Three Essays on the Economics of Coordination

By

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Abstract

This thesis is a collection of three essays about the economics of coordination. Coordination issues arise when, in presence of multiple equilibria, heterogeneously informed agents need to coordinate with each other towards a Pareto-superior outcome. Electoral outcomes, collective decision-making, currency attacks or political regime changes examples of coordination problems.

The first chapter provides a game theoretic analysis of group decision making, investigating how an agent's communication behavior is affected by different voting systems. I show that in an ideal state where communication is noisy but agents can communicate without opportunity costs, agents will always reach unanimous consensus regardless of which voting system governs the deliberative process. I further show that under the more realistic case in which communication involves opportunity costs, voting systems shape an agent's communication behavior. Specifically, when the opportunity costs of communication are low, a voting system based on unanimity approximates the results of the ideal state. Conversely, when communication involves high opportunity costs, a voting system based on majority is more desirable.

The second essay is an experimental test of the predictions developed in the first chapter. The experiment is designed to determine how different voting institutions influence the process of communication of collective decision bodies when communication can be costly. In contrast with the existing literature, I have found that different voting institutions induce different decision outcomes. In particular, a voting system based on unanimity fosters subjects' communication and information sharing. Once subjects choose to communicate, I also have observed that communication unambiguously improves the quality of the decision outcome across each voting rule.

The third and final essay provides a political regime-change interpretation of the organized crime phenomenon. Under the assumption that the a criminal organization in a society benefits of the support of individuals, I investigate the strategic interplay between a criminal organization and a large number of citizens who might be more inclined to support the criminal organization rather than reporting its illegal activities to the legal authority. Borrowing from the economic literature on coordination and regime change, I model a criminal organization as an autocratic regime and claim that illegal activities are used in order to raise citizens support.

CHAPTER 1

Choosing (All) Together

Caveat: A version of this dissertation essay coauthored with Jérôme Mathis (Toulouse School of Economics) and Simone M. Sepe (University of Arizona) is a work in progress.

1.1. Introduction

On September 12, 2002, George W. Bush forcefully addressed the U.N. General Assembly, alleging that the Iraqi government was culpable of, among others, violations of U.N. resolutions prohibiting the production of weapons of mass destruction. Because of these violations, the U.S. Government demanded the U.N. Security Council's authorization of military action against Iraq. Intensive negotiations among the permanent members of the Security Council immediately followed this request. These diplomatic efforts were justified by the so-called "great power unanimity" rule, which gives each permanent member of the Council the power to enforce maintenance of the status quo by vetoing the adoption of a certain proposal.

Based on the unspoken assumption that unanimity admits no nuances, most prior economic studies on voting systems have looked at the great power rule as the benchmark of unanimity (see, e.g., Feddersen and Pesendorfer (1998), Persico (2004), Austen-Smith and Feddersen (2006)). However, the "jury unanimity" system that characterizes U.S. jury trials contradicts such an assumption, requiring consensus by *all* agents (i.e., jurors) to implement *any* decision (i.e., both guilty and not-guilty verdicts).[1] This overlooked difference between one unanimity system and the other bears major consequences for the agents' incentives to report information. Because under the great power unanimity rule agents can unilaterally impose the maintenance of existing circumstances, agents who want to change such circumstances have incentives to manipulate their private information. In this respect, it is unsurprising that

[1] Article 31 of the Federal Rules of Criminal Procedure specifies that any jury verdict must be unanimous. See http://www.uscourts.gov/uscourts/RulesAndPolicies/rules/2010%20Rules/Criminal%20Procedure.pdf .

most of the complaints included in Bush U.N. address were later shown to be unsupported by solid evidence. Indeed, because of the no-military intervention bias arising from the great power unanimity rule, the Bush administration had incentives to exaggerate information to reduce the likelihood of a negative vote by the other permanent members of the U.N. Security Council.[2]

Conversely, under the jury unanimity system, failure to reach full consensus leads to a no-decision (i.e., mistrial). This implies that each agent can "force" other agents to extend the deliberative process by threatening to withhold her consensus. Accordingly, agents have no incentives to misreport or otherwise manipulate information that is useful to reach a decision. This promotes more accurate information aggregation and, therefore, better decision-making.

Moving from these observations, we claim, more generally, that voting rules shape deliberation and communication processes, influencing the way in which agents' private information is aggregated. We model this claim by considering a framework in which agents with private information are required to implement one of two alternative decisions in a noisy communication context. This implies that the agents' private information can only be estimated as long as the agents repeatedly interact among each other. We also let agents be heterogeneous in their *cognitive abilities*—that is, communication and intellectual skills—meaning that each agent has a different ability to convey her information to other agents as well as to process the information she receives by them.

As long as communication involves low opportunity costs, under this analytical framework optimal information aggregation develops along three dimensions. The first involves the agents' inference of the other agents' private information (*first order learning*). The second

[2]In February 2004, Hans Blix, the chief U.N. arm inspector during the Iraq disarmament crisis, explicitly accused the Bush administration of having exaggerated the threat of Iraqi weapons in order to gain support for a war against Iraq. Tellingly, he declared that "the intention [of the Bush admnistration] was to dramatize [the situation] just as the vendors of some merchandise are trying to exaggerate the importance of what they have."See http://archive.truthout.org/article/blix-says-bush-blair-insincere-salesmen-iraq. Nonetheless, it is worth remarking that the Bush administration was only able to obtain a resolution approving further U.N. inspections of Iraqi weapon sites. The subsequent decision to bring war against Iraq was, instead, unilaterally taken by the United States and the United Kingdom without the placet of the U.N. Security Council. See http://www.un.org/Depts/unmovic/documents/1441.pdf .

involves the agents' ability to assess the quality of the other agents' private information (*second order learning*). The third, and last, involves the truthfulness of the information reported during the deliberative process. For example, communication is likely to imply low opportunity costs—and, therefore, involve the three dimensions of optimal information aggregation described above—when the number of agents is small. Indeed, when available information is limited, the quality of the information aggregation process becomes crucial to exploit the full potential of such information. We show that in similar circumstances, a deliberative process based on the jury unanimity rule increases the probability of implementing the correct alternative by inducing truthful extended communication. Conversely, when the number of agents is large, communication may become too costly and, therefore, unfeasible. In these circumstances, a deliberative process based on majority rules—and as such mandating that private information be aggregated only through voting—is likely to dominate a deliberative process based on jury unanimity. This occurs because of two reasons. First, large sets of information supply to the lack of accurate information aggregation that is commonly experienced under a majority-based deliberative process employing voting without communication (Condorcet, 1785). Second, in context with no or coarse communication, a deliberative process based on jury unanimity systematically implements no-decisions at a high rate, whereas a majority-based deliberative process always ensures the implementation of one of the two alternatives. The following example provides an intuition of these results, offering a simplified version of the analysis developed under our analytical framework.

Example. Suppose that there are three agents—a, b, and c—who are required to make a decision over an alternative $A \in \{0, 1\}$ under a simple majority system. Social welfare is maximized when the agents select $A = \theta$, with $\theta \in \{0, 1\}$ representing the unknown state of nature. The realization of the state of nature is ex-ante equiprobable. Agents are benevolent, meaning that they want to implement a welfare-maximizing decision. Further suppose that $\theta = 0$ and that, before making a decision, each agent receives an informative signal, i.e., $x_i = 0 + \epsilon_i$, on the realized state of nature. The signal x_i is normally distributed with mean zero and variance σ_i^2, where $i \in \{a, b, c\}$ and $[\sigma_i^2]^{-1}$ measures each agent's cognitive ability. Lastly, an agent's signal and cognitive ability are private information.

The agents' payoffs are as follow. When the agents choose alternative $A = 1$ with $\theta = 0$ as well as when they choose $A = 0$ with $\theta = 1$, they receive a negative payoff of $-\frac{1}{2}$. Instead, when the agents choose the social maximizing alternative, they receive 0. Given this payoff structure, each agent will choose $A = 1$ (resp. $A = 0$) when her posterior belief about $\theta = 1$ (resp. $\theta = 0$) is higher (resp. lower) than $1/2$. Each agent forms her posterior belief based on her own private information (i.e.,(x_i, σ_i^2)). Table 1 shows how each agent will vote given the parametric values of signals and variances.[3]

Agent	a	b	c
Signal	$x_a = 0.1$	$x_b = 2$	$x_c = 3$
Variance	$\sigma_a^2 = 0.1$	$\sigma_b^2 = 1.5$	$\sigma_c^2 = 2$
Posterior	0.02	0.73	0.78
Vote	$v_a = 0$	$v_b = 1$	$v_c = 1$

TABLE 1.1.1. Simple Majority System.

As illustrated in Table 1, agent a votes for alternative 0, whereas agents b and c vote for alternative 1. Under a simple majority system, this implies that agents b and c can impose the choice of alternative 1 on agent a. Hence, assuming that agents b and c are sufficiently confident that alternative 1 is welfare-maximizing, they can prevent communication from taking place by implementing alternative 1.

Assume now that the agents are required to vote according to a jury unanimity rule. Under this different rule, agents b and c can no longer impose the implementation of alternative 1 on agent a. Instead, agent a can force them to engage in communication and exchange their private information. Thus, under a jury unanimity voting rule, each agent reports her private signal to the other agents and receives the other agents' private signals over time. Because

[3]For simplicity, in the simulation of Table 1 we impose the following method for updating beliefs: each agent updates her belief given the normally distributed signal she has originally received and does not take into account the distribution of variances. Similarly in Table 2 below, we impose that each agent updates her belief by (i) estimating the other agents' signals and variances through the frequentist approach; and (ii) updating her belief at the tenth stage of communication given the normally distributed signals she has received by the other agents, with the parameters of these signals being estimated through (i). In the model, instead, the agents are perfect Bayesian updaters.

communication is noisy and such noise comes from the agents' cognitive abilities, agent a receives the signal $x_{ab} = x_b + \eta_{ab}$ from agent b, where x_b is the message of agent b and $\eta_{ab} \sim \mathcal{N}\left(0, \frac{\sigma_a^2+\sigma_b^2}{2}\right)$ is the noise arising from communication between agents a and b. It is also worth remarking here the effect of second order learning. Under this effect, in the information aggregation process more weight is given to the signals (i.e., opinions) of agents with higher cognitive abilities. Thus although agent a starts the discussion with agents b and c as a minority, in a jury unanimity context her opinion becomes the most-decisive because she is the agent with the highest cognitive abilities.

As a result of this extended communication process, every agent has an estimate of the other agents' signals and cognitive abilities. Such a process lasts until the agents unanimously decide to halt it. In Table 2 we simulate how agents update their posterior beliefs after ten stages of communication, assuming that they have the same initial private information as in Table 1.

Agent	a	b	c
Signals	$x_a = 0.1$	$\bar{x}_a = 0.09$	$\bar{x}_a = 0.2$
	$\bar{x}_b = 0.9$	$x_b = 2$	$\bar{x}_b = 1.4$
	$\bar{x}_c = 2.1$	$\bar{x}_c = 0.99$	$x_c = 3$
Variances	$\sigma_a^2 = 0.1$	$\bar{\sigma}_a^2 = 0.12$	$\bar{\sigma}_a^2 = 0.26$
	$\bar{\sigma}_b^2 = 1.4$	$\sigma_b^2 = 1.5$	$\bar{\sigma}_b^2 = 1.8$
	$\bar{\sigma}_c^2 = 3.2$	$\bar{\sigma}_c^2 = 2.9$	$\sigma_c^2 = 2$
Posterior	0.01	0.31	0.35
Vote	$v_a = 0$	$v_b = 0$	$v_c = 0$

TABLE 1.1.2. Jury Unanimity System with Ten Stages of Communication.

As shown in Table 2, under a jury unanimity system each agent updates her posterior belief in accordance with both her private information and the other agents' private information. Specifically, each agent weighs the other agents' signals ($\bar{x}_i$) according to their (estimated) cognitive abilities ($[\bar{\sigma}_i^2]^{-1}$). As a result, the accuracy of each agent's posterior belief is improved and the agents select the social-welfare maximizing decision (i.e., $A = 0$).

We make two main contributions. First, we show that when communication among agents does not involve opportunity costs, agents always report their private information truthfully and vote sincerely (i.e., according to available information). In particular, with asymptotic communication, asymmetric information vanishes and all decision-relevant information becomes public. As a result, agents have the same posterior and reach unanimous consensus irrespective of which voting rule governs the deliberative process. This irrelevance result highlights the importance of extended communication as a means to improve the agents' estimate of private information and, therefore, implement better decision outcomes.

Second, we show that when communication among agents improves the estimate of private information but comes at an opportunity cost, voting rules shape agents' communication behavior. We formalize the tradeoff between the benefits of extended communication and higher opportunity costs by employing the concepts of ϵ-maximization and ϵ-equilibrium, under which the agents engage in extended communication if this delivers a benefit higher than ϵ. Based on this tradeoff, we show that when the opportunity cost the agents bear for engaging in extended communication is relatively low, a voting system based on unanimity is superior to any other voting system. This is because this system provides better aggregation of the agents' private information through truthful reporting and sincere voting in equilibrium.

In addition to the above contributions, we also conjecture that whenever communication is unfeasible, a voting system based on majority Pareto dominates a voting system based on unanimity. Indeed, because the benefits of unanimity is to induce extended communication, when communication is unfeasible such a system loses its comparative advantage.

These results shed some lights on the rational of different voting systems that are observed in institutions such as juries, boards of directors, and large deliberative assemblies (i.e., plenums). Unanimous consensus requirements governing the deliberative process of juries are efficient because the expected benefits arising from more accurate decisions dominates the expected social costs associated with the risk of a no-decision (i.e., mistrial). Conversely, in board decision-making, simple majority rules are desirable as most business entities and other board-ruled institutions cannot afford no-decision outcomes. Along the same line, in the case of plenums, simple majority rules are justified by the existence of a large number of agents and the increased costs of communication arising therefrom.

The remaining of this article is organized as follows. In Section 1.2, we review the related literature. In Section 1.3, we present our model. In Section 1.4, we show our main results. In Section 1.5, we discuss robustness extensions of our work. In section 1.6, we briefly conclude.

1.2. Related Literature

Our work contributes to several strands of literature. First, we contribute to the literature on group decision-making that examines communication as a means for aggregating dispersed information. DeGroot (1974) offers a pioneering statistical model on how groups of individuals reach consensus on the estimation of a common interest parameter. In his model, however, the process leading to consensus does not induce agents to improve the estimation quality of the common interest parameter. In contrast, in our model, truthful communication substantially improves the estimation quality of available information. In a related strand of literature, Ottaviani and Sørensen (2001, 2006-a, 2006-b) and Visser and Swank (2007) show that reputational concerns may result in distorted decision making when information can be manipulated. Similarly, Li, Rosen and Suen (2001) find that in small committees the existence of conflicting preferences may lead decision makers to manipulate their private information, which may limit the benefits of information aggregation. We add to these studies by highlighting that information manipulation is absent under the jury unanimity system, because agents always have the incentives to report their private information truthfully.

Second, we contribute to the literature examining the problem of strategic information transmission. The seminal work in this field is due to Crawford and Sobel (1982), who study such problem in the context of the relationship between a decision maker and an expert. Subsequent economic and political studies investigating this issue include, among others, Milgrom and Roberts (1986), Morgan and Krishna (2001, 2004), Prendergast (1993), and Aghion and Tirole (1997). A common result of these studies is that preference divergence between the expert and the decision-maker leads to partial disclosure of information. In contrast to these studies, we develop a model in which the experts *are* the decision makers, focusing on strategic information transmission among the decision makers (i.e., the agents) themselves.

Third, our work also contributes to the literature on optimal voting schemes, which examines how individuals make decisions when they have homogeneous preferences but different cognitive abilities. In a context where these abilities are known, Nitzan and Paroush (1982) and Shapley and Grofman (1984) show that the optimal voting rule gives more weight to agents with higher cognitive ability. We endogenize this result, showing that communication in the deliberative process induces each agent to assign more weight to the private information provided by better skilled agents.

Fourth, we contribute to the large literature considering the problem of strategic voting, which takes place when a voter anticipates that her vote is pivotal for the decision outcome. As shown by Austen-Smith and Banks (1996), Myerson (1998), and Feddersen and Pesendorfer (1998), in such a context there might be equilibria where agents vote against their private information. In particular, Feddersen and Pesendorfer (1998) find that unanimity may increase the probability of both type I error and type II error, because sincere voting is never a Nash equilibrium. Similarly, Persico (2004) finds that unanimity may distort agents' incentives to acquire information. As hinted to above, however, these studies focus exclusively on a unanimity system based on the great power unanimity rule. We add to such studies by showing that the results they obtain are reversed under an unanimity systems based on the jury unanimity rule.

Coughlan (2000) extends the result of Feddersen-Pesendorfer (1998) incorporating the possibility of pre-vote communication and mistrial. In line with our results, he finds that unanimity may be more efficient than majority rules in some specified circumstances. On their turn, Austen-Smith and Feddersen (2006) challenge Coughlan (2000) showing that his result does not hold when agents' preferences are private information. Mathis (2011), however, shows that when agents are able to provide verifiable evidence for their private information, the results of Austen-Smith and Feddersen (2006) are reversed, with unanimity inducing agents to provide higher-quality private information in the communication process and hence the approval of better decisions. Relatedly, Gerardi and Yariv (2007) show that any majority rule induces the same set of equilibrium payoffs absent verifiable information. Along the same line, Goeree and Yariv (2011) find experimental evidence that communication reduces the relative impact of different voting rules and improves the efficiency of information aggregation.

Finally, within this strand of literature, two studies that are most closely related to our work are Duggan and Martinelli (2001) and Lizzeri and Yariv (2012). Duggan and Martinelli (2001) provide conditions under which unanimity can be efficient. In their model, this results obtains from the possibility that the agents' signals may be infinitely precise and then correspond to the unknown state of nature with probability one. In contrast, in our model the efficiency of unanimity-based systems results from dynamic communication among the agents that allows them to asymptotically aggregate all the decision-relevant information. Lizzeri and Yariv (2012), instead, model committee deliberation as a dynamic information-acquisition game in which agents decide at each stage whether to acquire more public information. Considering agents with heterogeneous preferences and no private information, they find that unanimity leads to more extensive information acquisition and more accurate decision-making. We confirm their result in a context where agents have private information but homogenous preferences.

1.3. The Model

1.3.1. Setup. A group of $n > 2$ agents, indexed by $i \in N \equiv \{1, \dots, n\}$, are required to collectively choose an alternative $A \in \{0, 1\}$. The optimal choice depends on an underlying state of nature $\theta \in \{0, 1\}$, which is equiprobable and unknown by the agents. Matching the alternative with the underlying state of nature, i.e., $A = \theta$, maximizes social welfare.[4]

There are three possible decision outcomes: alternative A is implemented when at least k agents vote for such an alternative, with $k \in K \equiv \{k_0, k_1\}$ and $k_0 \in \{1, \dots, n\}$ (resp. $k_1 \in \{n - k_0 + 1, \dots, n\}$) being the voting rule for the alternative $A = 0$ (resp. $A = 1$). When neither of the alternatives receives at least k votes, the decision outcome is $\varnothing$, meaning that the agents fail to take a decision. Therefore, a decision D is an element of $\{0, 1, \varnothing\}$, where

[4] As long as alternatives can be ranked in a way that agents' preferences are single-peaked, our model extends to cases with a finite number of alternatives greater than two.

$D = \emptyset$ denotes that no alternative is implemented.[5] In particular, the no-decision outcome $D = \emptyset$ requires that $k_0 + k_1 > n + 1$, otherwise it never occurs, meaning that $D \in \{0, 1\}$.

This analytical framework can generalize any voting rule. A simple majority rule corresponds to $K = \left\{ \left\lfloor \frac{n}{2} + 1 \right\rfloor, \left\lfloor \frac{n}{2} + 1 \right\rfloor \right\}$, under which one alternative A is always selected when n is odd.

Unanimous rules, instead, can take several forms. For example, under the great power unanimity rule, where $K = \{1, n\}$ (resp. $K = \{n, 1\}$), only alternative $A = 1$ (resp. $A = 0$) requires $k = n$ votes, whereas the alternative $A = 0$ (resp. $A = 1$) can be unilaterally imposed by each agent. Therefore also in this case $D \in \{0, 1\}$. Under the jury unanimity rule, instead, $k = n$ votes are required to approve any alternative, i.e., $K = \{n, n\}$. This implies that agents are required to deliberate unanimously for either $A = 1$ (e.g., conviction) or $A = 0$ (e.g., acquittal). Instead, when neither of the alternatives receives n votes, $D = \emptyset$ is the decision outcome (e.g., mistrial). Hybrid forms such as $K = \{k_0, k_1\}$, with at least one element equal to n and the other strictly higher than 1, are also possible.

Super-majority rules can be either asymmetric or symmetric. Asymmetric super-majority rules correspond to $K = \left\{ n - k^S + 1, k^S \right\}$, with $\left\lfloor \frac{n}{2} + 1 \right\rfloor < k^S < n$. This is the case of, for example, rules governing committee deliberations under which a committee can either approve a project (i.e., $A = 1$) with k^S votes or maintain the status-quo (i.e., $A = 0$). In this case, the decision outcome is binary (i.e., $D \in \{0, 1\}$). Instead, symmetric super-majority rules are characterized by $K = \left\{ k^S, k^S \right\}$, with $\left\lfloor \frac{n}{2} + 1 \right\rfloor < k^S < n$ for each alternative. This system corresponds to, for example, special jury systems under which jurors are required to acquit or convict with at least k^S votes. When neither of the alternatives receives k^S votes, mistrial (i.e., $D = \emptyset$) is the decision outcome.

1.3.1.1. *Preferences.* Our main research question is how strategic deliberation allows to aggregate dispersed information. Hence, we assume benevolent agents with homogeneous preferences. Benevolence implies that the decision outcome that maximizes agents' utility is socially optimal. As standard in the literature, we represent agents' utility with a loss function

[5] Being $k_0 + k_1 \geq n + 1$, the elements of the decision set are both mutually exclusive and collectively exhaustive.

under which selecting $A = \theta$ delivers to each agent a payoff equal to zero, whereas selecting $A \neq \theta$ or $\emptyset$ delivers a negative payoff. Specifically, for each agent i the utility function u maps any decision D and any state of nature θ in the following payoffs:

$$u(D, \theta) = \begin{cases} 0 & \text{if} \quad D = \theta \\ -q & \text{if} \quad (D, \theta) = (1, 0) \\ -(1-q) & \text{if} \quad (D, \theta) = (0, 1) \\ -\xi_\theta & \text{if} \quad (D, \theta) = (\emptyset, \theta) \end{cases}$$

where $q \in (0,1)$ (resp. $(1-q)$) measures the loss associated with the wrong decision of choosing $A = 1$ (resp. $A = 0$) when the state of nature is $\theta = 0$ (resp. $\theta = 1$) and ξ_θ measures the loss associated with the decision outcome $D = \emptyset$ when the state of nature is θ.

