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Coupon Bonds and Liquidation Triggers: A Real Option Approach

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to my parents and my sister and …

…to SRSLC

CONTENTS

INTRODUCTION

Numerous empiric studies show that the interval between the beginning of financial distress and liquidation in the United States is included between 1 and 3 years (this gap is different around the world but it exists everywhere). However, many pricing models of corporate securities have the assumption that financial distress or default lead to immediate liquidation of firm assets.

These models assume that the liquidation or reorganization occurs when a defined bankruptcy trigger is met. This assumption is supported by common covenants, which give the debt-holders the right to ask for liquidation/reorganization if the firm asset's value falls below a pre-defined threshold.

I implement a structural model in which liquidation starts only if a time variable exceed a pre-defined grace period, in order to capture the effects of the time gap between the beginning of the financial distress and the moment of liquidation on cumulative default probabilities, recovery rate, yield spreads and on the corporate securities value.

In practice, I build a real option model in which the asset value is exogenous, there are a threshold barrier, a grace period and the firm's capital structure is complex. Indeed, there are two types of debts: a junior debt and a senior debt. Both bonds could be Zero Coupon Bond or Coupon Bond with maturity equal to 10 years.

The mechanism of the model take into account (through the presence of a pre-defined grace period) the characteristic of the US bankruptcy Code and reproduce the Chapter 7 case (I hypothesize that my representative B-rated firm can not continue its business when the asset value stay for 1 year below the threshold barrier).

The contribution of this approach is that I consider the presence of coupons and liquidation cost and at the same time the main common funding forms: equity, senior debt and junior debt. Other approach/papers, that consider the existence of the grace period, do not consider liquidation cost and hypothesize the presence of a very simple capital structure with equity and one Zero Coupon Bond. In addition, I use parameters' value equal to those found in literature and, consequently, I do not apply any type of calibration approach.

Therefore, I can compare cumulative default probabilities, recovery rate and yield spread computed through my approach with those empirically found, in order to understand the reliability of my model. At the same time, solving the model numerically, I can make a comparative statics and, consequently, I can evaluate how the parameters of the model affect the probability of default, the recovery rate, the yield spread and the value of the assets.

In order to evaluate the reliability of my framework, I use as benchmark the average cumulative default frequencies for B-rated debt as function of horizon, as given by Moody's (2011), the recovery rate found by Altman & Kishore (1996), the average yield spread over treasury bond of similar maturity for B-rated bonds, as reported by Huang & Huang (2003).

In Chapter 2, where I present a first version of my model in which I introduce liquidation cost, two debts and the grace period, the results that I get are mixed. Indeed, my cumulative default probabilities are lower than empirical data, my recovery rate is close to Huang & Huang (2003) data, while the recovery rate is almost coincident with the value found empirically. At the same time, I show that old models (hit and default) fail. In fact, using these models, I get a good approximation of cumulative default probabilities, while the recovery rate is very far from empirical value.

When I introduce coupons in Chapter 3 (and in Appendix A), I obtain results very satisfying, since the evolution of my cumulative default probabilities is compliant with empirical default probabilities and my recovery rate and my yield spread are almost coincident with those found in literature. I do not achieve the same outcomes if I adapt old models to my framework.

Indeed, if I include coupons, liquidation cost and two type of bond in old structure, the results are completely different from those empirically computed.

In summary: I am able to capture some stylized facts (such as cumulative Dps, recovery rate and yield spreads); I do that using real ingredients, since I use parameters' value (volatility, risk free rate, dividend yield, grace period, etc.) found in a couple of empirical research; and I could assert that the presence of grace period and the introduction of coupons are crucial ingredients for matching empirical data.

US Bankruptcy Code

The procedural aspects of the bankruptcy process are governed by the Federal Rules of Bankruptcy Procedure and local rules of each bankruptcy court. The Bankruptcy Code and Bankruptcy Rules (and local rules) set forth the formal legal procedures for dealing with the debt problems of individuals and businesses.

Indeed, six basic types of bankruptcy cases are provided for under the Bankruptcy Code, but only two of these filings are available to corporations: Chapter 7 and Chapter 11.

Chapter 7

In Chapter 7, a Court-appointed trustee liquidates the firm's assets and uses the proceeds to pay the holders of claims (creditors) in accordance with the provisions of the Bankruptcy Code. Company stops all operations and goes completely out of business. Indeed, some firms are so far in debt or have other problems so serious that can not continue their business operations. Administrative and legal expenses are paid first, and the remainder goes to creditors.

Secured creditors will have their collateral returned to them. If the value of the collateral is not sufficient to repay them in full, they will be grouped with other unsecured creditors for the rest of their claim.

Bondholders and other unsecured creditors will be notified of the Chapter 7, and should file a claim in case there is money left for them to receive a payment.

Stockholders do not have to be notified of the Chapter 7 case because they generally do not receive anything in return for their investment. In the unlikely event that creditors are paid in full, stockholders will be notified and given an opportunity to file claims.

Chapter 11

Chapter 11 is used by corporations that desire to continue operating business and repay creditors trough a court-approved plan of reorganization. In this case: 1) the creditor may seek an adjustment of debts, either by reducing the debt or by extending the time for repayment, or may seek a more comprehensive reorganization; 2) Management continues to run the day-to-day business operations but all significant business decisions must be approved by a bankruptcy court.

In practice, the steps of this procedure are the following:

1) Company prepares a plan of reorganization and the U.S. Trustee, the bankruptcy arm of the Justice Department will appoint one or more committees to represent the different stakeholders in working with the company to develop this plan;
2) The plan must be accepted by creditors, bondholders and stockholders;
3) Company prepares a disclosure statement and reorganization plan and files it with the court;
4) SEC (Security and Exchange Commission) reviews the disclosure statement to be sure it is complete;

5) the bankruptcy court confirms the plan confirmation.

Summary

When a firm is in bankruptcy there are four primary groups that could influence the future of this firm: Managers, Shareholders, unsecured Creditors[1] and secured Creditors.

Usually, Managers, Shareholders and unsecured Creditors prefer reorganization over liquidation, while secured Creditors could be harmed by reorganization.

Whit the 1978 Act, creditors have the possibility to propose their own plan of reorganization after managers have had 180 days to suggest their plan. The problem is that each class of creditors must be in favour of this plan. In addition, shareholders could vote for this approval if their claims are impaired.

Conclusion

It is demonstrated that old models (or hit and default models), very popular in 1980s and 1990s, are not able to match the empirical data, even if they are naïf models. Indeed, in a couple of research it was used a calibration approach in order to get results similar to empirical outcomes.

In addition, it is verified that in reality default and liquidation are distinct events and the default threshold isn't an absorbing barrier.

Therefore, I build a more complex and realistic model with less degrees of freedom, in which I take into account a grace period in order to study the economic implications of this element.

Using this approach and adding coupons bonds instead of ZCB, I am able to fit empirical data (cumulative default probabilities, recovery rate and yield spreads) without any type of

[1] Employees with past wages due and customers with deposits have a priority over other unsecured creditors.

calibration. In fact, I capture these stylized facts with real ingredients, since I choose the parameters' value found in a couple of empirical research.

I think that is possible to extend this model considering a liquidation state variable that is dependent of the severity of distress or different dynamics of risk–free rate or introducing other parameters (such as taxes).

CHAPTER 1

1. Literature review

In much of the continuous-time debt pricing literature, it has typically been assumed that default is tantamount to liquidation.

In reality, most of the companies which default go into a period of reorganization and may or may not be liquidated.

Indeed, using US data, Gilson, John, & Lang (1990) investigate the incentives of financially distressed firms to restructure their debt privately rather than through formal bankruptcy. In their sample of financially distressed companies, about half successfully restructure their debt outside of Chapter 11. They find that only about 5% of the bankruptcies in Chapter 11 are converted into Chapter 7 liquidations.

Using data on distressed UK companies, Franks & Sussman (2005) also find that: the typical debt structure is close to a corner solution, with the liquidation rights almost entirely concentrated in the hands of the main bank; while the banks' typical response to distress is an attempt to rescue the firm (rather than liquidate it automatically), they are very tough in their bargaining with the distressed firm; concentrating the liquidation rights helps to resolve co-ordination failures.

In addition, numerous empiric studies show that the interval between the beginning of financial distress and liquidation in the United States is included between 1 and 3 years (this gap is different around the world but it exists everywhere).

In fact, Covitz, Han & Wilson (2006) find that time in default is significantly related to whether the bankruptcy is triggered by litigation, the industry and macroeconomic conditions at the time of default, and the change in these conditions over the duration of the default. They, in addition, show that there has been a significant decline in the length of time spent in default for US public companies, for approximately 36 months in late 1980s to 12 months between 1993 and 2002;

Thorburn (2000) provides some first, large-sample evidence on the Swedish auction bankruptcy system. Compared to U.S. Chapter 11 cases, the small-firm bankruptcy auctions examined here are substantially quicker (2.5 months), have lower costs, and avoid deviations from absolute priority. Three-quarters of the firms are auctioned as going concerns, which is similar to Chapter 11 survival rates. Moreover, based on market values, creditors in going-concern auctions recover a similar fraction of face value as creditors of much larger firms in Chapter 11 reorganizations. The evidence presented suggests that the auction bankruptcy system is a surprisingly efficient restructuring mechanism for small firms.

Recently, some debt pricing models have therefore attempted to separate the notions of default and liquidation and/or take into account this time gap between the beginning of the financial distress and the moment of liquidation.

The rest of the chapter is structured as follow. The next paragraph describes real option literature. Paragraph 1.2 shows the evolution of the theoretical pricing (structural) models of corporate securities from Black & Scholes (1973) and Merton (1974) to Bruche (2011), that are based on different type of liquidation hypothesis. Finally I present some empirical studies used to test the reliability of these structural models.

1.1 Real Option Literature

In the last few decades it has been recorded in the financial markets a significant growth in both volume and complexity of the contracts that are traded in the over-the-counter market.

Black & Scholes (1973) derived a theoretical valuation formula for options based on the principle that “it should not be possible to make sure profits by creating portfolios of long and short positions in options and their underlying stocks" if options are correctly priced in the market.

Black & Scholes model is still widely used for options pricing, even though a couple of empirical papers have shown that the model does not explain the underlying asset price process.

A large number of models have been proposed to address the empirical weakness of the classic Black-Scholes approach. These extended models have been developed though three dimensions: univariate diffusion models, Stochastic volatility models and Jump models.

The *univariate diffusion* models are models in which it is relaxed the assumption of Geometric Brownian motion. In this category are included: the constant elasticity of variance models of Cox & Ross (1976) and Cox & Rubinstein (1985); the leverage models of Gesken (1979) & Rubinstein (1983); and the implied binomial and trinomial trees models such as Derman & Kani (1994) and Dupire (1994). At the beginning univariate diffusion were used in order to capture time-varying volatility and the leverage effect in a simple fashion. Then, these models were built to match observed cross-sectional option pricing patterns at any instant, in order to price over the counter exotic options.

Stochastic volatility models in which the instantaneous volatility of assets returns evolve stochastically over time as a diffusion process (Hull & White, 1987), as a regime switching process (in Naik, 1993) or as jump diffusion process (in Duffie, Pan & Singleton, 2000). The advantage of adopting stochastic volatility models is that these models are consistent with the stochastic and mean-reverting evolution of implicit standard deviations.

Jump models relax the diffusion assumption for asset prices such as Merton (1976) and Bates (1991). These models were used to match volatility smiles[2] and smirks[3].

[2] A plot of implied volatility vs. strike price will form an U-shaped curve similar to the shape of a smile.
[3] Implied volatility for options at the lower strikes are higher than the implied volatility at higher strikes.

Other models have combined features of these three approaches. Stochastic generalizations of binomial tree models are surveyed by Skiadopoulos (2000) while the affine class of distributional models creates a structure that nests particular specifications of the stochastic volatility and jump approaches.

All these models are more realistic than the Black and Scholes model. However, it is not sure if they improve pricing correctness and hedging performance. Indeed, each model relaxes several hypothesis of the Black and Scholes model, but the extra risk introduced raises the issue of how to price these risk.

In the 1980s it was very complicated to test the different approaches, because option data were not available until the introduction of option trading on centralized exchanges.

The introduction of some developments such as the Monte Carlo approach of Scott (1987), the higher dimensional finite difference approach of Wiggins (1987) and, principally, the Fourier inversion approaches of Stein & Stein (1991) and Heston, (1993), have facilitated the comparison between alternative option pricing models.

There are, therefore, a couple of empirical papers that test the performance of option pricing model, including Bakshi, Cao & Chen (1997 and 2000), Bates (1996 and 2000) and Dumas, Fleming & Whaley (1998), among others.

Empirical test are realized in this way: parameters of the model are estimated so that the model prices for some european options match the price that are observed in the market at a specific time. The resulting model is used to price other options later and then these prices are compared with those observed from the market.

However, Bakshi, Cao & Chen (1997) show that instantaneous option price evolution is not fully captured by underlying asset price movements, precluding the riskless hedging predicted by univariate diffusion models.

Bates (2000) points out that the standard assumption of independent and identically distributed returns in jump model implies these models converge towards BS model option prices at longer maturities, in contrast to the still pronounced volatility smiles and smirks (or reverse skew) at those maturities.

Bakshi, Cao & Chen (2000) find that adding jumps or stochastic interest rate does not improve the unconstrained stochastic volatility model's assessments of how to hedge option price movements. At the same, it is demonstrated that unconstrained stochastic volatility models price options better after jumps are added.

1.2 Structural Models literature

The evolution of the structural models from Black & Scholes (1973) and Merton (1974) to Bruche (2011) is presented here.

1.2.1 Default at maturity

In order to determine the value of corporate securities is essential the modelling of default. The classic approach is based on the idea that default is possible in the event that the total value of the firm is less than total amount of the debt.

For the first time, in fact, Black & Scholes (1973) pointed out that equity value is similar to the price of European call options, with a strike price equal to the total value of the debt at maturity.

They build a model in which there is a company that had common stock and bonds outstanding. In addition, they suppose that: bonds are zero coupon bonds, giving the holder the right to a fixed sum of money, if the firm can pay it, with a maturity of 10 years; bonds contain no restrictions on the company except a limitation that the company can not pay any dividends; the company plans to sell all assets it holds at the end of 10 years, pay off the bond holders if possible, and pay any remaining money to the stockholders.

In according with this model: the bondholders own the company's asset, while the stockholders have an option for buying the asset back; and the total value of equity at the maturity is equal to the maximum between zero and the difference between value of the company's asset minus the face value of the bonds.

At the same time, Merton (1974) developed a theory for pricing bonds when there is a significant probability of default, using the same hypothesis: "On the maturity date T, the firm must either pay the promised payment of B (total debt) to the debtholders or else the current equity will be valueless".

In practice, the value of corporate debt depends on the required rate of return on riskless, the various provisions and restrictions contained in the agreement and the probability that the firm will be unable to satisfy some or all of the indenture requirements.

Finally, Galai & Masulis (1976) combine the option pricing model with the capital asset pricing model in order to find a more complete model of security pricing. They show that this combination of models leads to a number of insights regarding stock risk and changes in corporate asset structure and capital structure.

In these models there is a common assumption that defaults and financial distress bring to immediate liquidation of firm assets.

1.2.2 Market Value and distress threshold

Black & Cox (1976), for the first time, introduced the effects of safety covenants on the value and behaviour of the firm securities. In particular, they considered the possibility of bankruptcy at any time or rather the hypothesis for which if the firm's value drops to a specified level, that is variable over time, then the bondholders could force the firm into bankruptcy and, consequently, get the ownership of the assets.

The authors also took into account the subordination of the claims of one class of bondholders (junior debt) to those of a second class (senior bond) and treated these two different type of assets as two options with different exercise prices.

Brennan & Schwartz (1978), on the other hand, studied the effects of corporate income taxes on the relationship between capital structure and valuation. They showed that the issue of additional debt have two effects on the value of the firm and, in particular, it increases the tax savings to be enjoyed so long as the firm survives and reduces the probability of the firm's survival for any given period.

In order to solve the model, they assumed (boundary condition) that the firm files for bankruptcy at any time if the value of its assets is less than the par value of outstanding bonds.

In addition, Mello & Parsons (1992) show how to adapt a contingent claims model to reflect the incentive effects of the capital structure and to measure the agency costs of debt. They suppose that the value of the firm is an endogenous function of an underlying stochastic variable describing the firm's product market and of the management's choice of operating and investment decisions.

Also in this model there is the hypothesis that default leads to an immediate liquidation of ths firm's assets.

In order to compute the Corporate Debt Value and define Optimal Capital Structure, Leland (1994) considered two different approaches the case in which bankruptcy is determined endogenously; or the case in which debt remains outstanding without time limit unless bankruptcy is triggered by the value of the firm's assets falling beneath the principal value of debt. This latter situation could be representative of a situation in which there is a long term debt with a protective covenant stipulating that the asset value of the firm is bigger than the principal value of the debt; or a continuously renewable line of credit with fixed interest rate and fixed amount of borrowing and at each instant the debt will be rolled over if the value of the assets is sufficient to repay the loan's principal (if not there is bankruptcy).

In this framework, however, Leland assume that the face value of debt remains static through time and that if bankruptcy occurs, a fraction $0 < a < 1$ of value will be lost to bankruptcy costs, leaving debtholders with value $(1 - a)*V$ and stockholders with nothing.

Longstaff & Schwartz (1995), moreover, for valuing risky corporate debt, developed a model that incorporates both default and interest rate risk, allows for deviations from strict absolute priority rules if the firm defaults and, as in Black and Cox (1976), includes a threshold value K for the firm at which financial distress occurs. If the asset value is greater than K, the firm continues to be able to meet its contractual obligations. If the asset value reaches K there is a financial distress and some form of corporate restructuring takes place.

Ericsson & Reneby, (2001) showed that many corporate securities can be viewed as portfolio of three basic claims (with simple valuation formulae): a down-and-out call option, a down-and-out binary option and a unit down-and-in claim. They assumed that default occurs at any time prior to maturity if the value of the assets falls below a constant or at debt maturity if the value of the assets is smaller than the total amount of debt.

The second case is redundant when, as in Black and Cox and in Leland, the models are characterized by a stationary (perpetual) capital structure.

Then, Leland & Toft (1996) develop a model of optimal leverage and risky corporate bond prices for arbitrary debt maturity. Bankruptcy is determined endogenously and will depend on the maturity of debt as well as its amount. Both value and flow conditions that characterize the bankruptcy point are presented. They show that bankruptcy can occur at asset values that may be either lower or higher than the principal value of debt. And a cash flow shortfall relative to required debt service payments need not result in default-it may be optimal for equity holders to raise further funds to avoid bankruptcy.

In order to estimate the tax advantage to debt and to determine optimal capital structure policy, Goldstein, Ju & Leland (2001) develop a theoretical model that provide a state variable which is invariant under capital structure change and account for cash payouts, since payout affects the probability of future bankruptcy.

They suppose, anyway, that the management chooses the bankruptcy level in the best interest of the equityholders and that default brings to an immediate liquidation of the asset.

Finally, Morellec, (2001) investigates the impact of asset liquidity on the values of corporate securities and the firm's financing decisions. The model endogenously determines both the default threshold and the sales curve that maximize equity value. Because of the limited liability principle, shareholders have the option to default on their obligations. The optimal exercise policy for this option is to default when the firm has negative economic net worth.

1.2.3 Not Immediate Liquidation

Recent theoretical works on capital structure and securities valuation suggest that liquidation happens only if the asset's value are smaller than a particular value (distress threshold) for an interval exceeding a pre-defined grace period.

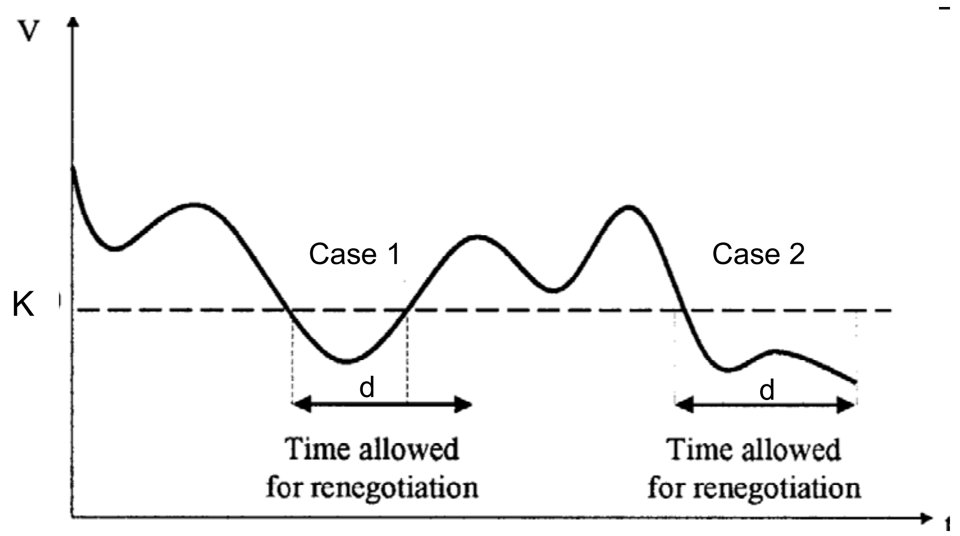
For example Mella-Barral (1999) assumes that liquidity problems do not have any influence on the default point as the debtors can keep on issuing equity to avoid a default. Subsequently in his model, default is endogenous and is the point where it is optimal for the debtors to irreversibly exchange their current claim for a residual claim which they will get on bankruptcy.

Hege & Merra-Barral (2000) extend Mella-Barral (1999) by taking into account multiple creditors.

Then, Fan & Sundaresan (2000) suggest that when the firm is in default, borrowers stop making the contractual coupon and start servicing the debt strategically until the firm's asset value goes back above the distress threshold. They show that Bankruptcies are often resolved using exchange offers of different types, such as delayed or missed interest or principal payments, extension of maturity, debt-equity swap, debt holidays, etc. Essentially all these distressed exchanges and delayed payments can be considered for them as a value redistribution between equity and debt holders. This assumption is based on Moody's Investors Service (1998), where are analyzed all defaults during 1982-1997. This research report that about half of long-term public bond defaults resulted in bankruptcy. Of the defaults, 43% were accounted for by missed payments and 7% by distressed exchanges.

François & Morellec (2004), on the other hand, suppose that default can lead either to liquidation of the firm's assets or to renegotiation of the debt contract. In bankruptcy, the firm incurs costs of financial distress, and the cash flows it generates are shared among claimholders. Moreover, for firms that choose to renegotiate their claims under the court's protection, the default date is the starting point of a period during which the parties involved in the process (the claimholders and the court) observe the evolution of the value of the firm's assets. The firm emerges from financial distress if the value of its assets shows signs of recovery during the observation period. Otherwise, liquidation is pronounced at the end of the period. Shareholders hold a Parisian down-and-out call option on the firm's assets. That is, shareholders have a residual claim on the cash flows generated by the firm's assets unless the value of these assets reaches the default threshold and remains below that threshold for the exclusivity period. In practice, they hypothesize that there isn't any liquidation (fig 1, Case 1), if the value of the firm's assets (V) dips below the threshold (K) and then bounces above this value before the conclusion of the grace period (d),. The liquidation occurs only if the value of the firm's asset is below the threshold K for a period bigger than d (fig.1, Case 2).

Figure 1 – ***The liquidation criterion***



Moraux (2004) consider the model developed by François and Morellec and show that under this type of liquidation procedure, debtholders may never receive coupons while the firm is never liquidated. Indeed, every time the asset's value goes above the default threshold, the "distress clock" is reset to zero. In order to eliminate this problem, He consider the cumulative excursion time below the default threshold, or better he assume that liquidation is declared when the total time (cumulated) under the threshold is larger than d.

Finally, D.Galai, Raviv, & Wiener (2007) developed a model in which liquidation is driven by a state variable that accumulates with time and the severity of distress. In particular, in this model recent and severe distress events have a larger impact on the decision to liquidate a firm's asset, while old distress events have a slight effect on the liquidation decision because the nature of the firm could have been changed in this period. Instead, mild financial distress does not lead to immediate liquidation.

1.2.4 Renegotiations

Debtholders, in practice, don't force to liquidate the firm's assets immediately when financial distress arrives.

Bebchuk & Chang (1992) and Bebchuk (2002) show that when an insolvent company files for reorganization, an "automatic stay" prevents debtholders from seizing assets until a reorganization plan is adopted.

Indeed, for political and social considerations, bankruptcy laws favour firm continuation and, in particular, it is possible to have *an out-of-court renegotiations* or *a legal bankruptcy protection*, like Chapter 11 of the U.S. Bankruptcy Code or Concordato Preventivo of the Italian legge fallimentare, that authorize to renegotiate outstanding debt.

In this latter case, the supervising court would convert the bankruptcy proceedings to a Chapter 7 (liquidation of the firm's asset), if there is no agreement on a reorganization plan between Debtholders and Equityholders. Before this conversion, it is impossible for debtholders to receive any value from the company unless they agree with the equityholders on the division of the firm's assets.

Out-of-court renegotiations

For the first time, Anderson & Sundaresan (1996) study the design and valuation of debt contracts in a general dynamic setting under uncertainty. Their framework is an extensive form game determined by the terms of a debt contract and applicable bankruptcy laws. Debtholders and equityholders behave non-cooperatively and the firm's reorganization boundary is determined endogenously.

Mella-Barral & Perraudin (1997), on the other hand, consider endogenous bankruptcy and they model the strategic behaviour of debtors. In their models, the debtors act strategically and always try to pay as low a coupon as possible. In good times when the liquidation value of the firm is high, the debtors will not pay lower than the contracted amount as they would realise that it would then be in the creditors' interest to reject their offer and liquidate the firm. However, the debtors might underperform the debt contract even if the firm is not experiencing any liquidity problems. They will do this when the liquidation value of the firm is not sufficiently high and thus when subsequently it would be not in the creditors' interest to reject the offer. Thus in their models, the debtors might default continuously and they will continue to do so until the creditors finally reject the offer. At this point the firm will be liquidated.

In Anderson and Sundaresan's model and Mella-Barral and Perraudin's model, endogenous bankruptcy point will in general be different from the default point.

Finally, Christensen, Flor, Lando & Miltersen, (2000) consider a dynamic model of the capital structure of a firm with callable debt that takes into account that equity holders and debt holders have a common interest in restructuring the firm's capital structure in order to avoid bankruptcy costs. Far away from the bankruptcy threat the equity holders use the call feature of the debt to replace the existing debt in order to increase the tax advantage to debt. When the bankruptcy threat is imminent, the equity holders propose a restructuring of the existing debt in order to avoid bankruptcy. This proposal makes both debt holders and equity holders better off and re-optimize the firm's capital structure. Both the lower and upper restructuring boundaries are derived endogenously by the equity holders' incentive compatibility constraints.

Renegotiation under Chapter 11

Franks & Torous (1989) and Longstaff (1990) develop contingent claims models that analyze the impact of Chapter 11 on debt values.

In particular, Franks & Torous (1989) describe the rights of the debtor-in-possession in Chapter 11. Chapter 11 provides the debtor-in-possession with a valuable option, and they have shown how that option may be priced into risky debt. Using simulation, the authors have compared the risk adjusted rates of interest with the option to enter Chapter 11 with the risk adjusted rates without that option.

Longstaff (1990) derive a closed form expression for the price of calls and puts that are extendible by either the option holder or the option writer and show that many types of corporate reorganizations, such as Chapter 11 bankruptcy, can be viewed as the exercise of an implicit extension privilege.

In these two papers, the authors model Chapter 11 as the right to extend (once) the maturity date of the debt. The longer this extension privilege, the more valuable it is to shareholders and hence the larger the credit spread on corporate debt.

1.2.5 Recent Evolution

Broadie, Chernov & Sundaresan (2007) build a model in which the firm may choose to default (Chapter 11) prior to completely destroying the equity value. This decision may still lead to liquidation (Chapter 7), or it may result in recovery from default. In order to have this framework they introduce two endogenous threshold along with a grace period. The first barrier lead to the Chapter 11 filing, and the second barrier determines the liquidation's decision. The company, however, is allowed to stay in Chapter 11 for no more than the duration of a grace period d. If the company spends more time than d in default, or if the value of unlevered assets reaches the second barrier, then the firm is liquidated and there are proportional costs of Liquidation.

Carr & Wu (2008) assume that there is a default corridor. They suppose that the stock price stays above a barrier $B > 0$ before default, but drops below a lower barrier $A < B$ at default and stays below A thereafter. They have, implicitly, the same dynamics for the asset value (V). In practice, in this model Carr and Wu hypothesize that when the default occurs the asset value decreases, due to the liquidation cost.

Naqvi, (2008) develops a continuous time asset pricing model of debt restructuring and values equity and debt by taking into account the fact that in practice the default point differs from the liquidation point. This separation allows him to delegate the liquidation decision to the creditors whilst default is triggered by the managers. The study identifies an agency cost of debt whereby the creditors liquidate the firm prematurely relative to the first best threshold. In this model default occurs because of liquidity problems and the critical default point is determined exogenously. Naqvi assume that debt service is met out of cash flows and that the firm cannot issue additional equity or debt to avoid a default. This is not a very stringent assumption as it might first appear. In practice, debt covenants frequently restrict the issue of additional debt with senior or equal status. Similarly, loan indentures quite often forbid the liquidation of firm's assets by owners as this could potentially undermine collateral values.

Bruche & Naqvi (2010) develop a model where equityholders decide when to default while bondholders choose when to liquidate. In practice, creditors do not liquidate the firm immediately upon default and could accept reduced coupon payments. Only in case of deterioration of firm's fundamental, they decide to liquidate. In this framework, the time between the default and the liquidation is not related to a particular grace period but is completely random.

