\gamma} = 20MeV$, the predicted branching ratio is $BR(K_S \to \pi^0\pi^0\gamma\gamma) = 4.7 \times 10^{-9}$. This decay is too small to be observed in current experiments.

$K^+ \to \pi^+\pi^0\gamma\gamma$

There is another interesting decay mode, $K^+ \to \pi^+\pi^0\gamma\gamma$, with two pions in the final state. This decay is different with others not only because there are charged particle involved, but also because this decay is the only one mode which has bremsstrahlung amplitude at $\mathcal{O}(p^2)$. At $\mathcal{O}(p^4)$, there are tree diagrams with both anomalous and non-anomalous contribution. Anomalous contribution depend on combination $a_3 - \frac{1}{2}a_2$, while the non-anomalous contribution depend on the combinations $N_{14} - N_{15} - N_{16} - N_{17}$

and $N_{14} - N_{15} - 2N_{18}$. The branching ratio is about $\mathcal{O}(10^{-10})$ [32].

2.4 One photon in final state

The amplitude for $K(P) \to \pi_1(p_1) + \pi_2(p_2) + \gamma(q)$ is decomposed into an electric amplitude $E(x_i)$ and a magnetic amplitude $M(x_i)$:

$$A(K \to \pi_1\pi_2\gamma) = \frac{1}{M_K^3}\epsilon^\mu(q)^*[E(x_i)(p_1q\,p_{2\mu} - p_2q\,p_{1\mu}) + M(x_i)\epsilon_{\mu\nu\rho\sigma}p_1^\nu p_2^\rho q^\sigma], \tag{2.30}$$

with

$$x_i = \frac{P \cdot p_i}{M_K^2} \qquad (i = 1, 2), \qquad x_3 = \frac{P \cdot q}{M_K^2}, \qquad x_1 + x_2 + x_3 = 1.$$

The invariant amplitudes $E(x_i)$, $M(x_i)$ are dimensionless. Summing over the photon helicity, the differential decay rate can be written as ($r_i = M_{\pi_i}/M_K$)

$$\begin{aligned}\frac{d\Gamma}{dx_1dx_2} =& \frac{M_K}{4(4\pi)^3}(|E(x_i)|^2 + |M(x_i)|^2)\cdot \\ & \big[(1 - 2x_3 - r_1^2 - r_2^2)(1 - 2x_1 + r_1^2 - r_2^2)(1 - 2x_2 + r_2^2 - r_1^2) \\ & - r_1^2(1 - 2x_1 + r_1^2 - r_2^2)^2 - r_2^2(1 - 2x_2 + r_2^2 - r_1^2)^2\big].\end{aligned} \tag{2.31}$$

There is no interference between E and M as long as the photon helicity is not measured.

$K_{L,S} \to \pi^0\pi^0\gamma$

When CP is conserved, the amplitude of K_L is purely electric, while K_S is purely magnetic. As for $K_L \to \pi^0\pi^0\gamma$, there is no tree diagrams at $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$, even one loop contribution is vanish [39, 53]. Therefore, the leading contribution comes from counterterms at $\mathcal{O}(p^6)$, which give a branching ratio $BR(K_L \to \pi^0\pi^0\gamma)_{\mathcal{O}(p^6)} = 7 \times 10^{-11}$ [64], while Heiliger and Sehgal [53] give a bigger prediction $BR(K_L \to \pi^0\pi^0\gamma)_{HS} = 1 \times 10^{-8}$ by relating $K_L \to \pi^0\pi^0\gamma$

to decay $K_L \to \pi^+\pi^-\gamma$ and they also show that $BR(K_S \to \pi^0\pi^0\gamma)_{HS} = 1.7 \times 10^{-11}$.

$K_S \to \pi^+\pi^-\gamma$

At $\mathcal{O}(p^2)$, the amplitude is determined by bremsstrahlung, and at $\mathcal{O}(p^4)$, contributions [20,21,65] from both counter terms and one loop are all purely electric. In addition, the 27-plet [65] was also considered recently. Experimental branching ratio [67] shows that

$$BR(K_S \to \pi^+\pi^-\gamma)_{E_{\gamma>50MeV}} = (1.79 \pm 0.05) \times 10^{-3}$$

$K_L \to \pi^+\pi^-\gamma$

The bremsstrahlung contribution from $\mathcal{O}(p^2)$ is suppressed because $K_L \to \pi^+\pi^-$ violates CP and it is much small, so in this decay, the magnetic amplitude is dominant if we assume CP conserving. The dominant contribution at $\mathcal{O}(p^4)$ occurs in the magnetic amplitude and it is due to the anomaly. There is no local contribution at this order, and the one loop contributions was proved to be very small. On the other side, experiments indicate there is a sizeable magnetic amplitude beyond $\mathcal{O}(p^4)$. At $\mathcal{O}(p^6)$, the dominant contribution is from a reducible anomalous amplitude, and also vector meson exchange in strong sector and model dependent weak sector enter at this level. Experimental branching ratio is $BR(K_L \to \pi^+\pi^-\gamma) = (4.15 \pm 0.15) \times 10^{-5}$.

$K^+ \to \pi^+\pi^0\gamma$

This decay has several properties, as $K_L \to \pi^+\pi^-\gamma$, the bremsstrahlung contribution was suppressed, the dominating contribution at $\mathcal{O}(p^4)$ is due to anomaly , with small one loop contribution. But, in this mode, the vector meson exchange is small and can be neglected at $\mathcal{O}(p^6)$.

The details of all decays discussed in this chapter can be found in review paper [18], the recent review paper [15] and references in it.

Chapter 3

Vector meson exchange in Radiative nonleptonic Kaon decays

3.1 Vector meson exchange in ChPT

In 1960, J.J. Sakurai [76] predicted vector mesons to couple with photon and then they played an important role in hadronic physics when he studied the interaction between photons and hadronic matter. After Chiral perturbation theory [41, 81] was build, all resonances were studied in chiral perturbation theory in which all matter fields carry non-linear realizations of chiral $SU(3)$, and the contribution of vector meson to the effective Lagrangian in strong sector are worked out [30]. Vector meson contributions saturate those coupling constants of the $\mathcal{O}(p^4)$ effective Lagrangian.

$$
\begin{aligned}
&L_1^V = \frac{G_V^2}{8M_V^2}, \quad L_2^V = 2L_1^V, \quad L_3^V = -6L_1^V, \\
&L_9^V = \frac{F_V G_V}{2M_V^2}, \quad L_{10}^V = -\frac{F_V^2}{4M_V^2} \\
&L_4^V = L_5^V = L_6^V = L_7^V = L_8^V = 0
\end{aligned}
\tag{3.1}
$$

where F_V and G_V are couplings which can be determined from decay $\rho^0 \to e^+e^-$ and $\rho \to \pi\pi$ respectively. Experimental data $\Gamma(\rho^0 \to e^+e^-) = (6.9 \pm 0.3)KeV$ shows that $|F_V| = 154MeV$, vector meson dominance in decay $\rho \to \pi\pi$ with chiral loops contribution reduce to $|G_V| = 53MeV$. In another paper [29], it is showed that the use of antisymmetric tensor fields to describe spin-1 resonances is equivalent to vector field formulations to $\mathcal{O}(p^4)$ in the strong sector.

On the other side, resonances also contribute to coefficients of the $\mathcal{O}(p^4)$ weak chiral Lagrangian [31] and the contributions obey linearly independent relations.

$$
\begin{aligned}
&N_1^V + N_2^V + 2N_3^V = 0, \quad 2N_3^V - 3N_4^V = 0 \\
&5N_1^V - N_2^V + 3N_{15}^V = 0, \quad N_1^V - 2N_2^V + 3N_{14}^V + 3N_{16}^V - 6N_{18}^V = 0 \\
&N_{28}^V - N_{30}^V = 0, \quad N_{16}^V - N_{18}^V + N_{27}^V = 0 \\
&N_{18}^V - N_{37}^V = 0, \quad 2N_{19}^V - N_{25}^V = 0, \quad N_{29}^V - N_{34}^V + N_{35}^V = 0
\end{aligned}
\tag{3.2}
$$

Vector meson exchange in higher order contribution to Kaon decay was studied [38, 66, 77]. Sehgal [77] pointed out that there is a potentially large contribution from vector meson exchange diagram shown in Figure (3.1).