A straightforward interpretation of these payoffs suggests that each agent i prefers $A = 1$ (resp. $A = 0$) to $A = 0$ (resp. $A = 1$) whenever she believes that the probability that $\theta = 1$ is greater (resp. lower) than q. We assume that $\xi_0 \in (0, q)$ and $\xi_1 \in (0, 1-q)$ so that at state $\theta = 1$ (resp. $\theta = 0$), $D = 1$ (resp. $D = 0$) is preferred to $D = \emptyset$, which, on its turn, is preferred to $D = 0$ (resp. $D = 1$). We also impose $\xi_1 \geq \frac{(1-q)(q-\xi_0)}{q}$ so that whatever the beliefs of an agent about the occurrence of state θ, she always prefers one of the two alternatives to the decision outcome $D = \emptyset$.

1.3.1.2. *Deliberation.* The deliberative process is a dynamic game, where each stage of deliberation is indexed by $t \in \{0, 1, 2, \ldots\}$.

Private Information. At $t = 0$, Nature randomly chooses θ with equal probability and sends to each agent i a normally distributed signal about the underlying state θ. This signal can be interpreted as, for example, the information that a board member receives before joining a board meeting or the evidence that is made available to a juror during a trial. Nature also assigns to each agent i a cognitive ability, identified by the variance of the signal that each agent receives, σ_i^2. As standard in the statistical literature, we pose that variances are distributed according to an inverse-gamma distribution $\Gamma^{-1}(\alpha, \beta)$, with α and β shape and scale parameters respectively. Each signal and its variance are private information of each agent i. Thus, the signal received by each agent i is

$$x_i^0 = \theta + \varepsilon_i^0$$

where (i) ε_i^0 is *i.i.d.* across all agents, (ii) $\varepsilon_i^0 \backsim \mathcal{N}(0, \sigma_i^2)$, and (iii) $[\sigma_i^2]^{-1}$ is the agent i's precision.[6] Each agent i knows her σ_i^2, but does not know σ_j^2 for each $j \neq i$. Being x_i^0 and σ_i^2 private information of each agent i, we define $h_i^0 \equiv (x_i^0, \sigma_i^2)$ as the initial private information of agent i.

Because of the normality assumption (i.e., $x_i^0|\theta, \sigma_i^2 \sim \mathcal{N}(\theta, \sigma_i^2)$ for all $i \in N$), the monotone likelihood ratio property (MLRP) applies. Defining the likelihood ratio as $\lambda(x_i^0) \equiv \frac{\mathbb{P}\left(x_i^0|\theta=1,\sigma_i^2\right)}{\mathbb{P}\left(x_i^0|\theta=0,\sigma_i^2\right)} = e^{\frac{\left(x_i^0-\frac{1}{2}\right)}{\sigma_i^2}}$, the function $\lambda(\cdot)$ measures agent i's relative degree of confidence that the state of nature is $\theta = 1$. Hence, $\lambda'(\cdot) > 0$ implies that a higher x_i^0 provides stronger evidence that the state of nature is $\theta = 1$.

Voting and Communication Decision. At $t \geq 1$, after observing their own private information, the agents—sequentially—(i) vote, (ii) decide whether to extend communication, and, if so, (iii) communicate by exchanging messages.

The rule governing extended communication is intrinsically related to the underlying voting rule. Communication does not take place (or continue) when at least $\min\{k_0, k_1\}$ agents so decide. Indeed, because $\min\{k_0, k_1\}$ agents are empowered to implement an alternative, communication is halted once this alternative is implemented. For example, under the rule $K = \{1, n\}$, each agent can unilaterally impose the decision outcome $D = 0$. Under the same rule, an agent willing to implement $A = 0$ can also halt communication. Similarly, under the rule $K = \{k_0, k_1\}$, k_0 agents willing to implement $A = 0$ can unilaterally halt communication when $k_0 < k_1$. Instead, under the rule $K = \{n, n\}$, the agents can halt communication only upon unanimous deliberation.

The purpose of communication is to allow the aggregation of the agents' private information. In particular, communication should allow each agent i to estimate: (i) the private

[6] We refer to cognitive ability to identify either the signal's variance, σ_i^2, or its precision $[\sigma_i^2]^{-1}$. Better cognitive ability refers to lower variance and, then, higher precision.

signal received by each agent $j \neq i$ (*first order learning*) and (ii) the cognitive ability of each agent $j \neq i$ (*second order learning*). However, communication is imperfect, meaning that private information is transmitted with noise. In particular, noise in communication arises from the interaction of agents with different cognitive abilities in receiving and transmitting information. Over time, this interaction allows each agent to update her belief that the state of nature is equal to θ conditional on (i) and (ii) above.

Messages. At the communication stage $t \geq 1$, each agent j simultaneously sends a public message $m_j^t \in \mathbb{R} \cup \{\emptyset\}$ to agents $i \neq j$, where $m_j^t = \{\emptyset\}$ means that agent j remains silent at stage t. Because agents have heterogeneous precisions, communication between j and i reflects both agent j's private information about θ and a noise in communication between agent i and agent j. Hence, the message that agent i receives from each $j \neq i$ writes as

$$x_{ij}^t = m_j^t + \eta_{ij}^t$$

where (i) m_j^t is the public message sent by agent $j \neq i$, (ii) $\eta_{ij}^t \sim \mathcal{N}\left(0, f(\sigma_i^2, \sigma_j^2)\right)$ is the noise in communication, and (iii) $f(\sigma_i^2, \sigma_j^2)$ is a bounded function invertible in each argument. More specifically, η_{ij}^t reflects the precision in communication between agent j and agent i, where such precision is measured by $\left[1/f\left(\sigma_i^2, \sigma_j^2\right)\right]$. Summing up, each *communication signal* x_{ij}^t received by agent i at every $t \geq 1$ is affected by: (i) the message m_j^t sent from agent j, (ii) the cognitive ability of the receiver i (i.e., $[\sigma_i^2]^{-1}$), and (iii) the precision of the sender j (i.e., $[\sigma_j^2]^{-1}$).[7]

Because the goal of communication is aggregating all decision-relevant information—that is $h^0 \equiv (h_i^0)_{i \in N}$, with $h_i^0 = (x_i^0, \sigma_i^2)$—and messages are unidimensional, each agent i needs to choose a *discussion topic* that is able to convey her bidimensional private information to the other agents. To this end, each agent i could, for example, alternate topics that focus on reporting her initial private signal x_i^0 with others focusing on reporting her precision $[\sigma_i^2]^{-1}$. However, we are inclined to exclude that agents can report their precision $[\sigma_i^2]^{-1}$ because this would be against social norms. As an alternative, agents could report their updated beliefs

[7] For simplicity, we assume that each agent's abilities to process and to convey information are perfectly correlated. This assumption is innocuous.

as such beliefs evolve during the deliberation process. However, because posterior beliefs are based on noisy messages and updated beliefs are also sent and received with noise, if agents exchanged messages on their posterior beliefs they would end up with a worse inference of the other agents' private information. Then, it is reasonable to impose that all the agents select their discussion topics so to exclusively convey information on their initial signals. In this way, as communication continues over time, every agent i is able to estimate each agent j's decision-relevant information, i.e., h_j^0.

1.3.1.3. *Actions and Timing.* Each date $t \geq 1$ is split into three phases $\{t_v, t_c, t_m\}$ of consecutive actions: (i) at $t_v \geq 1$, every agent i votes $v_i^{t_v} \in \{0,1\}$ for an alternative $A \in \{0,1\}$; (ii) at $t_c \geq 1$, every agent i asks to either halt or extend communication by choosing $c_i^{t_c} \in \{0,1\}$, where $c_i^{t_c} = 0$ (resp. $c_i^{t_c} = 1$) means that agent i requests to halt (resp. extend) communication; and (iii) at $t_m \geq 1$, every agent i reports a message $m_i^{t_m} \in \mathbb{R} \cup \{\emptyset\}$. At each phase, all agents move simultaneously. For expositional clarity, when we unambiguously refer to a particular action taken at any time $t \geq 1$, we will label this time simply as t.

Therefore, at every $t \geq 1$, each agent i chooses an action $a_i^t \equiv (v_i^t, c_i^t, m_i^t) \in \{0,1\}^2 \times \mathbb{R} \cup \{\emptyset\}$. The following figure summarizes the timing of the game.

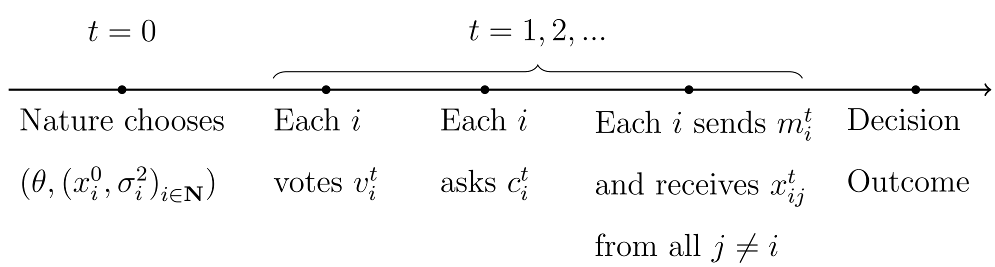


FIGURE 1.3.1. Timing of the Game.

1.3.2. Strategies and Solution Concept. In this section we focus on agents' deliberative behavior defining, for each agent: (i) the updating of beliefs, (ii) the voting strategy, (iii) the extending communication strategy, and (iv) the reporting message strategy. Let $\mathcal{B} \equiv \mathcal{U} \times \mathcal{V} \times \mathcal{C} \times \mathcal{M}$ denote the set of possible behaviors for each agent. Specifically, for each agent, $\mathcal{U}$ contains all possible methods for updating beliefs, $\mathcal{V}$ is the set of all possible voting strategies, $\mathcal{C}$ is the set of all possible extending communication strategies, and $\mathcal{M}$ is the set of all possible reporting message strategies. Thus, each agent i's deliberative behavior is specified by a collection $B_i = (U_i, V_i, C_i, M_i) \in \mathcal{B}$ where each element refers to (i), (ii), (iii), and (iv) above.

1.3.2.1. *Histories.* The dynamic structure of the game is such that the agents' strategies at any $t \geq 1$ are a function of past moves. Formally, this can be represented by letting $\mathcal{H}_i^0$ denote the set of all possible initial private information and, for $t \geq 1$, $\mathcal{H}_i^t$ denote the set of all possible histories of moves observed by agent i's from time 1 to time t. A history $h_i^0 \in \mathcal{H}_i^0$ contains then initial private information of each agent i, i.e., x_i^0 and σ_i^2. A history $h_i^t \in \mathcal{H}_i^t$, instead, contains the following elements: (i) tn votes, (ii) tn extended communication requests, (iii) t messages sent from agent i to all other $j \neq i$, and (iv) $t(n-1)$ signals that agent i received from every agent $j \neq i$.

By convention, $\mathcal{H}_i^{t_v} \equiv \mathcal{H}_i^{t-1}$; $\mathcal{H}_i^{t_c} \equiv \mathcal{H}_i^{t_v} \times \{0,1\}^n$; $\mathcal{H}_i^{t_m} \equiv \mathcal{H}_i^{t_c} \times \{0,1\}^n$ and $\mathcal{H}_i^t \equiv \mathcal{H}_i^{t_m} \times (\mathbb{R} \cup \emptyset)^n$. Let also $\mathcal{H}_i^l \equiv \overset{+\infty}{\underset{t=1}{\cup}} \mathcal{H}_i^{t_l}$, with $l \in \{v, c, m\}$.

Each history h_i^t is agent i's private information.

1.3.2.2. *Beliefs' Updating.* Before each action phase, all agents update their beliefs. Let $p_i^{t_l} \equiv \mathbb{P}\left(\theta = 1 | h_i^{t_l}\right)$ be agent i's *posterior belief* at date t_l, with $l \in \{v, c, m\}$ and $h_i^{t_l}$ denoting agent i's information that is available at date t_l.

1.3.2.3. *Voting Strategy.* At every stage $t_v \geq 1$, each agent i votes for one alternative. A *(pure) behavioral voting strategy* for agent i is a function from $\mathcal{H}_i^v$ to $\{0,1\}$ that maps a history $h_i^{t_v}$ into a vote $v_i^t\left(h_i^{t_v}\right) \in \{0,1\}$.[8] A voting strategy for agent i is *sincere* if at any t and for every $h_i^{t_v} \in \mathcal{H}_i^{t_v}$, $v_i^t\left(h_i^{t_v}\right) = 1$ only when $p_i^{t_v} > q$, and $v_i^t\left(h_i^{t_v}\right) = 0$ otherwise.

1.3.2.4. *Extended Communication Strategy.* At every stage $t_c \geq 1$, after having observed the voting outcome, each agent i asks either to halt or continue communication. A *(pure) behavioral stopping strategy* for agent i is a function from $\mathcal{H}_i^c$ to $\{0,1\}$ that maps a history $h_i^{t_c}$ into a halting decision on communication $c_i^t\left(h_i^{t_c}\right) \in \{0,1\}$.

1.3.2.5. *Reporting Message Strategy.* At every stage $t \geq 1$, whenever the communication is extended, each agent i chooses her reporting message. A *(pure) behavioral reporting strategy*

[8] To simplify our analysis, we restrict the model to pure strategies.

for agent i is a function from $\mathcal{H}_i^m$ to $\mathbb{R}$ that maps a history h_i^{tm} into a message $m_i^t\left(h_i^{tm}\right) \in \mathbb{R} \cup \{\emptyset\}$. A reporting strategy for agent i is *truthful* if $m_i^t\left(h_i^{tm}\right) = x_i^0$, for all $h_i^{tm} \in \mathcal{H}_i^m$.

1.3.2.6. *Equilibrium concept.* As solution concept, we use the perfect Bayesian equilibrium sustained by weakly undominated strategies, which we refer to as the *equilibrium*.[9]

1.4. The Informational Value of Consensus

1.4.1. The Efficient Decision Rule. In the situation where all decision-relevant information h^0 are common knowledge, agents collectively select $A = 1$ (resp. $A = 0$) when $\mathbb{P}(\theta = 1|h^0) > q$ (resp. $\mathbb{P}(\theta = 1|h^0) \leq q$). When, instead, communication is noisy, agents only have an estimate of the decision-relevant information h^0. This estimation is determined by the behavior of all agents $B \equiv (B_i)_{i \in N}$, that in turn depends on the voting rule K, which combine to characterize a deliberative process defined as follows.

Definition 1.1. A *deliberative process* $\mathcal{P} \equiv (K, B)$ is a voting rule K and behavior $B \equiv (B_i)_{i \in N}$ that for each agent i and any decision-relevant information set h^0 induces a probability $\mathbb{P}(\cdot|h^0)$ over all possible decision outcomes D.

With this definition, we can now identify the following particular class of deliberative processes. Let $\mathfrak{P}$ be the class of deliberative processes that implement alternative $A = 1$ (resp. $A = 0$) when $\mathbb{P}(\theta = 1|h^0) > q$ (resp. $\mathbb{P}(\theta = 1|h^0) \leq q$) with probability one for almost every h^0. This implies that any deliberative process belonging to $\mathfrak{P}$ selects, almost everywhere, the efficient alternative given all decision-relevant information. We can now state the following result.

Proposition 1.2. *A deliberative process $\mathcal{P}$ maximizes every agent's ex-ante payoff if and only if it belongs to $\mathfrak{P}$. Furthermore, if $\mathcal{P} = (K, B) \in \mathfrak{P}$, then B is an equilibrium.*

Proof. See Appendix. □

[9] Weakly undominated strategies eliminate some implausible and non-interesting equilibria . For example, consider the case in which it is common knowledge that all agents prefer $A = 0$ to $A = 1$, regardless of their posterior belief. Under a unanimity-based voting system, any strategy profile where at least two agents always vote for $A = 1$ is an equilibrium because no unilateral deviation can change the final outcome.

As a direct consequence of Proposition 1.2, we have the following result.

COROLLARY 1.3. *A deliberative process* $\mathcal{P}$ *is ex-ante Pareto efficient if and only if it belongs to* $\mathfrak{P}$.

The proof of this corollary comes straightforwardly from the maximization statement of Proposition 1.2: because all agents' ex-ante payoffs are maximized under the deliberative process $\mathcal{P}$, ex-ante Pareto efficiency follows.

We identify, now, a particular behavior, denoted as B^*, such that any deliberative process $\mathcal{P} = (K, B^*)$, for every K, belongs to the class $\mathfrak{P}$.

DEFINITION 1.4. The behavior B^* is such that, at every $t \geq 1$, each agent i:

(a) (*sincere voting*) votes $v_i^t = 1$ if $p_i^{tv} > q$ and $v_i^t = 0$ otherwise;

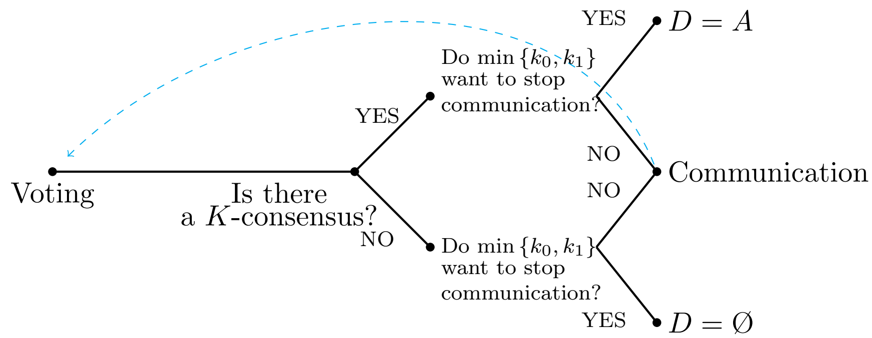
(b) asks to extend communication when her expected gain from making all decision-relevant private information publicly available is strictly positive;

(c) (*truthful communication*) reports $m_i^t = x_i^0$; and

(d) updates her beliefs according to Bayes' rule whenever possible.

Figure 2 summarizes the structure of the deliberative process (K, B^*), where Δ_i^t denotes the expected utility gain of agent i at stage t from making all decision-relevant private information publicly available.

Deliberative Process:



Behavior ($\forall i, \forall t \geq 1$): Voting | Extending Communication | Communication

$v_i^t = 1$ if $p_i^t > q$ | "Yes" if $\Delta_i^t > 0$ | $m_i^t = x_i^0$

$v_i^t = 0$ if $p_i^t \leq q$ | "No" if $\Delta_i^t = 0$

FIGURE 1.4.1. Deliberative Process (K, B^*) at Date $t \geq 1$.

As stated by the following result, the deliberative process (K, B^*) describes an ideal deliberation where the agents always reach unanimous consensus over an alternative A.

PROPOSITION 1.5. *For every K, the deliberative process (K, B^*) belongs to $\mathfrak{P}$.*

PROOF. See Appendix. □

Consistent with the findings of Coughlan (2000), Gerardi-Yariv (2007), and Visser et al. (2007), this result establishes that when communication does not involve any opportunity cost for the agents, B^* is an equilibrium behavior for whatever voting rule K. This equilibrium behavior induces a deliberative process that maximizes every agents ex-ante payoff and is ex-ante Pareto efficient. Under this deliberative process, all agents are able to estimate all decision relevant information (i.e., h^0) by extending communication at the limit (when $t \to \infty$). As a result, they always implement the alternative A that best approximates the state θ in light of h^0, meaning that $D = \emptyset$ is a decision outcome that occurs with probability zero. Indeed, as communication evolves from some $t \geq t'$, the size of each agent i's private information reduces, implying that the distance between each agent i and agent $j \neq i$ posterior beliefs reduces as well. Asymptotically, for each agent i, $\lim_{t\to+\infty} p_i^t = \mathbb{P}(\theta = 1|h^0)$. This is equivalent to say that because decision-relevant information becomes publicly available, the decision to halt communication and implement an alternative A is always taken unanimously by the agents. Indeed, under the behavior B^*, before the agents' posterior beliefs converge to the same limit point, there still exists a positive amount of private information in the deliberative process. Hence, whenever one agent asks to extend communication, the other agents will always agree.[10]

When we depart from this ideal state, the willingness to extend communication imposes opportunity costs on the agents. This implies that unanimous consensus is not guaranteed. Therefore, a deliberative process is defined as a function of the underlying voting rule. In the next section, we will then consider the case where agents exchange the benefits of extended communication against the opportunity costs arising therefrom.

1.4.2. The ϵ-Deliberative Process. To design the tradeoff between the benefits of extended communication and the opportunity costs arising therefrom, we introduce a notion of relative efficiency, i.e., ϵ-efficiency, where ϵ can be interpreted as the distance from the Pareto frontier. Along the same line, we also employ the concepts of ϵ-maximization and ϵ-equilibrium. This allow us to restrict our analysis to the implementation of ϵ-efficient decision rules.

Definition 1.6. We pose that:

(i) $x \in X$ ϵ-*maximizes* a function $f : X \to \mathbb{R}$ if $f(x) \geq f(x') - \epsilon$, for any $x' \in X$;

(ii) a deliberative process (K, B) is (ex-ante) ϵ-*efficient* if at $t = 0$ the expected payoff of any agent i cannot be increased by an amount higher than ϵ without decreasing at least one agent's expected payoff; and

(iii) a strategy profile $(a_i^t)_{i\in N}$ is an ϵ-*equilibrium* if, at any $t \geq 1$, for all $i \in N$ and $h_i^t \in \mathcal{H}_i^t$,

$$\mathbb{E}_{\left(a_i^t\right)_{i\in N}}\left[u(.,.)\,|h_i^{t_l}\right] \geq \mathbb{E}_{\left(\tilde{a}_i^t, a_{-i}^t\right)}\left[u(.,.)\,|h_i^{t_l}\right] - \epsilon$$

for any unilateral deviation $\tilde{a}_i^t$.

[10] To some extent, we can thus envisage this ideal state as a pre-political state without voting institutions, where Rawlsian decision-makers "rationally" and spontaneously reach unanimous decisions in drawing the architecture of society (Rawls, 1971).