Bruche (2011) presents a continuos-time structural model where there is a liquidation of a defaulting firms only if debtholders, coordinated or uncoordinated, attempt to enforce claim against these firms. In the model, coordinated creditors can have incentives to liquidate prematurely, in the sense that firm value would be higher if the firm was liquidated later. Uncoordinated creditors care about payoffs in an asset grab game. If legal costs of grabbing assets are low, they can have incentives to grab assets too early. Features of Chapter-7 type bankruptcy codes that affect creditor coordination change the payoffs in the asset grab game such that grabbing assets becomes less attractive, protecting debtors. This leads to later liquidation. The level of debt has, anyway, an effect on when the firm is liquidated, both in the case in which creditors are coordinated as well as in the case where creditors are uncoordinated.

1.2.6 Comparison with empirical data

Jones, Mason & Rosenfeld (1984) show that the credit yield spreads predicted by Merton (1974) are far below the empirically observed corporate Treasury yield spreads, but a couple of authors point out that extensions of the Merton Model within the structural framework that incorporate some realistic economic consideration can explain the observed yield spreads.

Anderson, Sundaresan & Tychon (1996) believe that incorporating strategic default by equity holders who try to extract concessions from bond holders can explain why corporate-Treasury credit spreads should be high.

Collin-Dufresne & Goldstein (2001) propose a structural model of default with stochastic interest rate. This model is able to show that firms with good credit quality are likely to issue more debt, which leads to credit that are comparable to the observed high yield spreads for long-maturity bonds issued by such firm.

Duffie & Lando (2001) study the implications of imperfect information for term structures of credit spreads on corporate bonds (short maturity bonds). With imperfect information about the firm’s value credit spreads remain bounded away from zero as maturity goes to zero and are higher than those generated with perfect information.

Zhou, (2001) develop a model that incorporates jump risk into the default process. Whit this jump risk, a firm could default immediately because of a rapid drop in its value. He show that his model explain a number of empirical regularities regarding default probabilities and recovery rates and it is able to match the size of credit spreads on corporate bonds.

Cooper & Davydenko (2004) propose a method of extracting expected returns on debt and equity from corporate bond spreads. In practice, They propose to predict expected default losses on any corporate bond based on its yield spreads, given information on leverage, equity volatility and equity risk premia. In line with historical default rates, Cooper and Davydenko find that only a small fraction of the spread for highgrade debt is due to expected default loss. For lower-grade debt, this component is larger, and their approach provides a method for adjusting yields to give expected debt returns. They find that the expected default component of the spread varies significantly within ratings categories, so using average figures for ratings categories for individual companies may be misleading.

Valuation of the capacity of different structural model to forecast real data

Many papers evaluates the capacity of structural models to forecast default rates, yield spread or cumulative default probabilities.

Huang and Huang (2003) show that several structural models make quite similar predictions on yield spreads if each of the models is calibrated to match historical default loss experience data. They conclude that additional factors (illiquidity and taxes) must be important in explaining the difference between the empirically-observed yield spreads and the predicted spreads. In addition, Huang and Huang point out that for investment grade bonds of all maturities, credit risk accounts for only a small fraction of the observed corporate Treasury yield spreads, while for junk bonds credit risk accounts for a much larger fraction of the observed corporate Treasury yield spreads.

Then, Eom, Helwege, & Huang (2004) test five structural models of corporate bond pricing. They find that all models have substantial spread prediction errors, but their errors differ sharply in both sign and magnitude. In particular, the average error is a rather poor summary of a model's predictive power, as the dispersion of predicted spreads is quite large. All models tend to generate extremely low spreads on bonds that the models consider safe and to generate very high spreads on the bonds considered to be very risky.

Leland (2004), instead, compare structural models' abilities to predict observed default rates on corporate bonds (as reported by Moody's, 2001). In particular, Leland focus on two sets of structural model that have been widely used in academic and practical applications: those with "exogenous default boundary" that reflects only the principal value of debt, and those with an "endogenous default boundary" where default is chosen by management to maximize equity value. He finds that both the endogenous and exogenous have under-predicted default probabilities at shorter time horizons and fits reasonably well the default probabilities, especially for longer time horizons.

The author obtains these results through the calibration of the models and, although this exercise provides acceptable levels of volatility, the predicted default probabilities are very sensitive to this variable.

In addition, Teixeria (2007) tests empirically the performance of three structural models of corporate bond pricing (Merton, Leland and Fan and Sundaresan). He find that the first two models overestimate bond prices, while Fan and Sundaresan model reveals an extremely good performance. When considering the prediction of credit spreads, the three models underestimate market spreads but, again, Fan and Sundaresan has a better performance.

Tarashev (2008) use firm-level data in order to evaluate the degree to which five structural credit-risk models account for the level and intertemporal evolution of actual default rates. He finds that probabilities of default implied by the models tend to match the level of default rates. In addition, his models explain a substantial portion of the variability of default rates over time.

Finally, Schaefer & Strebulaev (2008) study the ability of structural model to predict the hedge ratios of corporate bonds against the equity of the underlying firm. They show that even the simplest structural model is capable of capturing the extent to which a change in the value of corporate assets affect the value of corporate debt.

CHAPTER 2 – A Real Option Model with two ZCBs

2 Brief introduction

In this chapter, I try to replicate some stylized fact. In particular, I seek to reproduce the trend of cumulative default probabilities, recovery rate and yield spread for B-rated firm, building a real option model.

I start from some classical structural models, but I introduce some elements that are able to make more realistic this framework. Specifically, I take into account the difference between liquidation and default through a grace period, the presence of more complex capital structure (as in reality) and the existence of liquidation cost.

In addition, I use real ingredients since I use parameter's value found in a couple of empirical papers and, consequently, I do not have any type of calibration approach.

In the next chapter I will suppose also that one of the bond issued by the firm is a Coupon Bond, while in this construction I assume that both Bonds are zero coupon bonds as in a couple of classic structural models.

My framework, therefore, has less degree of freedom (versus old models) and it is characterized by an empirical modeling/test of the grace period.

The results, here, are mixed. Indeed, I am not able to fit the evolution of cumulative default probabilities, but I find a recovery rate that is very close to empirical data, while my yield spread is not very far from empirical data.

I show, however, that also using classical structural model the results are not very satisfying, because with these models it is possible to reproduce the trend of cumulative default probabilities, but it is impossible to find value of recovery rate and of yield spreads that are comparable to those empirically found.

It is interesting, finally, to point out that in chapter 3 my results will be really satisfying and, therefore, I will show that the combination of the presence of a grace period and of coupons is crucial for reaching these results.

2.1 Structural Model

I build a real option model to estimate the value of various corporate securities (Senior Debt, Junior Debt and Equity) under a wide array of bankruptcy procedures. This type of model also generates quantitative predictions of default probabilities (or expected default frequencies) for bonds. I will use these cumulative Default Probabilities data in order to verity if my results are in line with the real data.

According to the Black & Cox (1976) model, the default event allows the creditor (Senior Bondholders in this case) to force immediate liquidation through its safety covenants. In this framework, I assume that liquidation is declared when the asset value of the firm falls below distress threshold for a period that goes beyond the pre-determined grace time (denoted by *d*). So the firm goes in bankruptcy if, at any time before the debt maturity, the asset value is less than a threshold level (K) for a period exceeding *d* or if the value of the assets falls below the Debt Face Value (F) at maturity.

Therefore, in my model, as in François and Morellec (2002) and Moraux (2002) models, default and liquidation are distinct events and the default threshold isn't an absorbing barrier.

In this model, as in most of existing structural models, shareholders have a residual claim on the cash flows generated by the firm's assets unless the value of these assets reaches the default threshold and remains below that threshold for the grace period.

The Bondholders receive the debt's face value if the firm is not prematurely liquidated and the asset value at the end of the period (V_T) is greater than the debt's face value. In the event of Liquidation or if V_T is smaller than the debt's face value they get the remaining assets of the firm less the eventual liquidation cost, assigned in accordance with the debt's priority / seniority.

I use standard structural approach assumptions: assets are continuously traded in an arbitrage-free and complete market with riskless borrowing or lending at a constant rate r.

2.1.1 Model assumptions

2.1.1.1 Asset Value

As in most of existing structural models, I assume that the firm asset value evolves according to a diffusion process with a constant volatility, it doesn't depend on the capital structure and it is described by the following equation:

$$dV_t = (r_t - \delta_t)\, V_t\, dt + \sigma_v\, V_t\, dW_t \,, \tag{2.1}$$

where:

W_t denotes a standard Brownian motion;

V_t is the firm asset value;

t represents anytime between 0 and T;

r_t is the riskfree interest rate;

δ_t is the rate at which cash is paid out to the firm's shareholders;

σ_v is the volatility of the firm's asset value process.

In this framework, the process is a risk neutral process and all parameters are assumed constant trough time.

2.1.1.2 Debt

The firm has a total debt F, of which part is senior debt (SD) and the remaining part is junior debt (JD):

$$SD = \gamma * F \qquad (2.2)$$

$$JD = (1- \gamma) * F \qquad (2.3)$$

where γ is a parameter that measure the percentage of senior debt on total debt

The maturity of the Debt is equal to T and the Senior Debt and the Junior Debt are Zero-Coupon Bonds.

2.1.1.3 Liquidation Costs

There are some liquidation costs and these costs are equal to:

$$LC = \rho * V_t \qquad (2.4)$$

where ρ is a parameter that measure liquidation cost

In practice, liquidation costs are a fraction of asset value V_t at the moment of the default.

I assume that the fraction ρ is constant and, therefore, it is independent from the severity of the distress.

In addition, I suppose that liquidation costs are paid if the default occurs before T and also if default occurs at T.

2.1.1.4 The threshold level

In academic and/or in practical applications, as showed above, it has been used models with an "exogenous default boundary" or models with an "endogenous default boundary". Consistent with Longstaff and Schwartz (1995) and Leland (2004), my model is an "exogenous default boundary", due that the default boundary depends only upon the principal value of debt. Consequently, the threshold level K, which is time independent, is equal to:

$$K = \omega F \tag{2.5}$$

where $0 \leq \omega \leq 1$ and this parameter set the level of threshold as fraction of the face value of the total debt.

In practice, default boundary depends only on debt Principal F, and therefore it is not affected by debt maturity T, firm risk σ, payout rate δ, the riskless rate *r* or liquidation cost LC.

Usually, if ω is equal to 1, then the debt is without risk. In my case, this is not true because of the presence of the liquidation cost.

2.1.1.5 Random Variables

In order to determine the value of corporate securities, I define the following random variables:

$$g_t^k = \sup \{s \leq t \mid V_t = K_t\} \tag{2.6}$$

$$\theta^k_t = \inf \{t \geq 0 \mid t - g_t^k \geq d, V_t \leq K_t \} \tag{2.7}$$

where g_t^k is the last time before t that the value of the firm's assets crossed the threshold value K, and θ^k_t is the liquidation time, i.e., the first time the value of the firm's assets spent d units of time consecutively below the default threshold.

When d = 0, default leads to an immediate liquidation and my model is a special case of the standard modelling of default and liquidation.

When $d \rightarrow \infty$, default never leads to liquidation, and my model is a special case of the standard modelling of default and renegotiation (as in Anderson and Sundaresan, 1996 or in Fan and Sundaresan, 2000).

2.2 The valuation of corporate securities

In this model, the firm has a market value of the asset equal to V_t, which is financed by Equity (S_t), and a Total Debt (F), which is composed of senior debt (presumably bank debt) and junior debt (bond or loans by shareholders). I assume that a percentage gamma of the total debt F is senior debt (SD) and a percentage (1-gamma) of the total debt F is junior debt (JD).

The debt contract gives senior bondholders the right to decide on the liquidation of the firm during the period [0,T], only if the asset value of the firm at the time t (Vt) is smaller than K (threshold level) for a period bigger than d (the grace period).

2.2.1 Equity Value

In case of liquidation equityholders, as residual claimants, do not receive anything. If there is not liquidation, at debt maturity T, equityholders receive the maximum between zero and the difference between the firm's asset value (V_T) and the face value of the total debt (F). Indeed, the Equity holders pay-off is represented by this equation:

$$S(V_T, T, g_t^k) = \max(V_T - F, 0) * 1_{\{\theta^k > t\}} = \begin{cases} V_T - F & \text{if } V_T > F \text{ and } \theta^k > T \\ 0 & \text{otherwise} \end{cases} \quad (2.8)$$

Where $1_{\{\theta^k > T\}}$ is an indicator function equal to 1 if there is not liquidation and, equal to 0 with liquidation.

At any time before the debt maturity, if bankruptcy has not occurred, the value of Equity holders claim is given by:

$$S_t(V_t, T, g_t^k) = e^{-r(T-t)} E^Q_t [\max(V_T - F, 0) * 1_{\{\theta^k > t\}}] \quad (2.9)$$

Where E^Q_t [.] represents the conditional expectation under risk neutral measure Q, considered the information present at time t.

2.2.2 Senior Bond Value

The value of the senior debt (SD) is equal to:

1) face value if the firm is not prematurely liquidated, and the asset value of the firm V_T is greater than the face value of SD plus liquidation cost.

2) V_T – LC if the firm is not prematurely liquidated, and V_T is smaller than the face value of SD plus liquidation cost;

3) face value if the firm is liquidated, and V_t is greater than the face value of SD plus liquidation cost;

4) V_t – LC if the firm is liquidated, and V_t is less than the face value of SD plus liquidation cost.

In summary, I define the possible values of SD, to be:

$$SD_t(V_t, T, g_t^K) = \begin{cases} SD & \text{if } V_T \geq SD+LC \text{ and } \theta^k > T \\ V_T - LC & \text{if } V_T < SD+LC \text{ and } \theta^k > T \\ SD & \text{if } V_t \geq SD+LC \text{ and } \theta^k \leq T \\ V_t - LC & \text{if } V_t < SD+LC \text{ and } \theta^k \leq T \end{cases} \quad (2.10)$$

The expression (7) may be rewritten as:

$$SD_t(V_t , T , g_t^K) = E^Q_t [SD * e^{-r(T-t)} * 1_{\{\theta^k > T\}} * 1_{\{VT \geq SD + LC\}}] + E^Q_t [(V_T - LC) * e^{-r(T-t)} * 1_{\{\theta^k > T\}} * 1_{\{VT < SD + LC\}}] + E^Q_t [SD * e^{-r(\theta k - t)} * 1_{\{\theta^k \leq T\}} * 1_{\{Vt \geq SD + LC\}}] + E^Q_t [(V_t - LC) * e^{-r(\theta k - t)} * 1_{\{\theta^k \leq T\}} * 1_{\{Vt < SD + LC\}}] \quad (2.11)$$

where:

1) $1_{\{\theta k \leq T\}}$ is an indicator function equal to 1 if there is liquidation and, equal to 0 without liquidation.

2) $1_{\{VT \geq SD + LC\}}$ is an indicator function equal to 1 if V_T is greater than SD + LC, and equal to 0 otherwise;

3) $1_{\{VT < SD + LC\}}$ is an indicator function equal to 1 if VT is smaller than SD + LC, and equal to 0 otherwise;

4) $1_{\{Vt \geq SD + LC\}}$ is an indicator function equal to 1 if V_t is grater than SD + LC, and equal to 0 otherwise;

5) $1_{\{Vt < SD + LC\}}$ is an indicator function equal to 1 if Vt is smaller than SD + LC, and equal to 0 otherwise;

2.2.3 Junior Bond Value

The Value of the junior debt is equal to:

1) JD if the firm is not prematurely liquidated, and the asset value of the firm V_T is bigger than the face value of Debt (F);

2) $(V_T - SD - LC)$ if the firm is not prematurely liquidated, and the asset value of the firm V_T is bigger than the face value of SD plus the liquidation cost but smaller than F;

3) 0 if the firm is not prematurely liquidated, and V_T is smaller than the face value of SD plus the liquidation cost;

4) (V_t – SD – LC) if the firm is liquidated, and V_t is bigger than the face value of SD plus the liquidation cost;

5) 0 if the firm is liquidated, and V_t is smaller than the face value of SD plus the liquidation cost.

In summary, I define the possible values of JD, to be:

$$JD_t(V_t, T, g_t^K) = \begin{cases} JD & \text{if } V_T \geq F \text{ and } \theta^k > T \\ (V_T - SD - LC) & \text{if } F \geq V_T \geq SD+LC \text{ and } \theta^k > T \\ 0 & \text{if } V_T < SD+LC \text{ and } \theta^k > T \\ (V_t - SD - LC) & \text{if } Vt \geq SD+LC \text{ and } \theta^k \leq T \\ 0 & \text{if } Vt < SD+LC \text{ and } \theta^k < T \end{cases} \quad (2.11)$$

The expression (9) may be rewritten as:

$$JD_t(V_t, T, g_t^K) = E^Q_t\,[JD * e^{-r(T-t)} * 1_{\{\theta^k > T\}} * 1_{\{VT \geq F\}}] + E^Q_t\,[(V_T - SD - LC) * e^{-r(T-t)} * 1_{\{\theta^k > t\}} * 1_{\{VT \geq SD + LC\}}] + E^Q_t\,[(V_t - SD - LC) * e^{-r(\theta k - t)} * 1_{\{\theta^k \leq t\}} * 1_{\{Vt \geq SD + LC\}}] \quad (2.12)$$

Where:

1) $1_{\{VT \geq F\}}$ is an indicator function equal to 1 if V_T is greater than F, and equal to 0 otherwise;

In table 2.1, I summarize the payoffs given to the different stakeholders and reported analytically above from (2.8) to (2.12).

Tab. 2.1 – ***Payoffs***

	Bond repayment or Liquidation at the end (T)			Liquidation before T	
PAYOFFS TO	**$V_T > F$ and $\theta^k > T$**	**$F \geq V_T \geq SD+LC$ and $\theta^k > T$**	**$V_T < SD+LC$ and $\theta^k > T$**	**$F \geq Vt \geq SD+LC$ and $\theta^k \leq T$**	**$Vt < SD+LC$ and $\theta^k \leq T$**
EQUITYHOLDERS	$V_T - F$	0	0	0	0
SENIOR BONDHOLDERS	SD	SD	V_T - LC	SD	V_t - LC
JUNIOR BONDHOLDERS	JD	$(V_t - SD - LC)$	0	$(V_t - SD - LC)$	0
TOTAL	**V_T**	**V_T - LC**	**V_T - LC**	**V_t - LC**	**V_t - LC**

2.3 Default Probabilities

The asset value process under real measure, shown in equation 2.1, and the default boundary specification allow me to compute default probabilities over the time interval (0,T].

I highlight that, in my model, the firm goes in bankruptcy if, at any time before the debt maturity, the asset value is less than a threshold level (K) for a period exceeding *d* or if the value of the assets falls below the Debt Face Value (F) at maturity.

Therefore, Default Probabilities are computed including both default at T an default at t < T and I assume that any time liquidation costs are paid (according to Leland, 1994).

Let Pr (t,T] denote the cumulative default probability over the time interval (t,T] calculated based on information available at time t. I have:

$$Pr\,(t,T] = Pr\,(Vt, \sigma, r, \Omega) \tag{2.13}$$

Where cumulative default probabilities depend on the Value of the Asset, the asset's volatility, the riskfree rate and on Ω that denotes a vector of additional structural parameters present in the model such as dividend yield.

I compare "my" theoretical (internal) default probabilities with the average cumulative default frequencies, as given by Moody's Investors Service (2011), in order to test the reliability of my framework.

It is interesting to point out, anyway, that default is less likely in the Merton Model than in my model while it is more likely in absorbing barrier model, since in Merton model, default never occurs before the zero-coupons bond matures at T; and in absorbing barrier model, default occurs as soon as the asset Value V_t hit the threshold level K.

2.4 Recovery Rate

The implied recovery rate, the fraction of original principal value received by bondholders in the event of default, is given by:

$$RR = (1-LC)*V_l / F \tag{2.14}$$

I use also this value in order to test the validity of my model. Indeed, it is known that the empirical default recovery rate on average is equal to 51 - 52% as reported by Moody's Investors service (2011) and by Altman and Kishore (2006).

2.5 Yield to maturity and Credit Spread

I am able to compute the yield to maturity and the yield spread of the two different obligations, that are present in my model. In particular:

$$YM_{Senior} = (F*\gamma/P_{SD})\hat{}(1/(T-t)) - 1 \tag{2.15}$$

$$YM_{Junior} = (F*(1-\gamma)/P_{JD})\hat{}(1/(T-t)) - 1 \tag{2.16}$$

$$CS_{junior} = YM_{junior} - r \tag{2.17}$$

I use also this value in order to test the consistency of my framework. Huang and Huang (2003), in fact, report the average yield spread (470 bps) over Treasury bond of similar maturity for B-rated bonds (junior), based on the Lehman bond index data from 1973 to 1993.

2.6 Numerical implementation

Since in most cases an analytical solution is not available, I use a Monte-Carlo simulation approach, that considers 150,000 sample paths for calculating bond prices, equity value, cumulative default probabilities, yield to maturity, credit spreads and recovery rate.

I concentrate my analysis on all companies with the same credit rating at given point in time (i.e. companies that have credit rating equal to B or BBB), since data on default probabilities provided by rating agencies are grouped by rating categories.

2.7 Parameters' choice

I assume that firm's capital structure is constituted by ordinary stock, senior debt and junior debt. Both the obligations have a maturity of 10 years and are Zero-Coupon Bonds.

As shown above, I suppose that the firm has a total debt F, of which (γ)*F is senior debt (SD) and (1-γ)*F junior debt (JD) (see formulas 2.2 and 2.3). Specifically, here I suppose that γ is equal to 0.5 and, therefore, the total amount of Senior debt and of Junior debt are equal.

Anyway, in order to study the impact of this parameter on the prices and on the default probabilities, I present a sensitivity analysis in paragraph 2.10.

In particular, I follow some recent papers in my parameter choices. Base case parameters are:

1) the asset value of the firm at t = 0 is equal to 100 (as Black and Scholes 1973);

2) the leverage ratio is equal to 0.657, in line with B-rated bonds according to Huang and Huang (2003);

3) the total debt face value is equal to 65,70%. This depends on the fact that the leverage ratio is equal to 0,657 and the firm asset value is equal to 100;

4) the continuously compounded constant rate is equal to 6% as in Galai, Raviv and Wiener (2007);

5) the pre-defined grace period is 1, in line with the value found by Covitz, Han and Wilson (2006);

6) the volatility of the asset of the firm is 29%, as pointed out in prior work (Strebulaev and Schaefer, 2008) for B-rated companies;

7) the liquidation cost is equal to 20% of the asset value of the firm at the moment of liquidation. This value is consistent with previous findings that bankruptcy costs are about 10% - 20% of firm value (Andrade & Kaplan, 1998);

8) the dividend yield is equal to Zero, typical value for B – rated bond (Galai, Raviv and Wiener, 2007);

I set the parameter ω (Threshold's coefficient) equal to 85%. In order to study the impact of this parameter on the prices and on the default probabilities, I present a sensitivity analysis in paragraph 2.10. It is interesting, anyway, to point out that in Leland (2004) and in Davydenko (2007) this parameter (ω) is set equal to 0.7 (more or less) for model without grace period. I choose an average between this value and 1, that is the maximum value that ω could reach.

Finally, I consider 10 periods (each period = 1 yr), and I simulate 150,000 price paths of the underlying asset under the risk neutral process for each period. I take 10 periods because, in this way, each period corresponds to one of the 10 years of the debt maturity (as in Stohs & Mauer, 1996).

Consequently, if the value of the asset of the firm is smaller than the threshold measure (K) in two consecutive periods, then the result is the liquidation of the firm. In the model, however, it is possible to have the liquidation of the firm even if the final asset value of the firm is less than 65.7 (the total debt face value) in t = T = 10.

In the following table, I summarize the value of parameters:

Tab. 2.2 – *Parameters*

simulazioni zcb		
Parameter	**Symbol**	**Value Assumed**
Time to Maturity	T	**10**
Pre-defined grace period	d	**1**
Percentage of SD on Total Debt	**γ**	**0,5**
Default free interest rate	r	**0,06**
Volatility of the asset of the firm	σ	**0,29**
Liquidation Cost	LC	**0,2**
Threshold's parameter	**ω**	**0,85**
Dividend yield	δ	**0**
Initial Value of Assets	V_0	**100**
Leverage Ratio	LR	**0,657**
Debt Face Value	F	**65,7**
Debt Coupon rate	i	**0**
dt=0.01;	dt	**0,01**
paths		**150.000**

I don't calibrate my model in order to match perfectly my cumulative Default probabilities with the real cumulative default probabilities. I choose to use the "real" (and/or empirical) value of the parameters. Following this approach, I could verify the reliability of my hypothesis and of my model trying to match my cumulative default probabilities with Moody's cumulative default probabilities.

2.8 Results vs. Empirical data

In table 2.3, I summarize some results. Here, I highlight that the probability that the Senior Bond is fully paid is 86.7%, while for the Junior Bond this probability is equal to 68.7%.

In the next subparagraph, I present details on cumulative DPs, recovery rate and yield spreads.

Tab. 2.3 – ***Some results***

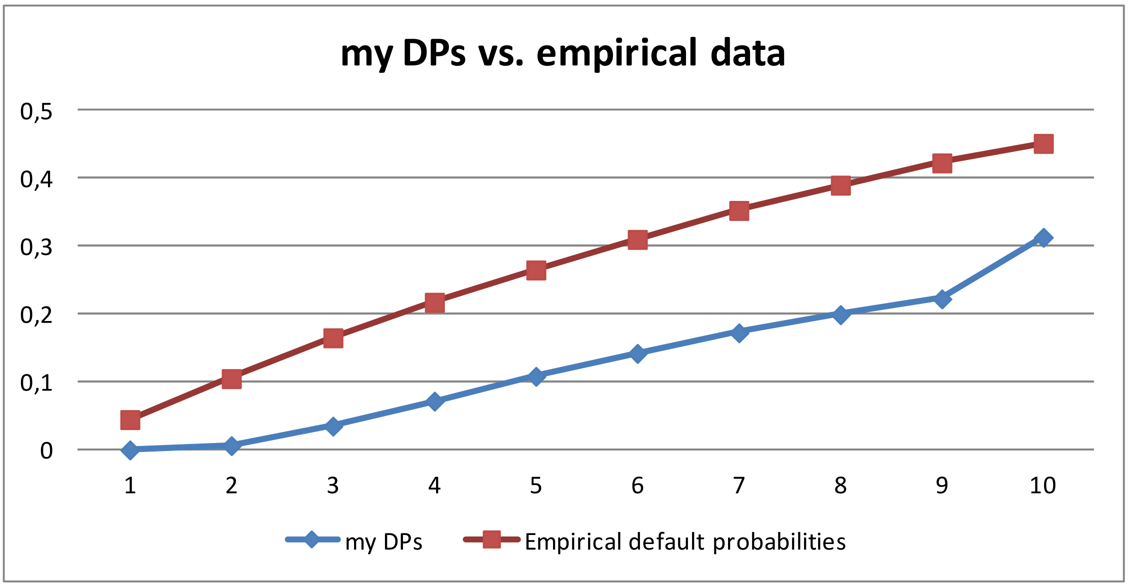
Liquidation Cost	1,83%
Average V at the default	43,37%
Yield to Maturity Senior	5,7%
Yield to Maturity Junior	9,6%
Yield Spread	**3,6%**
Cumulative default pr at T	31,3%
Cumulative default pr before T	24,38%
Pr. Senior paid in full	86,68%
Pr. Junior paid in full	68,67%
Recovery Rate	**52,80%**

2.8.1 Cumulative DPS

In order to test my hypothesis, I calculate the cumulative default probabilities present implicitly in the model and I compare these data with the cumulative DPs observed in literature and computed empirically.

In particular, I use the average cumulative default frequencies for B-rated debt as function of horizon, as given by Moody's (2011) for the period 1983 – 2010 as my parameter of comparison.

Figure 2.1 – ***Cumulative default probabilities***



Matching my data with Moody's data, it is evident that my DPs are very different from empirical data. I believe that it depends on the fact that usually bonds are coupon bonds while one of my main hypothesis (as in literature) is that bonds are ZCB.

In addition, I remind that I set my model on US market, while Moody's data are global data. In particular, it is important to point out that my d (grace period) is equal to 1 as in US, but I don't have any "global" reference.

2.8.2 Recovery Rate

My average Asset Value in case of default is equal to 43.4%, while the recovery rate (on average) is 52.8%. This value is not very different from the value empirically found in literature 51% - 52% (as above highlighted). I point out that this outcome is very close to empirical data principally thanks to the presence of liquidation cost.

2.8.3 Yield Spread

I find that the yield to maturity for the senior bond is equal to 5.7%, while for the junior bond this rate is equal to 9.6%. Therefore, the credit spread for junior bond (3.6%) is smaller than the value that is empirically found in literature (4.7%).