Therefore, it is necessary to construct an effective Lagrangian of vector meson exchange at $\mathcal{O}(p^6)$, and for radiative nonleptonic Kaon decays, the vertex $V \to P\gamma$, where V stands for vector meson, P for pseudoscarlar octet, is the

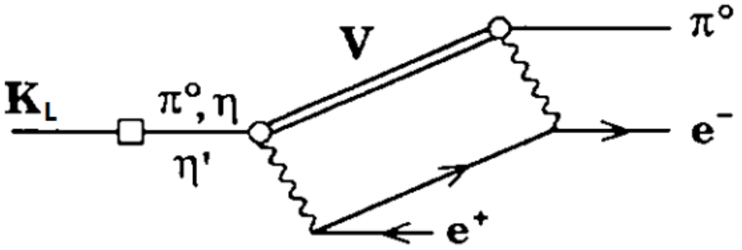

Figure 3.1: Vector meson exchange in $K_L \to \pi^0 e^+ e^-$

ingredient and it can be obtained from the Lagrangian [35]

$$\begin{aligned} &\mathcal{L}(VP\gamma)_{\mathcal{O}(p^3)} = h_V \epsilon_{\mu\nu\rho\sigma} < V^\mu \{u^{\hat{\nu}}, f_+^{\rho\sigma}\} > \\ &u_\mu = i u^\dagger D_\mu U u^\dagger, f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u. \end{aligned} \tag{3.3}$$

where $\hat{V}_\mu = \frac{1}{\sqrt{2}} \sum \lambda_i \hat{V}_\mu^i + \frac{1}{\sqrt{3}} \hat{V}_\mu^0$ are nonet vector field for vector mesons, and to be consistent with data for $V \to P\gamma$ decays, we can fix the coupling constant $|h_V| = (3.7 \pm 0.3) \times 10^{-2}$. Vector field transforms as $\hat{V}_\mu \to h(\phi) \hat{V}_\mu h(\phi)^\dagger$ under chiral transformation. Where $u(\phi)$ is an element of the coset space $SU(3)_L \times SU(3)_R / SU(3)_V$ and transform as $u(\phi) \to g_R u(\phi) h(\phi)^\dagger = h(\phi) u(\phi) g_L^\dagger$, and $U = u^2$.

Integrating out the heavy vector meson field between two $VP\gamma$ vertex, we arrive at the local effective Lagrangian for $PP\gamma\gamma$,

$$\mathcal{L}_6^V = \frac{h_V^2}{M_V^2} \left\langle \{u_\lambda, f_{+\mu\nu}\} \left(u^\lambda, f_+^{\mu\nu} + 2u^\mu, f_+^{\nu\lambda}\right) \right\rangle \tag{3.4}$$

where M_V is mass of vector meson, and in general we take $M_V = M_\rho$. For Kaon decays, we need to put a vertex from $\Delta S = 1$ weak nonleptonic Lagrangian on the initial or final mesons generated by this Lagrangian, and than this tree diagram is responsible for radiative nonleptonic Kaon decay with at least one pion in the final state. The lowest order CP conservation $\Delta S = 1$ weak chiral Lagrangian is

$$\mathcal{L}_2^{\Delta S=1} = G_8 F^4 \langle \mathbf{\Delta} u_\mu u^\mu \rangle, \qquad \mathbf{\Delta} = u \lambda_6 u^\dagger \tag{3.5}$$

where $|G_8| = 9 \times 10^{-6} GeV^{-2}$ is fixed from $K \to \pi\pi$ decays.

However, this is not the complete vector meson contribution at this level. For example, Ecker *etal* [35], pointed out that for $K^+ \to \pi^+\gamma^\star$ decays, the amplitude at $\mathcal{O}(p^4)$ is given in [36] and it has a counter term contribution involving renormalized coupling constant $w_1^r(\mu)$, $w_2^r(\mu)$ and $L_9^r(\mu)$ at given scale μ. $L_9^r(\mu)$ measures vector exchange wheres $w_1^r(\mu)$ and $w_2^r(\mu)$ come from direct weak couplings. However, a naive Vector meson dominance estimate for this decay would only include $L_9(\mu)$(form factor+weak transition) and gives a decay rate much bigger than the experimental data. The conclusion is that there is a strong cancellations between strong amplitudes with external weak transitions and direct weak amplitudes in general. Therefore, the question is how to estimate the direct weak contribution of vector meson exchange at $\mathcal{O}(p^6)$.

There are several models to do this job:

1. *Weak Deformation Model* [28, 35] assume that the long distance contribution of non leptonic $\Delta S = 1$ chiral Lagrangian can be achieved from the same level strong chiral Lagrangian by a deformation of the two 1-forms on coset space $SU(3)_L \times SU(3)_R/SU(3)_V$. Since we have the strong effective chiral Lagrangian for radiative non leptonic Kaon decays at $\mathcal{O}(p^6)$, the weak deformation induces a direct weak chiral Lagrangian at $\mathcal{O}(p^6)$. In the case of $K^+ \to \pi^+\gamma\gamma$, the contribution from WDM weak Lagrangian cancels out the contribution from vector meson exchange in strong sector. As for $K_L \to \pi^0\gamma\gamma$, the weak contribution is twice as large as the strong contribution and with opposite sign, and then it will affect the $\mathcal{O}(p^4)$ amplitude.

2. The second model, called as *Factorization Model* [73], assume that the chiral left handed current is given by the variation of low energy strong effective action in terms of Goldstone boson realization and external

fields. For $K \to \pi\gamma\gamma$, the Lagrangian involve the left handed currents coming from the $\mathcal{O}(p^2)$ strong chiral Lagrangian and from the effective Lagrangian (3.4) at $\mathcal{O}(p^6)$. Moreover, this model has an overall factor which is not given by the model and only can be fixed phenomenologically.

3. Other models like Bergstrom-Masso-Singer Model [5,6] and FMV [24] consider the different types of contribution to direct weak chiral Lagrangian.

There is one different model-independent way to estimate the contribution of vector meson exchange to direct weak chiral Lagrangian. In this method, one first constructs local operators at $\mathcal{O}(p^6)$ in ChPT, and then, restricting only on those relevant terms to the diphoton nonleptonic Kaon decay, three relevant operators remained which can be written in a simplified effective weak Lagrangian [7]

$$\mathcal{L}_6^W = \frac{\alpha G_8}{4\pi}\left(a_1 F_{\mu\nu}F^{\mu\nu}\langle\Delta\chi_+\rangle + a_2 F_{\mu\nu}F^{\mu\nu}\langle\Delta u^\lambda u_\lambda\rangle + a_3 F_{\mu\lambda}F^{\mu\sigma}\langle\Delta\{u^\lambda, u_\sigma\}\rangle\right) \quad (3.6)$$

where $\chi_+ = u^\dagger M u^\dagger + uMu$ and $M = diag(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$. a_i are coefficients which are expected to include all possible higher order contribution from vector meson exchange, unitarity correction and unknown new physics beyond standard model. Under assumption of CP conservation, all a_i's give contribution to $K_L \to \pi^0\gamma\gamma$, $K^+ \to \pi^+\gamma\gamma$, $K_S \to \pi^0\pi^0\gamma\gamma$, $K_S \to \pi^+\pi^-\gamma\gamma$ and $K^+ \to \pi^+\pi^0\gamma\gamma$, and moreover, a_1 can also contribute to $K_S \to \gamma\gamma$.

Using the experimental data of NA48 on $K_L \to \pi^0\gamma\gamma$, we can derive bounds on a_3 and a_2,

$$-1.0 < \frac{M_K^2}{F^2} a_3 < 0.07 \quad (3.7)$$

$$\frac{M_K^2}{F^2}(4a_2 + 2a_3) = 3.4 \pm 0.4 \quad (3.8)$$

also we can use NA48 result on $K_S \to \gamma\gamma$ to fit a_1 and then obtain

$$\frac{8m_K^2}{F^2} a_1 = 1.0 \pm 0.3 \quad (3.9)$$

With these bounded coefficients, the weak local Lagrangian(3.6) gives its contributions to those corresponding decay modes, and together with the contribution from strong $PP\gamma\gamma$ vertex with external weak transition, one get reliable contribution from vector meson exchange to diphoton and two pions nonleptonic Kaon decays.

3.2 Vector meson exchange in $K_S \to \pi^0\pi^0\gamma\gamma$

For $K_{L,S} \to \pi^0\pi^0\gamma\gamma$, there is no tree diagram at $\mathcal{O}(p^2)$ since there is no charged particle involved, while at $\mathcal{O}(p^4)$, $K_L \to \pi^0\pi^0\gamma\gamma$ has tree diagram composed of one vertex from $\mathcal{L}_2^{\Delta S=1}$ chiral Lagrangian and another vertex from Wess-Zumino-Witten anomaly functional. It was shown [27] that pion pole strongly dominate the overall rate and the rate at the regime of pion is sensitive to the value for cut on the invariant mass of photons.

On the other hand, $K_S \to \pi^0\pi^0\gamma\gamma$ does not involve anomaly under assumption of CP conservation, and there is no counter terms giving contribution to $\mathcal{O}(p^4)$ because all particles are neutral, so the sum of contributions from one loop diagrams to this process must be finite.

In this section, we give explicit expression for amplitudes of vector meson exchange contribute to decay $K_S \to \pi^0\pi^0\gamma\gamma$ at $\mathcal{O}(p^6)$, and try to estimate the size of the next to leading order contributions by comparing with leading order contributions from $\mathcal{O}(p^4)$.