Before presenting our main result, it is useful to briefly discuss the relevance of ϵ for our research. First, under Definition 1.6 - (iii), agents demand further communication to implement more accurate decision outcomes as long as this delivers them a benefit higher than ϵ. In this respect, similar to Wald (1947), ϵ can be interpreted as the opportunity cost of extended communication. Second, the trade-off imposed by $\epsilon > 0$ applies to any strategy contemplated by the agents' behavior. Hence, we refer to B^{ϵ} as the behavior under which a deviation from asking to extend communication delivers the agents a benefit lower than ϵ—consistent with Fudenberg and Levine (1986, p.263), who suggest that "if agents have sufficient inertia, they will not bother to realize small gains." Third, the measure of ϵ can be interpreted as the agents' rate of convergence toward the implementation of a decision outcome, which includes the implementation of $D = \emptyset$. Indeed, by definition of B^{ϵ}, a greater ϵ makes marginal further discussion less profitable. As a result, the deliberation process delivers decision outcomes more quickly. Fourth, we can associate the magnitude of ϵ to different deliberative contexts. In this sense, it is reasonable to assume that larger n corresponds to greater ϵ. Indeed, communication among a large number of agents is generally more tedious and implies higher opportunity cost than communication among a small number of agents. Finally, from a social point perspective, being ϵ the distance from Pareto efficiency, a large ϵ poorly supports the adoption of ϵ-efficiency as an approximation of Pareto efficiency.

We can now restrict our attention to the deliberative process (K, B^{ϵ}), providing the following proposition.

PROPOSITION 1.7. *Assume* $\epsilon \in (0, \max\{\xi_0, \xi_1\})$. *The deliberative process* (K, B^{ϵ})*:*

(i) ϵ*-maximizes every agent ex-ante payoff;*

(ii) is ex-ante ϵ*-efficient; and*

(iii) is such that B^{ϵ} *is an* ϵ*-equilibrium*

if and only if $K = \{n, n\}$.

PROOF. See Appendix. □

The intuition of the proof is as follows. When communication is valuable for each agent, i.e., when $\epsilon \in (0, \max\{\xi_0, \xi_1\})$, they are willing to communicate in order to better estimate decision-relevant information and reach a more accurate decision outcome. This makes the agents' ex-ante payoffs ϵ-maximal and the deliberative process ϵ-efficient.

To understand how the voting rule $K = \{n, n\}$ connects with Proposition 1.7, consider a non-unanimous voting rule $K = \{k_0, k_1\}$, with at least one element strictly lower than n, under which $\min\{k_0, k_1\}$ agents can halt communication. Specifically, take the extreme case of the great power unanimity rule, i.e., $K = \{1, n\}$, under which each agent is empowered to halt communication by imposing $A = 0$. In this case, the agents willing to implement the alternative $A = 1$ may find ϵ-profitable a deviation $B^{\epsilon\prime} \neq B^{\epsilon}$ under which they exaggerate their reporting strategies. Under particular histories, in fact, message misreporting reduces the probability that an agent halts communication by imposing $A = 0$.

This result offers a new perspective on unanimity-based deliberative processes as compared to the conclusions of the literature on voting strategy (see, e.g., Feddersen and Pesendorfer (1998), Persico (2003), Austen-Smith and Feddersen (2006)).

Proposition 1.7 holds when ϵ is relatively low. Conversely, when $\epsilon > \max\{\xi_0, \xi_1\}$, we can show that communication under unanimity becomes unfeasible. More specifically, whereas the sufficiency of Proposition 1.7 still holds, necessity is lost. Without communication the agents' behavior reduces to a voting strategy. This leads to two consequences. First, the voting rule $K = \{n, n\}$ loses its beneficial property of inducing more communication, making the implementation of the social welfare-maximizing alternative less likely and the decision outcome $D = \emptyset$ more likely. Therefore, a voting system that guarantees the implementation of one alternative A is more desirable in these circumstances. Second, being the vote the sole instrument to aggregate decision-relevant information, agents may find profitable to deviate from sincere voting. That is, they may engage in strategic voting, taking into account the probability of being pivotal at the voting stage (*see* Austen-Smith and Banks (1996)).

Based on these observations, we conjecture that without communication a deliberative process that is based on a simple majority rule Pareto-dominates a deliberative process (K, B^{ϵ}) based on the jury unanimity rule.

Conjecture. *Assume* $\epsilon > \max\{\xi_0, \xi_1\}$. *There exists a majority deliberative process* $(K, B(K))$ *with* $K = \left\{\left\lfloor \frac{n}{2} + 1\right\rfloor, \left\lfloor \frac{n}{2} + 1\right\rfloor\right\}$ *and* $B(K)$ *being an equilibrium, that ex-ante Pareto dominates any deliberative process* (K, B) *with* $K = \{n, n\}$.

The rational underlying the above conjecture can be illustrated as follows.

Consider the majority deliberative process $(K, B(K)) = \left(K = \left\{\left\lfloor \frac{n}{2} + 1\right\rfloor, \left\lfloor \frac{n}{2} + 1\right\rfloor\right\}, B^{\epsilon}\right)$ with n odd, under which the agents behave as under B^{ϵ} (i.e., vote sincerely). Being the ex-ante probability of belonging to the majority strictly higher than $1/2$ for each agent, the agents' ex-ante utility is likely to be higher under the majority deliberative process than under the deliberative process $(K = \{n, n\}, B^{\epsilon})$. This is because a majority system never implements $D = \emptyset$ and $\xi_1 \geq \frac{(1-q)(q-\xi_0)}{q}$. Additionally, the efficiency of such a system with respect to an unanimity system is increasing in the number of agents. Indeed, as $n \to \infty$, under the deliberative process $(K = \{n, n\}, B^{\epsilon})$, $\mathbb{P}(D = \emptyset)$ goes to 1. This means that the agents' utility-maximizing alternative is never implemented. This is in line with the result of the Condorcet jury theorem, under which simple majority is asymptotically optimal.

If we consider an alternative deliberative process of the form $(K, B(K))$, when communication is unfeasible, the agents' behavior reduces to a voting strategy. This implies that, in equilibrium, the agents condition their vote on being pivotal, which is the only circumstance where their vote may affect their expected payofsf (see, e.g., Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998)). Being the signal continuous, we focus on the case where agents follow a symmetric threshold strategy such as

$$v_i(x_i) = \begin{cases} 1 & \text{if} \quad x_i > \tilde{x}_i(K) \\ 0 & \text{if} \quad x_i \leq \tilde{x}_i(K) \end{cases}$$

where, for any given voting rule K, $\tilde{x}_i(K)$ is the threshold that makes agent i indifferent between the two alternatives. In particular, such threshold can be decomposed in two terms:

$$\tilde{x}_i(K) = x^S + x_i^D(K)$$

The first term (i.e., x^S) is the threshold that would make any agent indifferent between alternative 1 and alternative 0 under sincere voting. The second term $x_i^D(K)$, instead, reflects the distortion arising from being pivotal under a given voting rule K. Because under more restrictive voting rules, such as $K = \{n, n\}$, it is less likely that an alternative A is implemented, the agents strategically determine $x_i^D(K)$ to maximize their utility. This means that agents may vote against their private information. When $K = \left\{\left\lfloor \frac{n}{2}+1 \right\rfloor, \left\lfloor \frac{n}{2}+1 \right\rfloor\right\}$, such distortion is minimal, because the agents always select one alternative (provided that the number of agents is odd). However, in order to better understand the welfare properties of these two voting rules a tradeoff is to be taken into account. Although simple majority never implements $D = \emptyset$, it implements wrong decisions at an higher rate as compared to unanimity. At this juncture, two cases are in order. When the payoff associated with $D = \emptyset$ is the lowest, simple majority is obviously more desirable to unanimity. When, instead, $\xi_1 = \frac{(1-q)(q-\xi_0)}{q}$, the payoff associated with a no-decision is always an intermediate payoff between those associated with the two alternatives, implying that the welfare maximizing properties of simple majority are more ambiguous. Also in this case, however, we claim that from an ex-ante perspective simple majority performs better than unanimity. Indeed, the coordination effect arising from strategic voting under unanimity, although effective at reducing the probability that a no-decision be implemented, loses its accuracy when $x_i^D(K)$ is maximal. Furthermore, as the number of agents increase, also the distortion increases, meaning that as $n \to \infty$ agents' votes become uninformative.

In the Appendix, Example 1 provides a detailed explanation supporting this conjecture.

1.4.3. Discussion. In section 4.2. above, we have emphasized the importance of voting systems in aggregating dispersed information, identifying two extreme situations under which the desirability of a given voting rule depends on whether communication is feasible. In this section, we discuss how our theoretical results may contribute to further the understanding of certain deliberative institutions. Our aim is to verify to what extent ϵ-efficiency is a reasonable proxy for social welfare (i.e., Pareto efficiency) in situations in which communication is feasible. To pursue this goal, we begin by offering a compact visualization of the results of Proposition 1.5 and 1.7 in Figure 3 below.

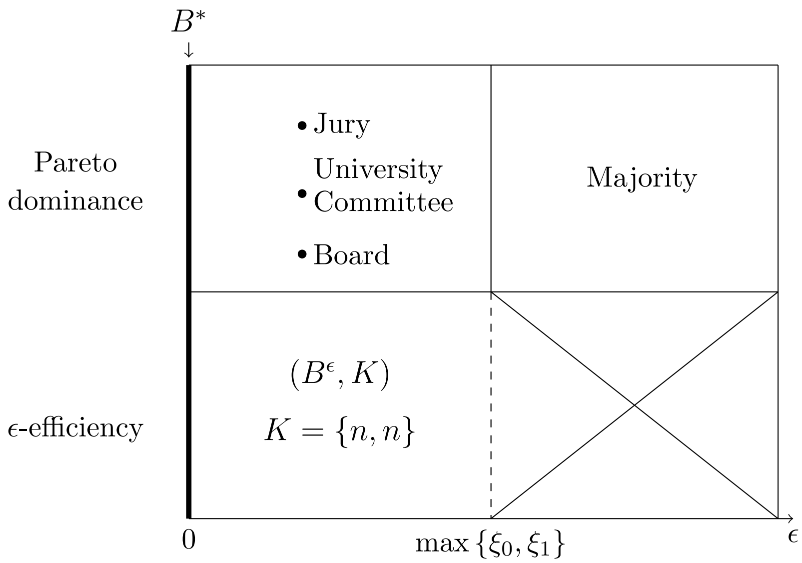


FIGURE 1.4.2. Welfare Analysis.

The bold vertical line in Figure 3 represents the ideal case in which $\epsilon = 0$, meaning that communication does not come at an opportunity costs for the agents. As stated in Proposition 2, under this condition we obtain that communication lasts until the private information of each agent becomes public. This, in turn, implies that the agents always reach unanimous consensus regardless of which is the voting rule.

Consider now the four boxes appearing in Figure 3. In the bottom-left box, we visualize the results of Proposition 1.7, which states that when an agent's deviation payoff from sincere voting, truthful reporting, and extended communication is contained in ϵ, the result of Proposition 1.5 can be approximated provided that the underlying voting rule is $K = \{n, n\}$. Indeed, under the conditions of Proposition 1.7, unanimous consensus over an alternative does not follow automatically as under Proposition 1.5. Instead, in choosing whether to transform their private information into public information, the agents face a tradeoff between obtaining more accurate information and facing higher opportunity costs. Under these circumstances, the voting rule $K = \{n, n\}$ provides the incentives to increase communication intensity and, therefore, transform private information into public information. Nonetheless, this system remains imperfect because the probability of implementing $D = \emptyset$ is increasing in ϵ.

The bottom-right box represents the case in which the deliberative process envisaged by Proposition 1.7 applies, but ϵ is so high as to prevent communication under the voting rule $K = \{n, n\}$. This implies that although the deliberative process is ϵ-efficient, it implements $D = \emptyset$ with high frequency. However, in accordance with the economic interpretation of ϵ provided above, ϵ-efficiency offers a poor approximation of Pareto efficiency under the magnitude of ϵ appearing in the bottom-right box of Figure 3. Hence, the case therein represented is not relevant for the purpose of this discussion.

The top-left and top-right boxes consider Pareto dominance as a criterion to asses the welfare validity of Proposition 1.7. In particular, in the top-right box, we conjectured that when communication is not affordable under unanimity, deliberative processes based on a majority rule are more desirable. This is because, absent communication, agents with homogeneous preferences but different private information are somewhat similar to agents with heterogeneous preferences and public information. Therefore, in the latter case, whereas a jury unanimity rule systematically fails to implement an alternative when communication is unfeasible, a simple majority rule always ensures that one alternative is implemented. This results is consistent with the actuality of collective decision-making institutions with large number of agents, such as plenums or electoral bodies. Indeed, deliberative processes that operate without communication and, at the same time, employ a jury unanimity rule are not observed in the real world.

Our major interest is then on the top-left box, under which communication is affordable. In this case, the interpretative problem is to understand which deliberative process (i.e., $(B^{\epsilon}, K = \{n, n\})$ or a majority deliberative process) better approximates Pareto efficiency. The answer to this question depends essentially on two dimensions. The first involves the extent to which communication may be biased by untruthful reporting and, therefore, fail to aggregate decision-relevant information. The second involves the social cost of implementing the decision outcome $D = \emptyset$ relative to implementing a wrong decision (the relative cost of $D = \emptyset$).

Concerning the first dimension, we distinguish situations characterized by "quantitative" private information (e.g., information based on data) and "qualitative"private information. In the case of quantitative information, it is reasonable to assume that each agent's private information is at least partially verifiable by other agents. This implies that a deviating behavior, such as reporting exaggerated information, is a less sever concern. Therefore, under these circumstances, a deliberative process based on unanimity loses its crucial comparative advantage, since other voting rules may also be efficient in inducing truthful communication.

As concerns the second dimension, when the expected social cost of implementing a wrong alternative is low relatively to that of implementing the decision outcome $D = \emptyset$, deliberative processes based on the jury unanimity rule are, in general, less desirable. Indeed, when a low payoff is associated with $D = \emptyset$ (e.g., when $D = \emptyset$ delivers to the agents the lowest payoff), the implementation of $D = \emptyset$—although ϵ-efficient—is clearly Pareto dominated by the implementation of any alternative A. In other words, because in such a case there is a deviating strategy that increases the agent's utility, the notion of ϵ-efficiency becomes a bad proxy for optimality.

On these premises, we can now proceed to consider some real-world applications.

Juries. The task performed by jurors is characterized by the individual and subjective assessment of the evidentiary basis disclosed at trial. This makes jurors' private information essentially qualitative. Because of this distinguishing feature, the implementation of a deliberative process that is suited to induce truthful communication increases the likelihood of optimal decision outcomes in jury contexts. Additionally, in such contexts, the relative cost of $D = \emptyset$ is low. Indeed, the institutional provision that a new trial can take place in case of mistrial makes the payoff associated with mistrial "intermediate" between a "right decision" payoff and a "wrong decision" payoff. Further, given the importance of the jurors' task, it is reasonable to assume that the opportunity cost of extended communication is not substantial as long as jurors' interaction can deliver better decisions. Overall, these observations suggest that the unanimity-based deliberative process carachterizing the U.S. jury system (and reflecting the paradigm $(B^{\epsilon}, K = \{n, n\})$) is ϵ-efficient and close to the Pareto frontier.

Appointments Committee. Similarly to jury contexts, the private information of appointments committees' members is generally more qualitative than quantitative. This circumstances makes incentives for truthful extended communication crucial to increase the likelihood of effective candidate assessments by committee members. Additionally, in appointment committees the cost of $D = \emptyset$ is also likely to be low. Indeed, hiring a bad candidate or failing to hire a good candidate are worse payoffs than, for example, postponing a hiring decision. Further, close to the case of a wrong jury decision, a wrong decision made by an appointment committee tends to be irreversible. This is because the hiring of a candidate often leads to a permanent appointment with the designating institution (consider, for example, the case of tenured professors). Likewise, the decision not to hire a candidate may result in the candidate's permanent appointment at a competing institution. Because of these reasons, in appointments committees a deliberative system based on unanimous consensus is likely to Pareto dominate alternative voting systems. This result is consistent with the practice of many educational institutions where social norms mandate that the operation of appointments committees be governed by unanimity rules.

Board. An essential task of corporate boards is to make investment decisions. Most often, in supporting the undertaking of one investment decision over the other, board members employ quantitative evidence that is partially verifiable. This kind of communication tends to make truthful communication a second-order problem. Additionally, the ability to make rapid decisions is often crucial for the success of business entities. Hence, in these contexts the relative cost of implementing $D = \emptyset$ is likely to be high. This observation is a fortiori verified whenever the ability to reverse a business decision is the key to limit expected losses, such as when, for example, the abandonment of a certain project allows an entity to limit the costs associated with the initial decision to begin that project. Further, individuals engaged in the business world are in general more time-constrained, meaning that the opportunity costs they face for extending communication tend to be high. Overall, these observations suggest that the reality of corporate boards where majority voting rules are the standard are consistent with the predictions arising from our theoretical analysis.

1.5. Robustness Extensions

To highlight that our framework can accommodate additional features we intentionally abstracted from in the above analysis, in this section we consider some robustness extensions of our baseline model. In particular, we explore the possibility that agents: (i) have heterogeneous preferences, (ii) do not know their cognitive ability $[\sigma_i^2]^{-1}$, (iii) may vote for the no-decision, and (iv) communicate sequentially.

Heterogeneous Preferences. To illustrate the possibility of heterogeneous preferences, we modify the basic model assuming that $q_1 \leq q_2 \leq \ldots \leq q_n$. In this case, even if we assume sincere voting and extended truthful communication, the agents may fail to reach unanimous consensus. This is because the agents' common posterior belief may lie between different measures of the loss associated with the wrong decision, i.e., $\lim_{t\to\infty} p_i^t \equiv p^* \in (q_1, q_n)$ for every i. In order to show to what extent this robustness extension is a concern for our model, we provide the following heuristic illustration.

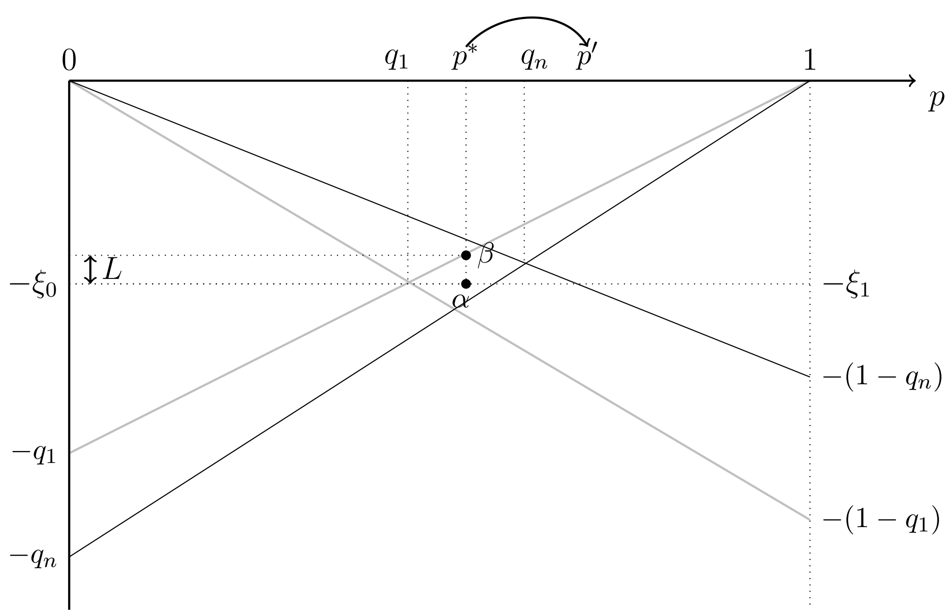


FIGURE 1.5.1. Heterogeneous Preferences.

Whether the behavior B^ϵ can still be supported as ϵ-equilibrium under $K = \{n, n\}$ depends essentially on the severity of the agents' heterogeneity in preferences. In this case, agent $i = 1$ (resp. agent $i = n$) could be induced to exaggerate her information in order to move p^* at the right of q_n.

To better understand this claim, suppose that at a date $t \geq 1$ all decision-relevant information h^0 has been estimated so that every agent agrees to halt communication. Also assume that, for $p^* \in (q_1, q_n)$, only agent 1 (whose payoff function is indicated in gray in the figure) prefers alternative 1. By reporting her information truthfully, agent 1 receives the payoff indicated by α, corresponding to the decision outcome $D = \emptyset$. Anticipating this result, however, she may exaggerate her reporting so as to inflate the other agents' posterior above q_n and convince them to implement her preferred alternative. This gives her the payoff indicated by β. Thus, agent 1 has an ϵ-profitable deviation as long as at date $t \geq 1$ there exists a $m_1^t \neq x_1^0$ such that $\mathbb{P}\left(\min_{j\neq 1} p_j > q_n | m_1^t, \cdot\right) \cdot \mathbb{P}\left(p^* \in (q_1, q_n)\, |h^0\right) \cdot L > \epsilon$, with (i) $\mathbb{P}\left(\min_{j\neq 1} p_j > q_n | m_1^t,\right)$ being the probability that agent 1 can move the other agents' posterior above q_n through misreporting; (ii) $\mathbb{P}\left(p^* \in (q_1, q_n)\, |h^0\right)$ being the probability that the agents' posterior belief be in the interval (q_1, q_n) when all decision-relevant information are estimated; and (iii) L being the difference between (a) the payoff that agent 1 receives when $D = 1$ is implemented and (b) the payoff she receives when $D = \emptyset$ is implemented.

This shows that Proposition 1.7 is not robust to high degree of heterogeneity in preferences, consistent with the idea that the importance of communication rests on aggregating decentralized private information, not preferences. Thus, in jury contexts, for example, the jury selection process is aimed at reducing the distance among the jurors' preferences and, therefore, the possible distortions that may arise from severe divergence in the jurors' preferences. When, instead, agents have an high degree of heterogeneity, other institutions such as deliberative systems based on majority voting rules are better suited to aggregate dispersed information.

Unknown Cognitive Abilities. In the baseline model, we have assumed that each agent knows her cognitive ability. When we relax this assumption and consider the case in which each agent does not know either her cognitive ability or that of the other agents, the results of Proposition 1.7 remain almost unchanged. To see why this is the case, consider the situation where agents behave in accordance with the deliberative process (K, B^{ϵ}), with $K = \{n, n\}$. Even when communication lasts for a high number of stages, each agent i is able to assess only the function $f\left(\sigma_i^2, \sigma_j^2\right)$ for each $j \neq i$, but not the single variances (including her own variance σ_i^2). This implies that the agents can never have the same assessment of the decision-relevant information h^0, even if they truthfully report information. This imperfect assessment may lead the agents' posteriors to be such that $q \in (p_{\min}, p_{\max})$, where $p_{\min} \equiv \min\{p_i\}_{i \in N}$ and $p_{\max} \equiv \max\{p_i\}_{i \in N}$. Under these conditions, a deliberative process based on jury unanimity will implement the decision outcome $D = \emptyset$. This situation, although less severe, parallels that arising from the existence of heterogeneous preferences discussed above. Therefore, the qualitative implications of Proposition 1.7 do not change as long as the heterogeneity of the agents' posteriors is not "too" severe.