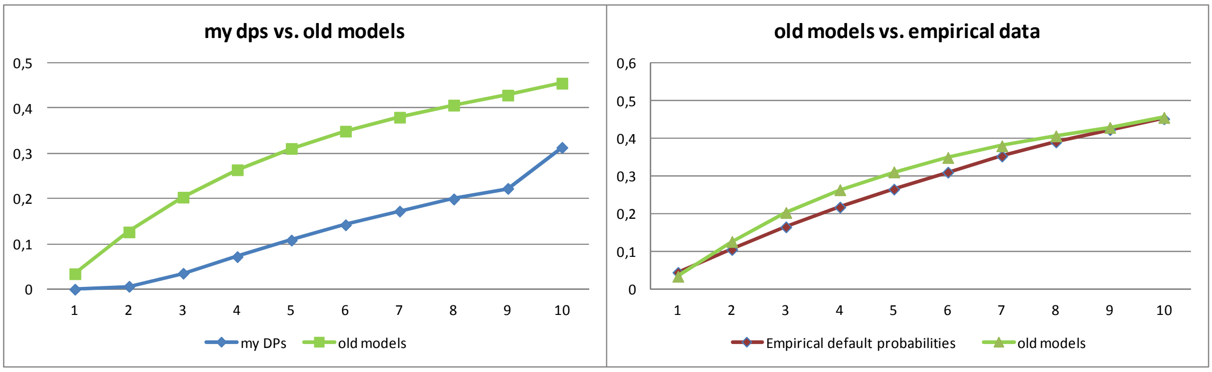
This difference, again, is attributable to the fact that in this framework there are not coupon bonds. Indeed, in chapter 3, with the introduction of coupons, I will get results more satisfying.

2.8.4 Empirical cumulative DPs vs. old models

I show below a comparison between the DPs found in my model and these probabilities computed using an adaptation of my model in which I assume that: the grace period is equal to 0.01; there aren't Liquidation costs; and there is only one type of Zero Coupon Bond.

With these hypothesis, indeed, I obtain a particular case (hit and default models) that replicate the "old" models present in literature.

Figure 2.2 – ***Cumulative default probabilities***



Analyzing the figure 2.2, I could point out that my DPs are smaller than the probabilities found with the "old models". These results, furthermore, are very close to empirical DPs, even if the last parallel is not practicable because the empirical data are computed for Coupon Bonds and global data.

This fact become evident if I compare the recovery rate computed through the hit and default models with those empirically found. Indeed, the recovery rate calculated empirically is equal to 52%, while in the hit and default model this parameter is equal to 83%.

Even if I consider an hit and default model with LC and two types of Bond, I have the same results. Indeed, I get: a recovery rate equal to 67%, that is very different from the empirical value of 52%; and a yield spread equal to 3%, that is very far from the 4.70% reported by the Lehman bond index.

The recovery rate is particularly high versus empirical data and this huge difference is dependent on the fact that in these model a firm default as soon as the asset value hit the barrier.

The credit spread is very different from the empirical data, because in these framework debts are always zero coupon bonds.

It is evident, therefore, that hit and default models (old models) does not fit empirical data, since the hypothesis of these models are not in line with the real world.

2.9 Hypothetical country

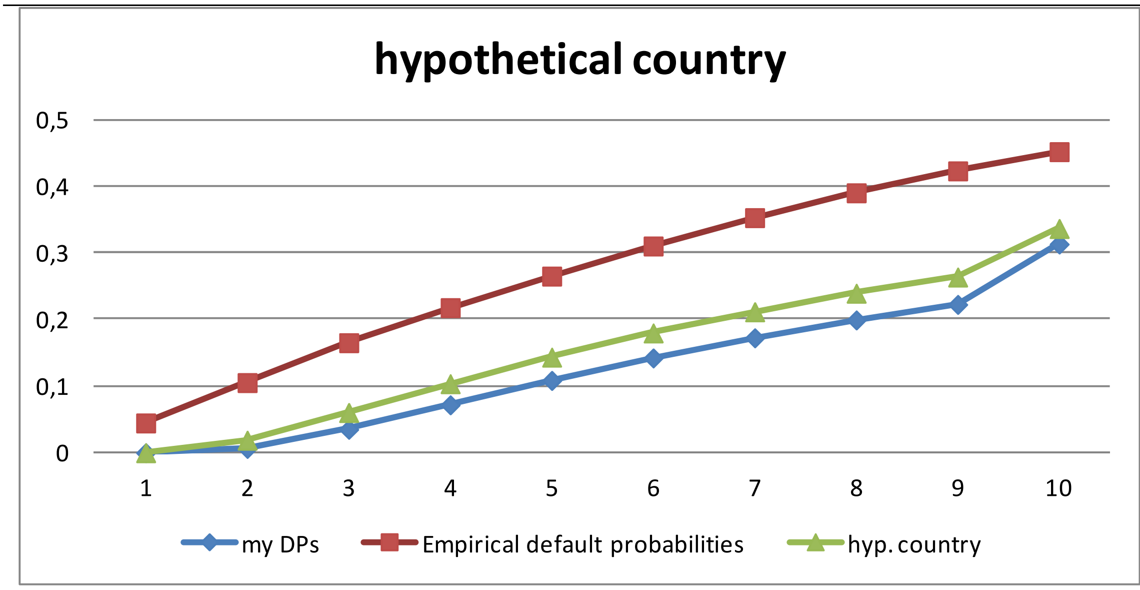
If I just change the level of the grace period from 1 to 0.65, I build an hypothetical case in which the grace period is more or less equal to the average between US data and Swedish data.

In this way, I try to point out that part of the difference between my results and empirical results are dependent on the fact that I use the US reference as level of grace period.

Indeed, this change produce a moderate variation and, consequently, my cumulative default probabilities become closer to empirical default probabilities (see figure 2.3).

I believe that the distance present between my results and empirical results is not equal to zero because in this framework there are not coupons. Indeed, in Chapter 3 I will get results more satisfying.

Figure 2.3 – ***Cumulative default probabilities***



2.10 Sensitivity analysis

I now consider how my results are sensitive to the choices of my parameters. I start from the base case, and then change various parameter choices in order to study the impact of these parameters on the asset value, on recovery rate and on cumulative DPs.

2.10.1 Grace period (d)

In my base case the initial grace period is set equal to one year (as in Covitz, Han and Wilson, 2006), but it is shown by different authors that under different legal system the average time in bankruptcy could be shorter or longer. Thoburn, for example, finds that in Sweden this period is much shorter and equal to 2.5 months, while in 1990s in the US the time in bankruptcy was longer than in the last years (18 – 24 months versus 12 months).

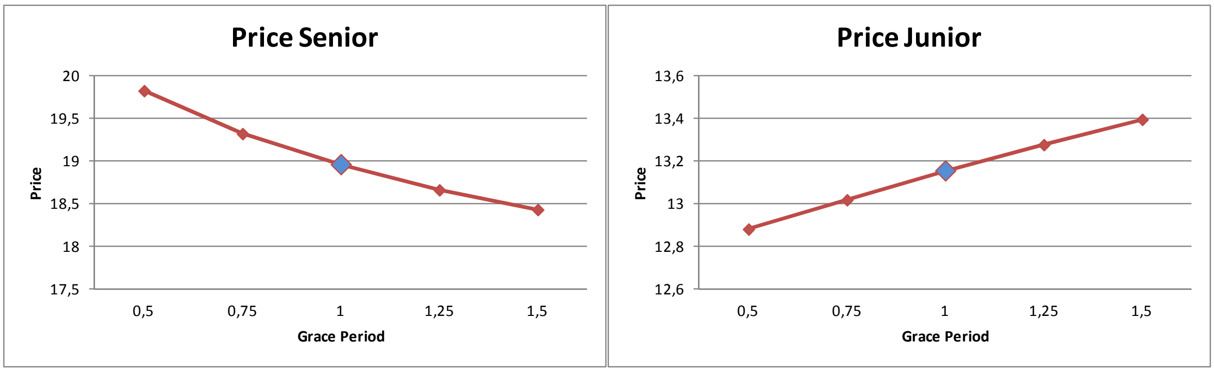
Therefore, I present a sensitivity analysis below in order to study the impact of the length of the initial grace period on the price of the assets, the recovery rate and the default probabilities.

An increase of the grace period extends the time in bankruptcy and decreases the capacity of the senior bondholders to extract value upon default (cfr. Figure 2.4).

Indeed, in this case decrease the probability that the senior bonds are fully paid. At the same time, go down the cumulative default probabilities (it is more likely that the asset value rebound up the threshold level), but the first effect prevail on the second effect.

Conversely, the Value of the Junior Bond and Equity price go up, due to the decrease of the DPs (cfr. Figures 2.4 and 2.5).

Figure 2.4 – ***Sensitivity analysis: grace period (d)***



As upon pointed out, when the grace period increases it is less likely the that the senior bonds are fully paid. This means, as shown in Figures 2.5 and 2.6, that the Value of Asset (V) at time of Default goes down and consequently the liquidation costs and recovery rate decrease (remind that the LCs are a percentage of V and that recovery rate is equal to (1-LC)*V(at_default)/F)).

Figure 2.5 – *Sensitivity analysis: grace period (d)*

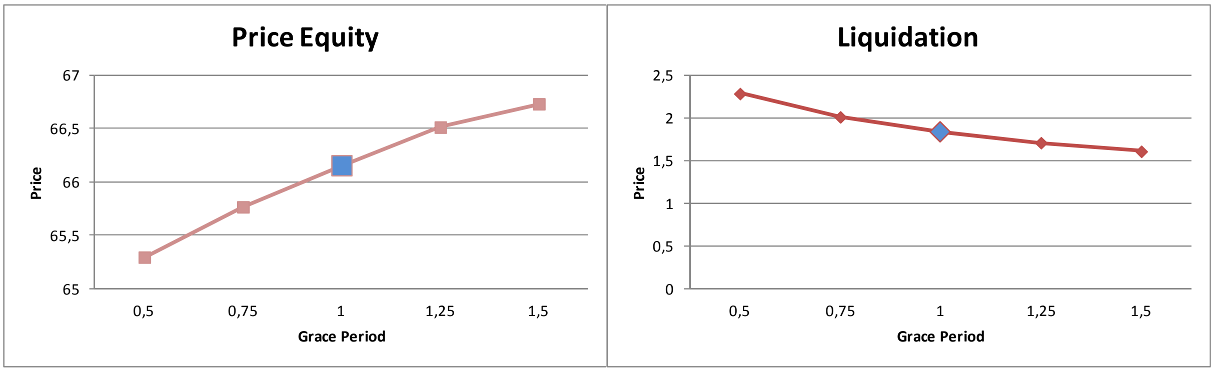
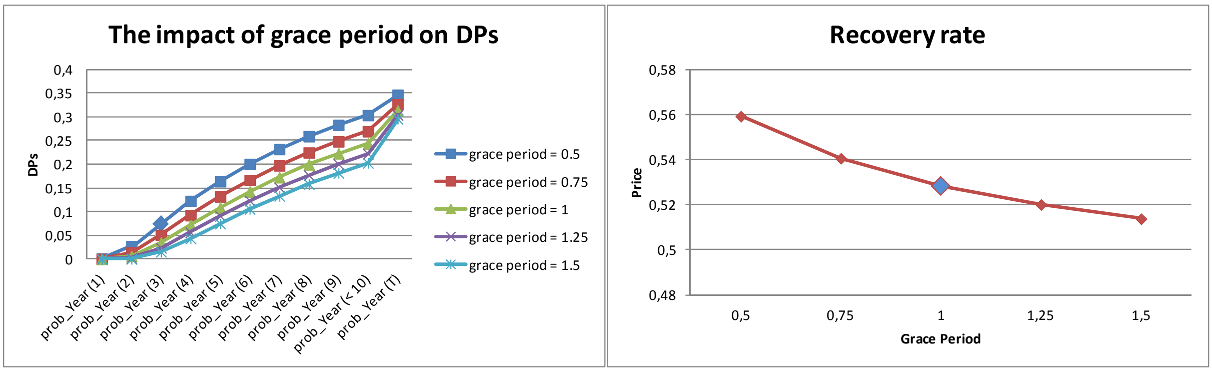


Figure 2.6 – Sensitivity analysis: grace period (d)



2.10.2 Volatility (sigma)

Figures 2.7 and 2.8 show that the price of the senior bond, liquidation costs and equity price increase with asset volatility, while the price of the junior bond steps down.

The trend of the value of equity is expected. Indeed, if there isn't a bankruptcy then the value of the asset (V) could be very high at the end of the process thanks to higher volatility.

At the same time, when the volatility of the asset goes up the model records an increase of the DPs . This event produces: an increase of the liquidation costs; and a reduction of the value of the junior bond, because it is more likely that the junior bondholders aren't fully paid. Consequently they have to sustain more often the liquidation costs.

Whereas the value of the senior bond, as above reported, steps up until the volatility is so high that the probability that the senior bondholders are fully paid decrease largely (because of the liquidation costs trend and DPs' variations).

Figure 2.7 – ***Sensitivity analysis: volatility***

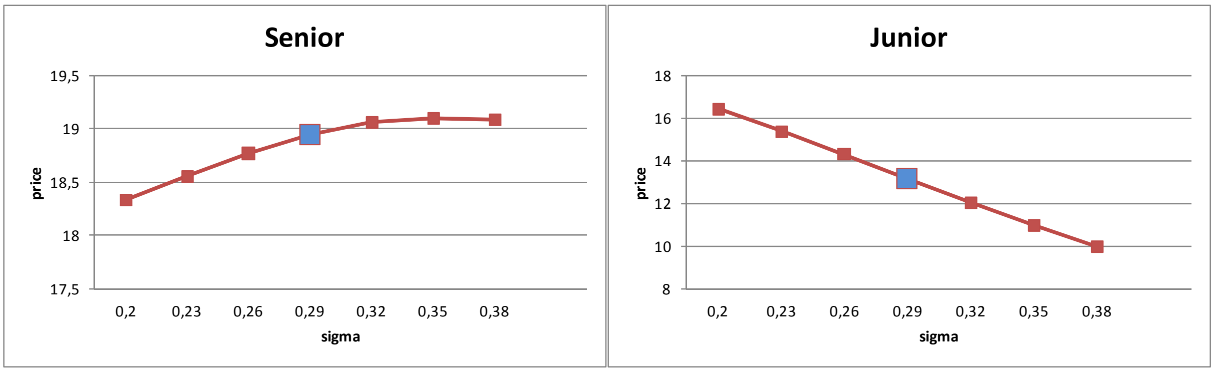
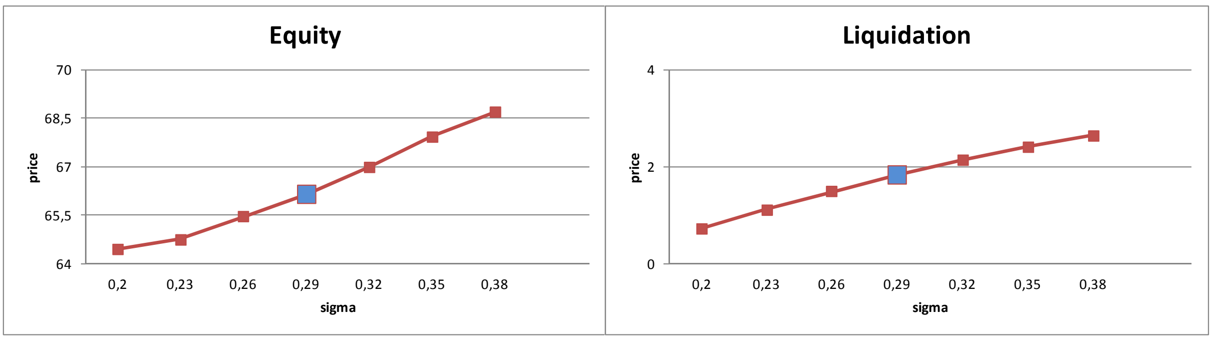
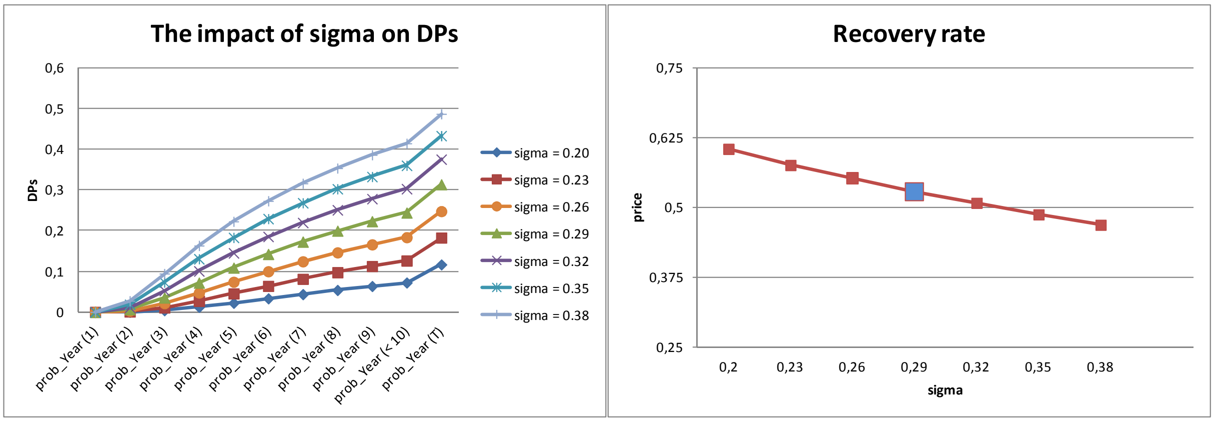


Figure 2.8 – ***Sensitivity analysis: volatility***



The trend of the recovery rate, finally, depends on the evolution of the liquidation costs and of the DPs (cfr. Figures 2.9).

Figure 2.9 – ***Sensitivity analysis: volatility***



2.10.3 The percentage of Senior Bond on the Total Debt

It is straightforward (and clear from figure 2.10) that the value of the two different type of Bonds depends on gamma (parameter that measure the percentage of the senior bond on the total debt) and that the Equity value does not change varying this parameter.

Also the Liquidation costs and DPs are constant (in this framework). Indeed, one of the main hypothesis of the model is that the threshold level depends only on the Total Debt.

It is interesting, however, to show how change the probability that the senior bond is fully paid (cfr. figure 2.11) varying gamma. When gamma is equal to 0.3 (30% of the total debt is Senior), this probability is very close to 1, while when gamma is equal to 0.7 this value is smaller than 0.75.

Conversely, the probability that the junior bond is fully paid is independent from the value of gamma, because it depends on the DPs (that are constant).

Figure 2.10 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***

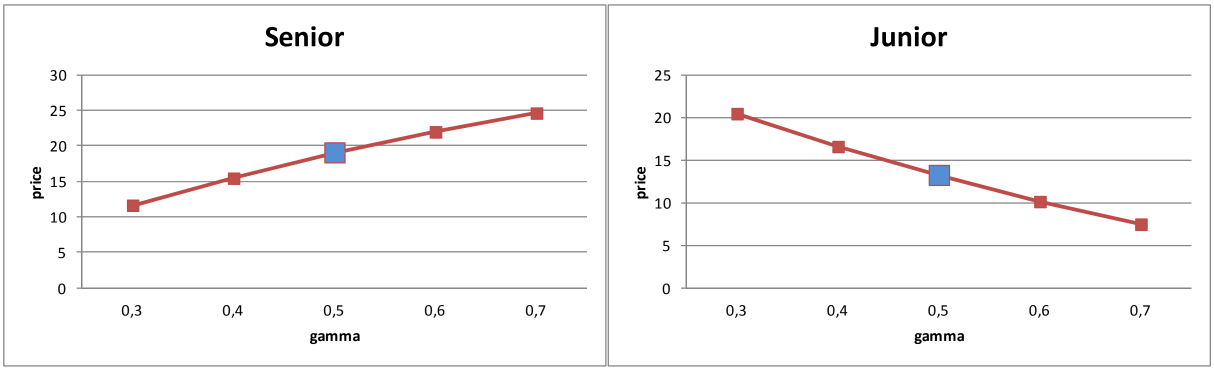
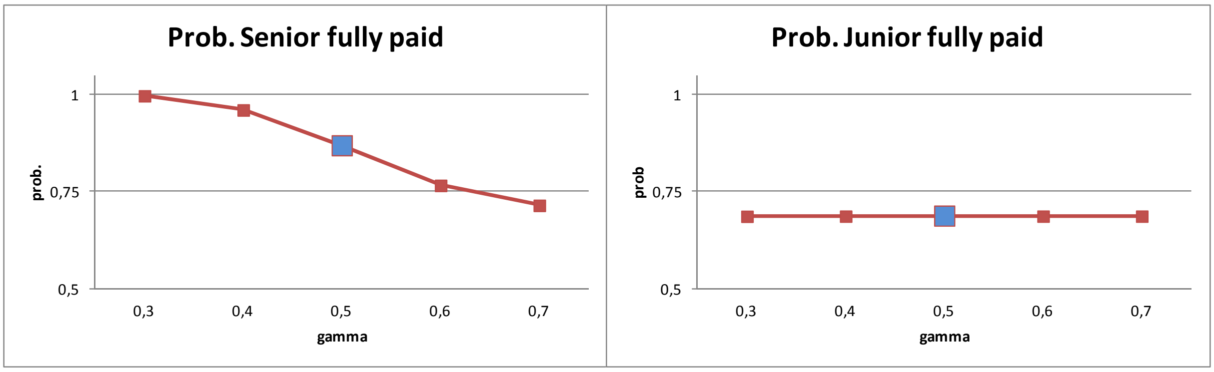


Figure 2.11 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***



2.10.4 Dividend yield (delta)

When a firm pays more dividend, the dynamic of the asset value (Brownian process) is more stable (V increases more slowly). It is more likely that the company fails and, therefore, the equity price decreases (cfr. Figure 2.13).

At the same time, the value of the junior bond steps down (figure 2.12), while the value of Senior Bond is more or less constant. It depends on the fact that the new process produces a decrease (very large) of the probability that junior bondholders is fully paid. Therefore, it is more probable that the junior bondholders have to sustain the liquidation costs (and then these costs step up).

Figure 2.12 – ***Sensitivity analysis: dividend yield***

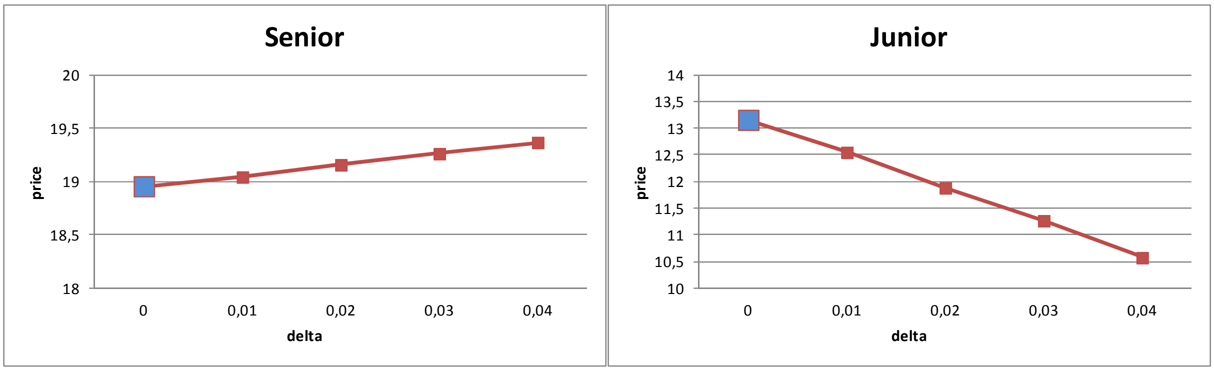
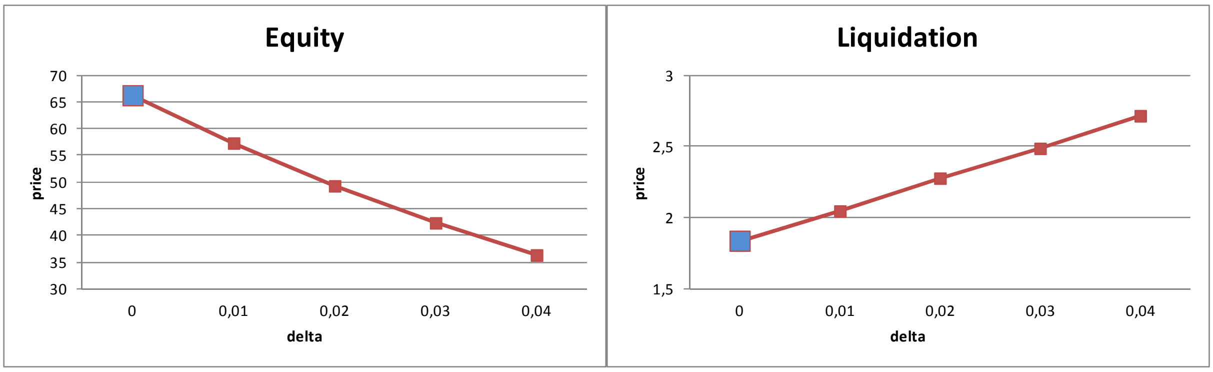
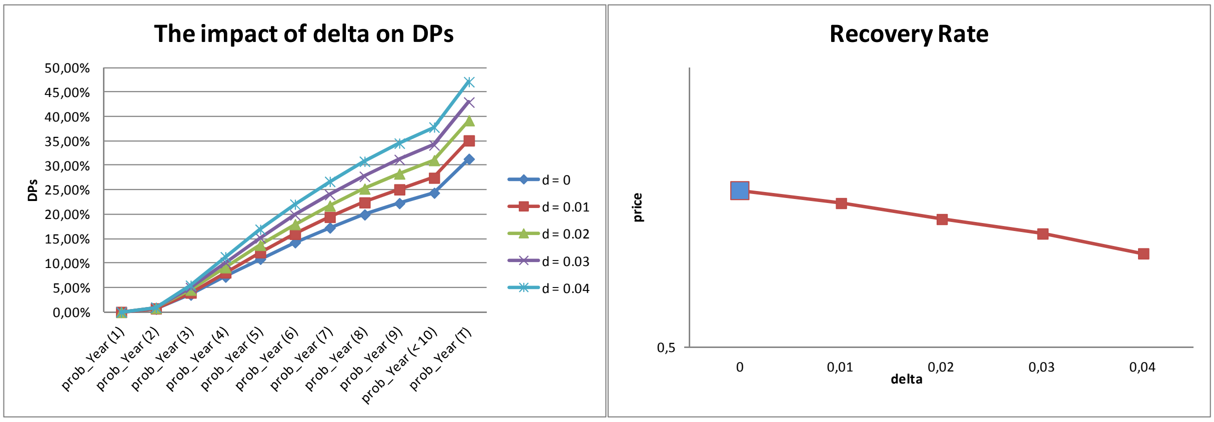


Figure 2.13 – ***Sensitivity analysis: dividend yield***



From figure 2.14 it is clear that Recovery Rate, finally, is stable because the growth of the liquidation costs and DPs are offset by the fact that the process is less variable (more often junior pays for the bankruptcy / default happens in junior's area).

Figure 2.14 – ***Sensitivity analysis: dividend yield***



2.10.5 Liquidation Costs

According to my base case, when there is liquidation, the bondholders (junior bondholders or senior bondholders depend on the situation) have to sustain the liquidation cost equal to 20%.

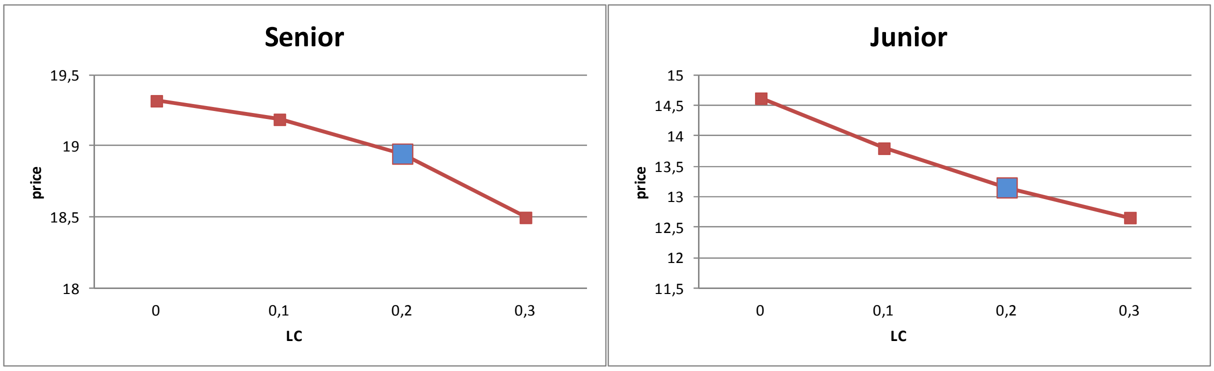
I show the impact of a different level of liquidation cost on the bond prices in Figure 2.15 However, there is no impact on the equity price. In case of liquidation: 1) equityholders do not receive anything; 2) they do not have to pay anything due to limited liability.

The larger price variation hits principally the junior debt price when the liquidation cost increases gradually from 0% to 20%.

Conversely, if the liquidation cost goes up from 20%, the impact is heavier on the price of the senior debt. Indeed, when the liquidation cost becomes bigger and bigger, it is most likely that:

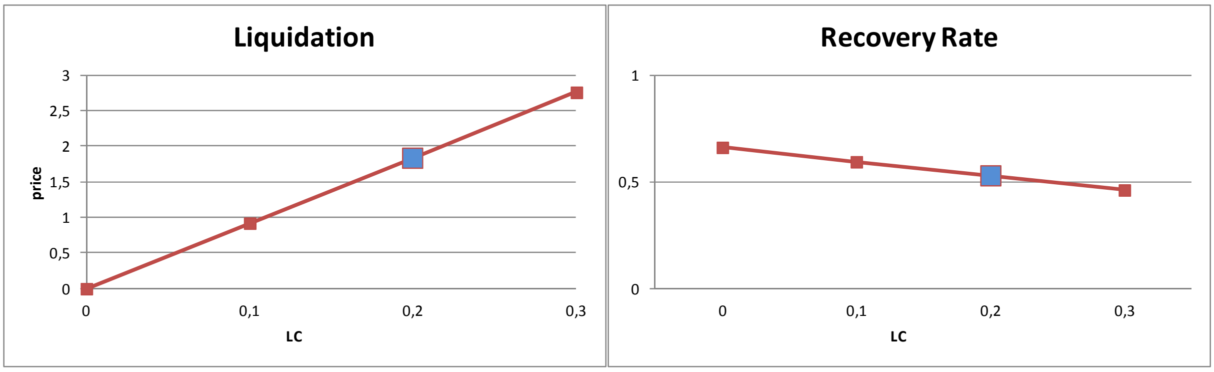
V_t - LC $\leq$ SD and consequently the senior debtholders have to sustain these costs.