3.2.1 Kinematics

The kinematics of $K_S \to \pi^0\pi^0\gamma\gamma$ could be described by five independent kinematic variables as follow.

$$
\begin{aligned}
&x_1 = p_1 \cdot q_1, \quad x_2 = p_1 \cdot q_2, \\
&y_1 = p_2 \cdot q_1, \quad y_2 = p_2 \cdot q_2, \\
&z = q_1 \cdot q_2, \quad x = x_1 + x_2, \quad y = y_1 + y_2
\end{aligned}
\tag{3.10}
$$

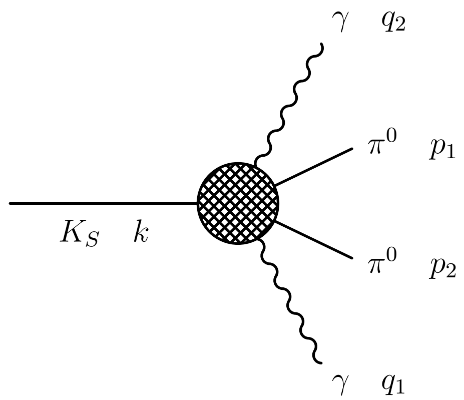

Figure 3.2: Momenta of initial and final states in $K_S \to \pi^0\pi^0\gamma\gamma$

And the amplitude can be decomposed to two parts under the assumption of CP conservation.

$$
\mathcal{A} = \epsilon_\mu(q_1)\epsilon_\nu(q_2)\left[\lambda_1(x_1, x_2, y_1, y_2, z)T_1^{\mu\nu} + \lambda_2(x_1, x_2, y_1, y_2, z)T_2^{\mu\nu}\right] \tag{3.11}
$$

where

$$
\begin{aligned}
T_1^{\mu\nu} &= q_2^\mu q_1^\nu - q_1 \cdot q_2 g^{\mu\nu} \\
T_2^{\mu\nu} &= z^2 p^\mu p^\nu + (p \cdot q_1)(p \cdot q_2) q_2^\mu q_1^\nu - z(p \cdot q_1) q_2^\mu p^\nu - z(p \cdot q_2) p^\mu q_2^\nu
\end{aligned}
\tag{3.12}
$$

where $p^\mu = p_1^\mu + p_2^\mu$. It is useful to rewrite $T_2^{\mu\nu}$ in a form which will be shown to be important at $\mathcal{O}(p^6)$.

$$T_2^{\mu\nu} = \mathcal{I}_1^{\mu\nu} + \mathcal{I}_2^{\mu\nu} + \mathcal{I}_3^{\mu\nu},$$

where

$$\begin{aligned}
\mathcal{I}_1^{\mu\nu} &= z^2 p_1^\mu p_1^\nu + (p_1\cdot q_1)(p_1\cdot q_2) q_2^\mu q_1^\nu - z(p_1\cdot q_1) q_2^\mu p_1^\nu - z(p_1\cdot q_2) p_1^\mu q_1^\nu \\
\mathcal{I}_2^{\mu\nu} &= z^2 p_2^\mu p_2^\nu + (p_2\cdot q_2)(p_2\cdot q_1) q_2^\mu q_1^\nu - z(p_2\cdot q_1) q_2^\mu p_2^\nu - z(p_2\cdot q_2) p_2^\mu q_1^\nu \\
\mathcal{I}_3^{\mu\nu} &= z^2 (p_1^\mu p_2^\nu + p_2^\mu p_1^\nu) + \left[(p_1\cdot q_1)(p_2\cdot q_2) + (p_1\cdot q_2)(p_2\cdot q_1)\right] q_2^\mu q_1^\nu \\
&\quad - z\left[(p_1\cdot q_1) q_2^\mu p_2^\nu + (p_2\cdot q_2) p_1^\mu q_1^\nu + (p_1\cdot q_2) p_2^\mu q_1^\nu + (p_2\cdot q_1) q_2^\mu p_1^\nu\right]
\end{aligned} \tag{3.13}$$

Because of the Bose symmetry, under exchange of $p_1 \leftrightarrow p_2$ and $q_1 \leftrightarrow q_2$, coefficients $\lambda_i(x_1, x_2, y_1, y_2, z)$ must satisfy

$$\lambda_i(x_1, x_2, y_1, y_2, z) = \lambda_i(x_2, x_1, y_2, y_1, z) = \lambda_i(y_1, y_2, x_1, x_2, z) \tag{3.14}$$

3.2.2 amplitude at $\mathcal{O}(p^4)$

In the work [39], Funck and Kambor listed all one-loop diagrams at $\mathcal{O}(p^4)$, classified them into seven types and showed all relevant amplitudes. We can reorganize these amplitudes into two types because there are charged π loops and K loops in all one loop diagrams by using the same labels. The total amplitude can be decomposed in this form:

$$\mathcal{A}(K_S \to \pi^0\pi^0\gamma\gamma) = \epsilon_\mu(q_1)\epsilon_\nu(q_2) \sum \left(A_i^{\mu\nu} + B_i^{\mu\nu}\right), \tag{3.15}$$

where $A_i^{\mu\nu}$ are contributions from charged π loop while $B_i^{\mu\nu}$ come from charged K loops.

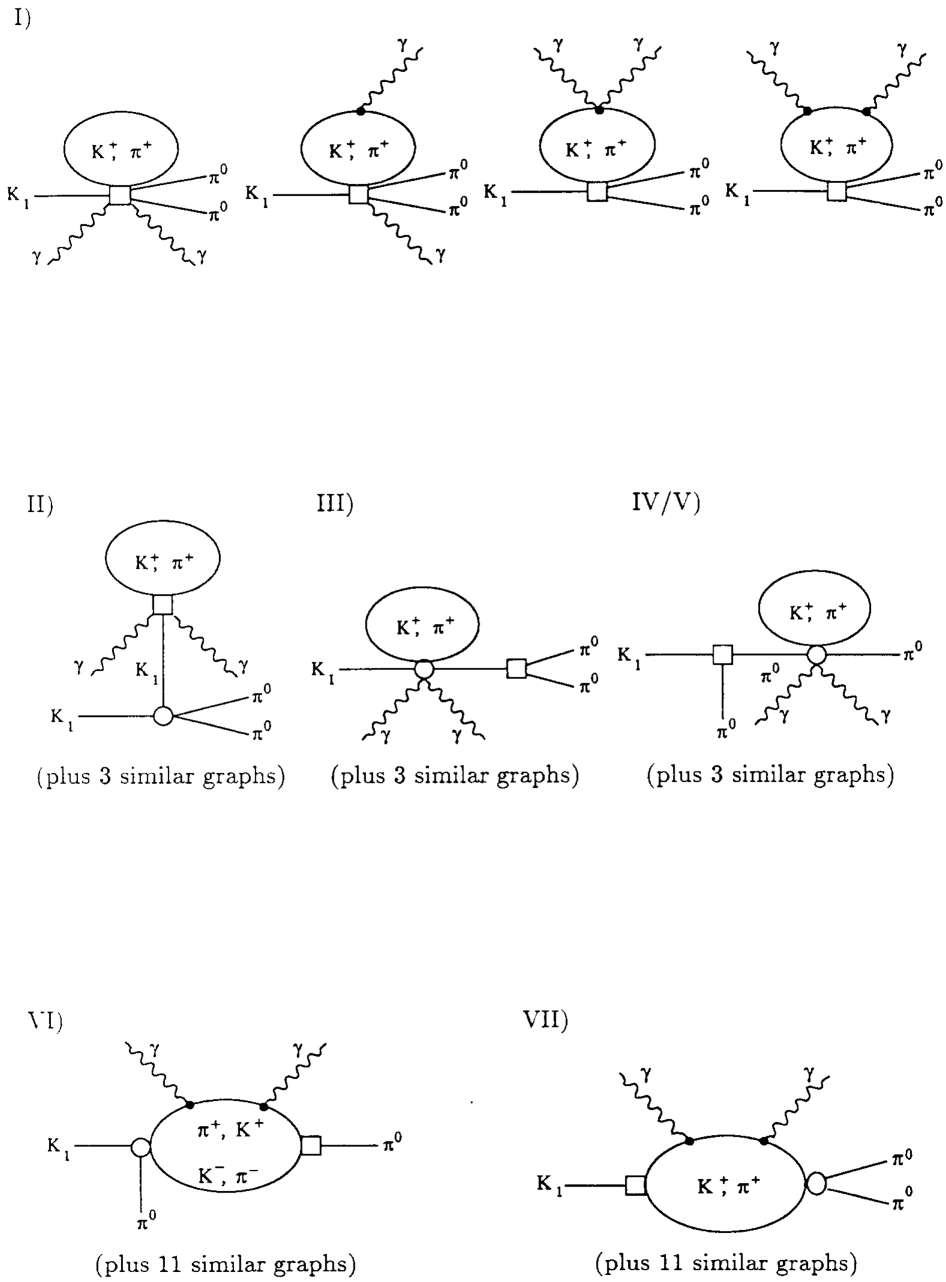

Figure 3.3: All one loop diagrams for $K_S \to \pi^0\pi^0\gamma\gamma$ at $\mathcal{O}(p^4)$