Voting for the No-Decision. The no-decision outcome $D = \emptyset$ is the event occurring under the voting rule $K = \{n, n\}$, when agents fail to reach unanimous consensus over an alternative. In the model, however, we do not allow the agents to vote explicitly for the the no-decision. We discuss here why this restriction does not change our results. For simplicity, we discuss the consequences of $v_i^t \in A \cup \{\emptyset\}$ on the voting rule $K = \{n, n\}$ and its impact on Proposition 3.

Defining $v_i^t = \emptyset$ as abstention lowering the quorum required to implement an alternative, the voting rule becomes $K = \{k', k'\}$, with $k' = n - v_{\emptyset}$ and $v_{\emptyset}$ being the number of agents choosing to abstain. This represents the case, for example, of decision bodies in which members are allowed to abstain at the voting stage. A different case arises when abstention is equivalent to a vote in favor of one of the two alternatives.[11] In this case, $v_i^t = \emptyset$ is equivalent to either $v_i^t = 1$ or $v_i^t = 0$. This means that the voting rule remains unchanged. That is, when the no-decision is an additional alternative that can be directly implemented by the agents, the voting rule set is expanded so to contain three elements, i.e. $K = \{n, n, 1\}$, where the third

[11]This is, for example, the case of the Italian Senate, i.e. the higher chamber of the Italian Parliament, where abstention is equivalent to a vote in favor of the status-quo.

element indicates the number of agents required to implement the alternative $D = \emptyset$. Such a K means that each agent can unilaterally implement the no-decision.

In all these cases, although there might exist communication histories under which an agent prefers to implement the no-decision outcome, from an ex-ante perspective the agents' expected payoff is always higher under the deliberative process $(K = \{n, n\}, B^{\epsilon})$ than under a deliberative process $(K = \{n, n, 1\}, B^{\epsilon})$. Indeed, being the deliberative process $(K = \{n, n\}, B^{\epsilon})$ the process that maximizes the agents' ex-ante utilities, every agent will rationally select that process. This conclusions makes the possibility for the agents to vote $v_i^t = \emptyset$ at best redundant, meaning that $v_i^t = \emptyset$ is never an equilibrium strategy.

Sequential Actions. When communication is public but the order of speech is sequential, our results hold. Under sequential moves, indeed, as long as the agents expect to engage in extended communication, their incentives to behave do not change in accordance with the order in which they (i) vote, (ii) ask to extend communication, or (iii) communicate. This is true, however, only as long as we exclude the possibility of herding behavior. This issue, which we rule out under the assumption that agents are benevolent (who, for instance, do not have career concerns), remains beyond the scope of our research.

1.6. Conclusions

We have analyzed how voting rules shape dynamic deliberative processes. We have found that voting rules are important communication devices.

When communication among agents is noisy but agents can costlessly communicate, voting rules are irrelevant because agents always have incentives to extend communication in order to turn their private information into public information. This produces unanimous consensus over decision outcomes. Because the agents anticipate this circumstance, truthful communication and sincere voting are supported as an equilibrium.

When, instead, communication among agents involve opportunity costs, voting rules shape communication strategies. Specifically, we have shown that when the opportunity costs of communication is relatively low, a unanimity-based deliberative process implements the decision outcome that approximates the maximization of social welfare. Instead, when communication involves high opportunity costs, an unanimity-based deliberative process is Pareto dominated by a majority-based deliberative process.

These results sheds some light on the existence of different voting institutions. When the cost associated with inaccurate decisions is high compared to the opportunity cost of extended communication—as happens in juries—an unanimity-based deliberative process is socially desirable. Conversely, a majority-based deliberative process is superior when the opportunity costs of extended communication are severe—as in plenums or large electoral bodies. Likewise, such a process is more desirable when the cost associated with a no-decision is high—as in corporate boards.

1.7. Appendix

Proof of Proposition 1.2.

First statement: Maximization

Necessity. Suppose, by contradiction, that there is a deliberative process $\mathcal{P}' \notin \mathfrak{P}$ that maximizes every agent's ex-ante payoff. From $\mathcal{P}' \notin \mathfrak{P}$ assume that there is a (non-negligible set of) decision-relevant information profile(s) $h^0 \equiv \left(h_j^0\right)_{j\in N} \in \mathcal{H}^0 \equiv \mathcal{H}_1^0 \times ... \times \mathcal{H}_n^0$ such that:

$$\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] > q \text{ with } \mathbb{P}_{\mathcal{P}'}\left[D = 1 \left| h^0\right.\right] < 1; \text{ or} \tag{1}$$

$$\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] < q \text{ with } \mathbb{P}_{\mathcal{P}'}\left[D = 0 \left| h^0\right.\right] < 1. \tag{2}$$

Assume (2) holds (a similar argument applies when (1) holds). Let $i \in N$. By assumption, there is a (non-negligible set of) profile(s) of histories $\left(h_j^t\right)_{j\in N}$ where, under the deliberative process $\mathcal{P}'$, $D = 1$ (or, resp. $D = 0$) is selected with a probability lower than 1 at date t, and $\mathbb{P}_{\mathcal{P}'}\left[h^0 \right| h_i^t] > 0$ with $\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] > q$ (resp. $\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] < q$). Under $\mathcal{P}'$, i's expected payoffs at history h_i^0 writes as

$$\begin{aligned} \sum_{(D,\theta)} u(D,\theta)\, \mathbb{P}_{\mathcal{P}'}\left[D,\theta \left| h_i^0\right.\right] &= \int_{\mathcal{H}^0} \sum_{(D,\theta)} u(D,\theta)\, \mathbb{P}_{\mathcal{P}'}\left[D,\theta \left| h^0, h_i^0\right.\right] d\mathbb{P}_{\mathcal{P}'}\left[\cdot\right| h_i^0] \\ &= \int_{\mathcal{H}^0} \sum_{(D,\theta)} u(D,\theta)\, \mathbb{P}_{\mathcal{P}'}\left[D \left| h^0\right.\right] \mathbb{P}\left[\theta \left| h^0\right.\right] d\mathbb{P}\left[\cdot\right| h_i^0] \end{aligned}$$

where the second equality comes from the fact that h_i^0 is an element of h^0, and that given h^0 (resp. h_i^0) the distribution of θ (resp. h^0) does not depend on $\mathcal{P}'$.

From the values taken by $u(D,\theta)$, from $\mathbb{P}\left[\theta = 0 \left| h^0\right.\right] = 1 - \mathbb{P}\left[\theta = 1 \left| h^0\right.\right]$ and from

$$\mathbb{P}_{\mathcal{P}'}\left[D = \varnothing \left| h^0\right.\right] = 1 - \mathbb{P}_{\mathcal{P}'}\left[D = 1 \left| h^0\right.\right] - \mathbb{P}_{\mathcal{P}'}\left[D = 0 \left| h^0\right.\right]$$

this expected payoff can be rewritten as

$$\int_{\mathcal{H}^0} \left(\Lambda_1 \mathbb{P}_{\mathcal{P}'}\left[D = 1 \left| h^0\right.\right] + \Lambda_0 \mathbb{P}_{\mathcal{P}'}\left[D = 0 \left| h^0\right.\right] - (\xi_1 - \xi_0)\, \mathbb{P}\left[\theta = 1 \left| h^0\right.\right] - \xi_0\right) d\mathbb{P}\left[\cdot\right| h_i^0] \tag{3}$$

with

$$\Lambda_1 \equiv (\xi_1 - \xi_0 + q)\,\mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] + \xi_0 - q \tag{4}$$

and

$$\Lambda_0 \equiv (\xi_1 - \xi_0 - (1-q))\,\mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] + \xi_0. \tag{5}$$

Observe that when $\mathbb{P}[\theta = 1 \,|h^0] > q$, we have $\Lambda_1 > \Lambda_0$ and, from $\xi_1 \geq \frac{(1-q)(q-\xi_0)}{q}$, the term Λ_1 is strictly positive. Similarly, when $\mathbb{P}[\theta = 1 \,|h^0] \leq q$, we have $\Lambda_0 \geq \max\{\Lambda_1, 0\}$.

Therefore, from $\mathcal{P}' \notin \mathfrak{P}$, (3) is strictly lower than

$$\int\limits_{\mathcal{H}^0} \left(\Lambda_1 \mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]>q\}} + \Lambda_0 \mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]\leq q\}} - (\xi_1 - \xi_0)\,\mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] - \xi_0\right) d\mathbb{P}\left[\cdot \right| h_i^0]$$

where $\mathbb{I}_E$ denotes the indicator function that takes value 1 if the event E is realized and 0 otherwise.

This is equal to

$$\int\limits_{\mathcal{H}^0} -q\left(1 - \mathbb{P}\left[\theta = 1 \left| h^0 \right.\right]\right) \mathbb{I}_{E_1} - (1-q)\,\mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] \mathbb{I}_{E_2} d\mathbb{P}\left[\cdot \right| h_i^0]$$

$$= \int\limits_{\mathcal{H}^0} u(1,0)\,\mathbb{P}\left[\theta = 0 \left| h^0 \right.\right] \mathbb{I}_{E_1} + u(0,1)\,\mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] \mathbb{I}_{E_2} d\mathbb{P}\left[\cdot \right| h_i^0] \tag{6}$$

with

$$E_1 \equiv \{\mathbb{P}[\theta = 1 \,|h^0] > q\} \quad \text{and} \quad E_2 \equiv \{\mathbb{P}[\theta = 1 \,|h^0] \leq q\}.$$

This corresponds to the ex-ante payoff that i would have obtained under any $\mathcal{P} \in \mathfrak{P}$, a contradiction.

Sufficiency. Let $i \in N$, $h^0 \in \mathcal{H}^0$, and $(\mathcal{P}, \mathcal{P}') \in \mathfrak{P}^2$. First, it can be easily checked that i's expected payoff writes as (6) and is invariant from any difference between $\mathcal{P}$ and $\mathcal{P}'$. Second, from the previous part of the proof (necessity), we already know that i's expected payoff is lower under any $\mathcal{P}'' \notin \mathfrak{P}$.

Second statement: Equilibrium

Let $\mathcal{P} \equiv (K, B^*) \in \mathfrak{P}$. Under B^*, the only off-the-equilibrium-path message is $\varnothing$. Assume that when a member chooses to remain silent his counterpart infer that his initial signal is $\frac{1}{2}$. Suppose, by contradiction, that there is a member $i \in N$ that has a profitable unilateral deviation B_i under $\mathcal{P}$. Denote by t the first date at which B_i differs from B_i^*. Under the deliberative process $\mathcal{P}$, i's expected payoff at any history $h_i^t \in \mathcal{H}_i^t$ writes as

$$\begin{aligned}\sum_{(D,\theta)} u(D,\theta)\,\mathbb{P}_{\mathcal{P}}\left[D,\theta\left|h_i^t\right.\right] &= \int_{\mathcal{H}^0}\sum_{(D,\theta)} u(D,\theta)\,\mathbb{P}_{\mathcal{P}}\left[D,\theta\left|h^0,h_i^t\right.\right] d\mathbb{P}_{\mathcal{P}}\left[\cdot\left|\,h_i^t\right.\right] \\ &= \int_{\mathcal{H}^0}\sum_{(D,\theta)} u(D,\theta)\,\mathbb{P}_{\mathcal{P}}\left[D\left|h^0,h_i^t\right.\right]\mathbb{P}\left[\theta\left|h^0\right.\right] d\mathbb{P}_{\mathcal{P}}\left[\cdot\left|\,h_i^t\right.\right]\end{aligned}$$

which, from $\mathcal{P} \in \mathfrak{P}$, can be rewritten as

$$\int_{\mathcal{H}^0}\sum_{\theta}\left(u(0,\theta)\,\mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]\leq q\}} + u(1,\theta)\,\mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]>q\}}\right)\mathbb{P}\left[\theta\left|h^0\right.\right] d\mathbb{P}_{\mathcal{P}}\left[\cdot\left|\,h_i^t\right.\right]. \tag{7}$$

Under the deliberative process $\mathcal{P}' \equiv (K, (B_{-i}^*, B_i))$, i's expected payoff at history h_i^t can be written as

$$\int_{\mathcal{H}^0}\sum_{\theta}\sum_{D} u(D,\theta)\,\mathbb{P}_{\mathcal{P}'}\left[D\left|h^0,h_i^t\right.\right]\mathbb{P}\left[\theta\left|h^0\right.\right] d\mathbb{P}_{\mathcal{P}'}\left[\cdot\left|\,h_i^t\right.\right]. \tag{8}$$

By comparing (7) and (8), and using the fact that $\mathbb{P}_{\mathcal{P}'}\left[\cdot\left|\,h_i^t\right.\right] = \mathbb{P}_{\mathcal{P}}\left[\cdot\left|\,h_i^t\right.\right]$ because $\mathcal{P}'$ does not differ from $\mathcal{P}$ before date t, the unilateral deviation B_i is profitable only if there is a (non-negligible set of) profile(s) $h^0 \in \mathcal{H}^0$ such that

$$\begin{aligned}&\sum_{\theta}\left(u(0,\theta)\,\mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]\leq q\}} + u(1,\theta)\,\mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]>q\}}\right)\mathbb{P}\left[\theta\left|h^0\right.\right] \\ <\;&\sum_{\theta}\sum_{D} u(D,\theta)\,\mathbb{P}_{\mathcal{P}'}\left[D\left|h^0,h_i^t\right.\right]\mathbb{P}\left[\theta\left|h^0\right.\right].\end{aligned}$$

There are two cases to consider depending on whether $\mathbb{P}\left[\theta=1\left|h^0\right.\right]$ is higher or lower than q. First, consider the case where $\mathbb{P}\left[\theta=1\left|h^0\right.\right] \leq q$. The previous strict inequality then writes as

$$
\begin{aligned}
(9) \qquad & u(0,1)\,\mathbb{P}\left[\theta=1\,\middle|\,h^0\right] \\
< \; & \left(u(0,1)\,\mathbb{P}_{\mathcal{P}'}\left[D=0\,\middle|\,h^0,h_i^t\right]+u(\varnothing,1)\,\mathbb{P}_{\mathcal{P}'}\left[D=\varnothing\,\middle|\,h^0,h_i^t\right]\right)\mathbb{P}\left[\theta=1\,\middle|\,h^0\right] \\
& +\left(u(1,0)\,\mathbb{P}_{\mathcal{P}'}\left[D=1\,\middle|\,h^0,h_i^t\right]+u(\varnothing,0)\,\mathbb{P}_{\mathcal{P}'}\left[D=\varnothing\,\middle|\,h^0,h_i^t\right]\right)\mathbb{P}\left[\theta=0\,\middle|\,h^0\right].
\end{aligned}
$$

From

$$\mathbb{P}_{\mathcal{P}'}\left[D=\varnothing\,\middle|\,h^0,h_i^t\right]=1-\mathbb{P}_{\mathcal{P}'}\left[D=1\,\middle|\,h^0,h_i^t\right]-\mathbb{P}_{\mathcal{P}'}\left[D=0\,\middle|\,h^0,h_i^t\right]$$

the right-hand side of (9) rewrites as

$$\Lambda_1\mathbb{P}_{\mathcal{P}'}\left[D=1\,\middle|\,h^0,h_i^t\right]+\Lambda_0\mathbb{P}_{\mathcal{P}'}\left[D=0\,\middle|\,h^0,h_i^t\right]-(\xi_1-\xi_0)\,\mathbb{P}\left[\theta=1\,\middle|\,h^0\right]-\xi_0$$

with Λ_1 and Λ_0 as in (4) and (5).

From $\mathbb{P}[\theta=1\,|h^0]\leq q$ we have $\Lambda_0\geq 0$. From $\mathbb{P}_{\mathcal{P}'}[D=0\,|h^0,h_i^t]\leq 1-\mathbb{P}_{\mathcal{P}'}[D=1\,|h^0,h_i^t]$ the right-hand side of (9) is then bounded above by

$$\mathbb{P}_{\mathcal{P}'}\left[D=1\,\middle|\,h^0,h_i^t\right]\left(\mathbb{P}\left[\theta=1\,\middle|\,h^0\right]-q\right)-(1-q)\,\mathbb{P}\left[\theta=1\,\middle|\,h^0\right]$$

which, from $\mathbb{P}[\theta=1\,|h^0]\leq q$, is lower than $-(1-q)\,\mathbb{P}[\theta=1\,|h^0]$. This last term corresponds to $u(0,1)\,\mathbb{P}[\theta=1\,|h^0]$. Therefore the right-hand side of (9) is bounded above by the left-hand side of (9), a contradiction.

A similar argument applies to the case $\mathbb{P}[\theta=1\,|h^0]>q$. *Q.E.D.*

Proof of Proposition 1.5. Let $\mathcal{P}=(K,B^*)$. First, we show that for t sufficiently large, the deliberative process $\mathcal{P}$ yields a unanimous consensus at the voting stage t and the consensus is on the alternative $A=1$ if $\mathbb{P}[\theta=1|h^0]>q$, and the alternative $A=0$ otherwise. Second, we show that communication evolves until a date t where there are at least k_0 (or, resp. k_1) players who vote for the alternative $A=0$ (resp. $A=1$) and believe that the realized initial history h^0 satisfies $\mathbb{P}[\theta=1|h^0]\leq q$ (resp. $\mathbb{P}[\theta=1|h^0]>q$) almost surely. Finally, we show that, for t sufficiently large every agent asks to halt communication.

First part: Consensus

At every $t \geq 1$, having observed h_i^t, each agent $i \in N$ computes the posterior belief $p_i^t = \mathbb{P}_{\mathcal{P}}\left[\theta = 1 | h_i^t\right]$. A unanimous consensus at the voting stage is reached at date t if either: for every $j \in N$, $p_j^t > q$ or for every $j \in N$, $p_j^t \leq q$. Let us show the following stronger condition under B^*:

$$\forall h^0 \in \mathcal{H}^0,\ \forall i \in N,\ \lim_{t \to +\infty} \mathbb{P}_{\mathcal{P}}\left[\theta = 1 \,\middle|\, h_i^t\right] = \mathbb{P}\left[\theta = 1 \,\middle|\, h^0\right]. \tag{10}$$

Let $h^0 \equiv (x^0, \sigma^2) \in \mathcal{H}^0$, with $x^0 \equiv (x_j^0)_{j \in N}$ and $\sigma^2 \equiv (\sigma_j^2)_{j \in N}$, and let $i \in N$, $t \geq 1$, and $h_i^t \in \mathcal{H}_i^t$. By denoting $\prod\limits_{j \in N \setminus \{i\}} \mathcal{H}_j^0$ as $\mathcal{H}_{-i}^0$, the probability $\mathbb{P}_{\mathcal{P}}\left[\theta | h_i^t\right]$ writes as

$$\begin{aligned}
\int\limits_{\mathcal{H}_{-i}^0} \mathbb{P}_{\mathcal{P}}\left[\theta \cap h_{-i}^0 \middle| h_i^t\right] dh_{-i}^0 &= \int\limits_{\mathcal{H}_{-i}^0} \mathbb{P}_{\mathcal{P}}\left[\theta \,\middle|\, h_{-i}^0, h_i^t\right] \mathbb{P}_{\mathcal{P}}\left[x_{-i}^0 \,\middle|\, \sigma_{-i}^2, h_i^t\right] \mathbb{P}_{\mathcal{P}}\left[\sigma_{-i}^2 \middle| h_i^t\right] dh_{-i}^0 \\
&= \int\limits_{\mathcal{H}_{-i}^0} \left(\prod_{j \in N \setminus \{i\}} A B_j^t C_j^t \right) dh_{-i}^0
\end{aligned}$$

with

$$A = \mathbb{P}\left[\theta \,\middle|\, h^0\right];\ B_j^t = \mathbb{P}_{\mathcal{P}}\left[x_j^0 \,\middle|\, \left(x_{ij}^k\right)_{k=1,\ldots,t}, \sigma_i^2, \sigma_j^2\right];\ \text{and } C_j^t = \mathbb{P}_{\mathcal{P}}\left[\sigma_j^2 \,\middle|\, \left(x_{ij}^k\right)_{k=1,\ldots,t}, \sigma_i^2\right]. \tag{11}$$

A corresponds to i's estimation of θ given all decision relevant information and writes as

$$\frac{\mathbb{P}\left[\theta \cap x^0 \cap \sigma^2\right]}{\mathbb{P}\left[x^0 \cap \sigma^2\right]} = \frac{\mathbb{P}\left[x^0 \middle| \theta, \sigma^2\right] \mathbb{P}\left[\theta \cap \sigma^2\right]}{\sum\limits_{\theta} \mathbb{P}\left[x^0 \cap \sigma^2 \cap \theta\right]} = \frac{\mathbb{P}\left[x^0 \middle| \theta, \sigma^2\right]}{\sum\limits_{\theta} \mathbb{P}\left[x^0 \middle| \theta, \sigma^2\right]},$$

which from $x_j^0 \middle| \theta, \sigma_j^2 \sim \mathcal{N}(\theta, \sigma_j^2)$ i.i.d. for every $j \in N$, can be rewritten as

$$\frac{\prod\limits_{j=1}^{n} \mathbb{P}\left[x_j^0 \middle| \theta, \sigma_j^2\right]}{\sum\limits_{\theta} \prod\limits_{j=1}^{n} \mathbb{P}\left[x_j^0 \middle| \theta, \sigma_j^2\right]} = \frac{e^{-\sum\limits_{j=1}^{n} \frac{\left(x_j^0 - \theta\right)^2}{2\sigma_j^2}}}{e^{-\sum\limits_{j=1}^{n} \frac{\left(x_j^0\right)^2}{2\sigma_j^2}} + e^{-\sum\limits_{j=1}^{n} \frac{\left(x_j^0 - 1\right)^2}{2\sigma_j^2}}}.$$