Figure 2.15 – *Sensitivity analysis: liquidation cost*



It is straightforward that Recovery Rate decrease when the Liquidation costs increase (cfr. Figure 2.16). Indeed, by construction the Recovery Rate is equal to ((1-LC)*V(at_default)/F) and here there are not variation of the DPs.

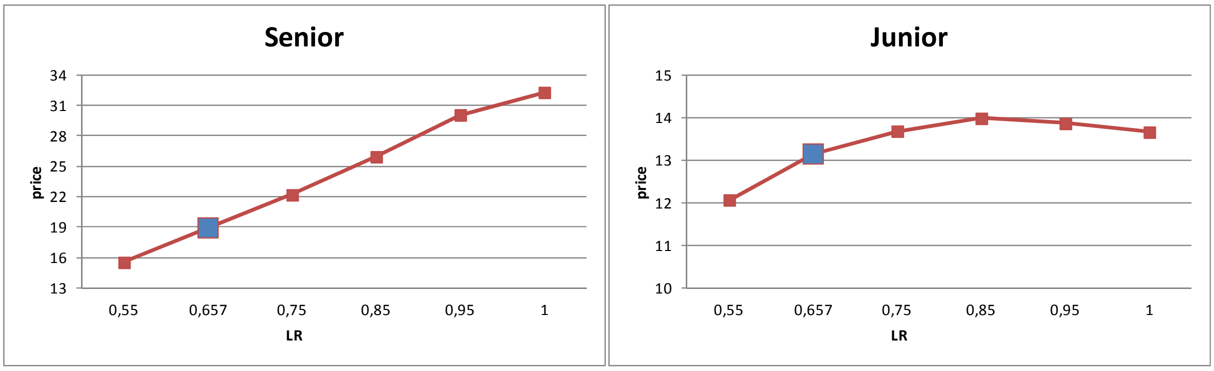
Figure 2.16 – *Sensitivity analysis: liquidation cost*



2.10.6 Leverage Ratio

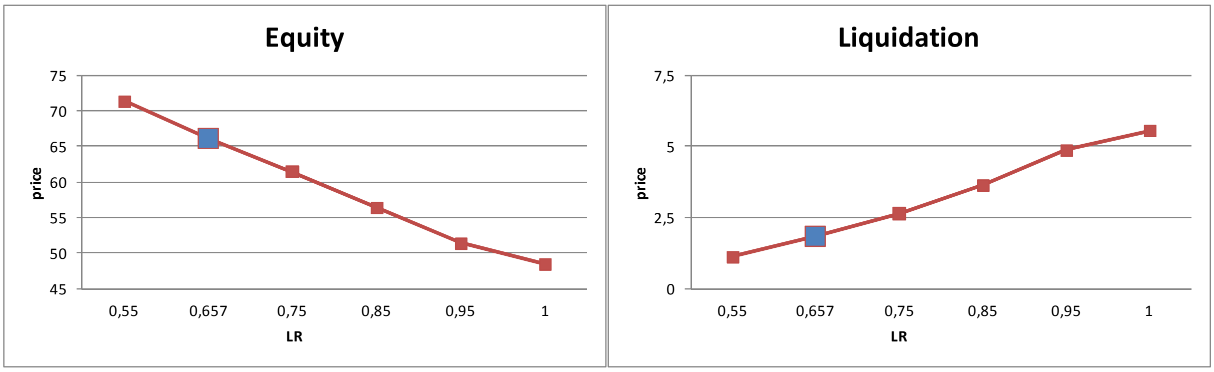
Figure 2.17 provides the evolution of bond values for different level of financial leverage ratio. Due to the rise of the Total Debt, there is an increase on the value of the senior and junior bonds. In reality, the price of the junior bond goes up until the effect of the total debt's growth is greater than the effect of the increase of the Liquidation costs and of the DPs.

Figure 2.17 – *Sensitivity analysis: leverage ratio*



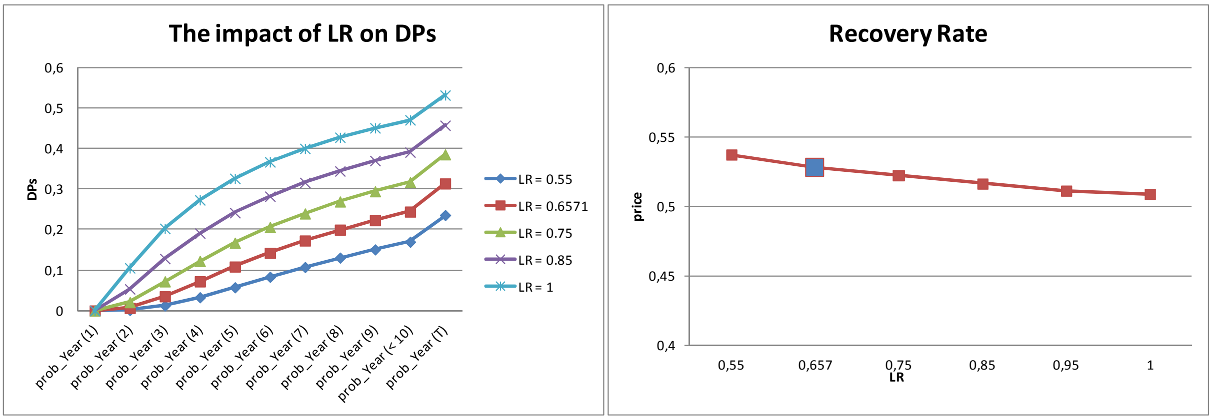
In line with my expectation, equity goes down (cfr. Figure 2.18), considered that the shareholders are the residual claimants. Liquidation costs step up, due to the fact that these costs are function of: 1) Liquidation Value; 2) Threshold level; 3) Total Debt (F).

Figure 2.18 – *Sensitivity analysis: leverage ratio*



When LR rises then the threshold level is higher and, therefore, it is more likely that V_t could below the barrier. This implies that cumulative default probabilities increase with LR (cfr. Figure 2.19).

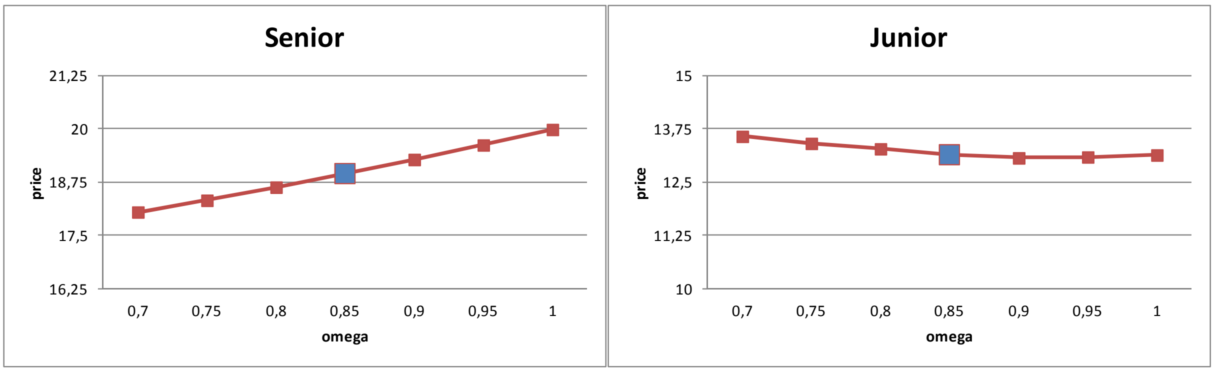
Figure 2.19 – *Sensitivity analysis: leverage ratio*



2.10.7 Threshold's parameter (Omega)

The following figures (2.20 and 2.21) show that the Price of the Senior Bond and the Liquidation Costs go up when there is a positive variation of omega (Threshold's parameter). This results are in with the expectation because when omega increases the barrier is higher.

Figure 2.20 – *Sensitivity analysis: Threshold's parameter*



It is interesting to highlight from figure 2.20 that the price of the Junior Bond decreases when omega passes from 0.7 to 0.9 and then increase. It depends on the fact that the liquidation costs' rise offset the effect related to the barrier's variation.

Cumulative DPs (cfr. Figure 2.22) step up when the threshold barrier is higher and, consequently, equity decreases because the equity holders are residual claimants (cfr. Figure 2.21)

The recovery rate increases (cfr. Figure 2.22), but the variation is modest because of the liquidation costs' trend. In practice, the evolution of LC offset the benefits linked to an higher barrier.

Figure 2.21 – ***Sensitivity analysis: Threshold's parameter***

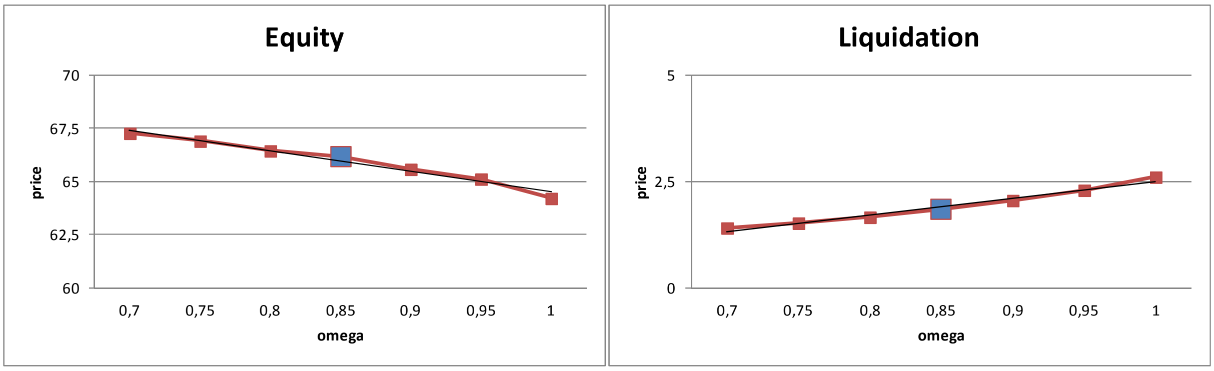
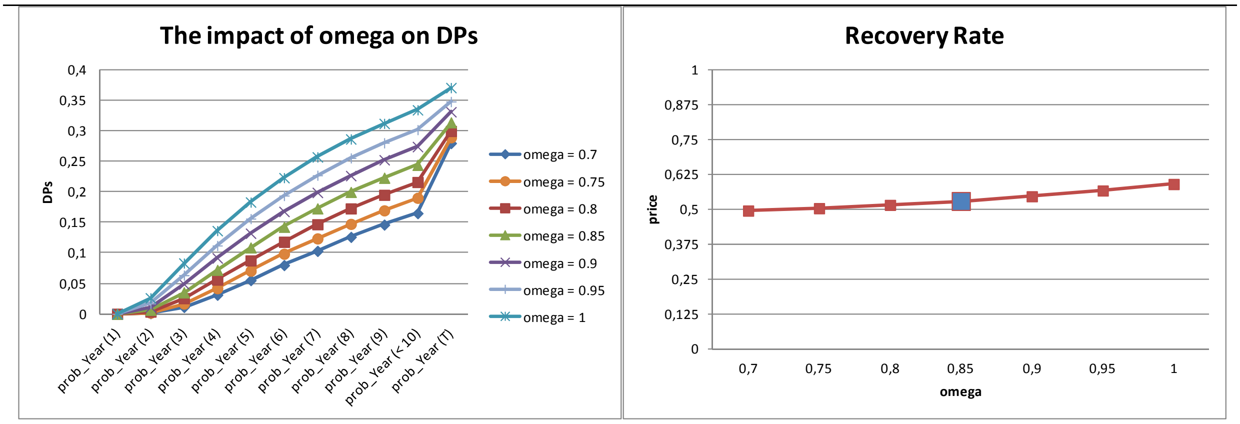


Figure 2.22 – ***Sensitivity analysis: Threshold's parameter***

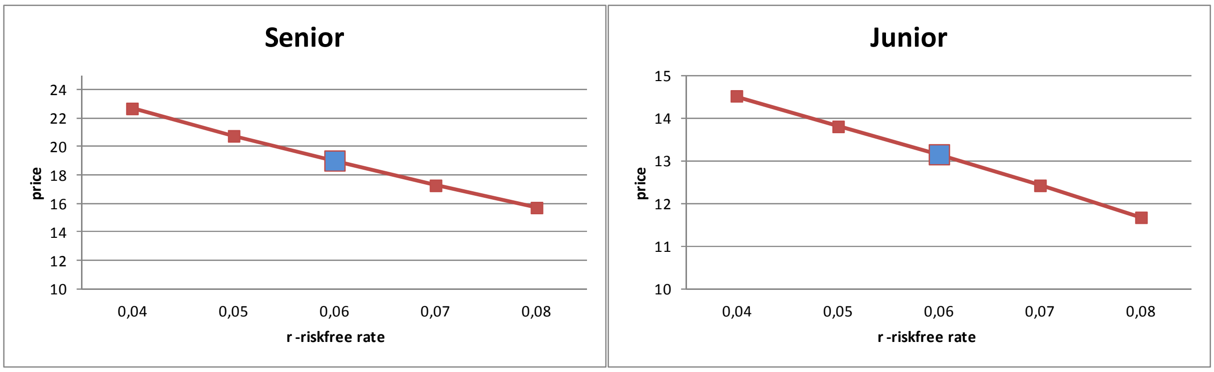


2.10.8 Free-risk Rate (r)

When the risk-free rate increases, there are two effects: the asset Value (V) appreciates with an higher instantaneous intensity and, consequently, the cumulative default probabilities decrease; and the discount rate is bigger.

The second effect is the principal effect on the value of bonds (cfr. Figure 2.23), while the first one is prevalent on the Equity value (cfr. Figure 2.24).

Figure 2.23 – *Sensitivity analysis: free-risk rate*



In this case, liquidation costs decline, while recovery rate increase slightly. These trends are produced by the Asset Value dynamic and by the fact that cumulative default probabilities go down.

Figure 2.24 – *Sensitivity analysis: free-risk rate*

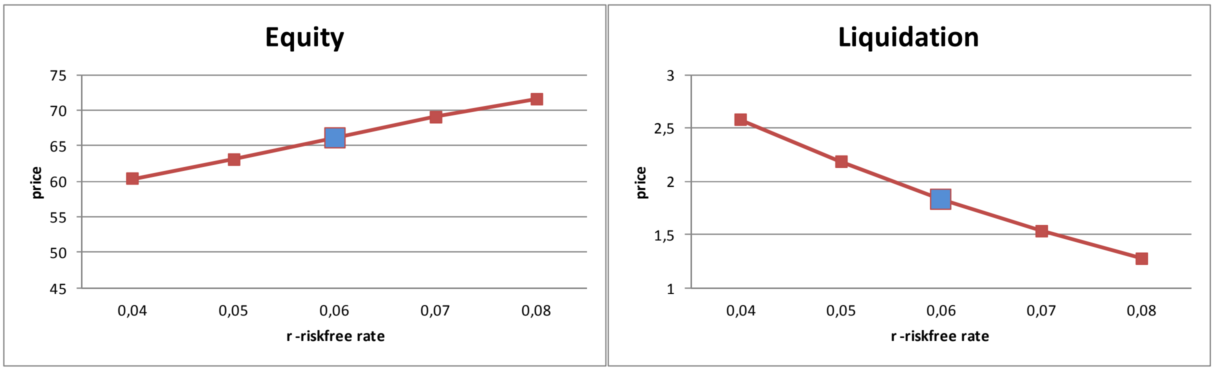
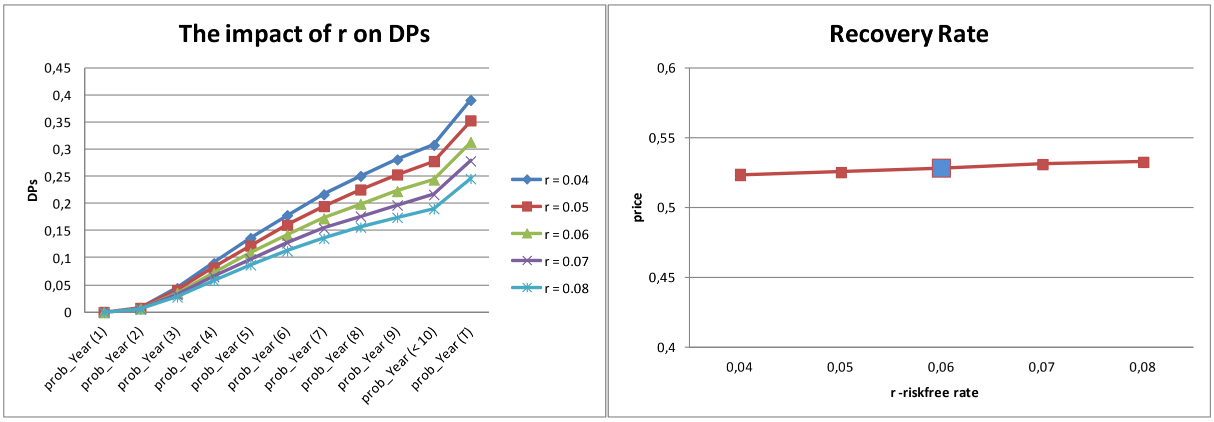


Figure 2.25 – *Sensitivity analysis: free-risk rate*



2.11 Conclusion

In this chapter, I present a model to compute the value of Cumulative Default probabilities, Recovery Rate, Yield Spread for B-rated firms and of different corporate securities (equity, senior debt and junior debt).

The model is built on the hypothesis that there are two different type of Bond (Senior and Junior) both Zero Coupon Bond and that the senior bondholders have the right to ask for liquidation only if the asset value of the firm is less than a pre-defined value (threshold) for an interval exceeding the grace period (d).

These assumptions make my model more realistic than the hit and default models, in which there is just one type of debt , there isn't liquidation cost and the grace period is equal to one day (d = 0.01).

The first result is that my "internal" cumulative DPs are different from empirical data (Moody's) and it depends principally on the fact that bonds, in reality, are coupon bonds while one of my main hypothesis (as in literature) is that bonds are ZCB. Indeed, when in Chapter 3 I add coupons to my model, I get results very satisfying and very close to empirical data.

Another element that influence this results is that I set my model on US market, while Moody's data are global data. It is important to point out that my d (grace period) is equal to 1 as in US, but we don't have any "global" reference for this parameter.

I remind, anyway, that if change the value of the grace period from 1 to 0.65 my results get closer to empirical data.

The recovery rate, computed through my framework, is 52.8% (on average). This value, however, is very close to the value empirically found in literature 51% - 52%, while my credit spread for junior bond (3.6%) is smaller than the value that is empirically found in literature (4.7%).

It is important, anyway, to point out that all results of the sensitivity analysis are in line with expectations and with the results historically found in literature.

At the same time, I verify the reliability of the hit and default models (old models), matching the same variables (cumulative DPs and recovery rate) with the empirical data. I get that cumulative Dps are very close to empirical DPs, but the recovery rate is equal to 83% while the empirically data is 52%.

This difference is dependent on the fact that the empirical data are computed for Coupon Bonds and are relative to global data.

Even if I consider an hit and default models with LC and two types of Bond, I have the same results. In fact, I get a recovery rate equal to 67%, that is very different from the empirical value of 52% and a yield spread equal to 3%, that is very far from the 4.7% reported by the Lehman bond index.

The results of this type of model are, therefore, not very satisfying. Indeed, these models fit relatively well cumulative default probabilities, but they are not able to find value of recovery rate and of yield spreads in line with those empirically found.

I conclude, therefore, that: 1) my model works theoretically and it is more realistic than old models, but the results are not comparable with the empirical data; 2) hit and default models (old models) present some problems, even if cumulative DPs are in line with empirical data.

CHAPTER 3 – A Real Option Model with a Junior Coupon Bond

3 Brief Introduction

In Chapter 2, I built a real option model different from classic structural models because I consider a grace period, two type of bonds and liquidation cost. Whit this new approach I wanted to fit some empirical data and in particular the trend of cumulative default probabilities, the value of the recovery rate and the value of the yield spreads.

The results were mixed since I was not able to replicate the evolution of cumulative default probabilities for B-rated firm.

Therefore, in this Chapter I introduce another hypothesis that is innovative respect to the classical real option model. In practice, I assume that one of the two bonds (the junior bond) is a coupon bond.

The outcomes are very satisfying, since I get cumulative default probabilities, a recovery rate and a yield spread very close to empirical data.

I could assert that: in my construction the presence of two type of bond and of liquidation cost have a positive impact on the value of recovery rate and yield spreads; and the presence of grace period and the introduction of coupons are crucial ingredients for fitting empirical data.

Instead, if I adapt the old models, considering two type of bonds and liquidation cost, I get results very different from empirical data and this difference does not change if I also introduce coupons in this framework.

3.1 Structural Model

In order to make my model more realistic, I change one of the structural model's main hypothesis. In this framework, indeed, I assume that one of two bond is a Coupons Bond. In particular, I suppose that the Junior Bond is a Coupons Bond, while the Senior Bond is still a Zero Coupon Bond.

This setting could replicate a real situation in which a firm has bank debt guaranteed and rolling (sort of ZCB) and another debt (Bond) with coupons issued on Financial Market.

The Coupon C is determined such that a time 0, Junior Debt is priced at par:

$$P_{JB} = (1 - \gamma) * F \tag{3.1}$$

Others assumptions are consistent with/to those introduced in Chapter 2. Here, I show again these hypothesis for simplicity:

- the default event allows the creditor (Senior Bondholders) to force immediate liquidation through its safety covenants;
- liquidation is declared when the asset value of the firm falls below distress threshold for a period that goes beyond the pre-determined grace time (denoted by *d*).
- shareholders have a residual claim on the cash flows generated by the firm's assets unless the value of these assets reaches the default threshold and remains below that threshold for the grace period.
- assets are continuously traded in an arbitrage-free and complete market with riskless borrowing or lending at a constant rate r.

3.1.1 Model assumptions

3.1.1.1 Asset Value

I assume again that the firm asset value evolves according to a diffusion process. This process has constant volatility, does not depend on the capital structure but, in this case, presents a jump because of the coupon payment. In practice, every time there is a coupons' payment the value of Asset V_T jump down and the dynamic restart from $(V_T – C)$.

The process is always a risk neutral process and all parameters $(r_t, \delta_t, \sigma_v)$ are assumed constant trough time.

3.1.1.2 Debt

The firm has a total debt F, of which part is senior debt (SD) and the remaining part is junior debt (JD):

$$SD = \gamma * F \tag{3.2}$$

$$JD = (1- \gamma) * F \tag{3.3}$$

where γ is a parameter that measure the percentage of senior debt on total debt

The maturity of the Debt is equal to T and I consider here the case where the Senior Debt is ZCB, while the Junior Debt is Coupon Debt.

3.1.1.3 Liquidation Costs

Liquidation costs are still a fraction of asset value V_t at the moment of the default and are equal to:

$$LC = \rho * V_t \qquad (3.4)$$

where ρ is a parameter that measure liquidation cost

3.1.1.4 The threshold level

My default boundary depends only upon the principal value of debt and it is not affected by my parameters and by coupon payments. Therefore, the threshold level K, which is time independent, is equal to:

$$K = \omega F \qquad (3.5)$$

where $0 \leq \omega \leq 1$ and this parameter set the level of threshold

In practice, the threshold level is constant between t and T and there aren't jumps because of the presence of the coupons.

3.1.1.5 Random Variables

In order to determine the value of corporate securities, I define the same random variables introduced in Chapter 2:

$$g_t^k = \sup \{s \leq t \mid V_s = K_s\} \qquad (3.6)$$

$$\theta^k{}_t = \inf \{t \geq 0 \mid t - g_t^k \geq d, V_t \leq K_t\} \qquad (3.7)$$

where g_t^k is the last time before t that the value of the firm's assets crossed the threshold value K, and θ^k_t is the liquidation time, i.e., the first time the value of the firm's assets spent d units of time consecutively below the default threshold.

3.2 The valuation of corporate securities

In this paragraph, I present the new pay-off of firm's stakeholders. In particular, it is clear that the presence of coupons impact on the condition/constrain of my equation.

It is necessary, however, to point out that in this framework when there is liquidation, junior bondholders receive the present value of coupons (*Σ Coupons$_t$*) that include the accrued interest.

3.2.1 Equity Value

In case of liquidation Equity holders, as residual claimants, do not receive anything. If there is not liquidation (if $V_T > F$ + last coupon), at debt maturity T, Equity holders receive the maximum between zero and the difference between the firm's asset value V_T and the face value of the total debt (F). The Equity holders pay-off, therefore, is represented by this equation:

$$S(V_T , T , g_t^k) = \max (V_T - F, 0)*1_{\{\theta^k > t\}} = \begin{cases} V_T - F & \text{if } V_T > F + \text{Coupon and } \theta^k > T \\ 0 & \text{otherwise} \end{cases} \quad (3.8)$$

At any time before the debt maturity, if bankruptcy has not occurred, the value of Equity holders claim is given by:

$$S_t (V_t , T , g_t^k) = e^{-r(T-t)} E^Q_t [\max (V_T - F, 0)*1_{\{\theta^k > t\}}] \quad (3.9)$$

3.2.2 Senior Debt

The value of the senior debt (SD) is equal to:

1) face value if the firm is not prematurely liquidated, and the asset value of the firm V_T is greater than the face value of Debt plus the last coupon;
2) face value if the firm is not prematurely liquidated, and the asset value of the firm V_T is smaller than the face value of Debt plus last coupon but greater than the face value of Debt plus liquidation cost;
3) V_T – LC if the firm is not prematurely liquidated, and V_T is smaller than the face value of SD plus liquidation cost;
4) face value if the firm is prematurely liquidated, and V_t is greater than the face value of SD plus liquidation cost;
5) V_t – LC if the firm is liquidated, and V_t is less than the face value of SD plus liquidation cost.

In summary, I define the possible values of SD, to be:

$$SD_t(V_t, T, g_t^K) = \begin{cases} SD & \text{if } V_T \geq F + \text{coupon and } \theta^k > T \\ SD & \text{if } V_T < F + \text{coupon, } V_T \geq SD + LC \text{ and } \theta^k > T \\ V_T - LC & \text{if } V_T < F + \text{coupon, } V_T < SD + LC \text{ and } \theta^k > T \\ SD & \text{if } V_t \geq SD + LC \text{ and } \theta^k \leq T \\ V_t - LC & \text{if } V_t < SD + LC \text{ and } \theta^k \leq T \end{cases} \tag{3.10}$$

The expression (3.10) may be rewritten as:

$$SD_t(V_t,T,g_t^K) = E^Q_t[SD*e^{-r(T-t)}*1_{\{\theta^k>T\}}*1_{\{VT\geq F\}}]+E^Q_t[SD*^{e-r(T-t)}*1_{\{\theta^k>T\}}*1_{\{VT<F+c\}}*1_{\{VT\geq SD+LC\}}$$

$$+E^Q_t[(V_T-LC)*e^{-r(T-t)}*1_{\{\theta^k>T\}}*1_{\{VT<F+coup\}}*1_{\{VT<SD+LC\}}]+E^q_t[SD*e^{-r(\theta k-t)}*1_{\{\theta^k\leq T\}}*1_{\{Vt\geq SD+LC\}}]$$

$$+E^Q_t[(V_t-LC)*e^{-r(\theta k-t)}*1_{\{\theta^k\leq T\}}*1_{\{Vt<SD+LC\}}] \tag{3.11}$$

where:

1) $1_{\{VT<F+c\}}$ is an indicator function equal to 1 if V_T is smaller than F + last coupon, and equal to 0 otherwise;

3.2.3 Junior Debt

The Value of the junior debt is equal to:

1) face value plus PVC (present Value of Coupons) if the firm is not prematurely liquidated, and the asset value of the firm V_T is bigger than the face value of SD plus the last coupon;
2) (V_T – SD – LC + PVC – last coupon) if the firm is not prematurely liquidated, and V_T is smaller than the face value of Debt but greater than the face value of SD plus liquidation cost;
3) PVC minus the last coupon if the firm is not prematurely liquidated, and V_T is smaller than the face value of SD plus liquidation cost
4) (V_t – SD – LC + *Σ Coupons$_t$*) if the firm is liquidated, and V_t is bigger than the face value of SD plus the liquidation cost;
5) *Σ Coupons$_t$* if the firm is liquidated, and V_t is smaller than the face value of SD plus the liquidation cost.