All contributions from charged pion loops are:

$$
\begin{aligned}
A_1^{\mu\nu} &= \frac{e^2 g}{32\pi^2 F^3}\left(-\frac{2}{3}M_K^2 + 2m_\pi^2 + \frac{4}{3}(x+y)\right)\frac{1}{z}F\left(\frac{2z}{m_\pi^2}\right)T_1^{\mu\nu} \\
A_2^{\mu\nu} &= \frac{e^2 g}{48\pi^2 F^3}(m_\pi^2 - 2z)\left(1 - \frac{3}{2}\frac{x+y}{M_K^2 - 2z}\right)F\left(\frac{2z}{m_\pi^2}\right)T_1^{\mu\nu} \\
A_3^{\mu\nu} &= \frac{ie^2 g}{96\pi^2 F^3}(x+y+4z)(1 - \frac{M_K^2 - m_\pi^2}{2(x+y+z+iM_K\Gamma_{K_S})}\frac{1}{z}F\left(\frac{2z}{m_\pi^2}\right)T_1^{\mu\nu} \\
A_4^{\mu\nu} &= -\frac{e^2 g}{48\pi^2 F^3}\left(\frac{3}{2}m_\pi^2 + y - 2z\right)\left(1 - \frac{M_K^2 - m_\pi^2}{y+z+i\frac{1}{2}m_\pi\Gamma_\pi}\right)\frac{1}{z}F\left(\frac{2z}{m_\pi^2}\right)T_1^{\mu\nu} \\
A_5^{\mu\nu} &= \frac{e^2 g}{48\pi^2 F^3}\left(\frac{3}{2}m_\pi^2 + x - 2z\right)\left(1 - \frac{M_K^2 - m_\pi^2}{x+z+i\frac{1}{2}m_\pi\Gamma_\pi}\right)\frac{1}{z}F\left(\frac{2z}{m_\pi^2}\right)T_1^{\mu\nu} \\
A_6^{\mu\nu} &= 0 \\
A_7^{\mu\nu} &= \frac{e^2 g}{12\pi^2 F^3}\big\{[M_K^2 - \frac{5}{4}m_\pi^2 - z - \frac{5}{4}(x+y)]\frac{1}{z}F\left(\frac{2z}{m_\pi^2}\right)T_1^{\mu\nu} \\
&\quad - 12(M_K^2 - m_\pi^2)\left(M_K^2 - m_\pi^2 - 2[x+y+z]\sigma^{\mu\nu}(p,q_1,q_2)\right)\big\}
\end{aligned}
\tag{3.16}
$$

where

$$
F(z) = \begin{cases} 1 - \frac{4}{z}(arcsin\frac{\sqrt{z}}{2})^2, & z \leq 4 \\ 1 - \frac{\pi^2}{z} + \frac{1}{z}\left(ln\frac{1-\sqrt{1-\frac{4}{z}}}{1-\sqrt{1+\frac{4}{z}}}\right)^2 + \frac{i2\pi}{z}ln\frac{1-\sqrt{1-\frac{4}{z}}}{1-\sqrt{1+\frac{4}{z}}}, & z > 4 \end{cases}
$$

is the loop function deduced firstly from decay $K_L \to \pi^0\gamma\gamma$. $\sigma^{\mu\nu}(p,q_1,q_2)$ is tensor four point loop integral and can be decomposed as $\sigma^{\mu\nu}(p,q_1,q_2) \equiv -\frac{a}{z}T_1^{\mu\nu} + \frac{b}{z^2}T_2^{\mu\nu}$.

On the other hand, all contributions from charged Kaon loops are:

$$
\begin{aligned}
B_1^{\mu\nu} &= \frac{e^2 g}{96\pi^2 F^3}(6M_K^2 - 2m_\pi^2 - 8(x+y) - 19z)\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu} \\
B_2^{\mu\nu} &= \frac{e^2 g}{96\pi^2 F^3}(M_K^2 - 2z)\left(2 - \frac{3(x+y)}{M_K^2 - 2z}\right)\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu} \\
B_3^{\mu\nu} &= \frac{e^2 g}{96\pi^2 F^3}(x+y+4z)\left(1 - \frac{M_K^2 - m_\pi^2}{2(x+y+z) + iM_K\Gamma_K}\right)\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu} \\
B_4^{\mu\nu} &= \frac{e^2 g}{192\pi^2 F^3}(y - 2z)\left(1 - \frac{M_K^2 - m_\pi^2}{y + z + i\frac{1}{2}m_\pi\Gamma_\pi}\right)\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu} \\
B_5^{\mu\nu} &= \frac{e^2 g}{192\pi^2 F^3}(x - 2z)\left(1 - \frac{M_K^2 - m_\pi^2}{x + z + i\frac{1}{2}m_\pi\Gamma_\pi}\right)\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu} \\
B_6^{\mu\nu} &= \frac{e^2 g}{32\pi^2 F^3}[M_K^2 - 2z - \frac{1}{2}(x+y)]\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu} \\
B_7^{\mu\nu} &= \frac{e^2 g}{96\pi^2 F^3}[-M_K^2 + 2z + \frac{5}{2}(x+y)]\frac{1}{z}F\left(\frac{2z}{M_K^2}\right)T_1^{\mu\nu}
\end{aligned}
\tag{3.17}
$$

Having all amplitudes of relevant Feynman diagrams, we can calculate decay spectra and total decay rate. As this process has background from $K_S \to \pi^0\pi^0$, $K_S \to \pi^0\pi^0\gamma$ and $K_S \to \pi^0\pi^0\pi^0$, it is important to do cuts in the invariant mass of the two photons($M_{\gamma\gamma} \geq \delta m$ and $|M_{\gamma\gamma} - m_\pi| \geq \delta m$, where $\delta m \in (0, 100MeV)$). The branching ratio $BR(K_S \to \pi^0\pi^0\gamma\gamma) \sim 5 \times 10^{-9}$ for different cuts.

3.2.3 amplitude at $\mathcal{O}(p^6)$

After considering all leading contributions at $\mathcal{O}(p^4)$, one need to evaluate the next to leading contributions at $\mathcal{O}(p^6)$ even though there is no complete framework of chiral Lagrangian at this level. We can only estimate those known processes such as unitarity correction and vector meson dominance. Since there is no phase space to generate 4 pions, we are pretty sure that we can concentrate on evaluating the contributions from vector meson exchange at this order.

As for $K_S \to \pi^0\pi^0\gamma\gamma$, we know there are two contributions from vector meson exchange: one is from the strong chiral Lagrangian sector, while another comes from effective weak chiral Lagrangian side. At first, we start to calculate the contribution from strong chiral Lagrangian sector.

Following effective Lagrangian (3.4), we list all possible vertices which give contribution to $K_S \to \pi^0\pi^0\gamma\gamma$.

(i) vertex of $K_S K_S \gamma\gamma$ from

$$-\frac{32e^2h_V^2}{9F^2M_V^2}\left(F_{\mu\nu}F^{\mu\nu}\partial_\lambda K_S\partial^\lambda K_S + 2F_{\mu\nu}F^{\nu\lambda}\partial_\lambda K_S\partial^\mu K_S\right) \tag{3.18}$$

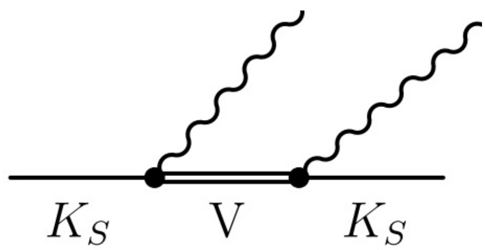

Figure 3.4: vertex for $K_S K_S \gamma\gamma$

(ii) vertex $\pi^0\pi^0\gamma\gamma$ from

$$\frac{80e^2h_V^2}{9F^2M_V^2}\left(F_{\mu\nu}F^{\mu\nu}\partial_\lambda\pi^0\partial^\lambda\pi^0 + 2F_{\mu\nu}F^{\nu\lambda}\partial_\lambda\pi^0\partial^\mu\pi^0\right) \tag{3.19}$$

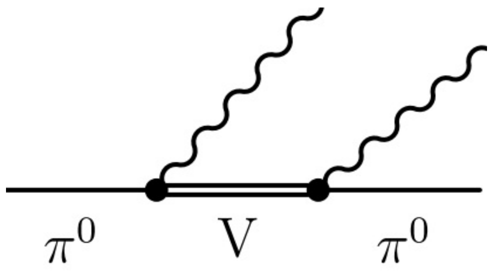

Figure 3.5: vertex for $\pi^0\pi^0\gamma\gamma$

(iii) vertex $\eta\pi^0\gamma\gamma$ from

$$\frac{16e^2h_V^2}{3\sqrt{3}F^2M_V^2}\big(F_{\mu\nu}F^{\mu\nu}(\partial_\lambda\eta\partial^\lambda\pi^0+\partial_\lambda\pi^0\partial^\lambda\eta)\\ +2F_{\mu\nu}F^{\nu\lambda}(\partial_\lambda\eta\partial^\mu\pi^0+\partial_\lambda\pi^0\partial^\mu\eta)\big) \tag{3.20}$$

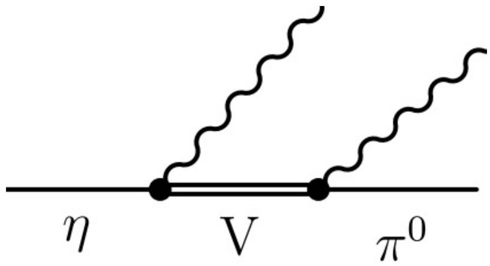

Figure 3.6: vertex for $\eta\pi^0\gamma\gamma$

Only these vertices are not enough since they do not involve flavor changing, therefore we need weak transition $K_S \to \pi^0(\pi^0,\eta)$ on one external leg to generate this process. We can have two types weak transition vertex from the lowest weak chiral Lagrangian (1.34).