B_j^t corresponds to i's estimation of x_j^0 given $\left(\sigma_i^2, \sigma_j^2\right)$ and the list of received messages $\left(x_{ij}^k\right)_{k=1,\dots,t}$. It writes as

$$\begin{aligned}
&\mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\dots,t}\middle| x_j^0, \sigma_i^2, \sigma_j^2\right] \sum_\theta \mathbb{P}\left[x_j^0\middle|\,\theta, \sigma_i^2, \sigma_j^2\right] \frac{\mathbb{P}\left[\theta\left|\sigma_i^2, \sigma_j^2\right.\right]}{\mathbb{P}\left[\left(x_{ij}^k\right)_{k=1,\dots,t}\middle| \sigma_i^2, \sigma_j^2\right]} \\
&\underset{x_j^0}{\propto} \mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\dots,t}\middle| x_j^0, \sigma_i^2, \sigma_j^2\right] \sum_\theta \mathbb{P}\left[x_j^0\middle|\,\theta, \sigma_j^2\right]
\end{aligned}$$

because $\mathbb{P}\left[\theta = 0\left|\sigma_i^2, \sigma_j^2\right.\right] = \mathbb{P}\left[\theta = 1\left|\sigma_i^2, \sigma_j^2\right.\right]$. From j's truthful reporting under $\mathcal{P}$ we have $x_{ij}^k\big|\, x_j^0, \sigma_i^2, \sigma_j^2 \sim \mathcal{N}\left(x_j^0, f\left(\sigma_i^2, \sigma_j^2\right)\right)$ i.i.d. for every $k = 1, ..., t$ and we obtain

$$\mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\dots,t}\middle| x_j^0, \sigma_i^2, \sigma_j^2\right] \propto_{x_j^0} \mathbb{P}_{\mathcal{P}}\left[\sum_{k=1}^t \frac{x_{ij}^k}{t}\middle| x_j^0, \sigma_i^2, \sigma_j^2\right]$$

so

$$B_j^t \propto_{x_j^0} \sum_\theta \mathbb{P}_{\mathcal{P}}\left[\sum_{k=1}^t \frac{x_{ij}^k}{t}\middle| x_j^0, \sigma_i^2, \sigma_j^2\right] \mathbb{P}\left[x_j^0\middle|\,\theta, \sigma_j^2\right].$$

Therefore

$$x_j^0\left|\left(x_{ij}^k\right)_{k=1,\dots,t}, \sigma_i^2, \sigma_j^2\right. \sim \mathcal{N}\left(\mu_t, \nu_t\right)$$

with

$$\mu_t \equiv \frac{\sigma_j^2}{\frac{f\left(\sigma_i^2,\sigma_j^2\right)}{t} + \sigma_j^2} \sum_{k=1}^t \frac{x_{ij}^k}{t} + \frac{\frac{f\left(\sigma_i^2,\sigma_j^2\right)}{t}}{\frac{f\left(\sigma_i^2,\sigma_j^2\right)}{t} + \sigma_j^2}\theta$$

and

$$\nu_t \equiv \left(\frac{1}{\sigma_j^2} + \frac{t}{f\left(\sigma_i^2, \sigma_j^2\right)}\right)^{-1}.$$

From $\left|f\left(\sigma_i^2, \sigma_j^2\right)\right| \le \sigma_i^2 + \sigma_j^2$ we have

$$\lim_{t\to+\infty} \mu_t = \lim_{t\to+\infty} \sum_{k=1}^t \frac{x_{ij}^k}{t} \qquad \text{and} \qquad \lim_{t\to+\infty} \nu_t = 0.$$

Hence

$$\lim_{t\to+\infty} B_j^t = \mathbb{I}_{\left\{x_j^0 = \lim\limits_{t\to+\infty} \sum\limits_{k=1}^t \frac{x_{ij}^k}{t}\right\}}.$$

C_j^t corresponds to i's estimation of σ_j given σ_i and the list of received messages $\left(x_{ij}^k\right)_{k=1,\ldots,t}$. It writes as

$$\begin{aligned}
&\int_{\mathbb{R}} \frac{\mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\ldots,t}\middle| x_j^0,\sigma_i^2,\sigma_j^2\right]\mathbb{P}\left[x_j^0\middle|\sigma_i^2,\sigma_j^2\right]\mathbb{P}\left[\sigma_j^2\,|\sigma_i^2\right]}{\mathbb{P}\left[\left(x_{ij}^k\right)_{k=1,\ldots,t}\middle|\sigma_i^2\right]}dx_j^0\\
\propto\ _{\sigma_j^2}\ &\int_{\mathbb{R}} \mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\ldots,t}\middle| x_j^0,\sigma_i^2,\sigma_j^2\right]\sum_\theta \mathbb{P}\left[x_j^0\middle|\theta,\sigma_i^2,\sigma_j^2\right]\mathbb{P}\left[\theta\middle|\sigma_i^2,\sigma_j^2\right]\mathbb{P}\left[\sigma_j^2\right]dx_j^0\\
\propto\ _{\sigma_j^2}\ &\sum_\theta \mathbb{P}\left[\theta\right]\int_{\mathbb{R}} \mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\ldots,t}\middle| x_j^0,\sigma_i^2,\sigma_j^2\right]\mathbb{P}\left[x_j^0\middle|\theta,\sigma_j^2\right]\mathbb{P}\left[\sigma_j^2\right]dx_j^0
\end{aligned}$$

because θ (resp. σ_j^2) is independently distributed from $\left(\sigma_i^2,\sigma_j^2\right)$ (resp. σ_i^2). Again, from j's truthful reporting under $\mathcal{P}$ we have $x_{ij}^k\big| x_j^0,\sigma_i^2,\sigma_j^2 \sim N\left(x_j^0, f\left(\sigma_i^2,\sigma_j^2\right)\right)$ i.i.d. for every $k=1,...,t$. So

$$\mathbb{P}_{\mathcal{P}}\left[\left(x_{ij}^k\right)_{k=1,\ldots,t}\middle| x_j^0,\sigma_i^2,\sigma_j^2\right]=\prod_{k=1}^t \mathbb{P}\left[x_{ij}^k\middle| x_j^0,\sigma_i^2,\sigma_j^2\right]=\frac{e^{-\sum_{k=1}^t \frac{\left(x_{ij}^k-x_j^0\right)^2}{2f\left(\sigma_i^2,\sigma_j^2\right)}}}{\left(2\pi f\left(\sigma_i^2,\sigma_j^2\right)\right)^{\frac{t}{2}}}$$

and from $\sigma_j^2\sim\Gamma^{-1}\left(\alpha,\beta\right)$ we have

$$\mathbb{P}\left[\sigma_j\right]=\frac{\beta^\alpha}{\Gamma\left(\alpha\right)}\frac{e^{-\frac{\beta}{\sigma_j^2}}}{\left(\sigma_j^2\right)^{\alpha+1}}\propto_{\sigma_j^2}\frac{e^{-\frac{\beta}{\sigma_j^2}}}{\left(\sigma_j^2\right)^{\alpha+1}}.$$

So

$$\begin{aligned}
C_j^t\ \propto\ _{\sigma_j^2}\ &\sum_\theta \mathbb{P}\left[\theta\right]\int_{\mathbb{R}}\frac{e^{-\sum_{k=1}^t \frac{\left(x_{ij}^k-x_j^0\right)^2}{2f\left(\sigma_i^2,\sigma_j^2\right)}}}{\left(f\left(\sigma_i^2,\sigma_j^2\right)\right)^{\frac{t}{2}}}\frac{e^{-\frac{\beta}{\sigma_j^2}}}{\left(\sigma_j^2\right)^{\alpha+1}}\frac{e^{-\frac{\left(x_j^0-\theta\right)^2}{2\sigma_j^2}}}{\sigma_j}dx_j^0\\
\propto\ _{\sigma_j^2}\ &\sum_\theta \mathbb{P}\left[\theta\right]\int_{\mathbb{R}}\frac{e^{-\left[\sum_{k=1}^t \frac{\left(x_{ij}^k-x_j^0\right)^2}{2f\left(\sigma_i^2,\sigma_j^2\right)}+\frac{2\beta+\left(x_j^0-\theta\right)^2}{2\sigma_j^2}\right]}}{\left(f\left(\sigma_i^2,\sigma_j^2\right)\right)^{\frac{t}{2}}\left(\sigma_j^2\right)^{\alpha+\frac{3}{2}}}dx_j^0.
\end{aligned}$$

Now consider the special case where $f\left(\sigma_i^2,\sigma_j^2\right)=\sigma_j^2$. C_j^t can be rewritten as

$$\sum_\theta \mathbb{P}\left[\theta\right]\int_{\mathbb{R}}\mathbb{P}\left[X_t\right]dx_j^0$$

with

$$\mathbb{P}\left[X_t\right] = \frac{1}{\left(\sigma_j^2\right)^{\alpha+\frac{t+1}{2}+1}} e^{-\frac{1}{\sigma_j^2}\left(\beta+\frac{1}{2}\left(x_j^0-\theta\right)^2+\frac{1}{2}\sum\limits_{k=1}^{t}\left(x_{ij}^k-x_j^0\right)^2\right)}$$

and

$$X_t \sim \Gamma^{-1}\left(\alpha+\frac{t+1}{2}, \beta+\frac{1}{2}\left(\left(x_j^0-\theta\right)^2+\sum_{k=1}^{t}\left(x_{ij}^k-x_j^0\right)^2\right)\right).$$

Thus

$$\mathbb{E}\left[X_t\right] = \frac{2\beta+\left(x_j^0-\theta\right)^2+\sum\limits_{k=1}^{t}\left(x_{ij}^k-x_j^0\right)^2}{2\alpha+t-1} \quad \text{and} \quad \mathbb{V}\left[X_t\right] = \frac{\mathbb{E}[X_t]^2}{\alpha+\frac{t-3}{2}}$$

so that

$$\lim_{t\to+\infty} \mathbb{E}\left[X_t\right] = \lim_{t\to+\infty} \frac{1}{t}\sum_{k=1}^{t}\left(x_{ij}^k-x_j^0\right)^2 = \sigma_j^2\ ,$$

because $x_{ij}^k \middle| x_j^0, \sigma_i^2, \sigma_j^2 \sim \mathcal{N}\left(x_j^0, f\left(\sigma_i^2, \sigma_j^2\right)\right)$ and, from $\lim\limits_{t\to+\infty} \mathbb{E}\left[X_t\right]^2 < +\infty$, we have $\lim\limits_{t\to+\infty} \mathbb{V}\left[X_t\right] = 0$. Hence, for $f\left(\sigma_i^2, \sigma_j^2\right) = \sigma_j^2$, we obtain

$$\lim_{t\to+\infty} C_j^t = \mathbb{I}_{\left\{\sigma_j^2 = \lim\limits_{t\to+\infty}\sum\limits_{k=1}^{t}\frac{\left(x_{ij}^k-x_j^0\right)^2}{t}\right\}}.$$

An analogous (but longer) argument applies when $f\left(\sigma_i^2, \sigma_j^2\right) \neq \sigma_j^2$ given that $f\left(\sigma_i^2, \cdot\right)$ is invertible and bounded.

Second part: Communication

Let D_t denotes the decision taken at date t when communication is not extended. Under $\mathcal{P}$ player i having observed h_i^t asks to extend communication if and only if

$$\Delta(h_i^t) \equiv \sum_{D,\theta}\int_{\mathcal{H}^0} \mathbb{P}_{\mathcal{P}}\left[D\left|h^0\right.\right]\mathbb{P}_{\mathcal{P}}\left[h^0\left|h_i^t\right.\right]\mathbb{P}\left[\theta\left|h^0\right.\right]\left(u\left(D,\theta\right)-u\left(D_t,\theta\right)\right)dh^0 > 0 \tag{12}$$

Under B^*, we have $\mathbb{P}_{\mathcal{P}}\left[D\left|h^0\right.\right] = 1$ if $\{D = 1$ and $\mathbb{P}\left[\theta = 1\left|h^0\right.\right] > q\}$ or $\{D = 0$ and $\mathbb{P}\left[\theta = 1\left|h^0\right.\right] \leq q\}$ and 0 otherwise. So $\Delta(h_i^t)$ rewrites as

$$\sum_{\theta}\int_{\mathcal{H}^0} \mathbb{P}_{\mathcal{P}}\left[h^0\left|h_i^t\right.\right]\mathbb{P}\left[\theta\left|h^0\right.\right]\left(u\left(1,\theta\right)\mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]>q\}}+u\left(0,\theta\right)\mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]\leq q\}}-u\left(D_t,\theta\right)\right)dh^0 \tag{13}$$

which is equal to

$$\int_{\mathcal{H}^0} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] \mathbb{P}\left[\theta = 0 \left| h^0\right.\right] \left(u\left(1,0\right) \mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]>q\}} - u\left(D_t, 0\right)\right) dh^0$$
$$+ \int_{\mathcal{H}^0} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] \mathbb{P}\left[\theta = 1 \left| h^0\right.\right] \left(u\left(0,1\right) \mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]\leq q\}} - u\left(D_t, 1\right)\right) dh^0.$$

There are two cases to consider, depending on whether D_t is equal to 1 or 0. If $D_t = 1$ then $\Delta(h_i^t)$ writes as

$$\int_{\mathcal{H}^0} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] \left(\mathbb{P}\left[\theta = 0 \left| h^0\right.\right] \left(u\left(1,0\right) \mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]>q\}} - u\left(1,0\right)\right) + \mathbb{P}\left[\theta = 1 \left| h^0\right.\right] u\left(0,1\right) \mathbb{I}_{\{\mathbb{P}[\theta=1|h^0]\leq q\}}\right) dh^0$$

which is equal to

$$\int_{\mathcal{H}^0_-} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] \left(\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] u\left(0,1\right) - \mathbb{P}\left[\theta = 0 \left| h^0\right.\right] u\left(1,0\right)\right) dh^0$$

where $\mathcal{H}^0_-$ denotes the set $\{h^0 \in \mathcal{H}^0 | \mathbb{P}\left[\theta = 1 \left| h^0\right.\right] \leq q\}$. From the payoff function $u\left(.,.\right)$ if $\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] < q$ then $\mathbb{P}\left[\theta = 1 \left| h^0\right.\right] u\left(0,1\right) - \mathbb{P}\left[\theta = 0 \left| h^0\right.\right] u\left(1,0\right) > 0$. Therefore $\Delta(h_i^t) = 0$ if and only if

$$\int_{\mathcal{H}^0_{-*}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] dh^0 = 0$$

where $\mathcal{H}^0_{-*}$ denotes the set $\{h^0 \in \mathcal{H}^0 | \mathbb{P}\left[\theta = 1 \left| h^0\right.\right] < q\}$.

A symmetric argument applies to the case where $D_t = 0$ so that communication evolves until there is a history h^t satisfying, for at least $\min\{k_0, k_1\}$ players $i \in N$, either

$$\left\{D_t = 1 \text{ and } \int_{\mathcal{H}^0_{-*}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] dh^0 = 0\right\} \qquad \text{or} \qquad \left\{D_t = 0 \text{ and } \int_{\mathcal{H}^0_{+*}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] dh^0 = 0\right\},$$

where $\mathcal{H}^0_{+*}$ denotes the set $\{h^0 \in \mathcal{H}^0 | \mathbb{P}\left[\theta = 1 \left| h^0\right.\right] > q\}$. By the martingale property of the beliefs if

$$\int_{\mathcal{H}^0_{+*}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t\right.\right] dh^0 = 0$$

then $\mathbb{P}_{\mathcal{P}}\left[\theta = 1 \left| h^0, h_i^t\right.\right] \leq q$ for almost every realized $\tilde{h}_0$. Hence if the decision D_t is taken at date t under $\mathcal{P}$ we have $D_t = 0$ if $\mathbb{P}_{\mathcal{P}}\left[\theta = 1 \left| \tilde{h}_0 \right.\right] \leq q$; and 1 otherwise, for almost every realized $\tilde{h}_0$.

Let us show now that there always exists a date t at which a decision is taken under $\mathcal{P}$. Assume, by contradiction, that there are some realized histories $\tilde{h}_0$ and $(h^t)_{t=1,2,\ldots} \in \bigcup_{t=1}^{+\infty} \mathcal{H}^t$ such that, at every $t \geq 1$, communication goes on. Under $\mathcal{P}$ we then have $\Delta(h_i^t) > 0$ for at least $\min\{k_0, k_1\}$ players $i \in N$. From (12), $\Delta(h_i^t)$ writes as

$$(u(1,0) - u(D_t,0)) \int_{\mathcal{H}^0_{+*}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t \right.\right] \mathbb{P}\left[\theta = 0 \left| h^0 \right.\right] dh^0 - u(D_t,0) \int_{\mathcal{H}^0_{-}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t \right.\right] \mathbb{P}\left[\theta = 0 \left| h^0 \right.\right] dh^0$$

$$+ (u(0,1) - u(D_t,1)) \int_{\mathcal{H}^0_{-}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t \right.\right] \mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] dh^0 - u(D_t,1) \int_{\mathcal{H}^0_{+*}} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t \right.\right] \mathbb{P}\left[\theta = 1 \left| h^0 \right.\right] dh^0.$$

For $\bar{\mathcal{H}}^0 \in \{\mathcal{H}^0_{+*}, \mathcal{H}^0_{-}\}$ and $\theta \in \{0,1\}$ we have

$$\begin{aligned}
\int_{\bar{\mathcal{H}}^0} \mathbb{P}_{\mathcal{P}}\left[h^0 \left| h_i^t \right.\right] \mathbb{P}\left[\theta \left| h^0 \right.\right] dh^0 &= \int_{\bar{\mathcal{H}}^0} \mathbb{P}_{\mathcal{P}}\left[\theta \cap h^0 \left| h_i^t \right.\right] dh^0 \\
&= \int_{\mathcal{H}^0} \mathbb{P}_{\mathcal{P}}\left[\theta \cap h^0 \left| h_i^t \right.\right] dh^0 \mathbb{I}_{\{h^0 \in \bar{\mathcal{H}}^0\}} = \mathbb{P}_{\mathcal{P}}\left[\theta \left| h_i^t \right.\right] \mathbb{I}_{\{h^0 \in \bar{\mathcal{H}}^0\}}.
\end{aligned}$$

From (10) when $\tilde{h}^0$ is the realized initial history we have

$$\lim_{t \to +\infty} \mathbb{P}_{\mathcal{P}}\left[\theta | h_i^t\right] \mathbb{I}_{\{\tilde{h}^0 \in \bar{\mathcal{H}}^0\}} = \mathbb{P}\left[\theta \left| \tilde{h}^0 \right.\right] \mathbb{I}_{\{\tilde{h}^0 \in \bar{\mathcal{H}}^0\}}.$$

Therefore

$$\begin{aligned}
\lim_{t \to +\infty} \Delta(h_i^t) = & \left(u(1,0) - u\left(\lim_{t \to +\infty} D_t, 0\right)\right) \mathbb{P}\left[\theta = 0 \left| \tilde{h}^0 \right.\right] \mathbb{I}_{\left\{\mathbb{P}\left[\theta=1 \left| \tilde{h}^0 \right.\right] > q\right\}} \\
& - u\left(\lim_{t \to +\infty} D_t, 0\right) \mathbb{P}\left[\theta = 0 \left| \tilde{h}^0 \right.\right] \mathbb{I}_{\left\{\mathbb{P}\left[\theta=1 \left| \tilde{h}^0 \right.\right] < q\right\}} \\
& + \left(u(0,1) - u\left(\lim_{t \to +\infty} D_t, 1\right)\right) \mathbb{P}\left[\theta = 1 \left| \tilde{h}^0 \right.\right] \mathbb{I}_{\left\{\mathbb{P}\left[\theta=1 \left| \tilde{h}^0 \right.\right] \leq q\right\}} \\
& - u\left(\lim_{t \to +\infty} D_t, 1\right) \mathbb{P}\left[\theta = 1 \left| \tilde{h}^0 \right.\right] \mathbb{I}_{\left\{\mathbb{P}\left[\theta=1 \left| \tilde{h}^0 \right.\right] > q\right\}}.
\end{aligned}$$

Now from (10) we have under $\mathcal{P}$, $\lim_{t\to+\infty} v_i(h_i^{t_v}) = \lim_{t\to+\infty} v_j(h_j^{t_v})$ which is equal to 1 if $\mathbb{P}\left[\theta = 1 \middle| \tilde{h}^0\right] > q$ and 0 otherwise. So the candidate D_t satisfies $\lim_{t\to+\infty} D_t$ is equal to 1 if $\mathbb{P}\left[\theta = 1 \middle| \tilde{h}^0\right] > q$ and 0 otherwise. Hence $\lim_{t\to+\infty} \Delta(h_i^t) = 0$ for every $i \in N$, a contradiction. *Q.E.D.*

Proof of Proposition 1.7.

Sufficiency. This part of the proof does not use the assumption that $\varepsilon < \max\{\xi_0, \xi_1\}$. (This result is valid for every $\varepsilon > 0$.) Let $\mathcal{P} \equiv (K, B^\varepsilon)$.

Statement i) $\varepsilon-$maximization

Let $i \in N$ and $h^0 \in \mathcal{H}^0$. Player i's expected payoff at history h_i^0 under $\mathcal{P}$ writes as

$$\mathbb{E}_{\mathcal{P}}\left[u\left(.,.\right)\middle|h_i^0\right] = \int_{\mathcal{H}_{\mathcal{P}}^D} \mathbb{E}_{\mathcal{P}}\left[u\left(.,.\right)\middle|\left(h_j^D\right)_{j\in N}\right]\mathbb{P}_{\mathcal{P}}\left[\left(h_j^D\right)_{j\in N}\middle|h_i^0\right] d\left(h_j^D\right)_{j\in N} \tag{14}$$

where $\mathcal{H}_{\mathcal{P}}^D$ denote the set of committee's profile of histories $\left(h_j^D\right)_{j\in N}$ at which a decision can be taken under $\mathcal{P}$.

By definition of $\mathcal{P}$, a decision is rendered under $K = \{n, n\}$ at $\left(h_j^D\right)_{j\in N} \in \mathcal{H}_{\mathcal{P}}^D$ only if i's expected gain from all decision-relevant private information being rendered public is lower than ε. That is

$$\mathbb{E}_{\mathcal{P}}\left[u\left(.,.\right)\middle|\left(h_j^D\right)_{j\in N}\right] + \varepsilon \geq \int_{\mathcal{H}^0} \mathbb{E}_{\mathcal{P}}\left[u\left(a^i\left(h^0\right),.\right)\middle|h^0\right]\mathbb{P}_{\mathcal{P}}\left[h^0\middle|\left(h_j^D\right)_{j\in N}\right] dh^0 \tag{15}$$

where $a^i\left(h^0\right)$ denotes the alternative that is selected by the committee when all decision-relevant information h^0 is public.

Therefore i's payoff is $\varepsilon-$maximal at $\left(h_j^D\right)_{j\in N}$ and from (14) i's ex-ante payoff is $\varepsilon-$maximal.

Statement ii) $\varepsilon-$efficiency

Straightforward from the first statement.