In summary, I define the possible values of JD, to be:

$$JD_t(V_t, T, g_t^K) = \begin{cases} JD + PVC & \text{if } V_T \geq F\text{+coupon and } \theta^k > T \\ (V_T - SD - LC + PVC - \text{coup.}) & \text{if } V_T < F\text{+coupon, } V_T \geq SD\text{+}LC \text{ and } \theta^k > T \\ PVC - \text{coupon} & \text{if } V_T < F\text{+coupon, } V_T < SD\text{+}LC \text{ and } \theta^k > T \\ (V_t - SD - LC + \Sigma\, Coupons_t) & \text{if } V_t \geq SD + LC \text{ and } \theta^k \leq T \\ \Sigma\, Coupons_t & \text{if } Vt < SD + LC \text{ and } \theta k < T \end{cases} \quad (3.12)$$

The expression (3.12) may be rewritten as:

$$JD_t(V_t,T,g_t^K) = E^Q_t[((JD)*e^{-r(T-t)}+PVC)*1_{\{\theta^k > T\}}*1_{\{VT\geq F+c\}}+E^Q_t[(((V_T-SD-LC-coupon)*e^{-r(T-t)}) + PVC)*1_{\{\theta^k > T\}}*1_{\{VT< F+c\}}*1_{\{VT \geq SD+LC\}}]+E^Q_t[((coupon)*e^{-r(T-t)}+PVC)1_{\{\theta^k \leq T\}}*1_{\{VT< F+c\}}*1_{\{VT< SD+LC\}}]+E^Q_t[(V_t - SD - LC + \Sigma\ Coupons_t)*e^{-r(\theta k-t)}*1_{\{\theta^k \leq T\}} *1_{\{Vt \geq SD+LC\}}]+E^Q_t[(\Sigma\ Coupons_t)*e^{-r(\theta k-t)}*1_{\{\theta^k \leq T\}} *1_{\{Vt < SD+LC\}} \quad (3.13)$$

where:

1) $1_{\{VT\geq F+c\}}$ is an indicator function equal to 1 if V_T is smaller than F + last coupon, and equal to 0 otherwise;

In table 3.1, I report the payoffs given to the different stakeholders.

Tab. 3.1 – ***Payoffs***

	Bond repayment or Liquidation at the end (T)			Liquidation before T	
PAYOFFS TO	**$V_{T} \geq$ F + Coupon and $\theta^k > T$**	**$V_T <$ F + Coupon, VT $\geq$ SD+LC and $\theta^k > T$**	**$V_T <$ F + Coupon, VT $<$ SD+LC and $\theta^k > T$**	**F $\geq$ Vt $\geq$ SD+LC and $\theta^k \leq T$**	**Vt $<$ SD+LC and $\theta^k \leq T$**
EQUITYHOLDERS	$V_T - F$	0	0	0	0
SENIOR BONDHOLDERS	SD	SD	V_T - LC	SD	V_t - LC
JUNIOR BONDHOLDERS	JD + PVC	(V_T – SD – LC + PVC - $coupon_T$)	PVC - $coupon_T$	(V_t – SD – LC + Σ $Coupons_t$)	Σ $Coupons_t$
TOTAL	**V_T + PVC**	**(V_T - LC) + (PVC -couponT)**	**(V_T - LC) + (PVC -couponT)**	**V_t - LC + Σ $Coupons_t$**	**V_t - LC + Σ $Coupons_t$**

3.3 Benchmark

Also, in this case, I am able to compute: default probabilities over the time interval (0,T]; the implied recovery rate RR = (1-LC)*V_1 / F; the yield to maturity and the yield spread of the two different obligations. I use this value in order to test the consistency of my model.

The outcomes, as pointed out below, are very satisfying. Indeed, I get a recovery rate and a yield spread in line with those empirically found and a trend of cumulative default probabilities very similar to empirical cumulative DPs with a perfect coincidence at the end of the process (at T = 10 years).

In addition, my results are better than those found using an adaption of old models (hit and default models).

3.4 Numerical implementation

I use, again, a Monte-Carlo simulation approach that considers 150,000 sample paths for calculating bond prices, equity value, cumulative default probabilities, and recovery rate. The solution found are stable up to the second digit.

3.5 Parameters' choice

I assume that firm's capital structure is constituted by ordinary stock, senior debt and junior debt. Both the obligations have a maturity of 10 years. Junior Debt is a Coupon Bond, while Senior Debt is a Zero Coupon Bond.

I utilize the same real value used in Chapter 2 for the parameters present in the Model. The two main differences are: the Coupon C is determined such that a time 0, Junior Debt is priced at par; and every time there is a coupons' payment the value of Asset V_T jump down and the dynamic restart from (V_T – C).

In tables 3.2, I summarize the parameters choice:

Tab. 3.2 – *Parameters*

Parameter	Symbol	Value Assumed
Time to Maturity	T	10
Pre-defined grace period	d	1
Percentage of SD on Total Debt	γ	0,5
Default free interest rate	r	0,06
Volatility of the asset of the firm	σ	0,29
Liquidation Cost	LC	0,2
Threshold's parameter	ω	0,85
Dividend yield	δ	0
Initial Value of Assets	V_0	100
Leverage Ratio	LR	0,657
Debt Face Value	F	65,7
Debt Coupon rate	i	0,1078
dt=0.01;	dt	0,01
paths		150.000

3.6 Results

Some results are reported in table 3.3, while considerations about cumulative DPs, recovery rate and yield spreads are presented in the next sections.

However, I point out, here, that the probability that the Senior Bond is fully paid is 76.9%, while for the Junior Bond this probability is equal to 53.28%. Also, in this framework, the Junior Bondholders pay more often the liquidation cost.

Tab. 3.3 – *Some results*

Liquidation Cost	2,65%
Average V at the default	41,95%
Yield Senior	5,49%
Yield Junior	11,07%
Cumulative default pr at T	46,72%
Cumulative default pr before T	38,13%
Pr. Senior paid in full	76,90%
Pr. Junior paid in full	53,28%
Recovery Rate	51,08%

3.6.1 Cumulative DPS

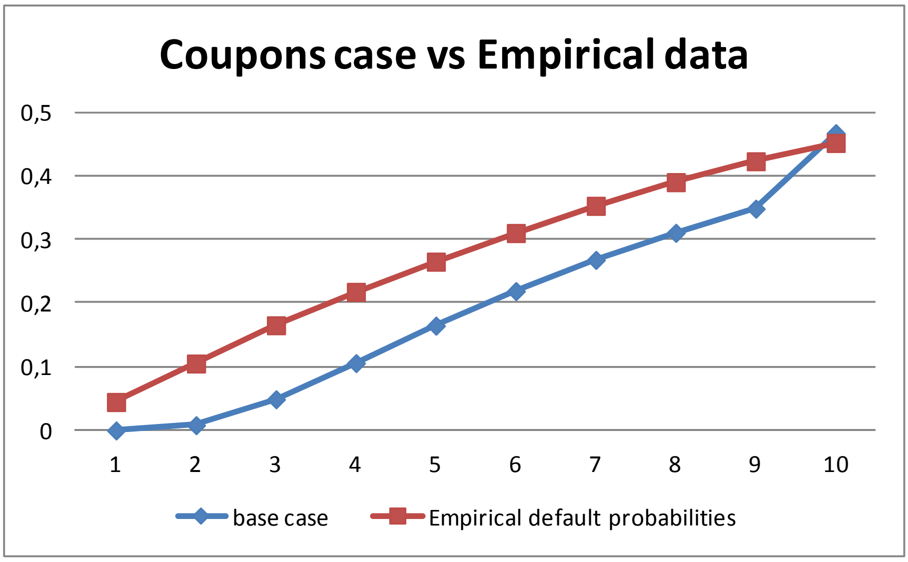
In order to test my hypothesis, I calculate the cumulative default probabilities present implicitly in the model and I compare these data with the cumulative DPs observed in literature (Moody's 2011).

Matching my data with Moody's data, it is evident that my DPs are really close to empirical data and that at the end (at T =10 years) these probabilities are coincident.

The introduction of coupons is a crucial assumption. Indeed, my results have an impressive improvement when I introduce this hypothesis.

The light difference depends on the fact that I set my model on US market, while Moody's data are global data. In particular, it is important to point out that my d (grace period) is equal to 1 as in US, but we don't have any "global" reference.

Figure 3.1 – ***Cumulative default probabilities***



3.6.2 Recovery Rate

The average Asset Value in case of default is equal to 41.95%, while the recovery rate (on average) is 51.05%. This value is in line with the value empirically found in literature 51% - 52% (as above highlighted).

The presence of coupon and the consequent growth of DPs produce a reduction of my recovery rate versus those found in chapter 2.

3.6.3 Yield Spread

I find that the yield to maturity for the senior bond is equal to 5.49%, while for the junior bond this rate is equal to 11.07%.

Therefore, the credit spread for junior bond (5.07%) is very close to the value that is empirically found in literature (4.7%).

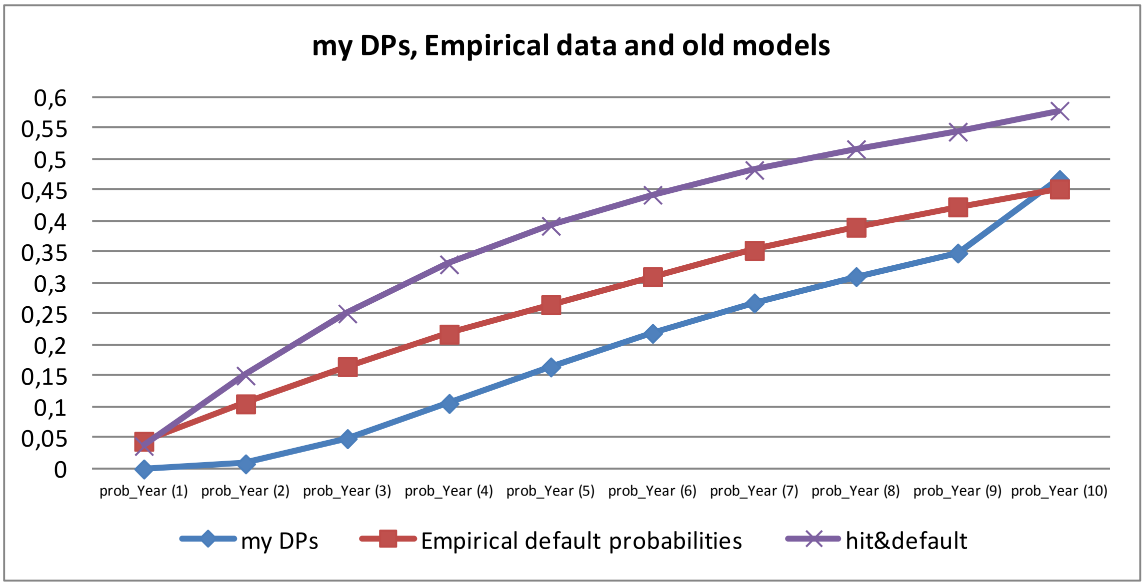
In practice, the values (for recovery rate and yield spreads) that I found implicitly from my model are equal to the values found in a couple of empirical research (Huang and Huang, 2003 and Altman and Kishore, 2006 etc.) and I get this value without any type of calibration.

3.6.4 Empirical cumulative DPs vs. old models

I show below a graph where:

- The blue line represents my results;
- The violet line represents the DPs computed through an adaption of my model in which I assume that the grace period is equal to 0.01 (such as in old models) and that there aren't Liquidation costs;
- The red line represents empirical data.

Figure 3.2 – ***Cumulative default probabilities***



Analyzing this figure, I could point out that the adaption of old models (g = 0.01 with two type of bonds, of which one Junior with coupons) give back results very different from Moody's data.

In practice, if I consider an adaption of old models with two type of debt and coupons, I get results very far from empirical data. This means that the presence of grace period represent the crucial assumption that allow me to match empirical data.

This fact is confirmed if I compare the recovery rate and yield spread computed through the hit and default model with those empirically found. Indeed, the recovery rate calculated empirically is equal to 52%, while in the hit and default model this parameter is equal to 83.8%; and the yield spread equal to 3.6% is far from the 4.70% reported by the Lehman bond index.

Even if I consider an hit and default models (old models) with LC, I have the same results. In fact, I get a recovery rate equal to 67.5%, that is still very different from the empirical value of 52%.

I could, therefore, assert that old models are not able to represent real world. Indeed, I showed in chapter 2 that without coupons these models do not fit recovery rate and yield spreads and, in addition, I highlight in this chapter that, if I include coupons, the main results do not change and on the contrary cumulative DPs become very different from empirical data.

3.7 Hypothetical country

Here, I present a variation of my model, in which I change just one variable and the coupon rate. In practice, in this particular setting I consider a grace period g = 0.65 (versus 1.0 in the base case) and, in addition, I vary the coupon rate in order to satisfy the hypothesis that the junior debt is priced at par.

Again, I choose the new level of the grace period equal to (more or less) the average between US data (1 year) and Swedish data (2.5 months).

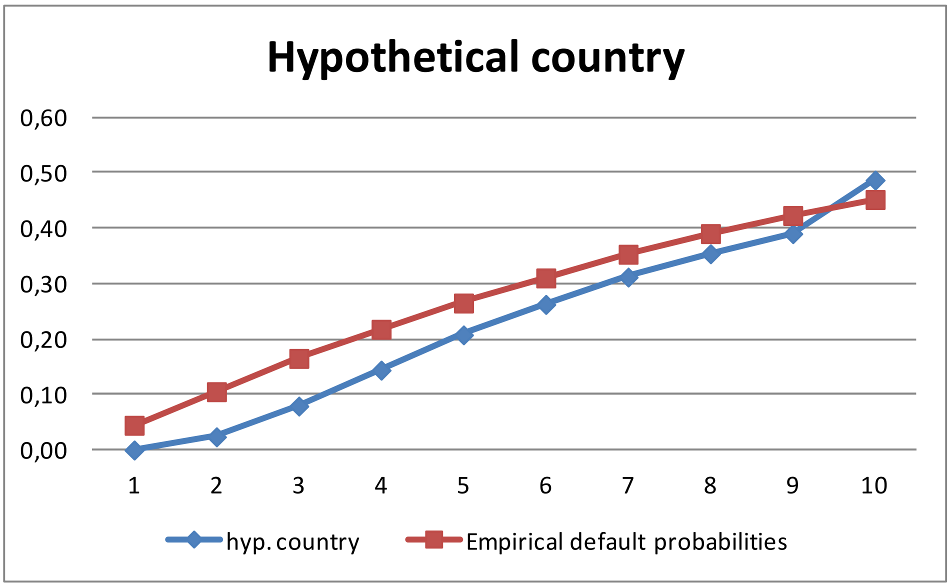
The outcomes are very satisfying due that new results match really well the empirical data.

This confirm that the difference between my data and empirical data is due mainly to the fact that my model is built on US data while empirical data (cumulative default probabilities) are global.

Indeed, it is important to remind that I don't have the global value for grace period, but I know that is equal to 1 in US.

I explain the light difference present at the end of the period between my data and real data such as a difference dependent on the fact that in my model it in not possible to renew the debt at the expiration date, while in the real world it is very likely to roll over the debt.

Figure 3.3 – ***Cumulative default probabilities***



3.8 Sensitivity analysis

I now consider how my results are sensitive to the choices of my parameters. I start from the base case, and then change various parameter choices in order to study the impact of these parameters on the asset value, recovery rate and cumulative DPs.

3.8.1 Grace period (g)

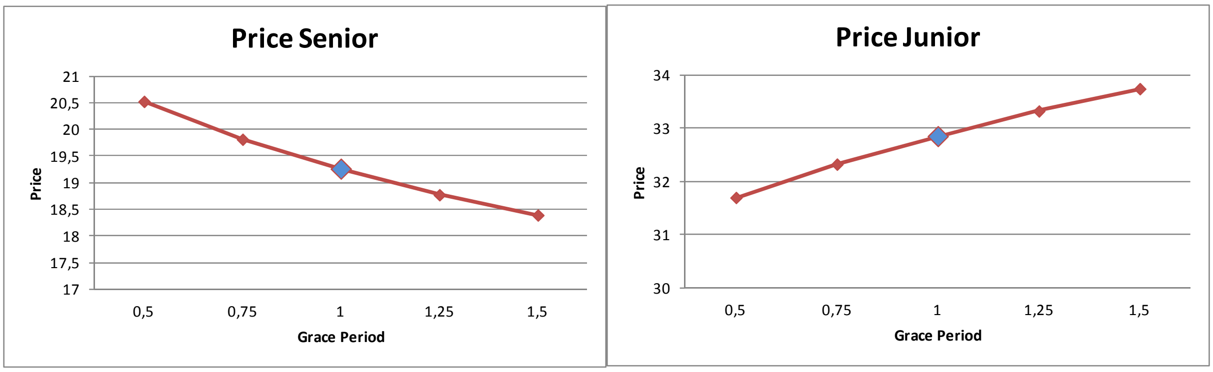
Here, I present a sensitivity analysis in order to study the impact of the length of the initial grace period on the price of the assets, the recovery rate and the default probabilities.

I remind that an increase of the grace period extends the time in bankruptcy and decreases the capacity of the senior bondholders to extract value upon default (cfr. Figure 3.4).

Indeed, in this case: 1) the probability that the senior bonds are fully paid decrease; 2) cumulative default probabilities go down (it is more likely that the asset value rebound up the threshold level; 3) the first effect prevail on the second effect.

Conversely, the Value of the Junior Bond (cfr. Figure 3.4) and Equity price (cfr. Figure 3.5) go up, because of the decrease of cumulative DPs.

Figure 3.4 – ***Sensitivity analysis: grace period***



It is less likely the that the senior bonds are fully paid. This means that the Value of Asset (V) at the Default goes down and consequently (cfr. Figures 3.5 and 3.6) the liquidation costs and recovery rate decrease (remind that the LCs are a percentage of V and that recovery rate is equal to (1-LC)*V(at_default)/F)).

Figure 3.5 – ***Sensitivity analysis: grace period***

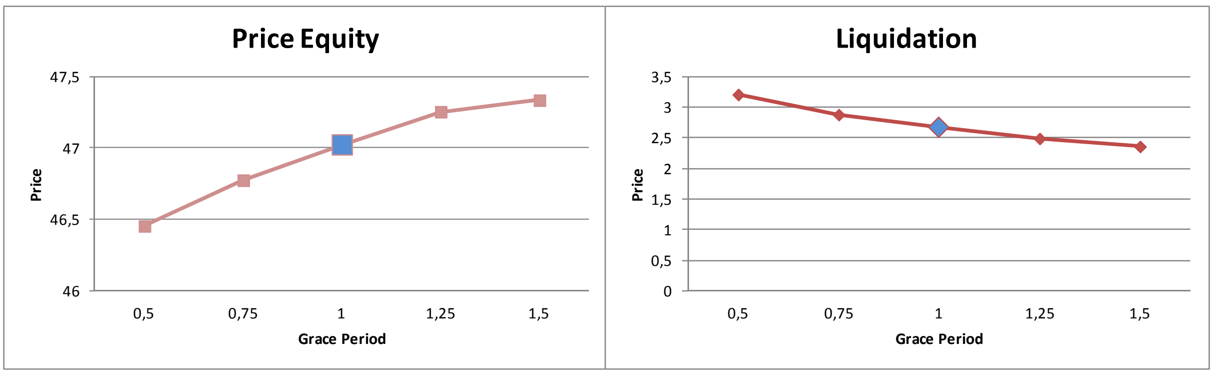
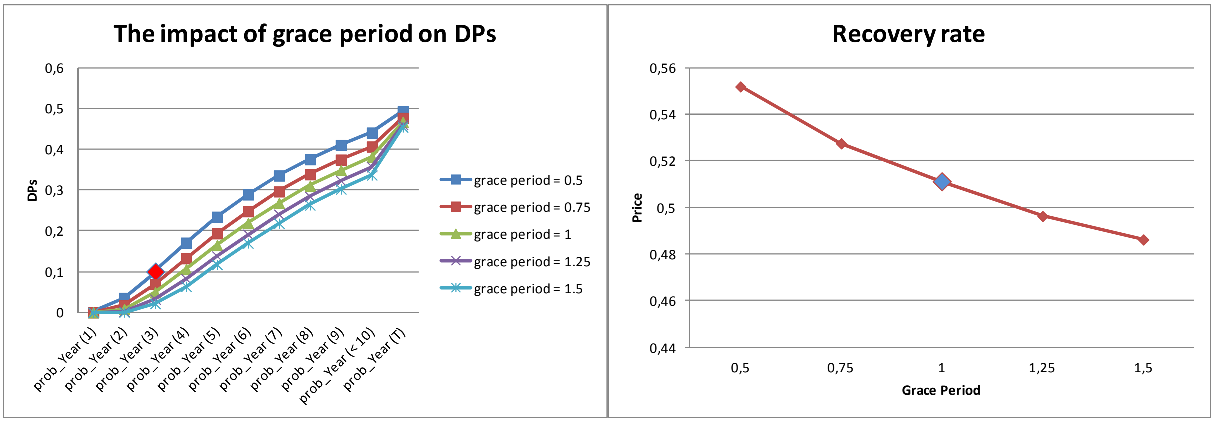


Figure 3.6 – ***Sensitivity analysis: grace period***



3.8.2 Volatility (sigma)

From Figure 3.8 it is evident that liquidation costs and equity price increase when asset volatility goes up, while from Figure 3.7 it is clear that the price of the junior bond steps down.

The trend of the value of equity is expected. Indeed, if there is not a bankruptcy then the value of the asset (V) could be very high at the end of the process.

At the same time, when the volatility of the asset goes up the model records an increase of the DPs. This event produces an increase of liquidation costs and a reduction of the value of the junior bond. It is more likely that the junior bondholders are not fully paid and consequently they have to sustain more often the liquidation costs.

Whereas the value of the senior bond (cfr. Figure 3.7) steps up until the volatility is equal to 0.29. From that point, the probability that the senior bondholders are paid in full decrease largely and consequently the value of this type of debt starts to decrease.

Figure 3.7 – ***Sensitivity analysis: volatility***

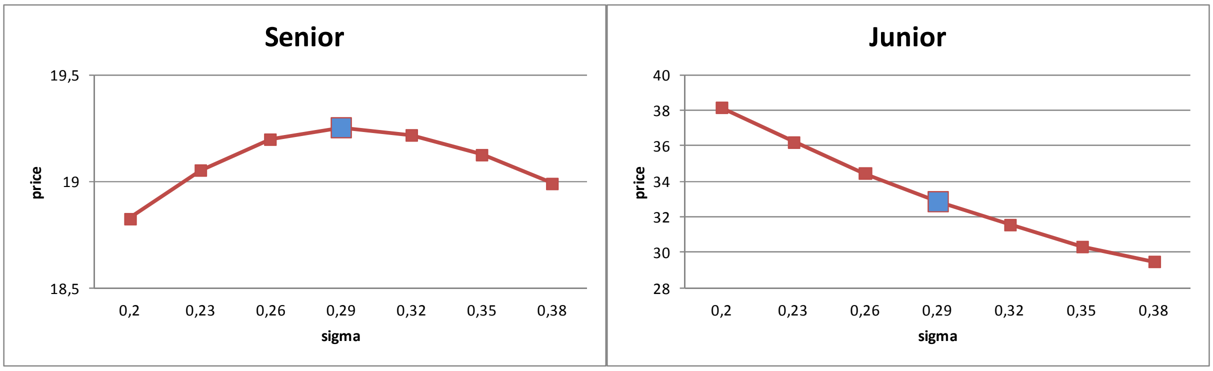
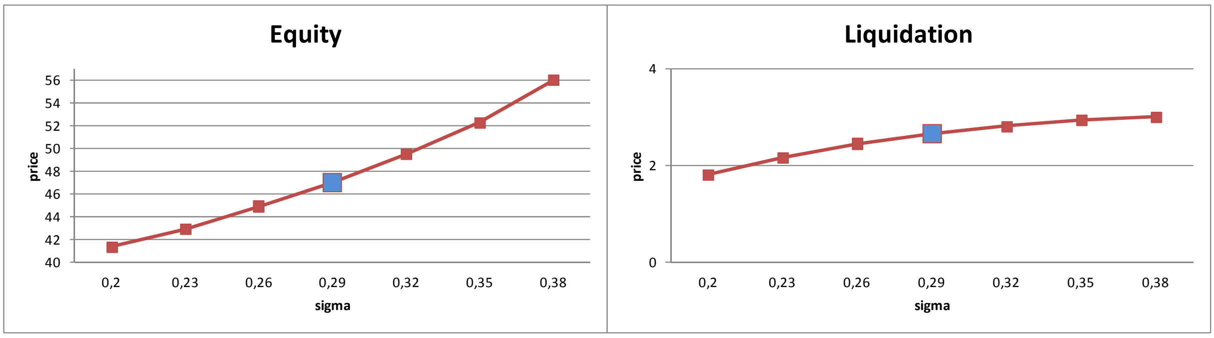
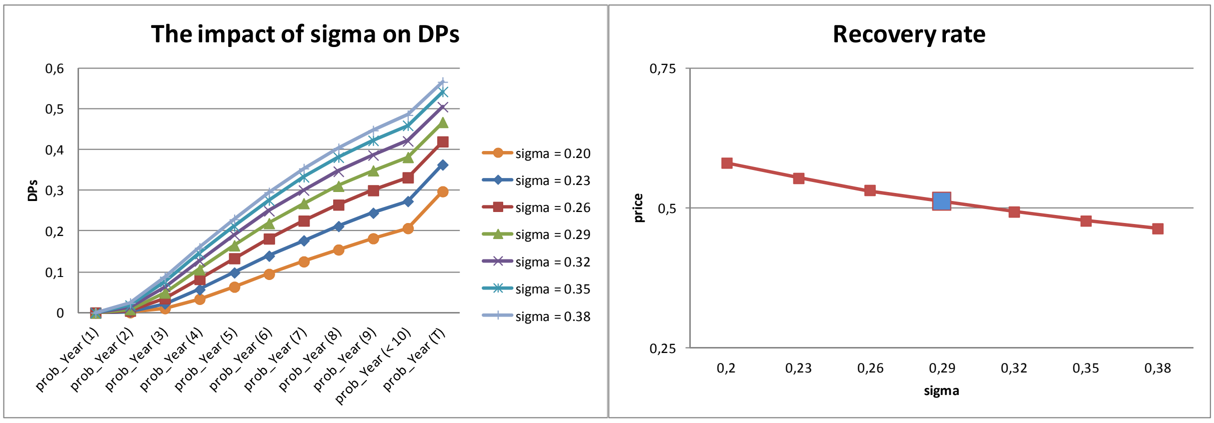


Figure 3.8 – ***Sensitivity analysis: volatility***



The trend of the recovery rate, finally, depends on the evolution of: 1) liquidation costs; 2) cumulative DPs.

Figure 3.9 – *Sensitivity analysis: volatility*

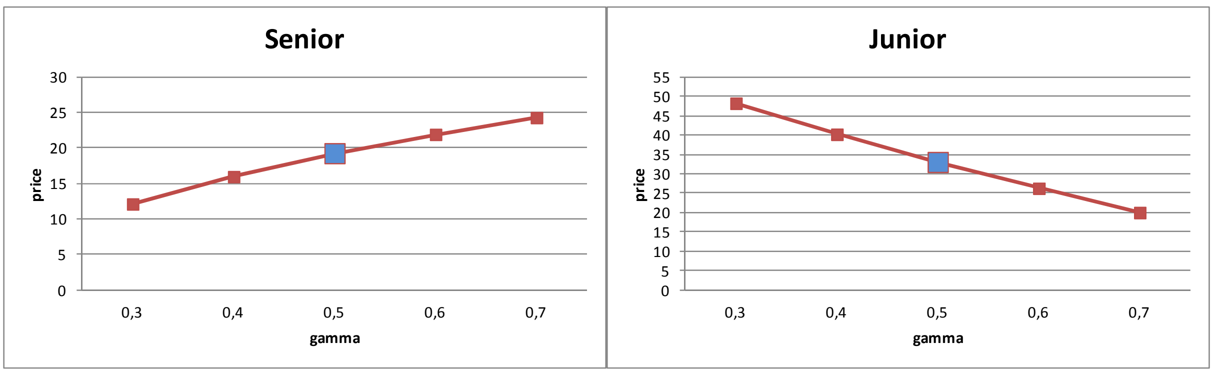


3.8.3 The percentage of Senior Bond on the Total Debt

It is straightforward that the value of the two different type of Bond depends on the percentage gamma of senior bond on the total debt.

Equity value goes up when the value of gamma increase (cfr. Figure 3.11). This pattern is dependent on the fact that the equityholders as residual claimants benefit from the reduction of coupons payment. Because of this reduction, cumulative default probabilities[4] and Liquidation costs go down while recovery rate increase slowly (cfr. Figures 3.11 and 3.12).

Figure 3.10 – *Sensitivity analysis: The percentage of Senior Bond on the Total Debt*



[4] In this framework when I change gamma, cumulative DPs are not constant as in Chapter 2.