1. The counterterms to $K_S \to \pi^0\pi^0$ from the lowest $\Delta S = 1$ weak chiral Lagrangian are

$$iG_8F(\partial_\mu\pi^0\partial^\mu\pi^0K_S - \pi^0\partial_\mu\pi^0\partial^\mu K_S) \tag{3.21}$$

and the corresponding Feynman diagram is

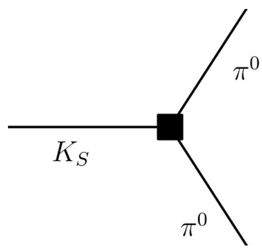

Figure 3.7: vertex for $K_S \to \pi^0\pi^0$

2. The counterterms to $K_S \to \pi^0\eta$ from the lowest $\Delta S = 1$ weak chiral Lagrangian are

$$iG_8F\left(-\frac{2}{\sqrt{3}}\partial_\mu\pi^0\partial^\mu\eta K_S + \sqrt{3}\eta\partial_\mu\pi^0\partial^\mu K_S - \frac{1}{\sqrt{3}}\pi^0\partial_\mu\eta\partial^\mu K_S\right), \quad (3.22)$$

and the corresponding Feynman diagram is

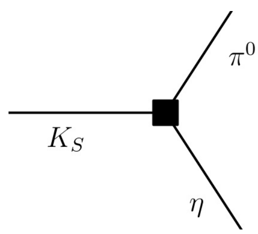

Figure 3.8: vertex for $K_S \to \pi^0\eta$

Since we now have all relevant vertices, we can list all possible Feynman diagrams for vector meson exchange in $K_S \to \pi^0\pi^0\gamma\gamma$ as follows.

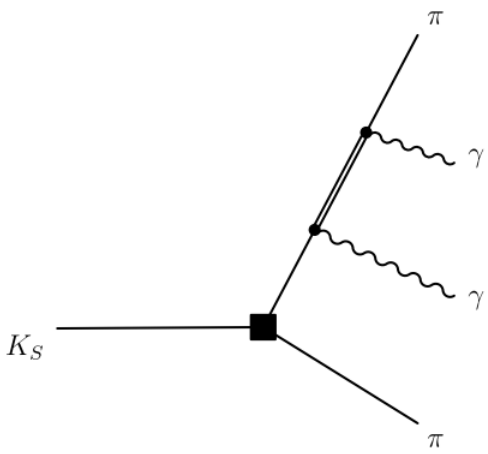

Figure 3.9: 4 similar diagrams for $K_S \to \pi^0\pi^0\gamma\gamma$

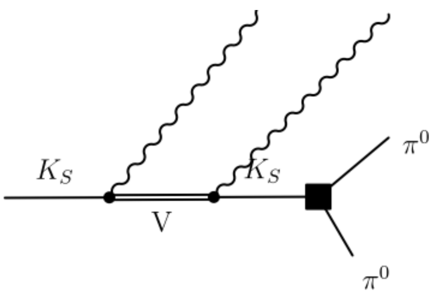

Figure 3.10: Another type diagram for $K_S \to \pi^0\pi^0\gamma\gamma$

From these Feynman diagrams, we can deduce the relevant amplitudes:

$$
\begin{aligned}
M_S^{\mu\nu} &= M_{S1}^{\mu\nu} + M_{S2}^{\mu\nu} \\
M_{S1}^{\mu\nu} &= \frac{160 G_8 e^2 h_V^2}{9 F M_V^2} \frac{2M_K^2 - 2m_\pi^2 - 2y - 2z}{y+z} \left(2m_\pi^2 + y - \frac{y_1 y_2)}{z} \right) T_1^{\mu\nu} \\
&- \frac{192 G_8 e^2 h_V^2}{9 F M_V^2} \frac{M_K^2 - m_\pi^2 - 3y - 3z}{2M_K^2 - 2m_\pi^2 - 3y - 3z} \left(2m_\pi^2 + y - \frac{y_1 y_2}{z} \right) T_1^{\mu\nu} \\
&+ \frac{160 G_8 e^2 h_V^2}{9 F M_V^2} \frac{2M_K^2 - 2m_\pi^2 - 2x - 2z}{x+z} \left(2m_\pi^2 + x - \frac{x_1 x_2}{z} \right) T_1^{\mu\nu} \\
&- \frac{192 G_8 e^2 h_V^2}{9 F M_V^2} \frac{M_K^2 - m_\pi^2 - 3x - 3z}{2M_K^2 - 2m_\pi^2 - 3x - 3z} \left(2m_\pi^2 + x - \frac{x_1 x_2)}{z} \right) T_1^{\mu\nu} \\
&+ \frac{160 G_8 e^2 h_V^2}{9 F M_V^2} \frac{2M_K^2 - 2m_\pi^2 - 2y - 2z}{y+z} \frac{1}{z} \mathcal{I}_2^{\mu\nu} \\
&- \frac{192 G_8 e^2 h_V^2}{9 F M_V^2} \frac{M_K^2 - m_\pi^2 - 3y - 3z}{2M_K^2 - 2m_\pi^2 - 3y - 3z} \frac{1}{z} \mathcal{I}_2^{\mu\nu} \\
&+ \frac{160 G_8 e^2 h_V^2}{9 F M_V^2} \frac{2M_K^2 - 2m_\pi^2 - 2x - 2z}{x+z} \frac{1}{z} \mathcal{I}_1^{\mu\nu} \\
&- \frac{192 G_8 e^2 h_V^2}{9 F M_V^2} \frac{M_K^2 - m_\pi^2 - 3x - 3z}{2M_K^2 - 2m_\pi^2 - 3x - 3z} \frac{1}{z} \mathcal{I}_1^{\mu\nu} \\
M_{S2}^{\mu\nu} &= -\frac{128 G_8 e^2 h_V^2}{9 F M_V^2} \frac{M_K^2 - m_\pi^2 - x - y - z}{x+y+z} \\
&\quad \left[\left(M_K^2 - x - y - 2z + \frac{xy}{z} \right) T_1^{\mu\nu} + \frac{1}{z} T_2^{\mu\nu} \right]
\end{aligned}
\tag{3.23}
$$

We find that there are contributions to $T_1^{\mu\nu}$, $T_2^{\mu\nu}$, $\mathcal{I}_1^{\mu\nu}$ and $\mathcal{I}_2^{\mu\nu}$ from vector meson exchange in strong chiral Lagrangian. As for $T_1^{\mu\nu}$ and $T_2^{\mu\nu}$, we are not surprised that there must be these contributions, while those terms related with $\mathcal{I}_1^{\mu\nu}$ and $\mathcal{I}_2^{\mu\nu}$ are more interesting, and we want to know if they are new structures at $\mathcal{O}(p^6)$. If one looks back in [35], one can see that it is clearly shown that the effective chiral Lagrangian of vector meson exchange between two strong coupling $VP\gamma$ vertex with a $\Delta S = 1$ weak transition on initial or final legs is not the full scenario of vector meson exchange in radiative Kaon decays, and those direct weak amplitudes are to be expected. As we mentioned before, we choose to use the relevant $\mathcal{O}(p^6)$ effective chiral

Lagrangian (3.6), which gives following counterterms for $K_S \to \pi^0\pi^0\gamma\gamma$.

$$\frac{iG_8\alpha}{4\pi F^3}F_{\mu\nu}F^{\mu\nu}\left\{a_1\frac{-2}{3}(M_K^2 - m_\pi^2)K_S\pi^0\pi^0 + a_2(K_S\partial_\lambda\pi^0\partial^\lambda\pi^0 - \partial^\lambda K_S\partial_\lambda\pi^0\pi^0)\right\}$$
$$+\frac{iG_8\alpha}{4\pi F^3}a_3F_{\mu\nu}F^{\nu\lambda}(2K_S\partial_\lambda\pi^0\partial^\mu\pi^0 - \pi^0\partial_\lambda K_S\partial^\mu\pi^0 - \pi^0\partial^\mu K_S\partial_\lambda\pi^0) \tag{3.24}$$