Statement iii) $\varepsilon-$equilibrium

Let $\mathcal{P} \equiv (K, B^\varepsilon)$. Under B^ε, the only off-the-equilibrium-path message is $\varnothing$. Assume that when a member chooses to remain silent his counterpart infer that his initial signal is $\frac{1}{2}$. Suppose, by contradiction, that there is a member $i \in N$ that has an $\varepsilon-$profitable unilateral deviation $B_i^{\varepsilon\prime}$ under $\mathcal{P}$. Denote by t the first date at which $B_i^{\varepsilon\prime}$ differs from B_i^{ε}. So there is $h_i^t \in \mathcal{H}_i^t$ such that

$$\mathbb{E}_{\mathcal{P}}\left[u\left(.,.\right)\left|h_i^t\right.\right] + \varepsilon < \mathbb{E}_{\mathcal{P}'}\left[u\left(.,.\right)\left|h_i^t\right.\right] \tag{16}$$

where $\mathcal{P}' \equiv \left(K, \left(B_i^{\varepsilon\prime}, B_{-i}^{\varepsilon}\right)\right)$. By definition of $\mathcal{P}$, for any history $\left(h_j^D\right)_{j\in N}$ at which there is a positive probability that a decision is rendered, we have (15); otherwise, communication would have been extended. Under $\mathcal{P}$ the set of such histories $\mathcal{H}_{\mathcal{P}}^D$ is necessarily non empty because, provided the length of communication is sufficiently large, the repeated truthful reporting with bounded noise of communication allows to approximate h^0 and reach a consensus on i's preferred outcome given h^0, which is $a^i\left(h^0\right)$. So,

$$\begin{aligned}
&\int\limits_{\mathcal{H}_{\mathcal{P}}^D} \mathbb{E}_{\mathcal{P}}\left[u\left(.,.\right)\left|\left(h_j^D\right)_{j\in N}\right.\right] d\mathbb{P}_{\mathcal{P}}\left[\cdot\left|h_i^t\right.\right] + \varepsilon \\
\geq &\int\limits_{\mathcal{H}_{\mathcal{P}}^D}\int\limits_{\mathcal{H}^0} \mathbb{E}\left[u\left(a^i\left(h^0\right),.\right)\left|h^0\right.\right] \mathbb{P}_{\mathcal{P}}\left[h^0\left|\left(h_j^D\right)_{j\in N}\right.\right] \mathbb{P}_{\mathcal{P}}\left[\left(h_j^D\right)_{j\in N}\left|h_i^t\right.\right] d\left(h_j^D, h_j^0\right)_{j\in N}.
\end{aligned} \tag{17}$$

The left-hand side of (17) is equal to the left-hand side of (16) so $\mathbb{E}_{\mathcal{P}'}\left[u\left(.,.\right)\left|h_i^t\right.\right]$ is strictly higher than the right-hand side of (17). But $\mathbb{E}_{\mathcal{P}'}\left[u\left(.,.\right)\left|h_i^t\right.\right]$ is bounded above by

$$\int\limits_{\mathcal{H}_{\mathcal{P}'}^D}\int\limits_{\mathcal{H}^0} \mathbb{E}\left[u\left(a^i\left(h^0\right),.\right)\left|h^0\right.\right] \mathbb{P}_{\mathcal{P}'}\left[h^0\left|\left(h_j^D\right)_{j\in N}\right.\right] \mathbb{P}_{\mathcal{P}'}\left[\left(h_j^D\right)_{j\in N}\left|h_i^t\right.\right] d\left(h_j^D, h_j^0\right)_{j\in N}$$

which is then strictly higher than the right-hand side of (17). Hence there is at least one profile $h^0 \in \mathcal{H}^0$ satisfying

$$\begin{aligned}
&\int\limits_{\mathcal{H}_{\mathcal{P}'}^D} \mathbb{P}_{\mathcal{P}'}\left[h^0\left|\left(h_j^D\right)_{j\in N}\right.\right] \mathbb{P}_{\mathcal{P}'}\left[\left(h_j^D\right)_{j\in N}\left|h_i^t\right.\right] d\left(h_j^D\right)_{j\in N} \\
> &\int\limits_{\mathcal{H}_{\mathcal{P}}^D} \mathbb{P}_{\mathcal{P}}\left[h^0\left|\left(h_j^D\right)_{j\in N}\right.\right] \mathbb{P}_{\mathcal{P}}\left[\left(h_j^D\right)_{j\in N}\left|h_i^t\right.\right] d\left(h_j^D\right)_{j\in N}.
\end{aligned}$$

This is equivalent to

$$\int\limits_{\mathcal{H}^D_{\mathcal{P}'}} \mathbb{P}_{\mathcal{P}'}\left[h^0 \cap \left(h^D_j\right)_{j\in N}\middle| h^t_i\right] d\left(h^D_j\right)_{j\in N} - \int\limits_{\mathcal{H}^D_{\mathcal{P}}} \mathbb{P}_{\mathcal{P}}\left[h^0 \cap \left(h^D_j\right)_{j\in N}\left| h^t_i\right.\right] d\left(h^D_j\right)_{j\in N} > 0$$

which can be rewritten as $\mathbb{P}_{\mathcal{P}'}\left[h^0 | \, h^t_i\right] - \mathbb{P}_{\mathcal{P}}\left[h^0 \, | h^t_i\right] > 0$. But $\mathbb{P}_{\mathcal{P}'}\left[h^0 | \, h^t_i\right] = \mathbb{P}_{\mathcal{P}}\left[h^0 | \, h^t_i\right]$ because $\mathcal{P}$ and $\mathcal{P}'$ do not differ prior to date t, a contradiction.

Necessity (preliminary). Assume that $K \neq \{n, n\}$ and $0 < \varepsilon < \max\{\xi_0, \xi_1\}$. We show that B^ε is not an $\varepsilon-$equilibrium. Specifically, assume without loss of generality that $K = \{n-1, n-1\}$, $\theta = 1$ and player i be such that $\sigma^2_i < \sigma^2_j$ for all $j \neq i$. Consider the history where, at stage $t = 1$, agent i's initial signal and cognitive ability are such that $\mathbb{P}\left(\theta = 1 | h^0_i\right) \leq q$, while for all $j \neq i$, signals and cognitive abilities are such that $\mathbb{P}\left(\theta = 1 | h^0_j\right) > q$. Assume, furthermore, that $\mathbb{P}\left(\theta = 1 | h^0\right) \leq q$.[12] Suppose the expected gain from extending communication to the next round is such that $\Delta_i\left(h^1_i\right) > \varepsilon$ and, for each $j \neq i$, $\Delta_j\left(h^1_j\right) > \varepsilon$. Under behavior B^ε, $v^1_i = 0$ and $c^1_i = 1$, while, for all $j \neq i$, $v^1_j = 1$ and $c^t_j = 1$. All agents, therefore, have the possibility to communicate.

Let $\mathcal{H}^*$ be the set of histories where under truthful communication, because of an unlucky realization of communication noise, every agent $j \neq i$ has observed an history such that $\mathbb{P}\left(\theta = 1 | h^2_j\right) > q$ and $\Delta_j\left(h^1_j\right) \leq \varepsilon$, while $\mathbb{P}\left(\theta = 1 | h^2_i\right) \leq q$ and $\Delta_i\left(h^1_i\right) > \varepsilon$. Under behavior B^ε, alternative $A = 1$ would be implemented, that is agent i's least preferred alternative. From an ex-ante perspective, therefore, agent i has an ε-profitable deviation from truthtelling at the communication stage $t = 1$.

Let $j^* \equiv \inf\limits_{j \neq i} \mathbb{P}\left(\theta = 1 | h^2_j\right)$, that is the player with the "mildest" belief towards $A = 1$, and let

$$\mathbb{P}\left(\theta = 1 | h^2_{j^*}\right) = q \tag{18}$$

define the posterior belief that makes agent j^* indifferent between the two alternatives. Assuming all agents $j \neq i$ are behaving according to B^ε, from agent i's perspective equation (18)

[12] A numerical example of such history is provided in Example 1 below.

rewrites as

(19)
$$\frac{\displaystyle\int\limits_{\mathcal{H}^*\left(\left\{x_j^0\right\}_{j\neq i},x_i^0,\left\{\sigma_j^2\right\}_{j\neq i},\sigma_i^2\right)}\int \exp\left\{A_{j^*}(\theta=1)-\frac{\left(m_i^1+\eta_{j^*i}^1-1\right)^2}{2f\left(\sigma_i^2,\sigma_{j^*}^2\right)}\right\}dF\left(\left\{x_j^0\right\}_{j\neq i},x_i^0,\left\{\sigma_j^2\right\}_{j\neq i},\sigma_i^2\right)dh^*}{\displaystyle\sum_\theta\int\limits_{\mathcal{H}^*\left(\left\{x_j^0\right\}_{j\neq i},x_i^0,\left\{\sigma_j^2\right\}_{j\neq i},\sigma_i^2\right)}\int \exp\left\{A_{j^*}(\theta)-\frac{\left(m_i^1+\eta_{j^*i}^1-1\right)^2}{2f\left(\sigma_i^2,\sigma_{j^*}^2\right)}\right\}dF\left(\left\{x_j^0\right\}_{j\neq i},x_i^0,\left\{\sigma_j^2\right\}_{j\neq i},\sigma_i^2\right)dh^*}=q$$

where $A_{j^*}(\theta)$ contain all j^*'s information about the history of plays that is not attributable to player i's message, for any θ. Using simple algebra, (19) rewrites as

$$\frac{1}{1+\displaystyle\int\limits_{\mathcal{H}^*\left(\left\{x_j^0\right\}_{j\neq i},x_i^0,\left\{\sigma_j^2\right\}_{j\neq i},\sigma_i^2\right)}\int \exp\left\{Bm_i^1+C_{j^*}\right\}dF\left(\left\{x_j^0\right\}_{j\neq i},x_i^0,\left\{\sigma_j^2\right\}_{j\neq i},\sigma_i^2\right)dh^*}=q \tag{20}$$

where $B=-\frac{1}{2f\left(\sigma_i^2,\sigma_{j^*}^2\right)}$ and C_{j^*} is a constant that does not depend on player i's message. Because the left-hand side of equation (20) is an increasing function of m_i^1 and it is bounded between 0 and 1, there exist a unique threshold message $\tilde{m}_i^1$ such that (20) holds. By reporting any message $m_i^1<\tilde{m}_i^1$, agent i induces agent j^* to vote for alternative $A=0$ and therefore has a profitable deviation from truthtelling. *Q.E.D.*

Example 1 (Preliminary). We provide here a numerical example of a history where one player has a profitable deviation from truthtelling. Assume without loss of generality that $K = \{n-1, n-1\}$ and $q = 0.5$, and suppose that a "dictator" (Player 1) represents $n-1$ agents that agree to select alternative the same alternative A, while agent i (Player 2) is the n-th agent who prefers a different alternative to be implemented.

Suppose there are six possible signals $\{a, b, c, d, e, f\}$, with the following table summarizing the probabilities of observing such signals

(21)

	$\theta = 1$	$\theta = 0$
$\mathbb{P}(a\|\theta)$	0.04	0.35
$\mathbb{P}(b\|\theta)$	0.06	0.28
$\mathbb{P}(c\|\theta)$	0.10	0.20
$\mathbb{P}(d\|\theta)$	0.20	0.20
$\mathbb{P}(e\|\theta)$	0.30	0.06
$\mathbb{P}(f\|\theta)$	0.30	0.01

Suppose $\varepsilon = 0.13$, player 1 receives signal b and player 2 receives signal e. From (21), we have that $p_1^0 \equiv \mathbb{P}(\theta = 1|b) \approx 0.18$ and $p_2^0 \equiv \mathbb{P}(\theta = 1|e) \approx 0.83$, which implies that $v_1^1 = 0$ and $v_2^1 = 1$. Having observed player 2's vote, player 1 infers that $x_2^0 \in \{d, e, f\}$ and player 2 infers that $x_1^0 \in \{a, b, c\}$. Player 1, then, updates his belief to $p_1^1 = \mathbb{P}(\theta = 1|b \bigcap \{x_2^0 \in \{d, e, f\}\}) \approx 0.5 = q$. Because, for $\varepsilon = 0.13$, player 1's incentive to extend communication is higher than ε, we have $c_1^1 = 1$.[13] Similarly, player 2 updates his belief to $p_2^1 = \mathbb{P}(\theta = 1|e \bigcap \{x_1^0 \in \{a, b, c\}\}) \approx 0.55$. Because the decision to extend communication is taken by the dictator, the game continues to the next round of communication.

From player 1 decision to extend communication, player 2 infers that $\mathbb{P}(\theta = 1|x_1^0 \in \{d, e, f\}) \approx q$. This further shows to player 2 that $x_1^0 \neq a$, from $\mathbb{P}(\theta = 1|a \bigcap \{d, e, f\}) = 0.35$, $\mathbb{P}(\theta = 1|b \bigcap \{d, e, f\}) \approx 0.5$ and $\mathbb{P}(\theta = 1|c \bigcap \{d, e, f\}) \approx 0.7$, we would have $\mathbb{P}(\theta = 1|a \bigcap d) \approx 0.18$, $\mathbb{P}(\theta = 1|a \bigcap e) \approx$

[13] Player 1's expected gain from extending communication at this stage, departing from the situation where $A = 0$ is implemented, is equal to0.1584, which is greater than 0.13.

0.36 and $\mathbb{P}(\theta = 1|a \bigcap f) \approx 0.77$. This implies that the expected gain from extending communication, departing from the situation where alternative $A = 0$ is chosen, is $\mathbb{P}(f|\{d, e, f\}) \cdot |2\mathbb{P}(\theta = 1|a \bigcap f) - 1| = 0.1188 < \varepsilon$. If player 1 observed $x_1^0 = a$, therefore, he would not have extended communication.

Player 2 faces now two possible scenarios, one where $x_1^0 = b$, the other where $x_1^0 = c$. In either case, player 2 would like to implement $A = 1$ because $\mathbb{P}(\theta = 1|b \bigcap e) \approx 0.52 > q$ and $\mathbb{P}(\theta = 1|c \bigcap e) \approx 0.71 > q$. Under truthful reporting, player 1 infers that $x_2^0 = d$ with probability

$$1 - \mathbb{P}\left(x_2^0 = e|x_{21}^1 = d, x_2^0 \in \{e, d, f\}\right) - \mathbb{P}\left(x_2^0 = f|x_{21}^1 = d, x_2^0 \in \{e, d, f\}\right) \approx 0.83$$

where $\mathbb{P}(x_2^0|x_{21}^1) = \frac{\mathbb{P}\left(x_{21}^1|x_2^0\right)\left(\mathbb{P}\left(x_2^0|\theta=1\right)+\mathbb{P}\left(x_2^0|\theta=1\right)\right)\frac{1}{2}}{\sum\limits_{x_{21}^1 \in \{d,e,f\}} \mathbb{P}\left(x_{21}^1|x_2^0\right)\left(\mathbb{P}\left(x_2^0|\theta=1\right)+\mathbb{P}\left(x_2^0|\theta=1\right)\right)\frac{1}{2}}$. Similarly, player 1 believes that $x_2^0 = e$ (resp. $x_2^0 = f$) with probability 0.14 (resp. 0.04). After the stage of communication, therefore, player 1 updated beliefs writes as

$$p_1^1 = 0.83\mathbb{P}\left(b \bigcap d\right) + 0.14\mathbb{P}\left(b \bigcap e\right) + 0.04\mathbb{P}\left(b \bigcap f\right) \approx 0.35 < q$$

which implies that $v_1^2 = 0$. Player 1's expected gain from extending communication, departing from the situation where $A = 0$ is chosen, becomes now

$$\begin{aligned}
&\mathbb{P}\left(x_2^0 = e|x_{21}^1 = d \bigcap x_2^0 \in \{e, d, f\}\right) \cdot \left(2\mathbb{P}\left(\theta = 1|b \bigcap e\right) - 1\right) \\
&+\mathbb{P}\left(x_2^0 = f|x_{21}^1 = d \bigcap x_2^0 \in \{e, d, f\}\right) \cdot \left(2\mathbb{P}\left(\theta = 1|b \bigcap f\right) - 1\right) \\
&\approx 0.02 < \varepsilon
\end{aligned}$$

Therefore, player 1 stops the communication and $A = 0$ is chosen. By reporting $m_2^1 = f$, player 2 decreases the probabilities that player 1 receives messages a, b, c, d and increases the probabilities that player 1 receives messages e, f, implying that player 2 has a profitable deviation from truthtelling. *Q.E.D.*

Example 2 (Preliminary version). To show that, absent communication, simple majority Pareto dominates unanimity, we need to prove that the expected utility of any given agent i is higher under simple majority. When $\varepsilon > \max\{\xi_0, \xi_1\}$, we can show that agents prefer not to communicate under unanimity. We will compare the ex-ante expected utility of any agent i under simple majority without communication, with the ex-ante expected utility obtained under unanimity. If unanimity is Pareto dominated by simple majority in this case, unanimity is also dominated by majority with communication.[14]

In this example, for computational convenience we assume n odd and that it is common knowledge that $\sigma_i^2 = \sigma^2$ for every $i \in N$

For a given voting rule K, the ex-ante expected utility of each agent i writes as

$$\begin{aligned}\mathbb{E}_K\left[u\left(D,\theta\right)|x_i\right] = &-\mathbb{P}\left(\theta=0\right)\left(\mathbb{P}_K\left[D=1|x_i\right]q + \left(1-\mathbb{P}_K\left[D=1|x_i\right]-\mathbb{P}_K\left[D=0|x_i\right]\right)\xi_0\right)\\ &-\mathbb{P}\left(\theta=1\right)\left(\mathbb{P}_K\left[D=0|x_i\right]\left(1-q\right) + \left(1-\mathbb{P}_K\left[D=0|x_i\right]-\mathbb{P}_K\left[D=1|x_i\right]\right)\xi_1\right).\end{aligned}$$

Before computing the respective probabilities of a wrong decision and a no-decision, we need to characterize the equilibrium voting strategy of each agent. Without communication, every agent conditions her vote on being pivotal. In particular, we focus on the case where agents follow threshold voting strategies, that is

$$v_i\left(x_i\right) = \begin{cases} 1 & \text{if} \quad x_i > \tilde{x}_i(K) \\ 0 & \text{if} \quad x_i \leq \tilde{x}_i(K) \end{cases} \tag{22}$$

where, for each agent i, $\tilde{x}_i(K)$ is the signal that solves $\mathbb{P}_K\left(\theta=1|\tilde{x}_i(K), \{i \text{ is pivotal}\}\right) = q$, that is

$$\frac{\mathbb{P}\left(\tilde{x}_i(K)|\left\{i \text{ is pivotal}\right\},\theta=1\right)\mathbb{P}_K\left(\left\{i \text{ is pivotal}\right\}|\theta=1\right)\mathbb{P}\left(\theta=1\right)}{\sum_\theta \mathbb{P}\left(\tilde{x}_i(K)|\left\{i \text{ is pivotal}\right\},\theta\right)\mathbb{P}_K\left(\left\{i \text{ is pivotal}\right\}|\theta\right)\mathbb{P}\left(\theta\right)} = q$$

[14]This is because, if communication is informative, the probabilities of mistake are lower under simple majority than under unanimity with no communication. When communication is not informative, on the other hand, the comparison between the two rules is equivalent to comparing majority and unanimity both without communication.

which is equivalent to

$$\frac{\mathbb{P}\left(\tilde{x}_i(K)|\theta=1\right)\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta=1\right)}{\sum\limits_{\theta}\mathbb{P}\left(\tilde{x}_i(K)|\theta\right)\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta\right)}=q$$

that is

$$\frac{\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta=1\right)}{\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta=1\right)+\exp\left\{\frac{1-2\tilde{x}_i(K)}{2\sigma^2}\right\}\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta=0\right)}=q$$

so

$$\tilde{x}_i(K)=x^S-x_i^D\left(K\right) \tag{23}$$

where x^S is the threshold under sincere voting with

$$x^S\equiv\frac{1}{2}\left(1-2\sigma^2\log\left(\frac{1-q}{q}\right)\right)$$

and $x_i^D\left(K\right)$ is the strategic distortion due to the voting rule with

$$x_i^D\left(K\right)\equiv\sigma^2\log\left(\frac{\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta=1\right)}{\mathbb{P}_K\left(\{i \text{ is pivotal}\}\,|\theta=0\right)}\right)$$

Because simple majority and unanimity are symmetric, we let $k=k_0=k_1$. Therefore, the probability of being pivotal $\mathbb{P}_k\left[\{i \text{ is pivotal}\}\,|\theta\right]$ writes as

$$\sum_{\substack{M\subseteq N\setminus\{i\}\\|M|=k-1}}\mathbb{P}\left[\Xi_1\bigcup\Xi_0|\theta\right] \tag{24}$$

with

$$\Xi_1\equiv\left(\bigcap_{j\in M}\{x_j>\tilde{x}_j^k\}\right)\bigcap\left(\bigcap_{j\notin M}\{x_j\leq\tilde{x}_j^k\}\right)=\prod_{j\in M}\mathbb{P}\left(x_j>\tilde{x}_j^k|\theta\right)\prod_{j\notin M}\mathbb{P}\left(x_j\leq\tilde{x}_j^k|\theta\right);$$

and

$$\Xi_0\equiv\left(\bigcap_{j\in M}\{x_j\leq\tilde{x}_j^k\}\right)\bigcap\left(\bigcap_{j\notin M}\{x_j>\tilde{x}_j^k\}\right)=\prod_{j\in M}\mathbb{P}\left(x_j\leq\tilde{x}_j^k|\theta\right)\prod_{j\notin M}\mathbb{P}\left(x_j>\tilde{x}_j^k|\theta\right).$$

Given that agents have homogeneous preferences, we restrict our attention to symmetric strategies, that is to the case where $\tilde{x_i}^k = \tilde{x}^k$, for every $i \in N$. Therefore, (24) rewrites as

$$\binom{n-1}{k-1}\left(\mathbb{P}\left(x_j > \tilde{x}^k|\theta\right)^{k-1}\mathbb{P}\left(x_j \leq \tilde{x}^k|\theta\right)^{n-k} + \mathbb{P}\left(x_j \leq \tilde{x}^k|\theta\right)^{k-1}\mathbb{P}\left(x_j > \tilde{x}^k|\theta\right)^{n-k}\right)$$

which is equal to

$$\frac{1}{2^{n-1}}\binom{n-1}{k-1}\left(\operatorname{erfc}\left(\frac{\tilde{x}^k-\theta}{\sqrt{2}\sigma}\right)^{k-1}\left(1+\operatorname{erf}\left(\frac{\tilde{x}^k-\theta}{\sqrt{2}\sigma}\right)\right)^{n-k} + \left(1+\operatorname{erf}\left(\frac{\tilde{x}^k-\theta}{\sqrt{2}\sigma}\right)\right)^{k-1}\operatorname{erfc}\left(\frac{\tilde{x}^k-\theta}{\sqrt{2}\sigma}\right)^{n-k}\right)$$

where $\operatorname{erf}(\cdot)$ and $\operatorname{erfc}(\cdot)$ denote the Gaussian error and complementary error functions, respectively. Hence, for a given voting rule k, every agent i votes according to (22), where $\tilde{x}^k$ is a fixed point that solves

$$\tilde{x}^k = x^S - \sigma^2 \log\left(\Xi(\tilde{x}^k)\right) \tag{25}$$

with

$$\Xi(\tilde{x}^k) \equiv \left(\frac{\operatorname{erfc}\left(\frac{\tilde{x}^k-1}{\sqrt{2}\sigma}\right)^{k-1}\left(1+\operatorname{erf}\left(\frac{\tilde{x}^k-1}{\sqrt{2}\sigma}\right)\right)^{n-k} + \left(1+\operatorname{erf}\left(\frac{\tilde{x}^k-1}{\sqrt{2}\sigma}\right)\right)^{k-1}\operatorname{erfc}\left(\frac{\tilde{x}^k-1}{\sqrt{2}\sigma}\right)^{n-k}}{\operatorname{erfc}\left(\frac{\tilde{x}^k}{\sqrt{2}\sigma}\right)^{k-1}\left(1+\operatorname{erf}\left(\frac{\tilde{x}^k}{\sqrt{2}\sigma}\right)\right)^{n-k} + \left(1+\operatorname{erf}\left(\frac{\tilde{x}^k}{\sqrt{2}\sigma}\right)\right)^{k-1}\operatorname{erfc}\left(\frac{\tilde{x}^k}{\sqrt{2}\sigma}\right)^{n-k}}\right).$$