Figure 3.11 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***

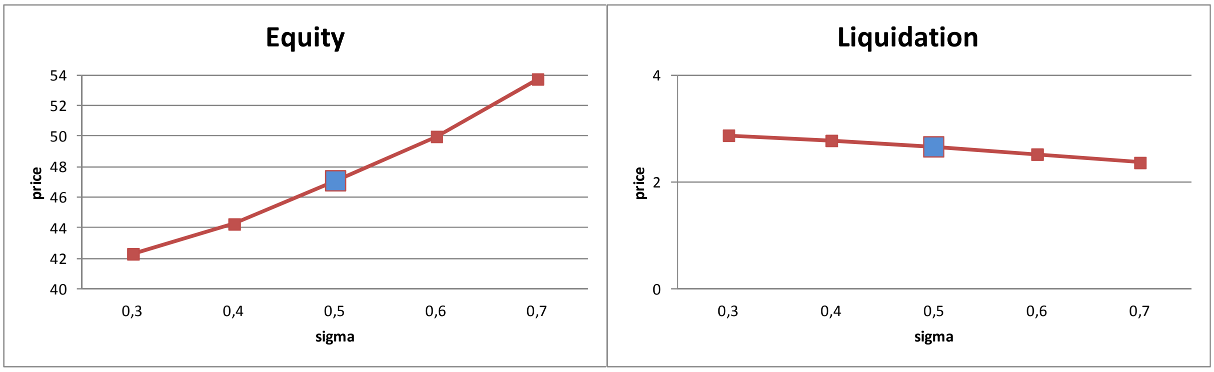
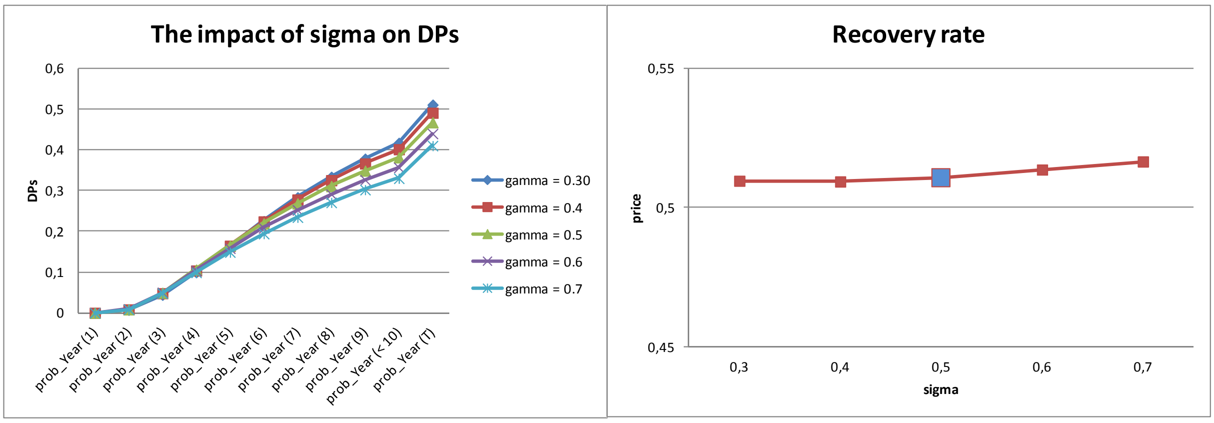


Figure 3.12 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***



3.8.4 Dividend yield (delta)

When a firm pays more dividend, the dynamic of the asset value (Brownian process) is more stable (V increases more slowly). It is more likely that the company fails and, therefore, the equity price decreases and Liquidation costs increase (cfr. Figure 3.14).

At the same time, the price of Senior Bond goes up, while the price of the junior bond steps down (cfr. Figure 3.13). It depends on the fact that the new process produces a decrease (very large) of the probability that junior bondholders is fully paid. Therefore, it is more likely that the junior bondholders have to sustain the liquidation costs (and then these costs step up).

The evolution of the value of the senior bond derives from the trend of cumulative default probabilities above specified and from the fact that the Value at the default that is constant thanks to the stability of the process.

Figure 3.13 – ***Sensitivity analysis: dividend yield***

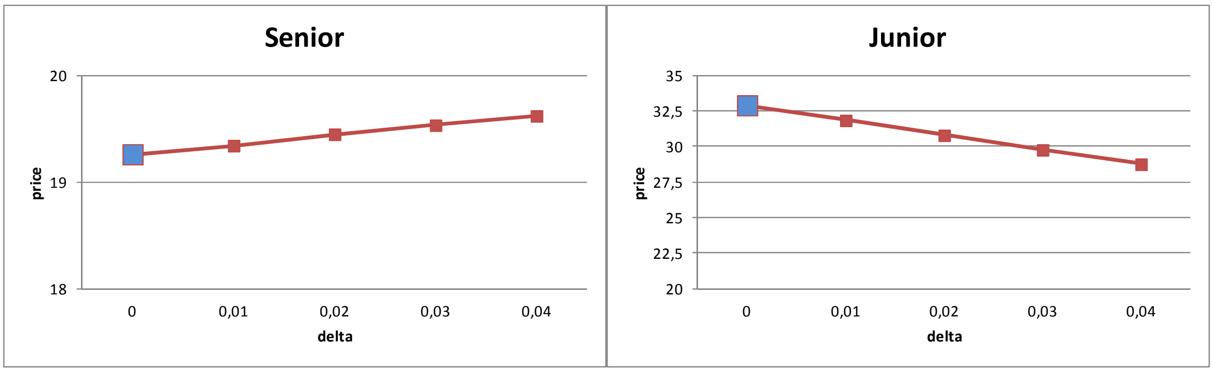
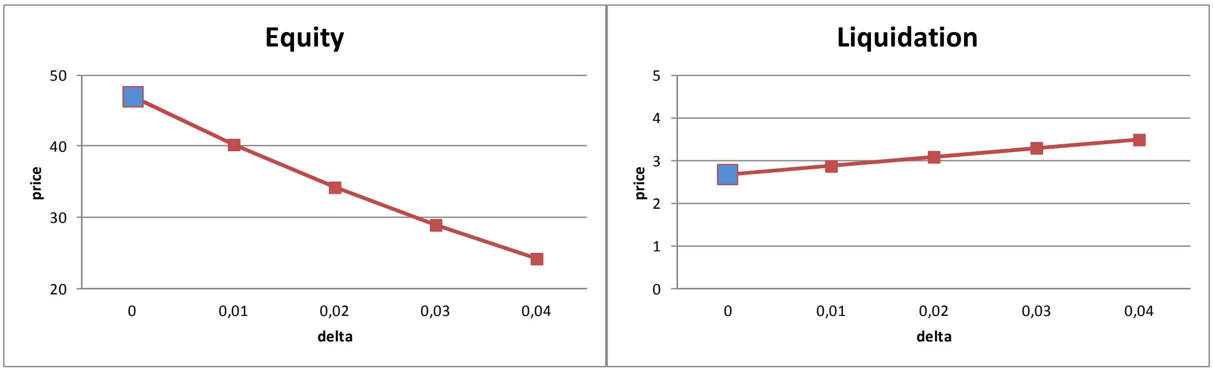
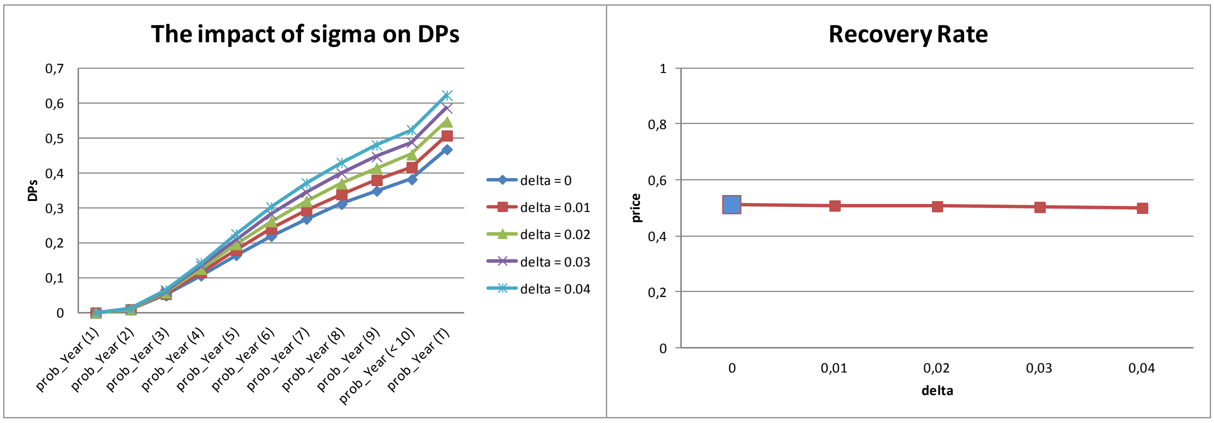


Figure 3.14 – ***Sensitivity analysis: dividend yield***



Recovery Rate is very stable, because the growth of the liquidation costs and DPs are offset by the fact that the process is less variable (cfr. Figure 3.15).

Figure 3.15 – Sensitivity analysis: dividend yield



3.8.5 Liquidation Costs

When there is liquidation, junior bondholders or senior bondholders have to sustain the liquidation cost. Therefore, the impact of variation in liquidation cost influence only the value of these two assets.

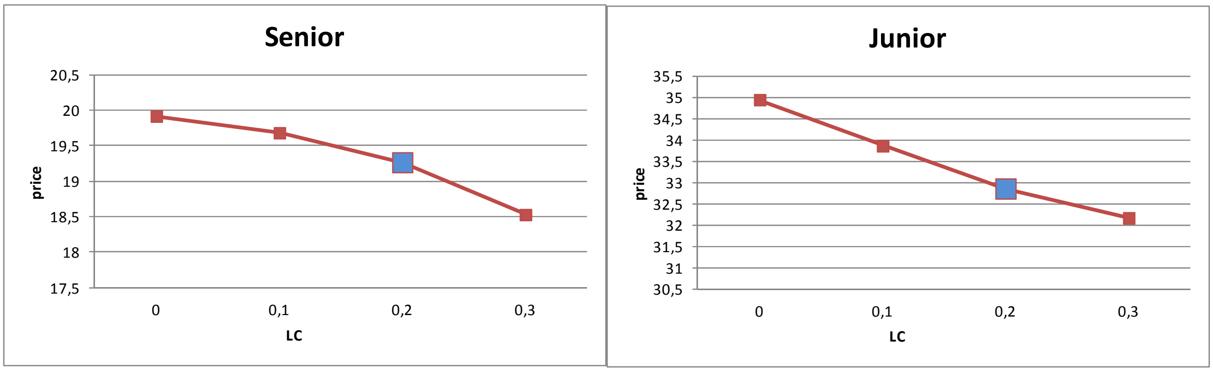
Here, as in Chapter 2, indeed, there is no impact on the equity price. In case of liquidation the equityholders don't receive anything and, at the same time, they don't have to pay anything due to limited liability.

From Figure 3.16 it is clear that Junior debt price is more affected by the variation of liquidation cost when this parameter increases gradually from 0% to 20%.

Conversely, if the liquidation cost goes up from 20% to 30%, the impact is heavier on the price of the senior debt. Indeed, when the liquidation cost becomes bigger and bigger, it is more likely that:

$V_t - LC \leq SD$ (the senior debtholders have to sustain these costs).

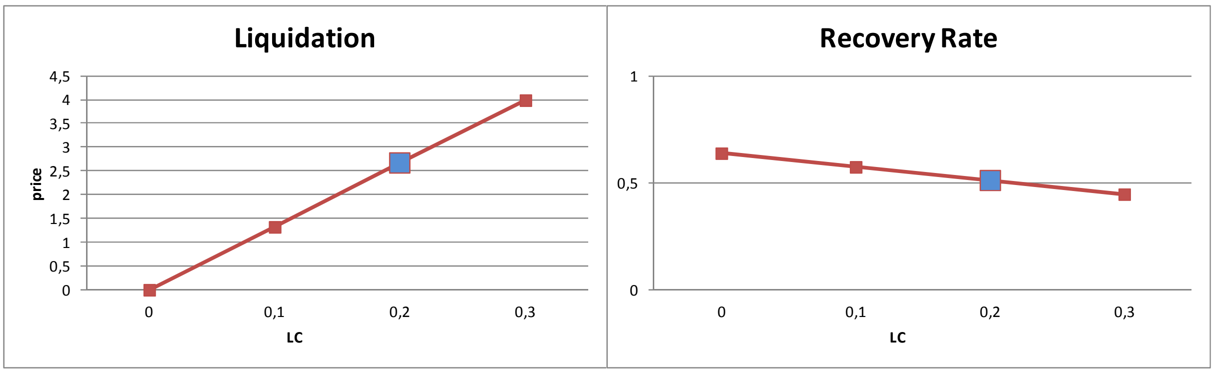
Figure 3.16 – ***Sensitivity analysis: liquidation cost***



Recovery Rate, on the other hand, decrease when the Liquidation costs increase. Indeed, Recovery Rate is equal to ((1-LC)*V(at_default)/F).

Finally, there are not variation of the cumulative Default Probabilities, because there is liquidation when the firm is already on default.

Figure 3.17 – ***Sensitivity analysis: liquidation cost***

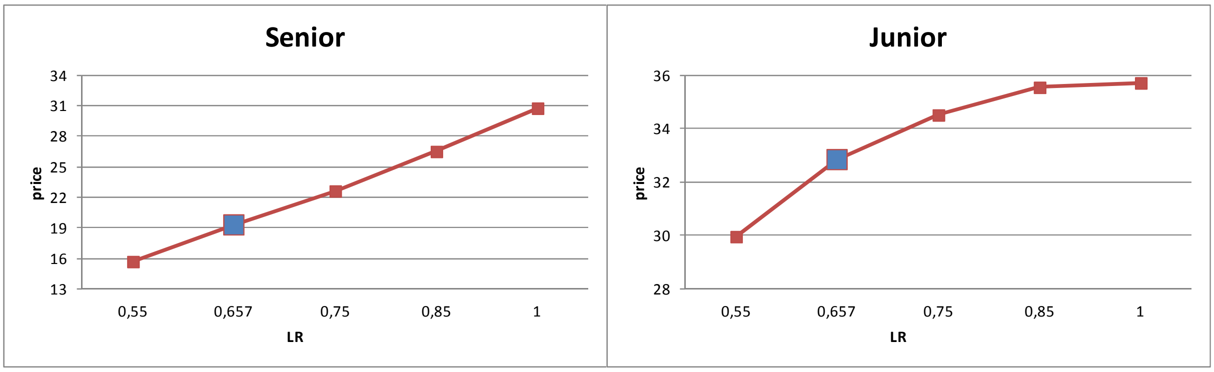


3.8.6 Leverage Ratio

When there is an increase of the value of the Total Debt, the price of the senior and junior bonds rise proportionally (cfr. Figure 3.18).

Clearly, the price of the junior bond goes up until the effect of the total debt's growth is offset by the increase of the Liquidation costs and of the cumulative Default Probabilities.

Figure 3.18 – ***Sensitivity analysis: leverage ratio***



I find, as expected, that the equity price goes down (cfr. Figure 3.19), while the cumulative Default Probabilities increase (cfr. Figure 3.20). In fact, when a firm has more debt, it is easier to have default.

Figure 3.19– ***Sensitivity analysis: leverage ratio***

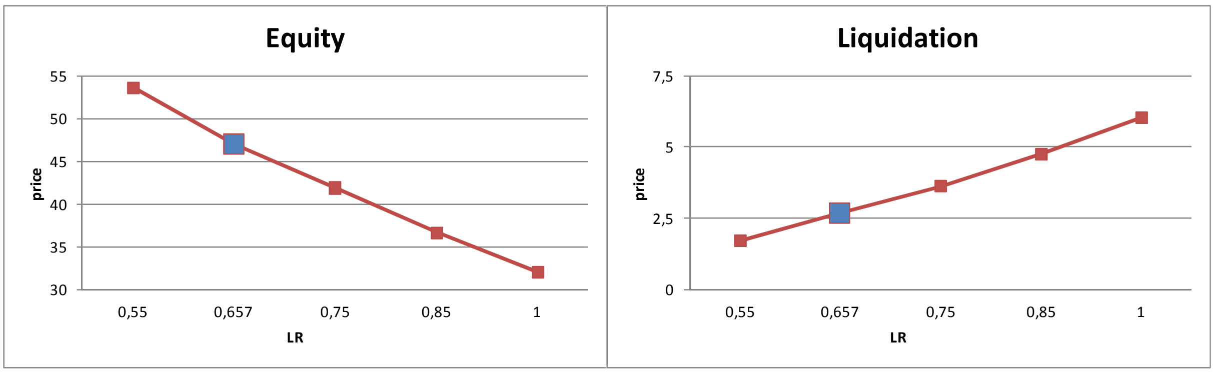
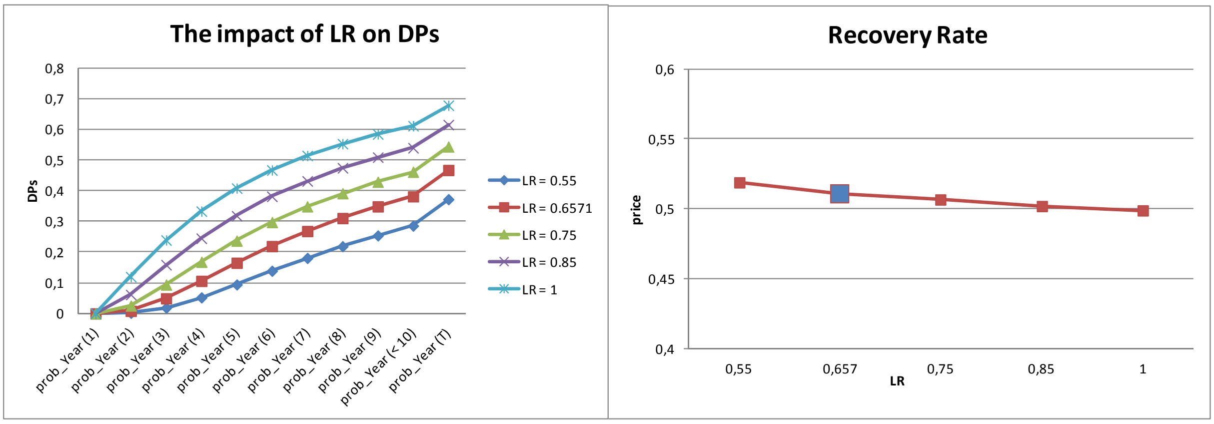


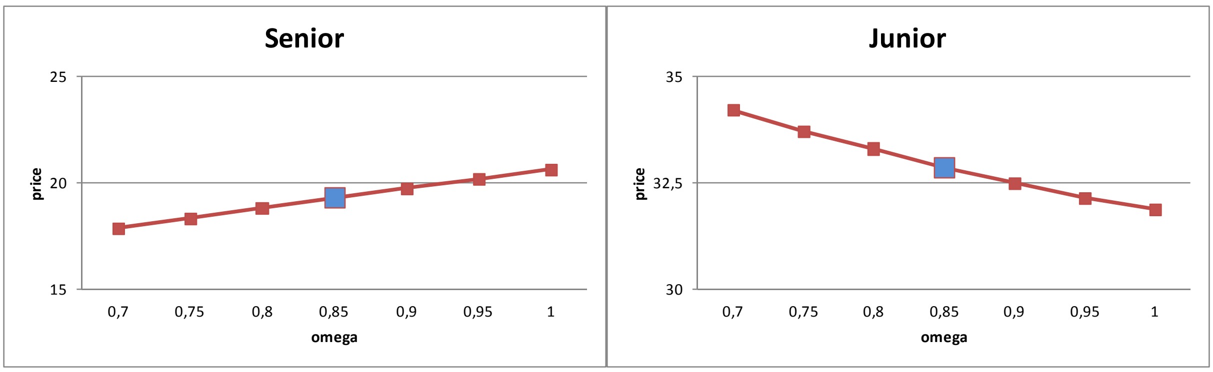
Figure 3.20 – Sensitivity analysis: leverage ratio



3.8.7 Threshold's parameter (Omega)

The Price of the Senior Bond (cfr. Figure 3.21) and the Liquidation Costs (cfr. Figure 3.22) go up when there is a positive variation of omega (Threshold's parameter). This results depend on the fact that when omega increases the barrier is higher. The Junior Bond (cfr. Figure 3.21), conversely, decreases when omega passes from 0.7 to 1 because liquidation costs' rise offset the benefit deriving from the barrier's variation.

Figure 3.21 – ***Sensitivity analysis: Threshold's parameter***



The DPs (cfr. Figure 3.23) step up when the threshold barrier is higher and consequently the value of equity goes down (cfr. Figure 3.22).

From Figure 3.23 it also clear that the recovery rate, on the other hand, increases but the variation is modest because of the liquidation costs' trend.

Figure 3.22 – ***Sensitivity analysis: Threshold's parameter***

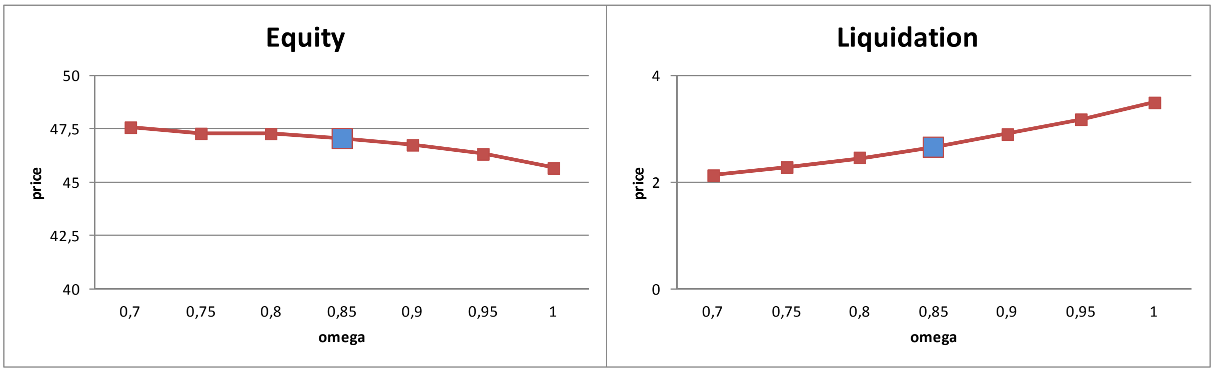
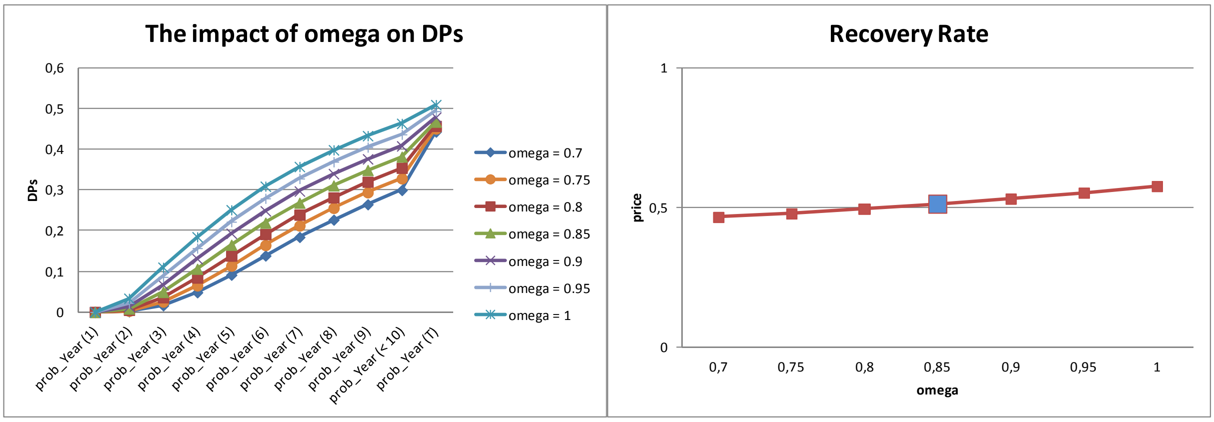


Figure 3.23 – Sensitivity analysis: Threshold's parameter

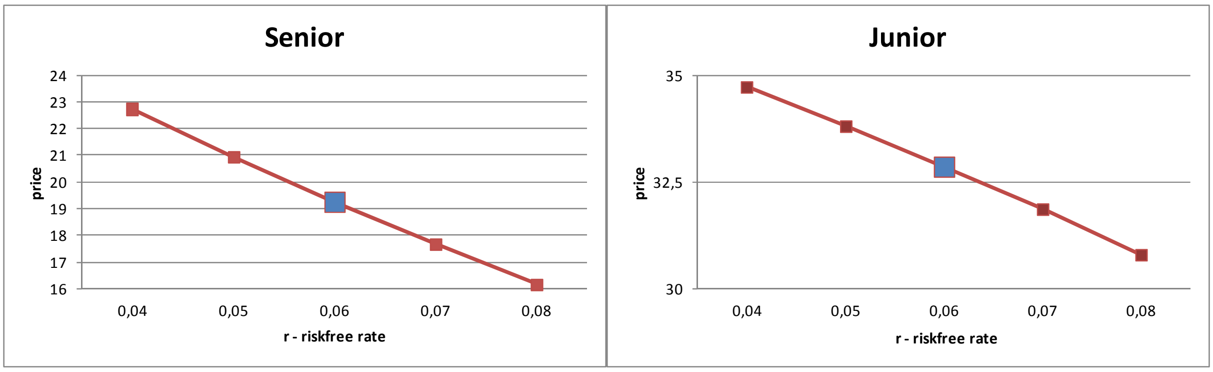


3.8.8 Free-risk Rate (r)

When the risk-free rate increases, there are two effects: the asset Value (V) appreciates with an higher instantaneous intensity and, consequently, the cumulative default probabilities decrease; and the discount rate became bigger and bigger.

The second effect is the principal effect on the value of bonds, as shown in figure 3.24, while the first one is prevalent on the Equity value (cfr. Figure 3.25).

Figure 3.24 – ***Sensitivity analysis: Free-risk Rate***



Liquidation costs decline, given that the cumulative default probabilities go down, while and recovery rate increase slightly thanks to the Asset Value dynamic (cfr. Figure 3.26).

Figure 3.25 – ***Sensitivity analysis: Free-risk Rate***

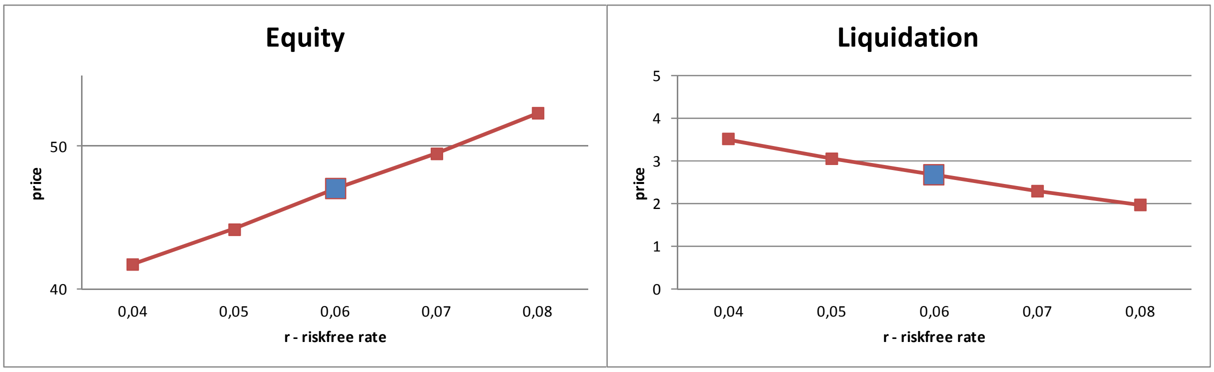
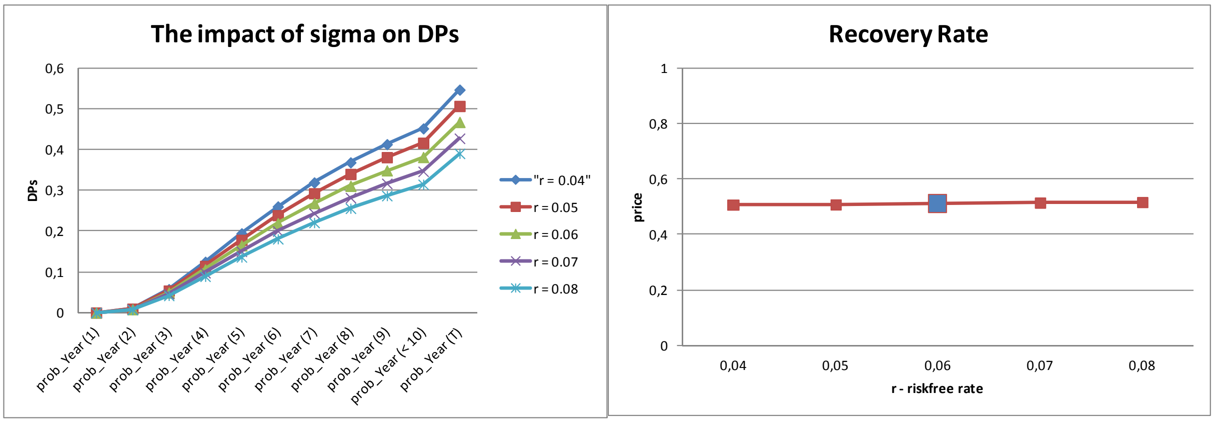


Figure 3.26 – ***Sensitivity analysis: Free-risk Rate***



3.9 Conclusion

In this chapter I present an evolution of the model built in Chapter 2. In particular, I suppose that one of the two type of debt is a coupon bond (junior debt).

With this change, I try to match better empirical data (Recovery Rate, Yield Spread and Cumulative Default probabilities) for B-rated firms.

The two main differences between this new version of my model and the version presented in Chapter 2 are that: the Coupon C is determined such that a time 0, Junior Debt is priced at par; and every time there is the coupons' payment the value of Asset V_T jump down and the dynamic restart from (V_t – C).

The first results is that: my cumulative DPs present a trend similar to those empirically found; and at the end of the period (after T = 10) these probabilities are coincident with the empirical data.

Second, the average Asset Value in case of default is equal to 41.95 %, while the recovery rate (on average) is 51.08%. This latter value, however, is in line with the value empirically found in literature 51% - 52% (as above highlighted).

Third, yield to maturity for the senior bond is equal to 5.49%, while for the junior bond this rate is equal to 11.07%. Therefore, the credit spread for junior bond (5.07%) is in line with the value that is empirically found in literature (4.7%).

Fourth, the probability that the Senior Bond is fully paid is 76.9%, while for the Junior Bond this probability is equal to 53.28%.

The interesting thing, however, is that I capture these stylized fact with real ingredients . All parameters' value are chosen using data found in a couple of empirical papers.