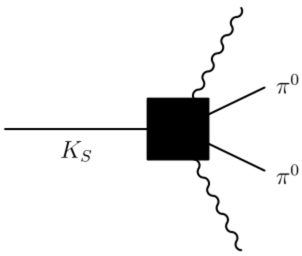

Figure 3.11: Feynman diagram for $K_S \to \pi^0\pi^0\gamma\gamma$ from local operators

From which, we deduce the amplitude of the weak chiral Lagrangian to $K_S \to \pi^0\pi^0\gamma\gamma$.

$$M_W^{\mu\nu} = \frac{G_8\alpha}{\pi F^3}\left\{\frac{2a_1}{3}(M_K^2 - m_\pi^2) + a_2(2M_K^2 - 2m_\pi^2 - 3x - 3y - 4z) + \frac{a_3}{2}(x+y)\right\}T_1^{\mu\nu}$$
$$-\frac{G_8\alpha}{\pi F^3}a_3(T_2^{\mu\nu} + \mathcal{I}_3^{\mu\nu}) \tag{3.25}$$

Putting amplitudes (3.23)from strong sector and amplitude (3.25) from weak sector together, we get the total amplitude $M_V^{\mu\nu} = M_S^{\mu\nu} + M_W^{\mu\nu}$ which represents vector meson exchange in $K_S \to \pi^0\pi^0\gamma\gamma$ at $\mathcal{O}(p^6)$

We observe that, for $K_S \to \pi^0\pi^0\gamma\gamma$, the cancellation between contributions from direct weak local operators and contributions from weak transition in external legs is not clear as $K^+ \to \pi^+\gamma\gamma$ and $K_L \to \pi^0\gamma\gamma$ because coefficients a_i only get bounds from experimental data. Taking the condition that the coefficient of $\mathcal{I}_i^{\mu\nu}$ must be equal because of gauge symmetries and Bose symmetries, we get relations between these coefficients

$$\begin{aligned}
&-\frac{G_8\alpha}{\pi F^3}a_3 \\
&=\frac{64G_8e^2h_V^2}{9FM_V^2z}\left[\frac{5(M_K^2-m_\pi^2-y-z)}{y+z}-\frac{3(M_K^2-m_\pi^2-3y-3z)}{2M_K^2-2m_\pi^2-3y-3z}\right] \\
&=\frac{64G_8e^2h_V^2}{9FM_V^2z}\left[\frac{5(M_K^2-m_\pi^2-x-z)}{x+z}-\frac{3(M_K^2-m_\pi^2-3x-3z)}{2M_K^2-2m_\pi^2-3x-3z}\right]
\end{aligned} \tag{3.26}$$

We get one equation from the last two lines of (3.26) immediately and one possible solution is $x=y$ which means $p_1\cdot(q_1+q_2)=p_2\cdot(q_1+q_2)$. Other solutions can be deduced from equation

$$18z^2-[30(M_K^2-m_\pi^2)-18(x+y)]z+10(M_K^2-m_\pi^2)^2+18xy-15(M_K^2-m_\pi^2)(x+y)=0 \tag{3.27}$$

Putting those solutions into (3.26), we have different expressions for coefficient a_3. It is interesting that other coefficients likes a_1 and a_2 can only be fixed from experimental data, while the a_3 can be obtained from gauge symmetry and Bose symmetry, and this coefficient is not approximately constant any more, but it gets a kinematic dependence. In the next section, we discuss the effect of different solutions to the final amplitude of $K_S\to\pi^0\pi^0\gamma\gamma$ at $\mathcal{O}(p^6)$.

3.2.4 Analysis

At first, we take the simplest solution $x=y$, therefore, coefficient a_3 has formula

$$a_3=-\frac{256\pi^2F^2h_V^2}{9M_V^2z}\left[\frac{5(M_K^2-m_\pi^2-y-z)}{y+z}-\frac{3(M_K^2-m_\pi^2-3y-3z)}{2M_K^2-2m_\pi^2-3y-3z}\right] \tag{3.28}$$

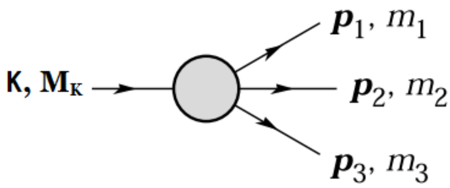

Figure 3.12: Variables for three body decays

To estimate this equation, we need a *Dalitz plot* analysis. At first,we need to treat two photons as an effective particle label as (p_3, m_3), where $p_3^2 = m_3^2 = 2z$, and then define $p_{ij} = p_i + p_j$ and $m_{ij}^2 = p_{ij}^2$. For a given value of m_{12}^2, the range of m_{23}^2 is determined by its values

$$\begin{aligned}(m_{23}^2)_{max} &= (E_2^* + E_3^*)^2 - \left(\sqrt{(E_2^*)^2 - m_2^2} - \sqrt{(E_3^*)^2 - m_3^2}\right)^2 \\ (m_{23}^2)_{min} &= (E_2^* + E_3^*)^2 - \left(\sqrt{(E_2^*)^2 - m_2^2} + \sqrt{(E_3^*)^2 - m_3^2}\right)^2\end{aligned} \quad (3.29)$$

where $E_2^* = \frac{m_{12}^2 - m_1^2 + m_2^2}{2m_{12}}$ and $E_3^* = \frac{M_K^2 - m_{12}^2 - m_3^2}{2m_{12}}$ are the energies of particles 2 and 3 in the m_{12} rest frame. For decay $K_S \to \pi^0\pi^0\gamma\gamma$, $E_2^* = \frac{m_{12}}{2}$ and $E_3^* = \frac{M_K^2 - m_{12}^2 - 2z}{2m_{12}}$. On the other hand, $m_{23}^2 = m_\pi^2 + 2y + 2z$, so we get a constrain on y:

$$E_2^*E_3^* - \sqrt{(E_2^*)^2 - m_2^2}\sqrt{(E_3^*)^2 - m_3^2} < y < E_2^*E_3^* + \sqrt{(E_2^*)^2 - m_2^2}\sqrt{(E_3^*)^2 - m_3^2} \quad (3.30)$$

From those two square roots, we get two conditions for m_{12}:

$$(E_2^*)^2 - m_2^2 \geq 0$$

$$(E_3^*)^2 - m_3^2 \geq 0$$

With the fixed invariant mass of photons $M_{\gamma\gamma}^2 = 2z$, one gets a clean range for m_{12}, and then fix the value of m_{12} in the range to give explicit bound on y by using equation (3.30). The last thing is to estimate what is the value of a_3 according to the equation (3.28).

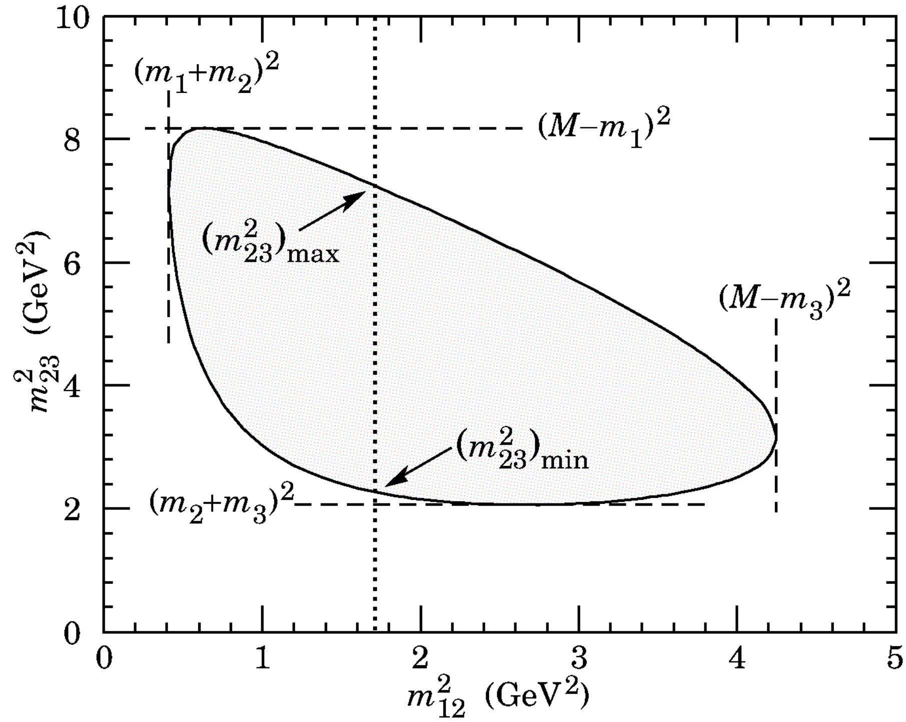

Figure 3.13: Dalitz plot for a three-body final state

We know that we need to perform cuts in the invariant mass of photons to exclude the background. And the cut around $M_{\gamma\gamma} = m_\pi$ does not affect decay $K_S \to \pi^0\pi^0\gamma\gamma$ too much as shown at $\mathcal{O}(p^4)$, so the first value we choose $M_{\gamma\gamma} = m_\pi = 135$, thereafter, the bound on m_{12} is $(270, 359)$. To make a clear description of the a_3 dependence on kinematics, we choose $m_{12} = 300$ and $m_{12} = 340$, and the range of y $(33108, 61317)$ and $(27200, 45520)$ with respectively, and then, the evolution of $\frac{M_K^2}{F^2}a_3$ are given in the following pictures.