Having defined the threshold strategy of each agent, we are now able to explicitly characterize the probabilities of implementing a wrong decision and a no-decision. For a given player i, the probability that $D = 1$ is selected writes as

$$\mathbb{P}_k\left[(D=1,\theta)\,|x_i\right] = \sum_{\substack{M\subseteq N\\|M|>k}} \mathbb{P}\left[\left(\bigcap_{j\in M}\{x_j > \tilde{x}_k\}\right)\bigcap\left(\bigcap_{j\notin M}\{x_j < \tilde{x}_k\}\right)\bigcap\theta\,\middle|\, x_i\right]$$

which is equal to

$$\frac{\mathbb{P}\left(x_i|\theta\right)}{\sum_\theta \mathbb{P}\left(x_i|\theta\right)}\sum_{|M|=k}^{n}\binom{n}{|M|}\mathbb{P}\left[\bigcap_{j\in M}\{x_j > \tilde{x}_k\}\,\middle|\,\theta\right]\mathbb{P}\left[\bigcap_{j\notin M}\{x_j < \tilde{x}_k\}\,\middle|\,\theta\right]$$

which, using $\mathbb{P}\left(x_i|\theta\right)=\exp\left\{-\frac{(x_i-\theta)^2}{2\sigma^2}\right\}$, rewrites as

$$\frac{1}{2^n\left(1+\exp\left\{(-1)^{\theta}\left(\frac{2x_i-1}{2\sigma^2}\right)\right\}\right)}\sum_{|M|=k}^{n}\binom{n}{|M|}\operatorname{erfc}\left(\frac{\tilde{x}_k-\theta}{\sqrt{2}\sigma}\right)^{|M|}\left(1+\operatorname{erf}\left(\frac{\tilde{x}_k-\theta}{\sqrt{2}\sigma}\right)\right)^{n-|M|}.$$

Similarly, the probability that $D=0$ is selected is given by

$$\mathbb{P}_k\left[(D=0,\theta)\,|x_i\right]=\frac{\frac{1}{2^n}\sum_{|M|=k}^{n}\binom{n}{|M|}\left(1+\operatorname{erf}\left(\frac{\tilde{x}_k-\theta}{\sqrt{2}\sigma}\right)\right)^{|M|}\operatorname{erfc}\left(\frac{\tilde{x}_k-\theta}{\sqrt{2}\sigma}\right)^{n-|M|}}{1+\exp\left\{(-1)^{\theta}\left(\frac{2x_i-1}{2\sigma^2}\right)\right\}}.$$

We now need to show that, for each agent i, the ex-ante expected utility is higher under the voting rule $K=\left\{\left\lfloor\frac{n}{2}+1\right\rfloor,\left\lfloor\frac{n}{2}+1\right\rfloor\right\}$ than under $K=\{n,n\}$. This reduces to show that $\Delta_K\equiv\mathbb{E}\left[\mathbb{E}_M\left[u\left(D,\theta\right)|x_i\right]-\mathbb{E}_U\left[u\left(D,\theta\right)|x_i\right]\right]$ is positive, that is

$$0\leq\mathbb{E}\left[Aq+B\left(1-q\right)+C\xi_0+D\xi_1\right]\tag{26}$$

with

$$A\equiv\mathbb{P}_U\left[(1,0)\,|x_i\right]-\mathbb{P}_M\left[(1,0)\,|x_i\right];\qquad B\equiv\mathbb{P}_U\left[(0,1)\,|x_i\right]-\mathbb{P}_M\left[(0,1)\,|x_i\right];$$

$$C\equiv1-\mathbb{P}_U\left[(1,0)\,|x_i\right]-\mathbb{P}_U\left[(0,0)\,|x_i\right];\ \text{and}$$

$$D\equiv1-\mathbb{P}_U\left[(0,1)\,|x_i\right]-\mathbb{P}_U\left[(1,1)\,|x_i\right].$$

These last four parameters can be rewritten as

$$A=\frac{\operatorname{erfc}\left(\frac{\tilde{x}_U}{\sqrt{2}\sigma}\right)^n-\sum_{|M|=\left\lfloor\frac{n}{2}+1\right\rfloor}^{n}\binom{n}{|M|}\left(1+\operatorname{erf}\left(\frac{\tilde{x}_M}{\sqrt{2}\sigma}\right)\right)^{|M|}\operatorname{erfc}\left(\frac{\tilde{x}_M}{\sqrt{2}\sigma}\right)^{n-|M|}}{2^n\left(1+\exp\left\{2x_i-1\right\}\right)};$$

$$B=\frac{\left(1+\operatorname{erf}\left(\frac{\tilde{x}_U-1}{\sqrt{2}\sigma}\right)\right)^n-\sum_{|M|=\left\lfloor\frac{n}{2}+1\right\rfloor}^{n}\binom{n}{|M|}\left(1+\operatorname{erf}\left(\frac{\tilde{x}_M-1}{\sqrt{2}\sigma}\right)\right)^{|M|}\operatorname{erfc}\left(\frac{\tilde{x}_M-1}{\sqrt{2}\sigma}\right)^{n-|M|}}{2^n\left(1+\exp\left\{1-2x_i\right\}\right)};$$

$$C=-A+\frac{\sum_{|M|=\left\lfloor\frac{n}{2}+1\right\rfloor}^{n}\binom{n}{|M|}\left(1+\operatorname{erf}\left(\frac{\tilde{x}_M}{\sqrt{2}\sigma}\right)\right)^{|M|}\operatorname{erfc}\left(\frac{\tilde{x}_M}{\sqrt{2}\sigma}\right)^{n-|M|}-\left(1+\operatorname{erf}\left(\frac{\tilde{x}_U}{\sqrt{2}\sigma}\right)\right)^n}{2^n\left(1+\exp\left\{2x_i-1\right\}\right)};\ \text{and}$$

$$D=-B+\frac{\sum_{|M|=\left\lfloor\frac{n}{2}+1\right\rfloor}^{n}\binom{n}{|M|}\left(\operatorname{erfc}\left(\frac{\tilde{x}_M-1}{\sqrt{2}\sigma}\right)\right)^{|M|}\left(1+\operatorname{erf}\left(\frac{\tilde{x}_M-1}{\sqrt{2}\sigma}\right)\right)^{n-|M|}-\left(\operatorname{erfc}\left(\frac{\tilde{x}_U-1}{\sqrt{2}\sigma}\right)\right)^n}{2^n\left(1+\exp\left\{1-2x_i\right\}\right)}.$$

Using simple algebra, (26) can be rewritten as

$$\Delta(x_i) = \frac{a + b \cdot \exp\{c \cdot x_i\}}{d + \exp\{c \cdot x_i\}} \tag{27}$$

where a, b, c, and d are all positive constants. Because x_i is normally distributed, the probability distribution function of $\Delta(x_i)$ is given by

$$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{\left(\frac{1}{c}\log\left(\frac{x_i d - a}{b - x_i}\right) - \theta\right)^2}{2\sigma^2} \right\} \frac{bd - a}{c(b - x_i)(dx_i - a)} \tag{28}$$

with $\frac{x_i d - a}{b - x_i} > 0$, $x_i \neq b$, $x_i \neq \frac{a}{d}$.[15]

We verify (26) through multiple numerical approximations. In particular, we report here the values of $\mathbb{E}[\Delta_k]$ that obtain varying the the threshold q from 0.5 to 0.7 to 0.9. In all the simulations, we assume the following parameters $\sigma = 0.6$, $\xi_0 = q(1-q)$ and $\xi_1 = \frac{(1-q)(q-\xi_0)}{q}$.

	$q = 0.5$	$q = 0.7$	$q = 0.9$
$n = 3$	0	0.0102687	0.0214383

TABLE 1.7.1. Numerical approximation of $\mathbb{E}[\Delta_k]$.

Because the numerical approximations become almost intractable when the number of agents grows, we further conjecture that $\Delta(x_i)$ is increasing in the number of agents. This is because the probability of implementing $D = \emptyset$ always increases under the unanimity rule, while it is always zero under majority rule (assuming an odd number of agents).

[15]The probability distribution function is obtained by integrating the density function of x_i over the appropriate area that is defined by the function $\Delta(x_i)$, that is by computing $F(\hat{x}) \equiv \mathbb{P}(\Delta(x_i) \leq \hat{x}) = \int_{-\infty}^{\frac{1}{c}\log\left(\frac{\hat{x}d-a}{b-\hat{x}}\right)} \frac{1}{\sqrt{2\pi^2}} \exp\left[-\frac{(x_i-\theta)^2}{2\sigma^2}\right] dx_i$ and then $F'(\hat{x})$, which is given by (28).

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CHAPTER 2

Voting with Costly Communication

Caveat: A version of this dissertation essay coauthored with Krista J. Saral (Webster University Geneva) and Simone M. Sepe (University of Arizona) is a work in progress.

2.1. Introduction

In many circumstances economic agents deliberate before taking collective decisions. Members of juries, university committees, boards or government agencies, typically have the possibility to communicate and discuss about the alternatives at stake before taking a decision. When agents have heterogeneous preferences for time, however, different voting institutions may induce the agents to heterogeneous behavior during the communication phase.

In a context of group decision-making, this paper empirically investigates the relationship between voting rules and communication behavior. We study, in particular, what is the impact of varying voting institutions on the communication behavior of agents who can, at a cost, endogenously choose to communicate before taking a decision.

Recent literature, both theoretical and experimental, has emphasized the role of communication in affecting group decision-making processes. In particular, some authors focused on communication as a mean to aggregate dispersed information (e.g., Coughlan (2000), Austen-Smith and Feddersen (2005, 2006), Gerardi and Yariv (2007), Mathis et al. (2013)), while others focused on communication as a way to aggregate heterogeneous preferences (e.g., Li and Suen (2001), Visser and Swank (2007), Lizzeri and Yariv (2012)). In this paper, following the first strand of the literature, we empirically analyze how agents behave under different deliberation rules, such as voting rules and costs of communication. We report observations from laboratory experiment where subjects face the tradeoff between the benefit of aggregating private information and the cost of communicating. Because agents' private information can only be estimated as long as the agents repeatedly interact among each other, we claim that the opportunity cost of communication influences the behavior of agents.

Specifically, we follow previous experimental literature on jury decision-making revolving our design around an abstract voting game that avoids any specific term such as “guilty”, “innocent”, “status-quo” or “alternative (Guarnaschelli et al. (2000)). The experiment is implemented using two boxes and colored balls, which represent two different states of nature. These states of nature represent either a guilty or an innocent defendant. “Box 1” (guilty) contains 7 blue and 3 red balls, while “box 2” (innocent) contains 7 red balls and 3 blue balls. Agents receive a private signal about the underlying state of nature (pick one ball from the randomly selected box) and then cast a vote for one of the alternatives. Before taking the decision, subjects have the possibility to communicate and then vote again. If the subjects choose not to communicate, instead, their payoffs will be determined according to the tally of the fist voting round. Payoffs are maximized when the decision outcome matches the underlying state of nature (i.e., when the group chooses the right box). In this experiment, we vary the voting and communication rules and we study the effect on the endogenous choice of communication, on the voting behavior and on the efficiency of the decision outcomes.

In contrast with the results highlighted by Goeree and Yariv (2011), we report that, when costly communication is available to subjects, different voting institutions induce different decision outcomes. In particular, we observe that the jury unanimity system, which requires unanimous consensus over the implementation of each alternative, is the unique voting rule that *always* induces the subjects to communicate with each other. On the contrary, the simple majority rule and the “standard” unanimity rule (which requires unanimous consensus only over one alternative), induce the subjects to strategically avoid communication.[1]

Our second result pertains to the effect of communication on the voting behavior. Once subjects choose to communicate, we observe that communication unambiguously improves the

[1]Somehow surprisingly, with the exception of Coughlan (2000), previous literature has ignored the institutional difference between a jury unanimity system and a standard unanimity system. In the first case, agents are required to implement one of the alternatives with unanimous consensus, otherwise no decision is implemented. In the second case, instead, unanimous consensus is required to implement one alternative, otherwise the status-quo (a “default” alternative) is preserved.

quality of the decision outcome across each voting rule. We report that the simple majority rule combined with communication is the most effective voting rule, with almost 88% of right decision outcomes. We also report that the jury unanimity rule produces wrong outcomes only in 4.5% of the cases, but it also produces a high number of indeterminate cases where no decision is taken (i.e., a mistrial).

These insights suggest that voting institutions have an impact on the communication behavior of decision-makers. From a policy perspective, therefore, the design of communication protocols is a fundamental tool to improve the efficiency of collective decision bodies.

The paper is structured as follows. In section 2.2 we compare this paper with the relevant literature on group decision-making. The experimental design is described in section 2.3. Section 2.4 describes the theoretical and behavioral predictions. We finally analyze our experimental observations in section 2.5, and briefly conclude in section 2.6.

2.2. Related Literature

This paper is related to the literature on group decision-making, pioneered by Condorcet (1785). Using a simple binary model with two states of nature and two alternatives, which resembles the standard problem of a jury, Condorcet argues that large groups make better decisions than individuals using a simple majority rule. This result, known as Condorcet Jury Theorem, is based on the assumption that voters behave naively and does not consider the possibility of agents' strategic behavior.

A flourishing theoretical literature followed after the Condorcet's model. Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998) and Myerson (1998) studied the implications of strategic behavior on the Condorcet jury theorem, emphasizing that in a Nash equilibrium agents may vote strategically against their private information. In particular, such strategic behavior arises under a standard unanimity voting rule.

A more recent literature has analyzed the role of pre-vote communication on the decision outcomes of collective decision bodies. Coughlan (2000) was the first to acknowledge that a jury unanimity rule - based on communication and allowing for the possibility of mistrial - may induce decision outcomes that are completely different from those induced by a standard unanimity rule. Similarly, Persico (2004), Austen-Smith and Feddersen (2006) and Gerardi and Yariv (2007) generalize this approach studying the effect of more general communication protocols over the efficiency of the decision outcomes. Lizzeri and Yariv (2012) and Mathis et al. (2013) extend the basic static model to a dynamic communication game and show that unanimity rules induce more informative and efficient decision outcomes.

The first experimental study of the Condorcet model is attributable to Guarnaschelli et al. (2000). Their main result confirms the theoretical prediction of Feddersen and Pesendorfer (1998) that the standard unanimity rule induces strategic voting. Other authors have experimentally investigated the effect of agents' biases on the efficiency of the decision outcomes (Battaglini et al. (2008), Ali et al. (2008), Morton et al. (2012)) or the effect of sequential voting (Battaglini et al. (2007)).

More recently, Goeree and Yariv (2011) extended the Feddersen and Pesendorfer (1998) model adding a free-form communication stage. They find that communication reduces institutional differences. This is because their communication protocol is not consistent with each voting rule.[2] In contrast with this result, we show that different voting rules have an impact on the communication behavior of subjects and lead, in turn, to different institutional outcomes.

2.3. Experimental design

This experiment is designed to determine how voting rules influence the process of communication in groups when communication can be costly. The design revolves around the standard voting game of 2 boxes (jars) with 10 balls in each (Guarnaschelli et al. (2000), Goeree and Yariv, (2011)). Box 1 has 7 blue and 3 red, while box 2 has 7 red and 3 blue. Using the jury metaphor, Box 1 (Box 2) indicates the state of the world where the defendant is guilty (innocent). There is a third unknown box whose distribution of balls matches either

[2]Specifically, Goeree and Yariv (2011) give to all subjects the possibility to communicate as long as they are willing to do so, regardless of the voting rule.

box 1 or box 2, with equal probability. The goal of the group is to try and collectively determine which known box the unknown box matches with a higher payoff to all members if the correct decision is made.

2.3.1. Setup. Before the game starts the computer selects with equal probability one of the two boxes. Each group consists of five randomly assigned members. Each subject in a group is asked to pick a ball from the unknown box. The subject clicks on the box and the color of the ball is revealed. The remainder of the balls remains gray. Once subjects observe their signal, they are then asked to place a vote for either Box 1 or Box 2.

2.3.2. Information. Given the composition of each box, the precision of the signal drawn from each subject is equal to $p = 0.7$. To determine a collective decision for the group, subjects cast a vote for one of the two boxes. In addition to voting, the groups will also have the opportunity to communicate if they choose to do so.

2.3.3. Voting Rules. The experiment tests three voting rules: majority rule, jury unanimity, and standard unanimity using a between subjects design. Departing from Goeree and Yariv (2011)'s approach, we also allow agents to endogenously choose whether to engage in a costly communication phase. Following Mathis et al. (2013), we define a voting rule as a couple of two thresholds $K = \{k_1, k_2\}$, where k_i is the number of votes required to select box $i \in \{1, 2\}$. The voting rules tested in this experiment, based on a group of $n = 5$, are $K_M = \{3, 3\}$ (i.e. majority), $K_J = \{5, 5\}$ (i.e. jury unanimity) and $K_U = \{5, 1\}$ (i.e. standard unanimity). In words, under majority rule, three of the five group members are required to select each of the two boxes. Under jury unanimity, all five members are required to agree to select either box. Under standard unanimity, instead, five members are required to select box 1, while only one member is required to implement box 2.

2.3.4. Communication Rules. In all treatments, free-form communication in a chat window will be allowed if the group members choose to communicate. To begin communication, the subjects must agree according to the voting rule in use to communicate.

Under majority rule, communication starts if at least three of the five subjects choose to do so, otherwise there is no communication. Similarly, once the subjects agree to communicate, at least three of the five members of the group must agree to stop communication. Under the jury unanimity rule, instead, while only one subject is required to start the communication phase, all members must agree to stop it. Finally, the standard unanimity requires all five members of the group to agree to begin communication, and just one member to stop it. Once the communication phase ends, subjects have the possibility to vote again and possibly change their initial vote.

If communication is agreed to, the subjects will enter into a new stage where they are reminded of their original signal, the potential boxes that the unknown box could be, and are then allowed to communicate via free-form chat with other members of their group. During the communication stage, there a "cost clock" continuously updates with the cost of communication. Subjects will end communication by the voting rule, or it will automatically end at 5 minutes. The cost of communication will be subtracted from final decision earnings at the end of each round.[3]

If communication is not agreed to, the outcome is determined by the initial vote and subjects are taken to a results screen for the round, which informs them of the true state of the world (the true box), the vote tally, the decision outcome, and their final payoff which is based on the true box and decision.

2.3.5. Preferences. The subjects share identical preferences that the unknown box matches the box voted in. If the correct box is chosen, the subjects will each receive a payoff of 100 ECU (Experimental Currency Unit). If the subjects select the wrong alternative, they receive a payment of 10 ECU. If the subjects fail to implement a decision - a possibility that only arises in the jury unanimity treatment - they will receive a payment of 15 ECU.

Because losses are possible in the costly treatments, we will endow the subjects with 500 ECU at the start of the experiment, which is a starting amount from which losses and profits

[3] A version of this experimental design with a costless communication treatment is forthcoming.

will be subtracted in each round. If a subject goes bankrupt, his endowment will be reset once, allowing him to continue to play.

In summary, each experimental session tested one voting rule. Within each session, subjects played for 10 rounds, with one practice round preceding each session. The experiments have been conducted at the Webster University Geneva campus. Table 2.3.5 summarizes the payoffs, cost parameters and number of observations for each treatment.[4]

Voting Rule	Majority		Jury		Standard	
True Box	Box 1	Box 2	Box 1	Box 2	Box 1	Box 2
Preferences						
Outcome = Box 1	100	10	100	10	100	10
Outcome = Box 2	10	100	10	100	10	100
Outcome = No Decision	n.a.	n.a.	15	15	n.a.	n.a.
(per-minute) cost of communication	20		20		20	
N. of subjects	15		10		15	
N. of rounds	10		10		10	

TABLE 2.3.1. Experimental design

2.4. Theoretical Predictions

This model is an extension of the basic Condorcet jury model. We follow Mathis et al. (2013) considering a group of $n \geq 3$ agents who are required to collectively implement one of two alternatives, $A \in \{\text{Box 1}, \text{Box 2}\}$. The optimal choice depends on an underlying state of nature $\theta \in \{\text{Box 1}, \text{Box 2}\}$, which is equiprobable and unknown by the agents. Optimality requires that $A = \theta$ or, using the jury metaphor, that a guilty (innocent) defendant is convicted (acquitted).

An alternative A is implemented when at least k agents vote for it, with $k \in K \equiv \{k_1, k_2\}$ being the voting rule. In case neither of the alternatives receives at least k votes, none of the two boxes is selected .

Subjects share the same utility function, as indicated by their payoffs reported in Table 2.3.5. Once the state of nature is chosen, each subject observes a signal $s \in \{\text{blue}, \text{red}\}$ of

[4]See Appendix 2.7 for the experimental instructions.

accuracy $p \equiv \mathbb{P}(s = \text{blue}|\theta = \text{Box 1}) = \mathbb{P}(s = \text{red}|\theta = \text{Box 2}) = 0.7$ and then casts a vote for one of the two alternatives.

After the first voting stage, agents have the possibility to communicate. Communication starts if at least $n - \min\{k_1, k_2\} + 1$ agents agree to do so. If subjects choose not to communicate, the alternative selected at the first voting stage is implemented. This implies that at the first voting stage, each agent strategy σ maps each possible signal $s \in \{\text{blue}, \text{red}\}$ into a probability of choosing Box 2. We focus on *symmetric responsive* equilibria, that is equilibria in which rational agents respond to their information and where agents with the same signal play the same strategy.