In addition, when I change the value of the grace period from 1 to 0.65 (equal to the average between the Us value and Swedish value) my DPS fit very well the empirical.

In this latter case (hypothetical country), I demonstrate that the difference between my data and empirical data is due mainly to the fact that my model is built on US data while empirical data (cumulative default probabilities) are global. Indeed, in my base case the key parameter d (grace period) is set equal to 1 (that represent the US value), while empirical data are global data and it is demonstrate that in a couple of Country it is possible to have a liquidation in less than 1 year.

I could assert that my model with coupons match very well empirical data. In particular when I consider the "hypothetical country", varying the level of d (grace period).

It is important, then, to point out that I made a couple of sensitivity analysis and the results are in line with expectations and with the results historically found in literature.

Finally, If I adapt old models to this framework, I get, again, that: cumulative Dps are very different from empirical DPs; the recovery rate calculated empirically is equal to 52%, while in the hit and default model this parameter is equal to 83.8%; the yield spread is equal to 3.6% and it is very far from the 4.70% reported by the Lehman bond index.

This happens, even if I consider an hit and default model with LCs. Indeed, I get a recovery rate equal to 67%, that is very different from the empirical value of 52%.

Therefore, I show again that old models are inefficient and not able to match empirical data. On the contrary, I could affirm that, with my approach, I am able to reproduce really well the real world and that it is crucial the combination of these two ingredients: the presence of the grace period and of coupons.

Appendix A – A Real Option Model with a Senior Coupon Bond

A. Brief Introduction

Here, I present a different adaption of my model in which I assume that the Senior Bond is the coupon bond. In this way, I want to show that the results found in chapter 3 do not depend on the hypothesis that the Junior Bond is the coupon bond.

In other word, I try to demonstrate that it is fundamental the presence of coupons in the model and it is not important which bond is the zero coupon bond and which one is the coupon bond. The outcomes are in line with expectations and confirm that it is crucial the presence of coupons. Indeed, I get a recovery rate and a yield spreads very close to empirical value and my cumulative default probabilities follow the trend of data found empirically.

A.1 Structural Model

As pointed out above, I assume that the Senior Bond is a Coupon Bond, while the Junior Bond is a Zero Coupon Bond. This setting could replicate a real situation in which a firm has a bank debt with coupons or a plain vanilla bond, while junior debt is a debt versus stakeholders.

The Coupon C is determined such that a time 0, Senior Debt is priced at par:

$$P_{SB} = (\gamma) * F \qquad \text{(a.1)}$$

A.2 The valuation of corporate securities

Here, I show the new pay-off of equityholders and of bondholders and I point out that, when there is liquidation, senior bondholders receive the present value of coupons (Σ $Coupons_t$) and this value include the accrued interest.

A.2.1 Equity Value

Also in this framework, in case of liquidation Equity holders, as residual claimants, do not receive anything. Therefore, if there is not liquidation, at debt maturity T, Equity holders receive the maximum between zero and the difference between the firm's asset value V_T and the face value of the total debt (F). The Equity holders pay-off is represented by this equation:

$$S(V_T, T, g_t^k) = \max(V_T - F, 0)*1_{\{\theta^k > t\}} = \begin{cases} V_T - F & \text{if } V_T > F + \text{Coupon and } \theta^k > T \\ 0 & \text{otherwise} \end{cases} \quad (a.2)$$

At any time before the debt maturity, if bankruptcy hasn't occurred, the value of Equity holders claim is given by:

$$S_t(V_t, T, g_t^k) = e^{-r(T-t)} E_t^Q[\max(V_T - F - \text{coupon}, 0)*1_{\{\theta^k > t\}}] \quad (a.3)$$

A.2.2 Senior Debt

The value of the senior debt (SD) is equal to:

- face value plus PVC (present Value of Coupons) if the firm is not prematurely liquidated, and the asset value of the firm V_T is greater than the face value of Debt plus the last coupon;
- face value plus PVC (present Value of Coupons) if the firm is not prematurely liquidated, and the asset value of the firm V_T is greater than the face value of Debt plus liquidation cost and last coupon;
- V_T – LC + PVC – last coupon if the firm is not prematurely liquidated, and V_T is smaller than the face value of SD plus liquidation cost and last coupon;
- face value + $\Sigma\ Coupons_t$ if the firm is liquidated, and V_t is greater than the face value of SD plus liquidation cost;

- $V_t - LC + \Sigma\ Coupons_t$ if the firm is liquidated, and V_t is less than the face value of SD plus liquidation cost.

In summary, I define the possible values of SD, to be:

$$SD_t(V_t, T, g_t^K) = \begin{cases} SD + PVC & \text{if } V_T \geq F + \text{coupon and } \theta^k > T \\ SD + PVC & \text{if } V_T < F + \text{coupon, } V_T \geq SD + LC + \text{coupon and } \theta^k > T \\ V_T - LC + PVC - \text{coup.} & \text{if } V_T < F + \text{coupon, } V_T < SD + LC + \text{coupon and } \theta^k > T \\ SD + \Sigma\ Coup_t & \text{if } V_t \geq SD + LC \text{ and } \theta^k \leq T \\ V_t - LC + \Sigma\ Coup_t & \text{if } V_t < SD + LC \text{ and } \theta^k \leq T \end{cases} \quad (a.4)$$

The expression (a.4) may be rewritten as:

$$SD_t\ (V_t,T,g_t^K) = E^Q_t[(SD*e^{-r(T-t)}+PVC)*1_{\{\theta^k>T\}}*1_{\{VT\geq F\}}]+E^Q_t[(SD*e^{-r(T-t)}+PVC)*1_{\{\theta^k>T\}}* *1_{\{VT<F+c\}}*1_{\{VT\geq SD+LC+c\}}+E^Q_t[((V_T-LC-c)*e^{-r(T-t)})+PVC)*1_{\{\theta^k>T\}}*1_{\{VT<F+c\}}*1_{\{VT<SD+LC\}}]+E^q_t[(SD + \Sigma\ Coup_t)*e^{-r(\theta k\ -t)}*1_{\{\theta^k\leq T\}}*1_{\{Vt\geq SD+LC\}}] +E^Q_t\ [(V_t - LC + \Sigma\ Coup_t\ )*e^{-r(\theta k\ -t)}*1_{\{\theta^k\leq T\}}*1_{\{Vt < SD+LC\}}] \quad (a.5)$$

where:

1) $1_{\{VT \geq SD + LC + c.\}}$ is an indicator function equal to 1 if V_T is greater than SD + LC + last coupon, and equal to 0 otherwise;

2) $1_{\{VT < SD + LC + c.\}}$ is an indicator function equal to 1 if V_T is smaller than SD + LC, and equal to 0 otherwise.

A.2.3 Junior Debt

The Value of the junior debt is equal to:

- face value if the firm is not prematurely liquidated, and the asset value of the firm V_T is bigger than the face value of TD plus the last coupon;

- $(V_T - SD - LC - coupon)$ if the firm is not prematurely liquidated, and V_T is smaller than the face value of Debt but greater than the face value of SD plus liquidation cost and last coupon;
- 0 if the firm is not prematurely liquidated, and V_T is smaller than the face value of SD plus liquidation cost and last coupon;
- $(V_t - SD - LC)$ if the firm is liquidated, and V_t is bigger than the face value of SD plus the liquidation cost and accrued interest;
- 0 if the firm is liquidated, and V_t is smaller than the face value of SD plus the liquidation cost and accrued interest.

In summary, I define the possible values of JD, to be:

$$JD_t(V_t, T, g_t^K) = \begin{cases} JD & \text{if } V_T \geq F+coupon \text{ and } \theta^k > T \\ (V_T - SD - LC - coupon) & \text{if } V_T < F+coupon,\ V_T \geq SD+LC+coupon \text{ and } \theta^k > T \\ 0 & \text{if } V_T < F+coupon,\ V_T < SD+LC+coupon \text{ and } \theta^k > T \\ (V_t - SD - LC) & \text{if } V_t \geq SD + LC + a.c. \text{ and } \theta^k \leq T \\ 0 & \text{if } Vt < SD + LC + a.c. \text{ and } \theta k < T \end{cases} \quad (a.6)$$

The expression (a.6) may be rewritten as:

$$JD_t(V_t,T,g_t^K) = E^Q_t[((JD)*e^{-r(T-t)})*1_{\{\theta^k > T\}}*1_{\{VT \geq F + c\}} + E^Q_t[((V_T - SD - LC - coupon)*e^{-r(T-t)})*1_{\{\theta^k > T\}}*1_{\{VT < F+coupon\}}*1_{\{VT \geq SD + LC + c\}}] + E^Q_t[(V_t - SD - LC)*e^{-r(\theta k-t)}*1_{\{\theta^k \leq T\}}*1_{\{Vt \geq SD + LC\}}]$$

(a.7)

In table a.1, I point out the payoffs given to the different stakeholders and computed analytically above from (a.3) to (a.7).

Tab. a.1 – *Payoffs*

	Bond repayment or Liquidation at the end (T)			Liquidation before T	
PAYOFFS TO	**$V_T \geq F$ + Coupon and $\theta^k > T$**	**$V_T < F$ + Coupon, $VT \geq$ SD+LC+coupon and $\theta^k > T$**	**$V_T < F$ + Coupon, $VT <$ SD+LC+coupon and $\theta^k > T$**	**$F \geq Vt \geq$ SD+LC+a.i. and $\theta^k \leq T$**	**$Vt <$ SD+LC+a.i. and $\theta^k \leq T$**
EQUITYHOLDERS	$V_T - F$	0	0	0	0
SENIOR BONDHOLDERS	SD + PVC	SD + PVC	V_T – LC + PVC - $coupon_T$	SD + Σ $Coup_t$ +a.i.	V_t – LC + Σ $Coup_t$
JUNIOR BONDHOLDERS	JD	(V_T – SD – LC – $coupon_T$)	0	(V_t – SD – LC)	0
TOTAL	**V_T + PVC**	**(V_T - LC) + (PVC -$coupon_T$)**	**(V_T - LC) + (PVC -$coupon_T$)**	**V_t - LC + Σ $Coupons_t$ + a.i.**	**V_t - LC + Σ $Coupons_t$ + a.i.**

A.3 Benchmark

I use, again, default probabilities over the time interval (0,T], the implied recovery rate RR = (1-LC)*V_1 / F and the yield spread as parameter in order to test the consistency of my model.

A.4 Numerical implementation

I use, also in this case, a Monte-Carlo simulation approach that considers 150,000 sample paths for calculating bond prices, equity value, cumulative default probabilities, and recovery rate and the solution are stable up to the second digit.

A.5 Parameters' choice

I assume that firm's capital structure is constituted by ordinary stock, senior debt and junior debt. Both the obligations have a maturity of 10 years. Junior Debt is a Zero Coupon Bond, while Senior Debt is a Coupon Bond.

I utilize the same real value used in Chapter 2 for the parameters present in the Model. The two main differences are: the Coupon C is determined such that a time 0, Senior Debt is priced at par; and every time there is the coupons' payment the value of Asset V_T jump down and the dynamic restart from (V_T – C).

Table a.2 report the parameters' choice.

Tab. a.2 – *Parameters*

simulazioni zcb		
Parameter	**Symbol**	**Value Assumed**
Time to Maturity	T	**10**
Pre-defined grace period	d	**1**
Percentage of SD on Total Debt	γ	**0,5**
Default free interest rate	r	**0,06**
Volatility of the asset of the firm	σ	**0,29**
Liquidation Cost	LC	**0,2**
Threshold's parameter	ω	**0,85**
Dividend yield	δ	**0**
Initial Value of Assets	V_0	**100**
Leverage Ratio	LR	**0,657**
Debt Face Value	F	**65,7**
Debt Coupon rate	i	**0,06475**
dt=0.01;	dt	**0,01**
paths		**150.000**

A.6 Results

From table a.3 it is clear that the probability that the Senior Bond is fully paid is 79.28%, while for the Junior Bond this probability is equal to 58.87%.

More details about default probabilities, recovery rate and yield spreads are pointed out from paragraph a.6.1 to paragraph a.6.4.

Tab. a.3 – *Some results*

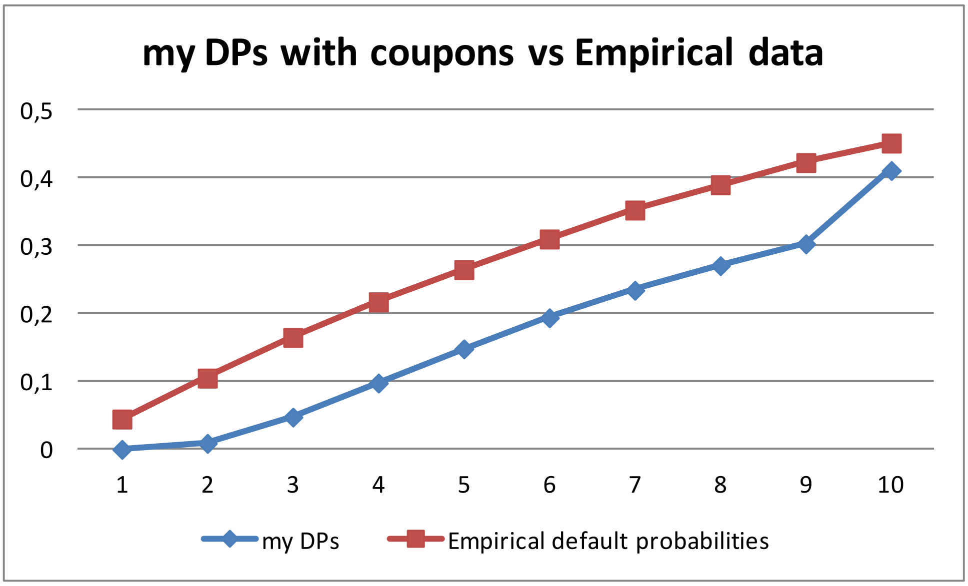
Liquidation Cost	2,36%
Average V at the default	42,37%
Yield Senior	6,58%
Yield Junior	**11,13%**
Cumulative default pr at T	**41,13%**
Cumulative default pr before T	33,14%
Pr. Senior paid in full	79,28%
Pr. Junior paid in full	58,87%
Recovery Rate	**51,59%**

A.6.1 Cumulative DPS

Again, I test my model, matching my cumulative default probabilities with the cumulative DPs observed in literature (Moody's, 2011).

From this comparison, it is evident that my DPs are not very far from empirical data and that at T = 10 these probabilities are very close. The light difference still depends on the fact that I set my model on US market, while Moody's data are global data. In particular, it is important to point out that my d (grace period) is equal to 1 as in US, but we don't have any "global" reference.

Figure a.1 – ***Cumulative default probabilities***



A.6.2 Recovery Rate

The average Asset Value in case of default is equal to 42.4%, while the recovery rate (on average) is 51.59%. This value is in line with the value empirically found in literature 51% - 52% (as above highlighted).

A.6.3 Yield Spread

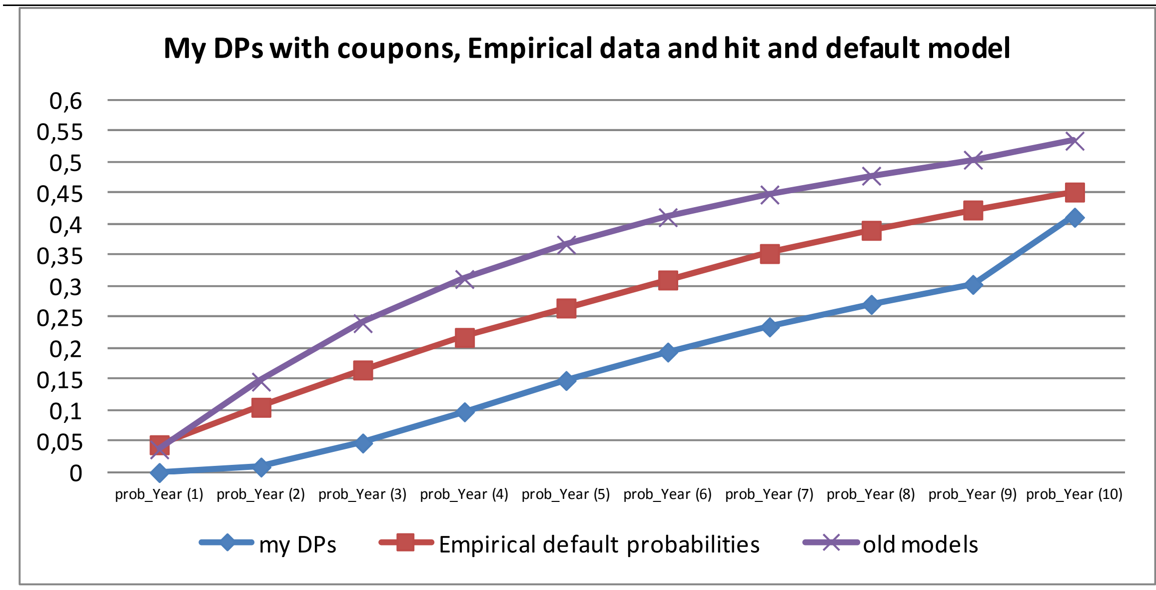
I find that the yield to maturity for the senior bond is equal to 6.58%, while for the junior bond this rate is equal to 11.13%. Therefore, the credit spread for junior bond (5.13%) is not very far / is in line with the value that is empirically found in literature (4.7%).

A.6.4 Empirical cumulative DPs vs. old models

I show below a graph where:

- The blue line represents my DPs;
- The violet line represents the DPs computed through an adaption of my model in which I assume that the grace period is equal to 0.01 (such as in old models), there aren't Liquidation costs;
- The red line represents empirical data.

Figure a.2 – ***Cumulative default probabilities***



Analyzing the figure, I could point out that the adaption of old models (g = 0.01 with two type of debt, of which one Junior with coupons) give back results very different from Moody's data.

This latter fact is confirmed if I compare the recovery rate and yield spread computed through the hit and default model with those empirically found. Indeed, the recovery rate calculated empirically is equal to 52%, while in the hit and default model this parameter is equal to 83.7%; the yield spread is equal to 0.4% and it is very far from the 4.70% reported by the Lehman bond index.

Even if I consider an hit and default models with LC, I have the same results. Indeed, I get a recovery rate equal to 67%, that is very different from the empirical value of 52%;

Also in this case, I could affirm that old models are not able to outline the real world.

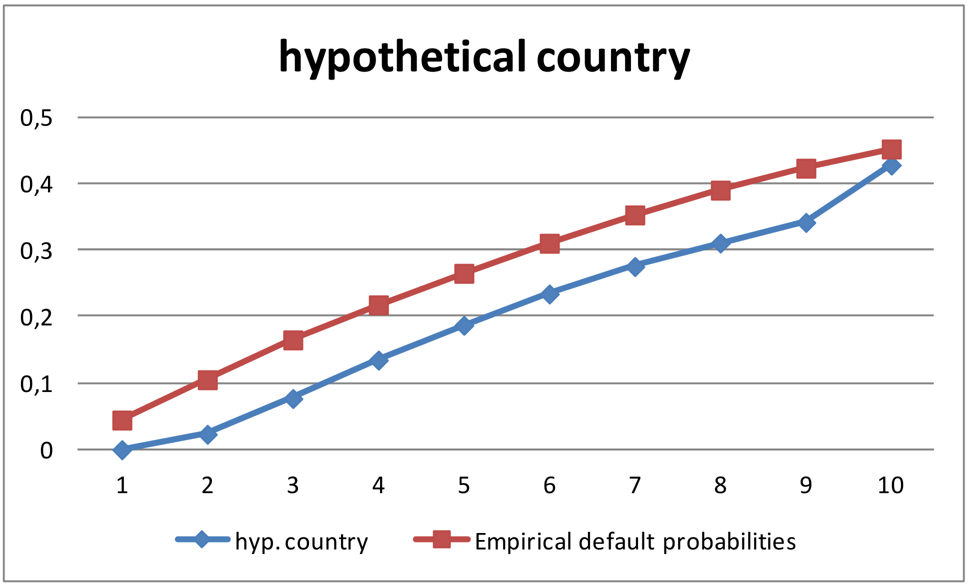
A.7 Hypothetical country

Also in this appendix, I present an hypothetical case, in which I change only one variable of my model and the coupon rate in order to satisfy the hypothesis that the senior debt is priced at par. In practice, I consider a grace period g = 0.65 (versus 1.0 in the base case), that is more or less in line with the average between the grace period found for US and the grace period found for Sweden.

The outcomes are very satisfying and they match empirical data very well.

These results, once again, confirm that the difference between my data and empirical data is due mainly to the fact that my model is built on US data and for that I use a grace period equal to 1, while empirical data (cumulative default probabilities) are global.

Figure a.3 – ***Cumulative default probabilities***



A.8 Sensitivity analysis

I now consider how my results are sensitive to the choices of my parameters. I start from the base case, and then change various parameter choices in order to study the impact of these parameters on the asset value, on recovery rate and on the DPs.

The results are in line with the expectations and with the sensitivity analysis reported in Chapter 2 and Chapter 3.

A.8.1 Grace period (d)

Figure a.4 – ***Sensitivity analysis: grace period***

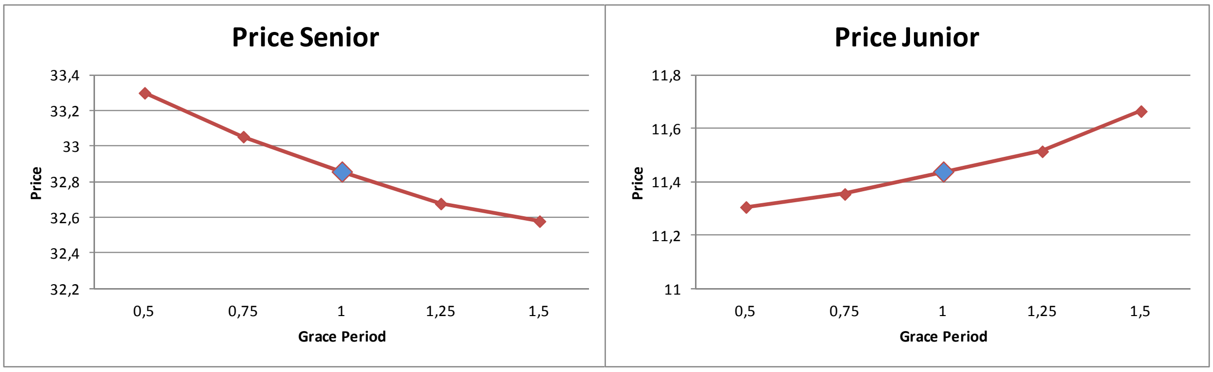


Figure a.5 – ***Sensitivity analysis: grace period***

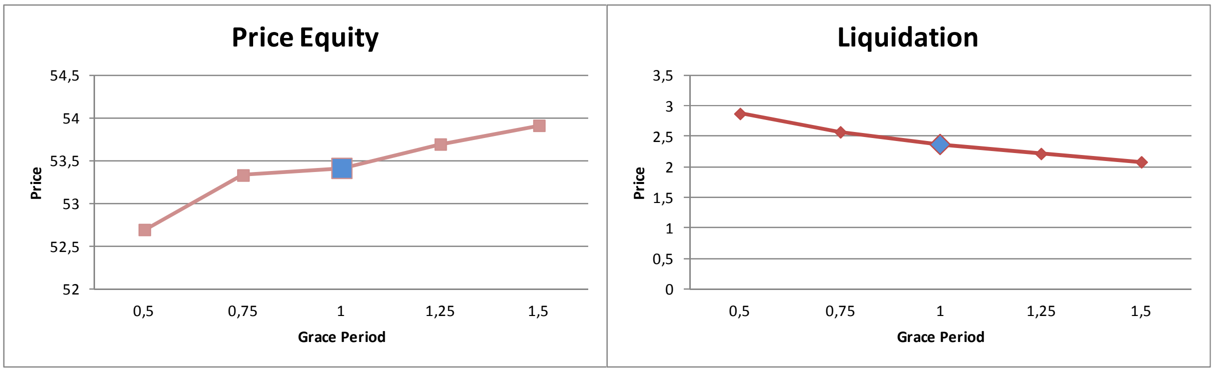
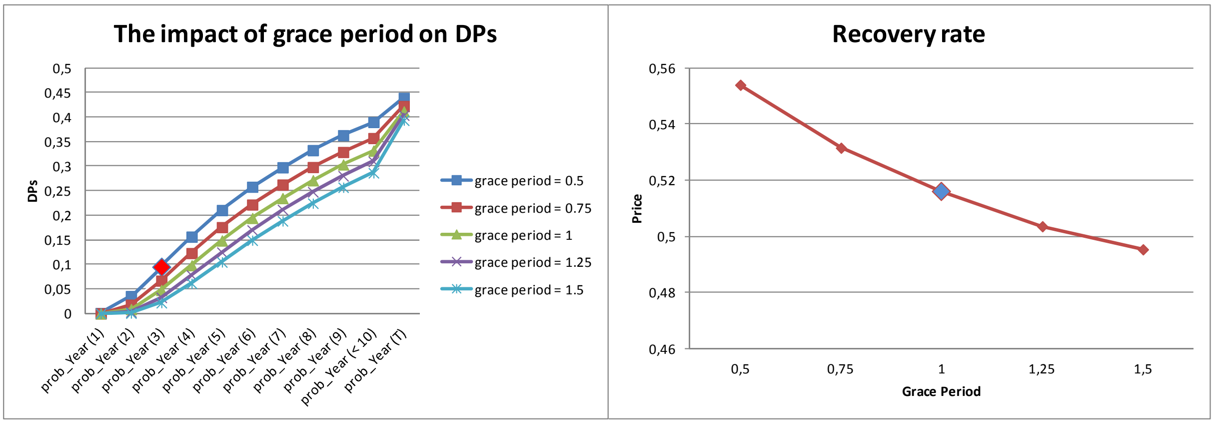


Figure a.6 – ***Sensitivity analysis: grace period***



A.8.2 Volatility (sigma)

Figure a.7 – ***Sensitivity analysis: volatility***

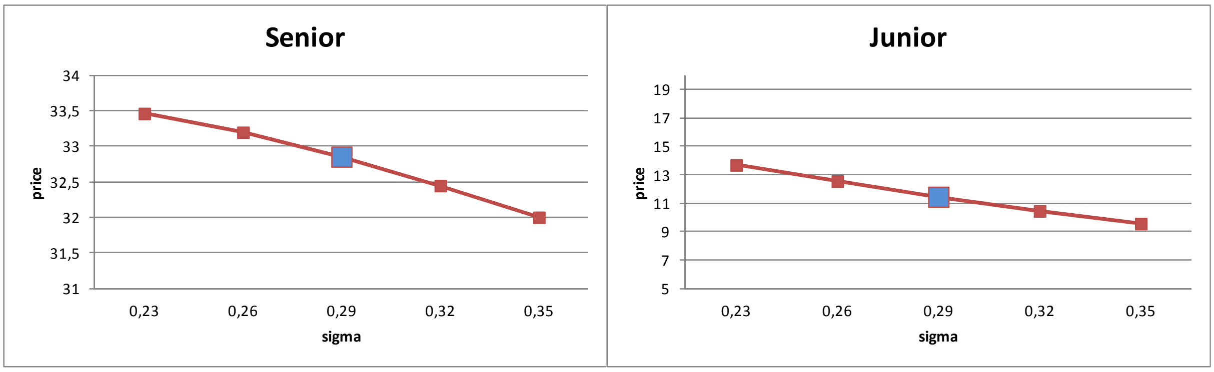
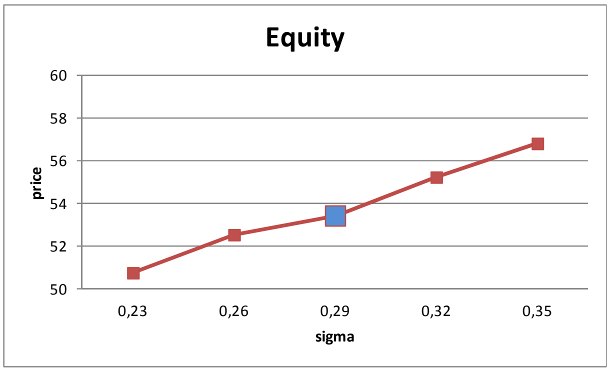


Figure a.8 – ***Sensitivity analysis: volatility***



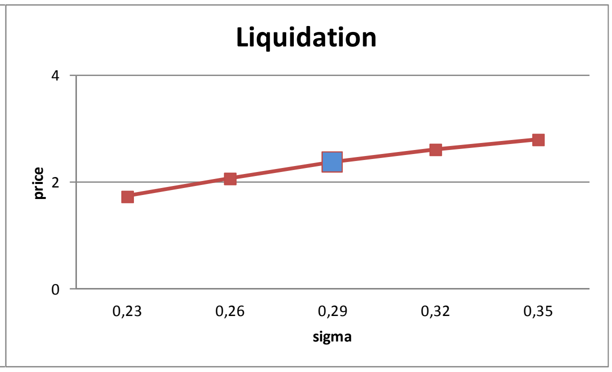
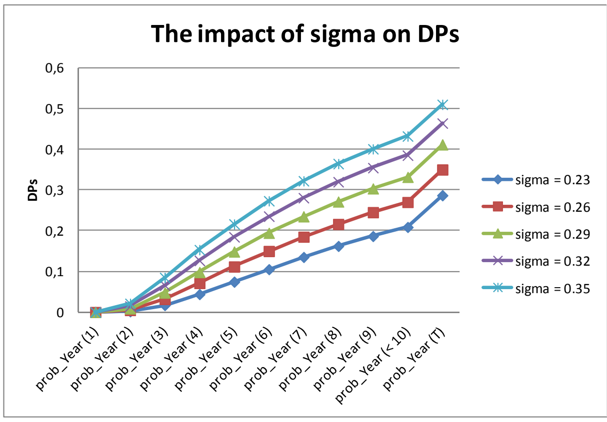
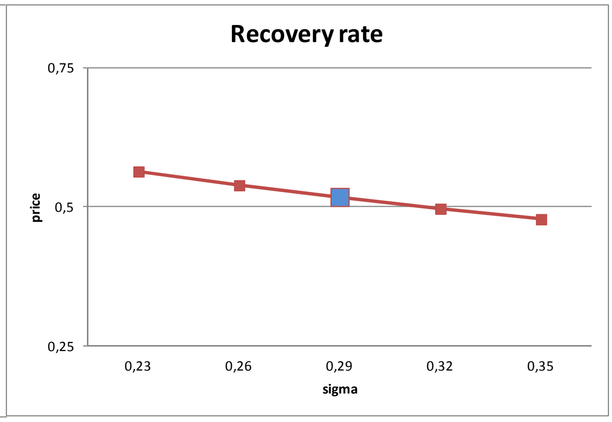