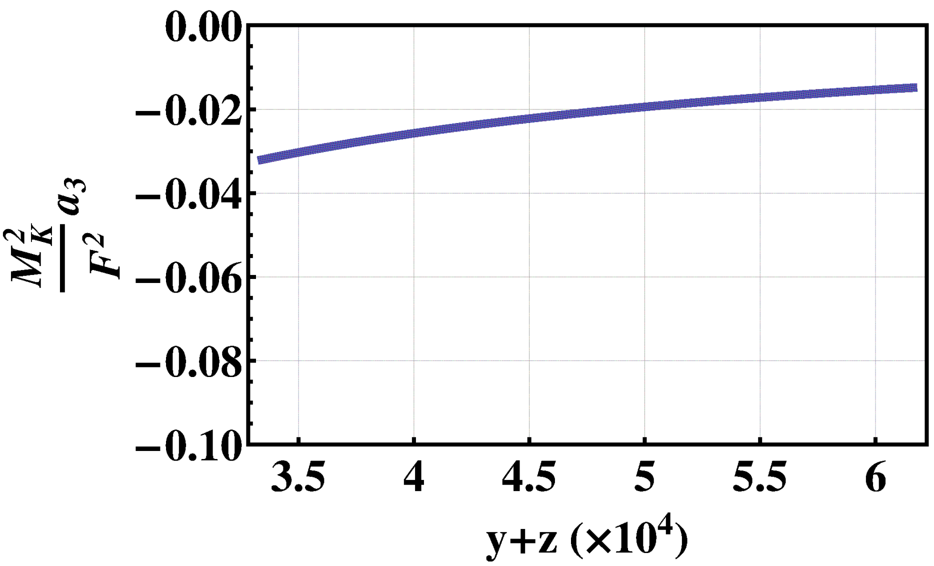

Figure 3.14: Plot of a_3 when y is $(33108, 61317)$ and $M_{\gamma\gamma} = 135MeV$

From Figure 3.14 and Figure 3.15, one notices that at the same kinematic region $(3.5 \times 10^4,\ 4.5 \times 10^4)$ of $y + z$, the coefficient $\frac{M_K^2}{F^2}a_3$ varies between -0.03 and -0.02.

To understand how much the coefficient $\frac{M_K^2}{F^2}a_3$ dependents on cut on the invariant mass of photons $M_{\gamma\gamma}$, we choose $M_{\gamma\gamma} = 145$ to get range of m_{12} $(270, 349)$, and again we choose $m_{12} = 300$ and $m_{12} = 340$ for comparing

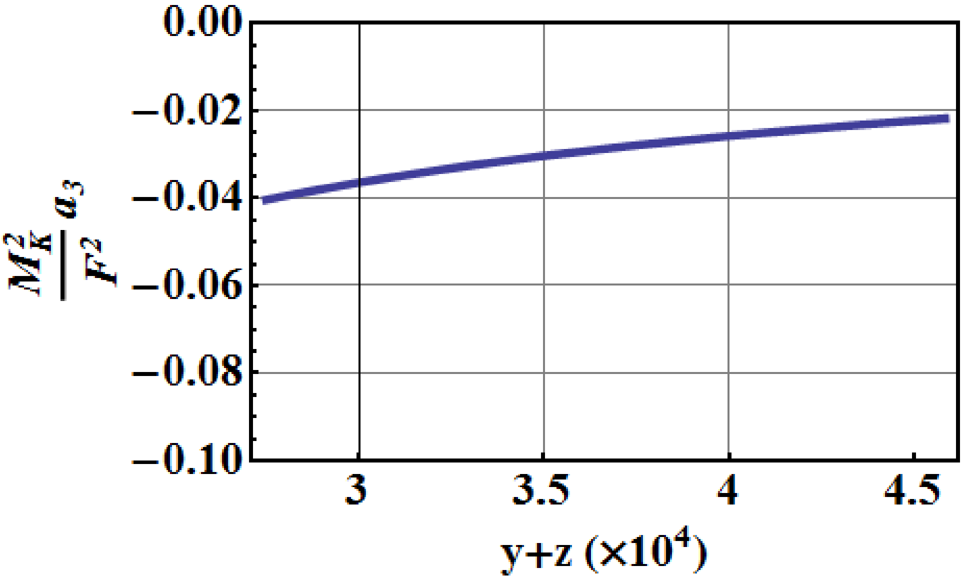

Figure 3.15: Plot of a_3 when y is $(27200, 45520)$ and $M_{\gamma\gamma} = 135MeV$

with result from cut $M_{\gamma\gamma} = 135$, and the range of $y + z$ is $(32872, 54452)$ and $(31022, 43382)$. Result are showed in Figure 3.16 and Figure 3.17:

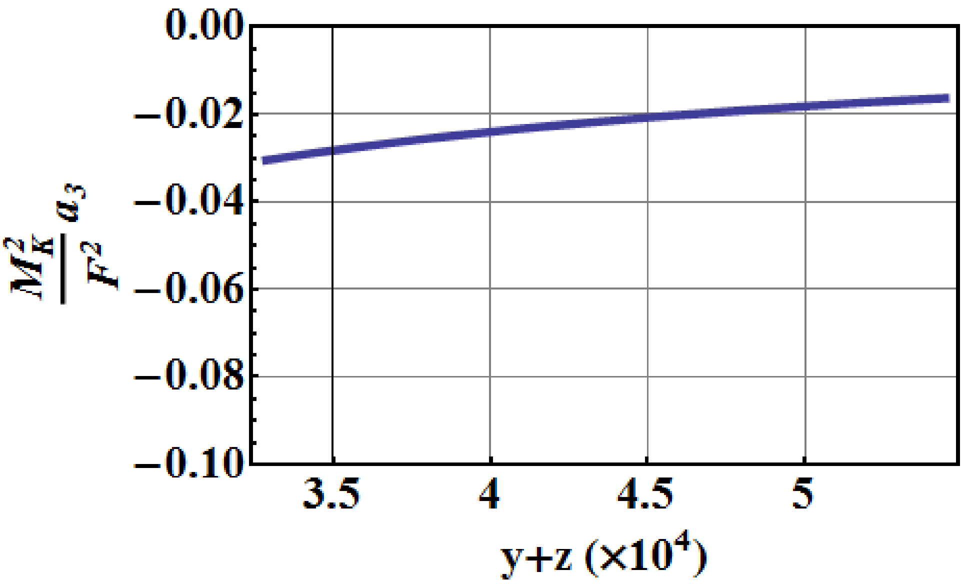

Figure 3.16: Plot of a_3 when y is $(32872, 54452)$ and $M_{\gamma\gamma} = 145$

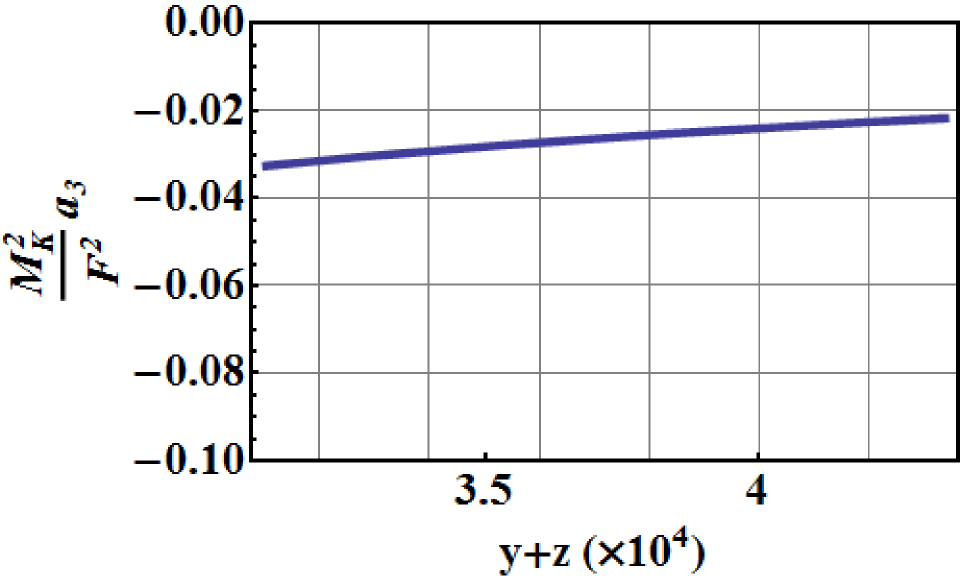

Figure 3.17: Plot of a_3 when y is $(31022, 43382)$ and $M_{\gamma\gamma} = 145$

Again, we see the almost same result that $\frac{M_K^2}{F^2} a_3$ varies between -0.03 and -0.02 when the kinematic range is $(3.5 \times 10^4,\ 4.5 \times 10^4)$. Up to now, we can draw a unambiguous conclusion that even though coefficient $\frac{M_K^2}{F^2} a_3$ is kinematic dependent, its value varies very slowly from -0.03 to -0.02 in the reasonable kinematic region, and clearly it is fit well with the conclusion (3.7) from experimental data of $K_L \to \pi^0 \gamma\gamma$. Together with constraint condition (3.8), another coefficient $\frac{M_K^2}{F^2} a_2$ can be given a concrete value 0.86 ± 0.1.