Figure a.9 – ***Sensitivity analysis: volatility***





A.8.3 The percentage of Senior Bond on the Total Debt

Figure a.10 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***

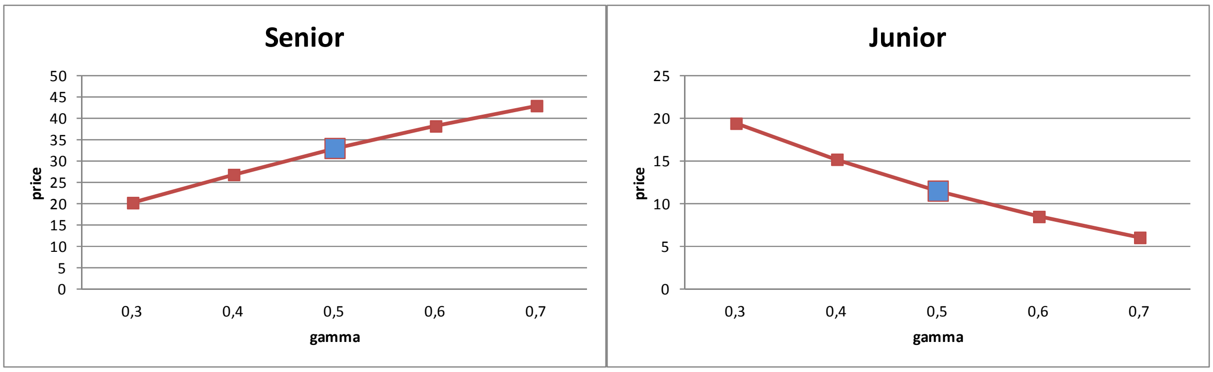


Figure a.11 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***

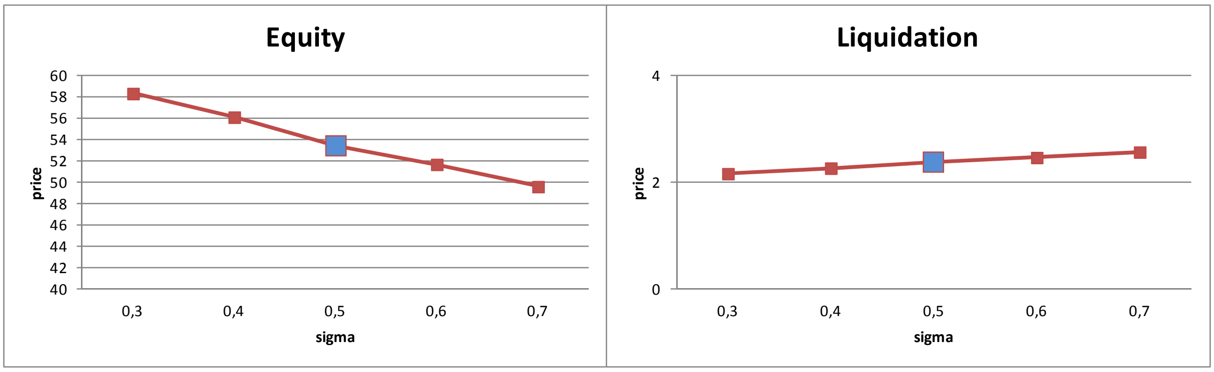
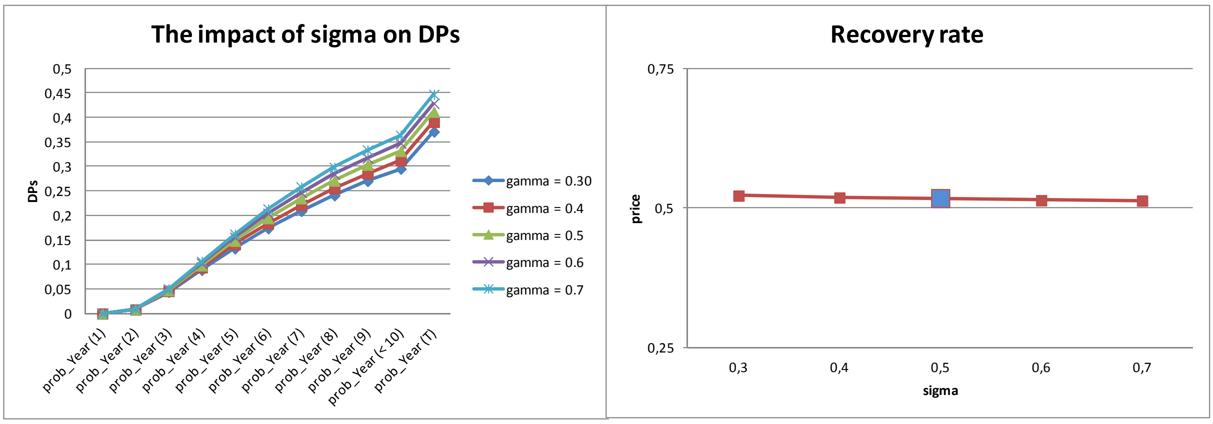


Figure a.12 – ***Sensitivity analysis: The percentage of Senior Bond on the Total Debt***



A.8.4 Dividend yield (delta)

Figure a.13 – ***Sensitivity analysis: dividend yield***

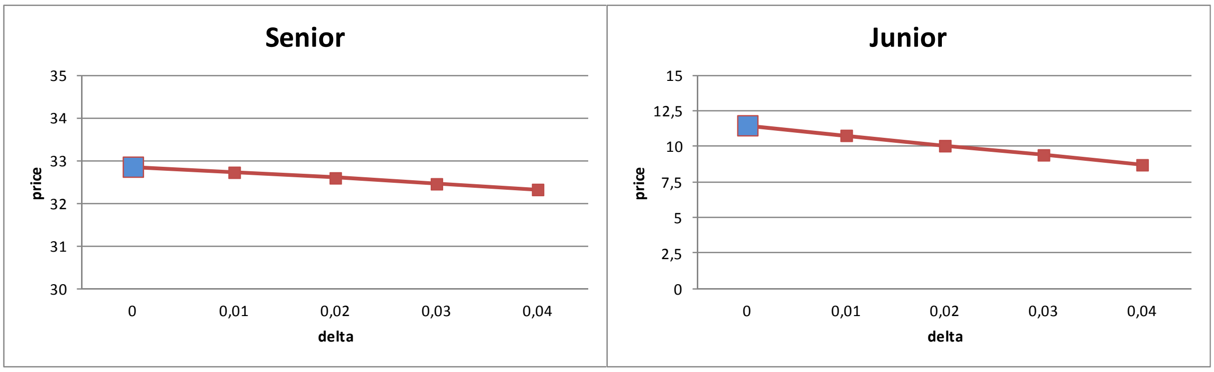


Figure a.14 - ***Sensitivity analysis: dividend yield***

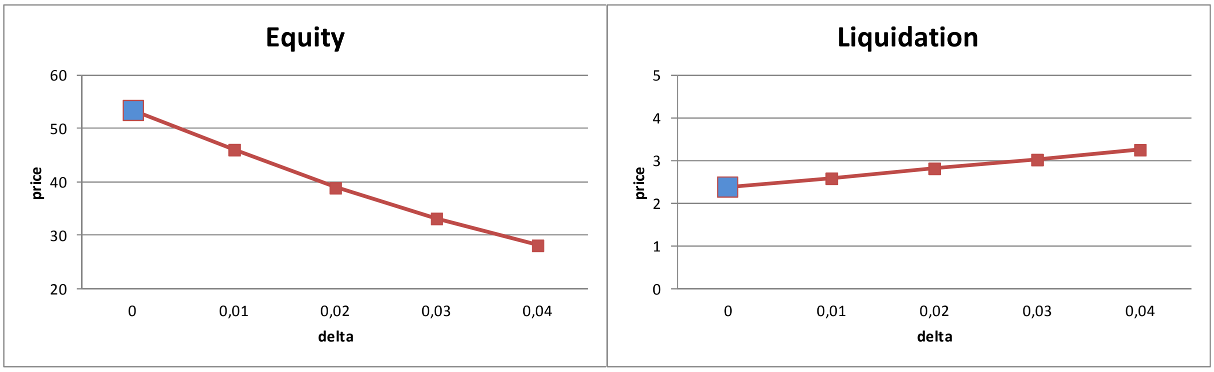
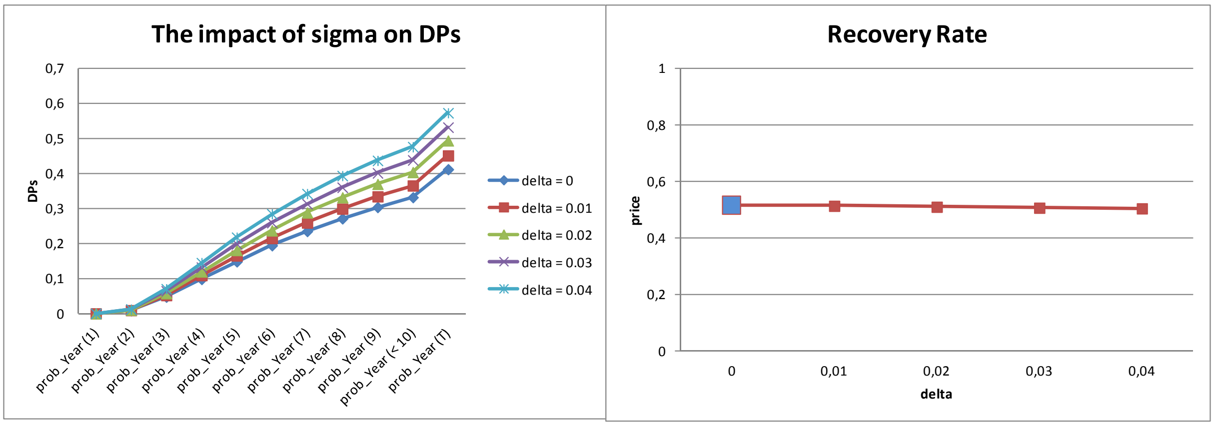


Figure a.15 – ***Sensitivity analysis: dividend yield***



A.8.5 Liquidation Costs

Figure a.16 – ***Sensitivity analysis: liquidation cost***

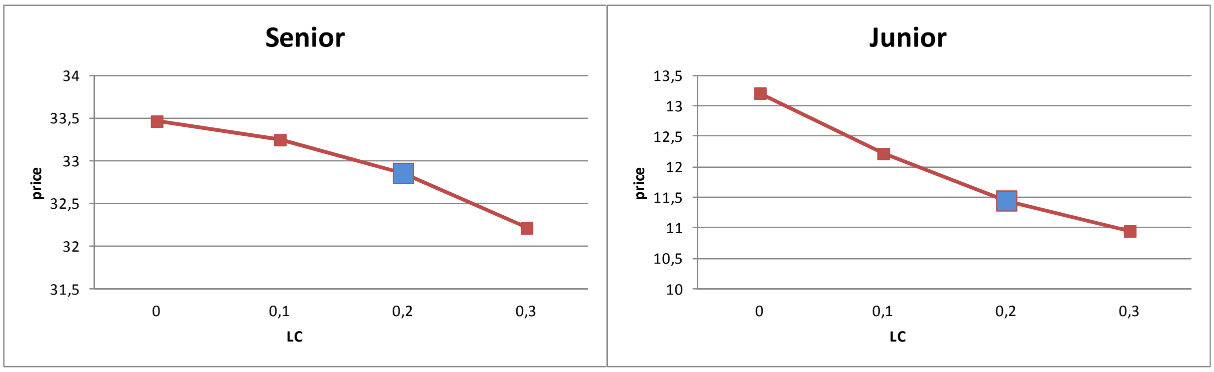
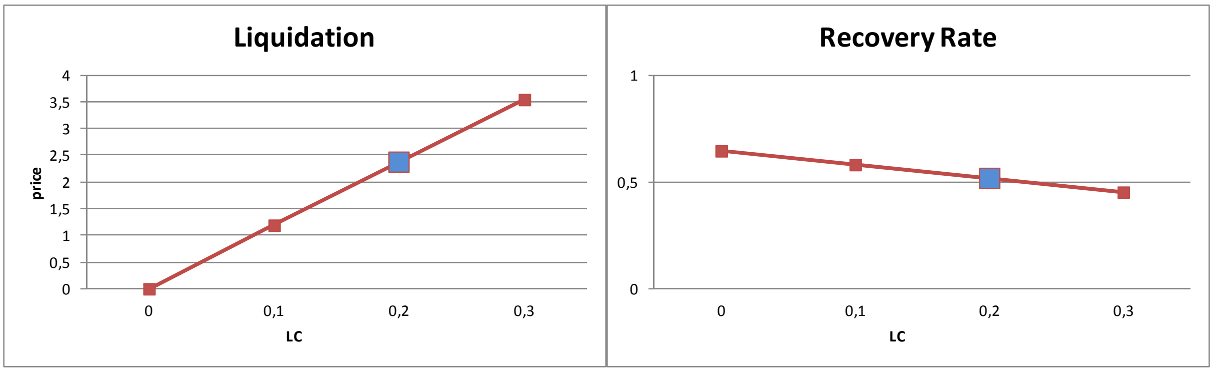


Figure a.17 – ***Sensitivity analysis: liquidation cost***



A.8.6 Leverage Ratio

Figure a.18 – ***Sensitivity analysis: leverage ratio***

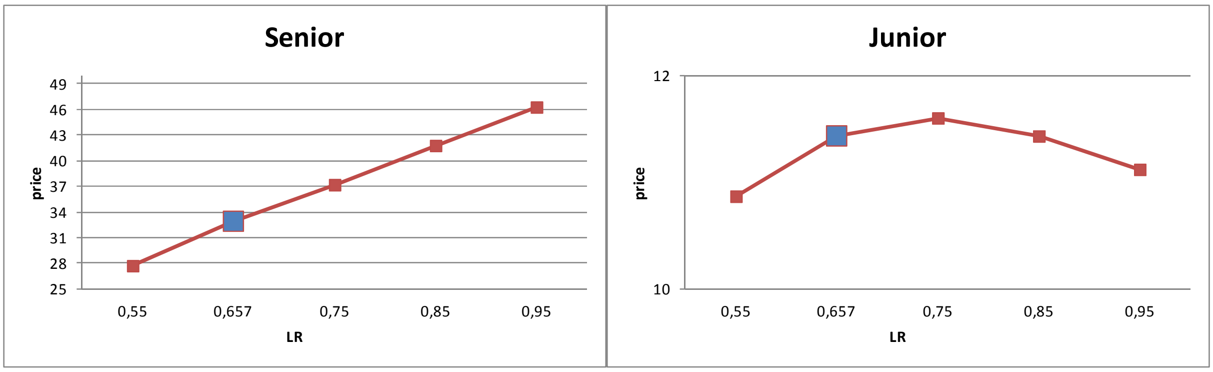


Figure a.19 – ***Sensitivity analysis: leverage ratio***

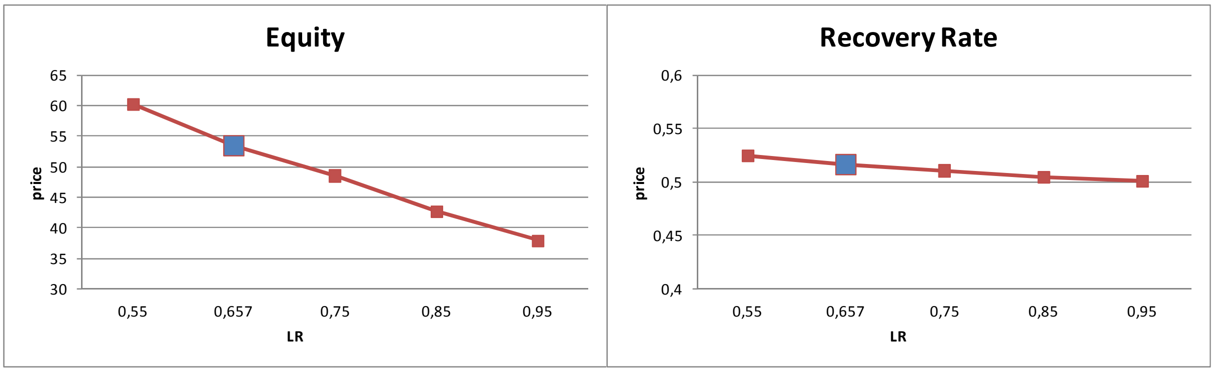
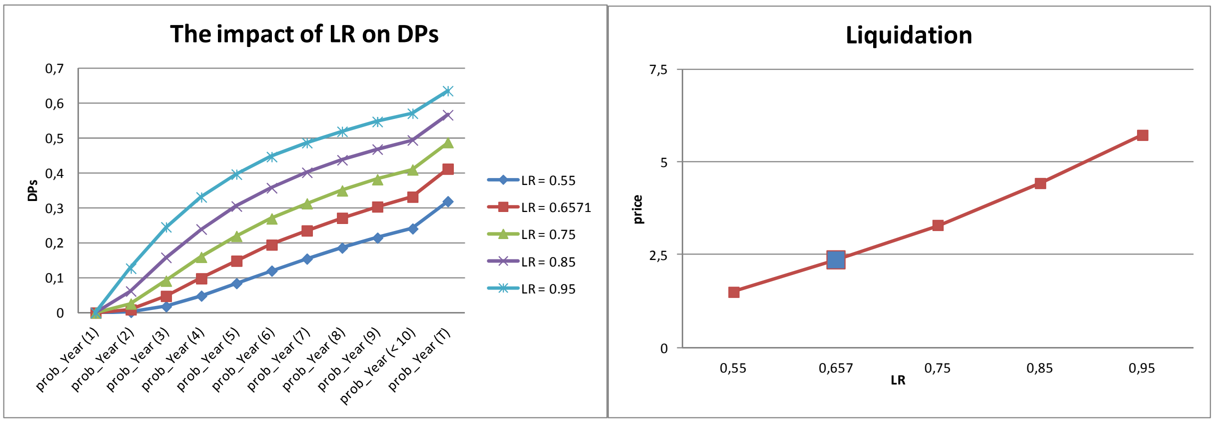


Figure a.20 – ***Sensitivity analysis: leverage ratio***



A.8.7 Threshold's parameter (Omega)

Figure a.21 – ***Sensitivity analysis: Threshold's parameter***

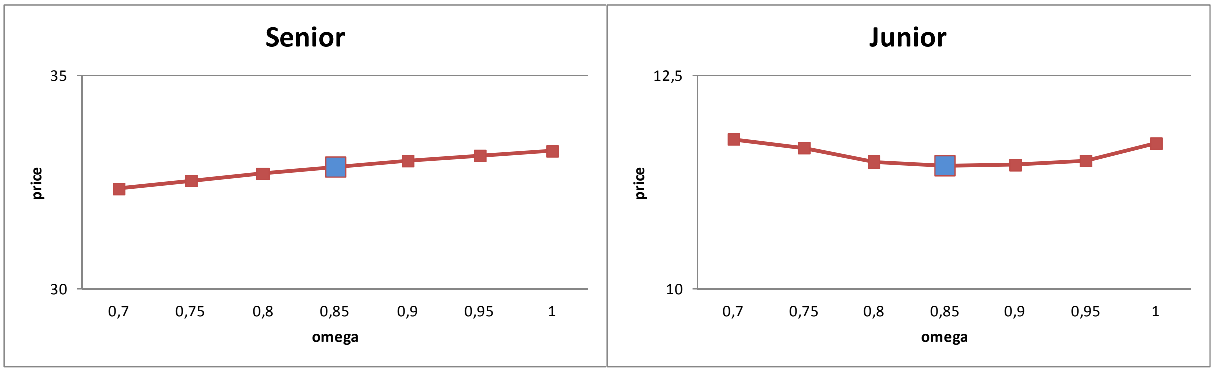


Figure a.22 – *Sensitivity analysis: Threshold's parameter*

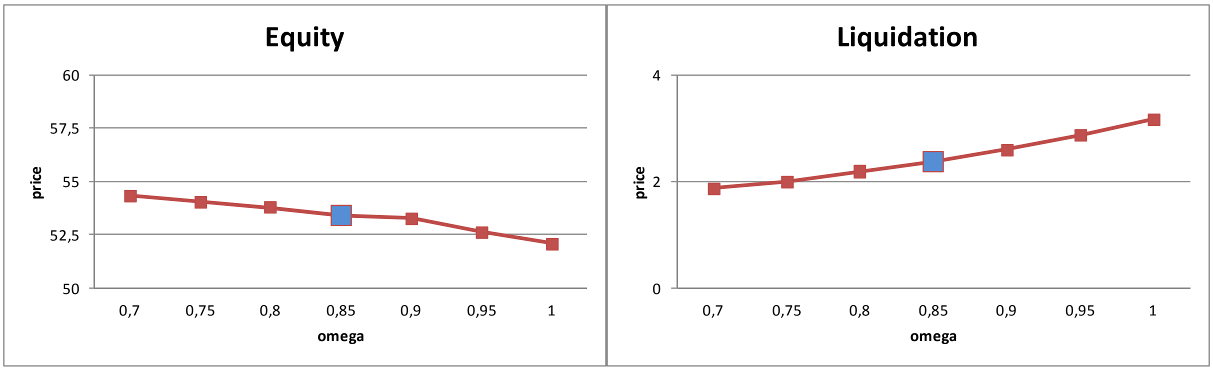
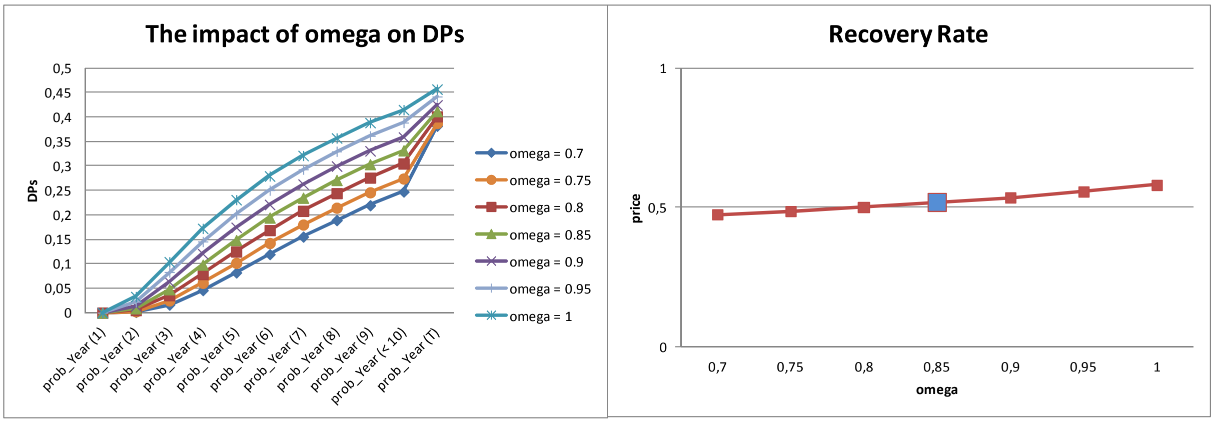


Figure a.23 – *Sensitivity analysis: Threshold's parameter*



A.8.8 Free-risk Rate (r)

Figure a.24 – ***Sensitivity analysis: Threshold's parameter***

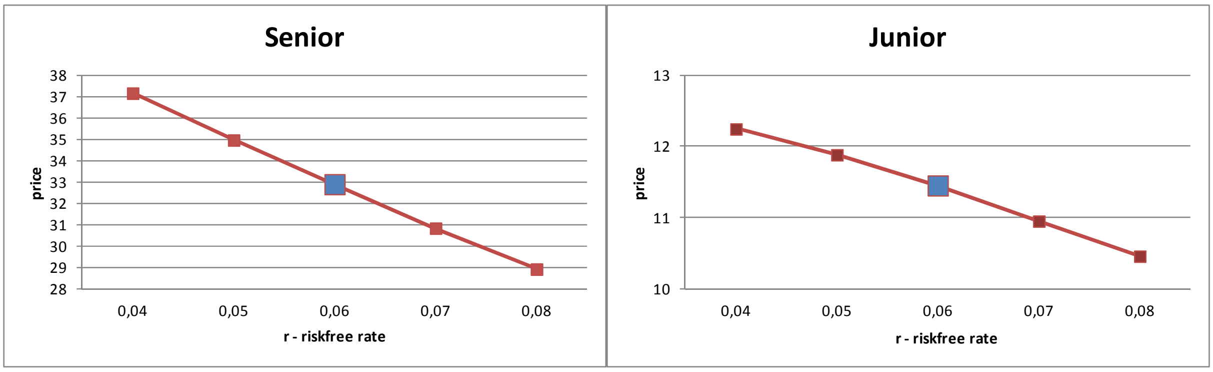


Figure a.25 – ***Sensitivity analysis: Threshold's parameter***

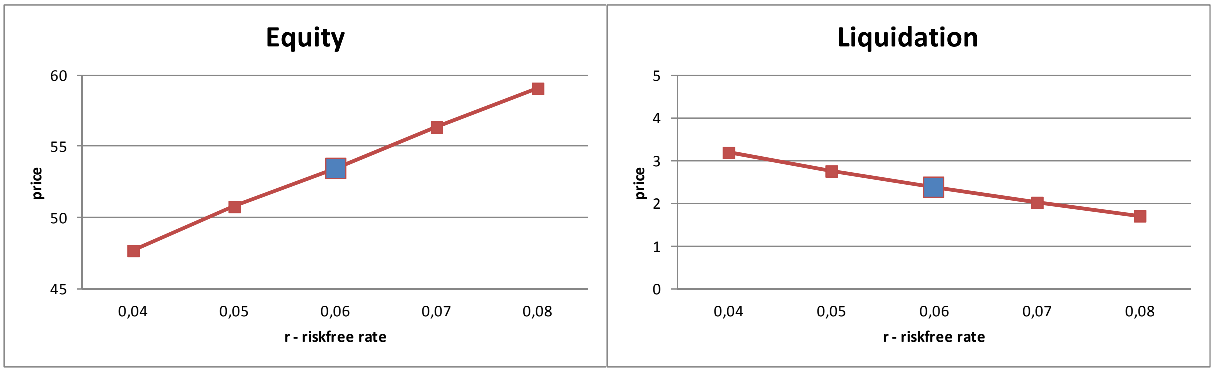
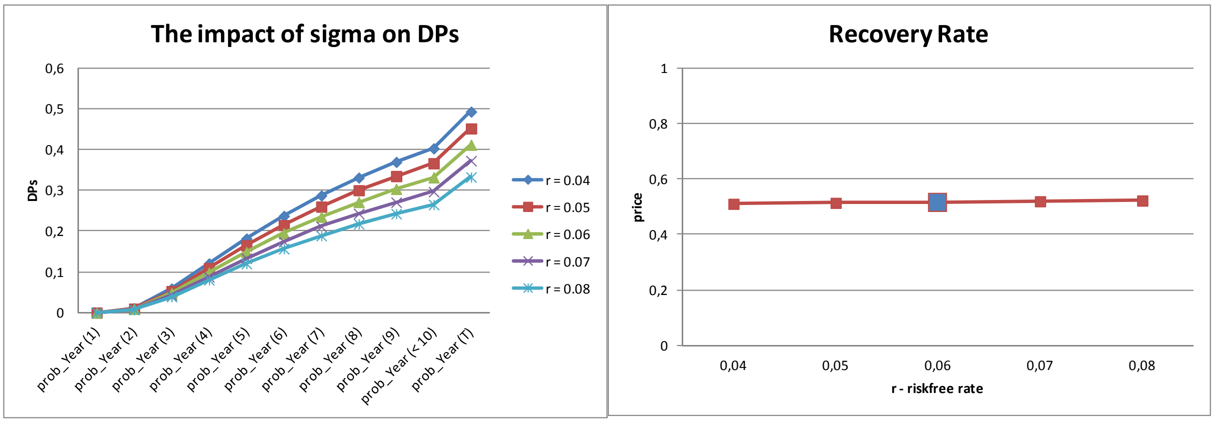


Figure a.26 – ***Sensitivity analysis: Threshold's parameter***



A.9 Conclusion

The results of this Appendix are in line with the results of Chapter 3. Therefore, I am sure that the reliability (or capacity of matching cumulative default probabilities, recovery rate and yield spread) of my model does not depend on the fact that the junior debt is a coupon bond. As pointed out above, the combination of the presence of coupons and of the grace period represents the crucial aspect of my model.

Therefore, I can assert that my framework fit really well empirical data because it is more compliance with reality than old models.

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