As for equation (3.27), using *Dalitz plot* analysis again, we consider invariant mass of photons $M_{\gamma\gamma} = m_\pi = 135 MeV$, and range of y is technically chosen as $(35000, 50000)$, we can plot the relation between x and y, clearly, it is shown that ratio $\frac{x}{y}$ varies around 3-3.5. Because we use the same procedure as the condition of $x = y$, the result of $\frac{M_K^2}{F^2} a_3 \in (-0.03, -0.02)$ is still hold. To estimate the contribution of vector mesons exchange at $\mathcal{O}(p^6)$, we only choose the largest contribution at $\mathcal{O}(p^4)$ and give a rough estimation. From the results of a_i, a_3 is so small that can be neglected immediately, and ratio of $\frac{a_1}{a_2}$ is close to $\frac{1}{8}$; in addition, a_1 has factor $\frac{2}{3}$ while a_2's factor is 2, so only

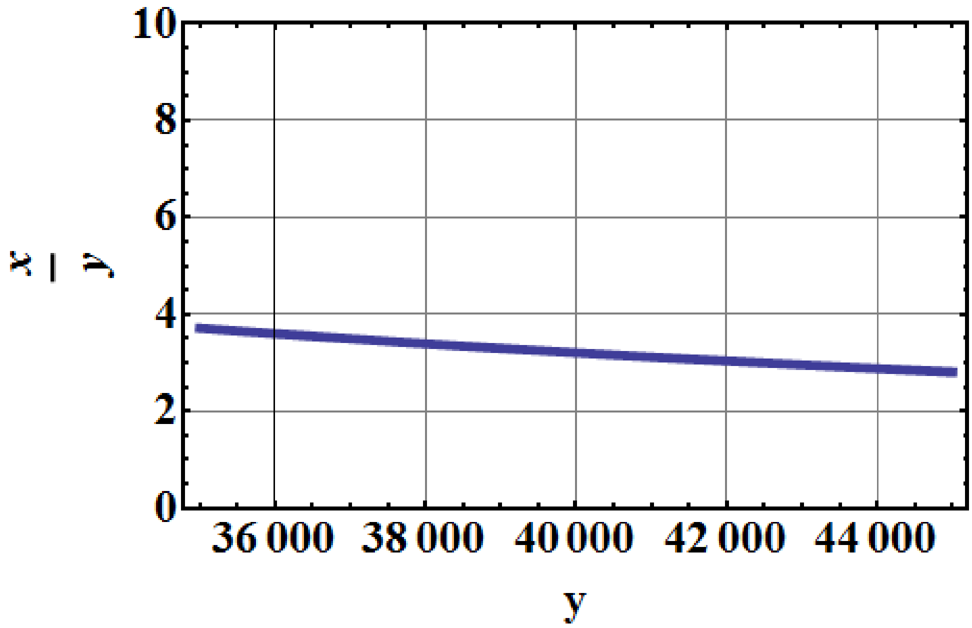

Figure 3.18: Ratio of $\frac{x}{y}$

considering contributions from a_2 is reasonable. Taking $M_{\gamma\gamma} = m_\pi$, and loop function $F(\frac{2z}{m_{pi}^2}) \sim -0.096$, and rude calculation shows that ratio between $\frac{2G_8\alpha}{\pi F}(\frac{M_K^2}{F^2}a_2)$ and form factor of the first term in $A_7^{\mu\nu}$ is about -0.66. It reveals that the contribution of vector meson exchange at $\mathcal{O}(p^6)$ cancels the biggest contribution at $\mathcal{O}(p^4)$ about 60%, and it means that decay $K_S \to \pi^0\pi^0\gamma\gamma$ is strongly suppressed by weak vector mesons exchange at $\mathcal{O}(p^6)$.

3.3 Conclusions

We have calculated the next to leading order contribution from vector meson exchange at $\mathcal{O}(p^6)$ to decay $K_S \to \pi^0\pi^0\gamma\gamma$ in chiral perturbation theory, and compared with leading order contribution from one loop digrams at $\mathcal{O}(p^4)$. As for vector meson exchange in strong chiral sector, it is clear that the effective chiral Lagrangian (3.4) and $\Delta S = 1$ weak chiral Lagrangian give a nice framework to describe all possible contributions. However, there is another type of vector meson exchange with weak interaction, and facts tends to a complicate condition since we do not know the full structure of weak vertices of vector meson with pseudoscalar particles and photon. Using a general way to estimate the contributions of weak vector meson exchange, we deduce the kinematic expression of coefficient a_3 in the relevant effective chiral Lagrangian at $\mathcal{O}(p^6)$ because of requirement of symmetries. Using *Dalitz plot* analysis, we plot the variation of a_3 in the possible kinematic range under different cuts of invariant mass of photons. These plots show that $\frac{M_K^2}{F^2}a_3 \in (-0.03, -0.02)$ and supports the conclusion that $\frac{M_K^2}{F^2}a_3$ prefers to be close to the boundaries -0.9 or 0.1 by diphoton invariant mass analysis from NA48. With this result and the constraint (3.8) from NA48, we get another coefficient, $\frac{M_K^2}{F^2}a_2$, close to 0.85. After fixing those relevant coefficients, we have found that the strong vector meson exchange give very tiny contributions, while weak vector meson exchange dominate at $\mathcal{O}(p^6)$, and the weak vector meson exchange suppress the biggest contribution at $\mathcal{O}(p^4)$ about 60%. This might be reason why we do not detect decay $K_S \to \pi^0\pi^0\gamma\gamma$ yet. The last result we achieved is that the kinematic parameter x and y are connected and $K_S \to \pi^0\pi^0\gamma\gamma$ can only happen at $x = y$ or $x \sim 3y$ region.

Chapter 4

Conclusions

ChPT is a powerful tool to study low energy pseudoscalar mesons physics. These pseudoscalar mesons with light mass are treated as Goldstone boson which is the product of spontaneous chiral symmetry breaking. The effective Lagrangian contains an infinite number of terms which are organized by power counting of momentum and mass. Each term in effective Lagrangian is derivative expansion of Goldstone field and is consistent with chiral symmetry, parity and charge conjugate. However, the coupling coefficient(also called chiral coupling) of each term is not constrained by chiral symmetry, and we can't derive these coefficients from the Lagrangian of standard model, because we don't understand the QCD confinement mechanism yet. Once we fixed these chiral couplings from experiments, we can make predictions to specific decay modes. By comparing with experimental data, we can improve our understanding of underlying symmetries.

In this thesis, we gave a short introduction to ChPT in chapter 1, and we showed its application to radiative nonleptonic Kaon decays in chapter 2. And then, in chapter 3, we studied the specific decay $K_S \to \pi^0\pi^0\gamma\gamma$ to $\mathcal{O}(p^6)$ order. At this order, the only known contribution to $K_S \to \pi^0\pi^0\gamma\gamma$ comes from vector meson exchange. Considering contributions of vector meson from strong sector and weak sector, we got one relation (3.26) which express the kinematic dependence of coupling coefficient a_3. Analysis showed that the coupling a_3 in effective Lagrangian (3.5) is close to zero. This result agree with the low diphoton invariant mass analysis [7] of NA48. Furthermore, we fixed another coupling a_2 by using one constrain relation (3.8) deduced from experimental data of NA48 on $K_L \to \pi^0\gamma\gamma$. With all fixed coupling, we made a clear prediction that vector meson exchange in weak sector is dominant for $K_S \to \pi^0\pi^0\gamma\gamma$ at $\mathcal{O}(p^6)$, and we found that the big cancellation between contributions at $\mathcal{O}(p^4)$ and weak vector meson exchange contribution at $\mathcal{O}(p^6)$, this result seems to explain why we still do not observe decay $K_S \to \pi^0\pi^0\gamma\gamma$ in current experiments.

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