Consider the case of the simple majority rule $K_M = \{3, 3\}$. In this case, there is a unique symmetric equilibrium where every agent selects the box that matches his signal, that is Box 1 (Box 2) when the signal is blue (red). Because a rational agent conditions his vote on his signal and on the event of being pivotal, indeed, a pivotal agent knows that - in a sincere voting equilibrium - two members of his group observed a blue signal while the other two members observed the red signal. These signals cancel out each other, inducing the pivotal agent to best respond by voting sincerely. A similar argument holds for the jury unanimity case $K_J = \{5, 5\}$.

Under the standard unanimity $K_U = \{5, 1\}$, instead, agents' incentive to vote sincerely disappear. To see why this is the case, notice that an agent is pivotal only when the other four agents voted for Box 1 and he has observed a red signal. Assuming that the other agents are voting sincerely, the event of being pivotal reduces the pivotal agent's posterior belief that the true state of nature is Box 2. Put it differently, being pivotal is equivalent to observe four blue signals and one red signal. A pivotal agent, therefore, is induced to mix his vote between Box 1 and Box 2 even when he observes a red signal. In turn, a sincere voting equilibrium does not exist under the standard unanimity voting rule, as first shown by Feddersen and Pesendorfer (1998).

When agents choose to communicate instead, because the final vote depends on the history of messages sent by each agent during the communication phase, there may be several equilibria of the game. Mathis et al. (2013) show that, under some circumstances, the jury unanimity rule is the unique voting rule that induces truthful reporting and sincere voting. Communication stops when at least $\min\{k_1, k_2\}$ agents decide to stop communication.

All the considerations above allow us to define the following behavioral hypotheses under the costly communication treatment. Let α_K be the probability that the agents choose to communicate under voting rule K and, similarly, let β_K be the sum of all agents' payoffs for each voting rule K.[5]

Behavioral Hypothesis 1 *Under costly communication,*

$$\alpha_J \geq \alpha_M \geq \alpha_U$$

and

$$\beta_J \geq \beta_M \geq \beta_U$$

Using the insights developed by Goeree and Yariv (2011), moreover, we expect that

Behavioral Hypothesis 2 *Under costless communication,*

$$\alpha_J = \alpha_M = \alpha_U$$

and

$$\beta_J = \beta_M = \beta_U$$

Behavioral hypothesis 1 establishes that, under costly communication, the jury unanimity rule induces more communication with respect to the other rules. Because the jury unanimity rule should also induce longer truthful communication and sincere voting behavior, the jury unanimity Pareto dominates the other two voting rules. This is because, even though the majority rule does not induce strategic voting behavior, agents communicate more under jury unanimity and therefore are able to aggregate more information.

[5]The parameter β_K can be interpreted as a measure of the efficiency of the voting rule K.

Behavioral hypothesis 2 is based, instead, on the results of Goeree and Yariv (2011). In their setting, because communication is costless, each voting rule induces the same communication and voting behavior. Moreover, Goeree and Yariv implicitly impose a communication rule that is equivalent to our communication rule under jury unanimity. Every subject, indeed, has the possibility to communicate as long as he want. This would not be possible under a majority voting rule, because an excluded minority may always want to communicate more to prevent the majority to implement one of the two alternatives.

2.5. Results

We discuss now the main results obtained when we vary the voting rule and allowing the subjects to communicate at a cost. We will compare these results with those obtained under costless communication in Goeree and Yariv (2011). We first focus on the communication behavior, then on the strategic voting behavior and finally on the efficiency of each voting rule.

2.5.1. Communication.

Table 2.5.1 shows the percent distribution of rounds in which the subjects agreed to communicate, for each voting rule. As predicted in behavioral hypothesis 1, subjects communicate more under the jury unanimity rule. Specifically, subjects *always* communicate under jury unanimity. Under majority rule (standard unanimity) instead, almost 60% (85%) of the time subjects prefer not to communicate. Because communication is costly, subjects are less willing to pay the cost of communication to aggregate information.

Communication Rule	(1) Majority	(2) Jury	(3) Standard
No	59.09	0	84.85
Yes	40.91	100.00	15.15
N	110	110	165

TABLE 2.5.1. Percent distributions of rounds in which subjects agreed to communicate, by voting rule.

Table 2.5.1 shows the percent distribution of rounds in which subjects are able to reach consensus or, in other words, all subjects agree on which alternative has to be selected. As intuition would suggest, communication helps the subjects to coordinate towards one of the alternatives. Under majority rule, for example, if the subjects choose to communicate, the frequency with which they reach a consensus switches from 8% to almost 67%. Similarly, under standard unanimity, consensus is reached more often when subjects choose to communicate (when subjects communicate, consensus is reached 60% of the time against 18% when communication is not agreed to). An overall inspection of table 2.5.1 shows that, once subjects agree to communicate, consensus is easier to reach under the majority voting rule, while it is almost equivalent under the two types of unanimity.

Consensus	(1)	(2)	(3)	(4)	(5)	(6)
Rule	Majority	Majority	Jury	Jury	Standard	Standard
Communication	No	Yes	No	Yes	No	Yes
No	92.31	33.33	0	40.91	82.14	40.00
Yes	7.69	66.67	0	59.09	17.86	60.00
N	65	45	0	110	140	25

Table 2.5.2. Percent distributions of rounds in which subjects reached a consensus over one alternative.

2.5.2. Voting Behavior. We focus now on the voting behavior of subjects. In particular, we focus on whether the subjects voted sincerely at each voting stage. Table 2.5.2 shows the percent distribution for each voting rule. Under the jury unanimity rule, subjects vote sincerely 76% of the time at the first voting stage. This probability becomes lower in the second voting stage, suggesting that communication helped the subjects to change their initial posterior belief. Under the majority rule, instead, subjects voted sincerely 73% of the time at the first voting stage, and almost 67% of the time at the second voting stage. Although the sincere voting probabilities are lower under the majority rule, communication allows the subjects to gather more information and then change their initial vote if necessary. This result remains constant across all the three voting rules.

Sincere Voting	(1)	(2)	(3)	(4)	(5)	(6)
Rule	Majority	Majority	Jury	Jury	Standard	Standard
Stage	1st	2nd	1st	2nd	1st	2nd
No	27.27	33.33	23.64	30.91	19.39	24.00
Yes	72.73	66.67	76.36	69.09	80.61	76.00
N	110	45	110	110	165	25

TABLE 2.5.3. Percent distributions of sincere voting by voting rule and by voting stage.

Under the standard unanimity, the ratio of sincere voting is surprisingly higher than under the other two voting rules. This result is in contrast with the theoretical predictions of Feddersen and Pesendorfer (1998), according to which strategic voting should emerge under a standard unanimity rule. In our experimental data, however, subjects vote sincerely with a very high probability. We conjecture that this behavior is explained by the fact that communication was almost never agreed to during the standard unanimity treatment. Because subjects were expecting not to communicate, subjects voted sincerely in the first voting stage after the initial rounds. In support of this view, the bottom-left panel of Figure 2.5.2 shows that, under the standard unanimity rule, the ratio of sincere voting over the total number of votes is close to 0.6 in the first two rounds and becomes (and remains) higher after the third round.

2.5.3. Efficiency. We conclude the analysis of the results focusing on the efficiency of each voting rule. Table 2.5.3 shows the percent distribution of decision outcomes, listed by each voting rule. Under the jury unanimity rule, the percent of wrong decision outcomes is much lower than under the other two voting rules. The jury unanimity, however, produces in almost 40% of the time a no-decision outcome - which corresponds, using the jury example, to a mistrial. Surprisingly, both the majority rule and the standard unanimity induce the subjects to select the right alternative in almost 73% of the cases.

FIGURE 2.5.1. Sincere voting ration over time, by voting rule.

Decision Outcome	(1)	(2)	(3)
Rule	Majority	Jury	Standard
Wrong	27.27	4.55	27.27
Right	72.73	54.55	72.73
No Decision	0	40.91	0
N	110	110	165

TABLE 2.5.4. Percent distribution of decision outcomes, by voting rule.

Table 2.5.3 highlights the percent distributions of decision outcomes, disaggregated by the subjects' decision to communicate and by voting rules. The accuracy of the decision outcomes becomes significantly higher when agents agree to communicate, consistently with the idea that the communication helps subjects to share private information. This is not true under the standard unanimity rule, where the percent of right decision outcomes falls from 75% (without communication) to 60% (with communication).

Decision Outcome Rule	(1) Majority	(2) Majority	(3) Jury	(4) Jury	(5) Standard	(6) Standard
Communication	No	Yes	No	Yes	No	Yes
Wrong	38.46	11.11	0	4.55	25.00	40.00
Right	61.54	88.89	0	54.55	75.00	60.00
No Decision	0	0	0	40.91	0	0
N	65	45	0	110	140	25

Table 2.5.5. Percent distributions of decision outcomes and decisions to communicate, by voting rule.

These results are in contrast with our behavioral hypothesis 2, which predicts that the jury unanimity rule should dominate, in terms of higher payoffs, the majority and the standard unanimity rules. Our experimental data show, instead, that the standard unanimity produced the highest number of correct decision outcomes. We believe, however, that such result is due to a "peculiar" series of realizations of the state of nature. In the standard unanimity treatment, indeed, in almost 80% of the cases the computer selected Box 2 as the "true box." Because the probability to select Box 2 is much higher than the probability to select Box 1 under the standard unanimity rule, the subjects realized high payoffs in most of the rounds. This effect should disappear if subjects were to play for a higher number of rounds.

The main insight emerging from this experimental data is that, in contrast with the predictions of Goeree and Yariv (2011) and Gerardi and Yariv (2007), different voting rules induce different communication behavior. In turn, when agents are allowed to endogenously choose whether to communicate, different voting rule produce different decision outcomes.

2.6. Conclusion

In a context of collective decision-making, this paper reports the result of the first experimental test of endogenously costly communication.

We acknowledge that different voting institutions trigger different voting and communication behavior in our subjects pool and, therefore, have different impact on the decision outcome.

Under a jury unanimity rule, subjects are always willing to communicate. We observe that the jury unanimity rule produces the lowest number of wrong decision outcome. We also observe, however, that this voting mechanism induces in many cases the subject to not implement any decision. When the subjects agree to communicate, finally, we have found that the simple majority rule induces the most accurate decision outcomes.

These results demonstrate that voting rules are communication devices that affect the deliberative process of collective decision bodies. From a policy perspective, therefore, it is important to understand the implications of institutional protocols aimed at affecting the communication behavior of decision-makers. Further research should test whether the results obtained here are robust to different experimental treatments such as, for instance, to the possibility of communicating at zero cost or to heterogeneity in subjects expertise or preferences.

2.7. Appendix

Experiment Instructions - Standard Unanimity.

General Information. The purpose of this experiment is to study how people make decisions in a particular situation. You will have the possibility to earn cash (or extra-credit points) during today's experiment. All amounts earned in this experiment are denominated in experimental points. These points will transform into CHF at the rate of 0.01 per point (or 1 point – 1 centime). You have been given an initial endowment of 500 points, which is equal to 5 CHF. All gains and losses that you will make during the experiment will be added or subtracted from your 500 initial endowment. You will be using the computer for the entire experiment and all interaction between you and the others will be through computer screens. Please do not talk directly with other subjects, or look at other screens during the experiment. If you have a question, please raise your hand and your question will be answered. Please turn to your computer screen as we begin.

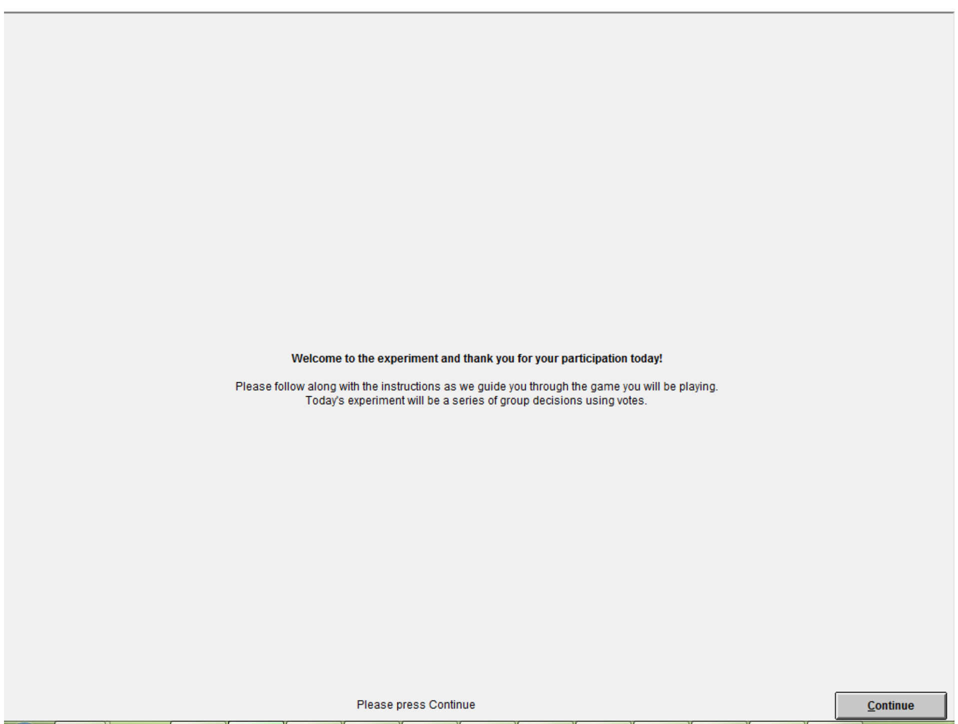

FIGURE 2.7.1. Welcome screen.

You should now see the first screen. Please wait until instructed to click buttons or enter information. You will now be able to follow along on your screen as I read. Welcome to the experiment and thank you for your participation today. Please follow the instructions carefully as we guide you through the game you will be playing. Today's experiment will be a series of group decisions using votes. Please click "Continue" to learn more about the basics of the experiment.

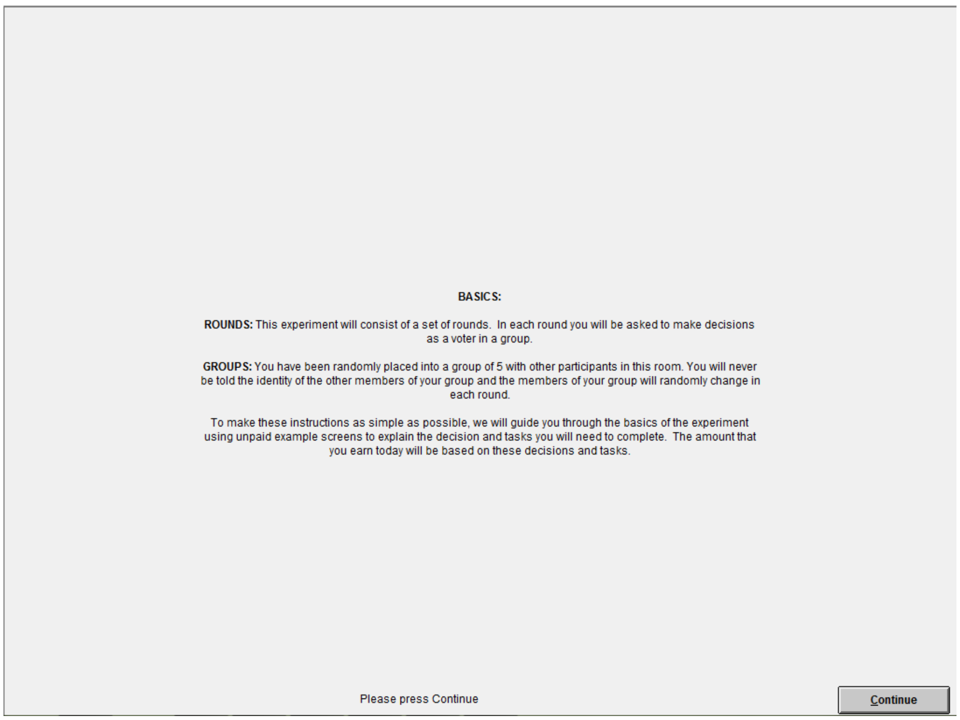

FIGURE 2.7.2. General information screen.

Basics. Rounds: This experiment will consist of a set of rounds. In each round you will be asked to make decisions as a voter in a group.

Groups: You have been randomly placed into a group of 5 with other participants in this room. You will never be told the identity of the other members of your group and the members of your group will randomly change in each round. To make these instructions as simple as possible, we will guide you through the basics of the experiment using unpaid example screens to explain the decision and tasks you will need to complete. The amount that you earn today will be based on these decisions and tasks. Please click "Continue" to be taken to the first example decision screen.

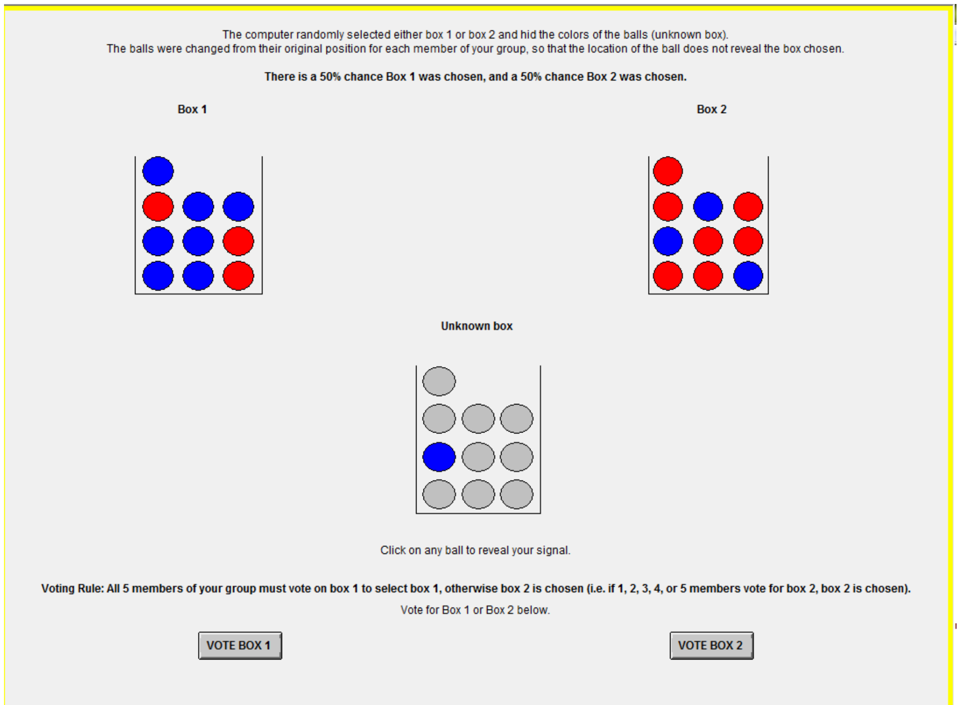

FIGURE 2.7.3. First voting round screen.

A sample of the first decision screen of a round is now displayed. All member of your group will see this same screen. On the top of this screen there are two boxes filled with 10 colored balls. The box on the left, Box 1, contains 7 blue and 3 red balls. The box on the right, Box 2, contains 7 red and 3 blue balls. Below this, in the middle of the screen there is an "unknown box" with 10 balls. The true color of these balls is currently hidden but the mixture of blue and red balls box matches either box 1 or box2. Whether or not the unknown box matches box 1 or box 2 was randomly determined by the computer with 50/50 probability (like a coin flip). Your Group's objective is to try and determine if the unknown box matches either box 1 or box 2. To reach a group decision, you and the other members of your group will vote for either box 1 or box 2. To help you make this decision, we will allow you to select one ball and reveal it's color. To see how this works, please select a ball in the unknown box by pointing the mouse to any ball and clicking. You should now see the color of the selected ball displayed on your screen. You can only choose 1 ball. Please note that the balls in the selected box have been shuffled for you. This means that the location of the colored ball that you see does not indicate the box chosen. The balls have also been shuffled differently for each member of your group, which means that even if you and another member choose the same ball location, it does not necessarily imply that the color will match. In summary – the location of a ball doesn't matter, only the color.

Group Decision. The box your group chooses will be based on a voting rule shown below the unknown box. The voting rule for your group is that all 5 members of your group must vote for box 1 for box 1 to be selected. If less than 5 members vote for box 1, then box 2 will be selected. In other words, if 1,2,3,4, or 5 members vote for box 2, then box 2 is selected. Box 1 can only be selected if all 5 members vote for it. For example, if 4 members of your group vote for box 1 and 1 member votes for box 2, then box 2 will be selected. For example purposes, please now vote for Box 1.

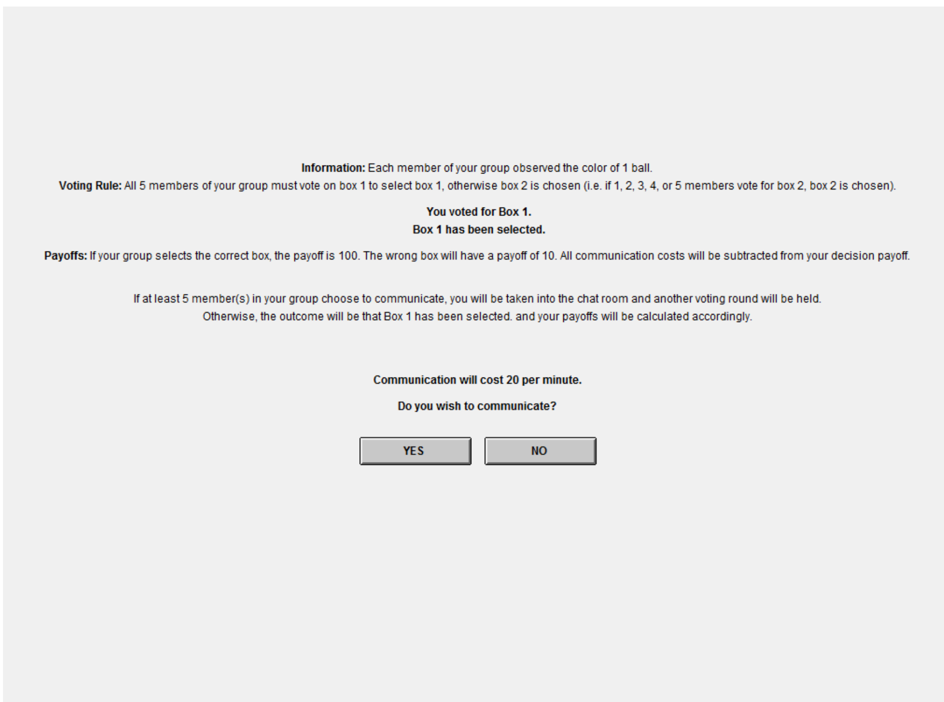

FIGURE 2.7.4. Group decision and endogenous choice of communication screen.

Now that you have all placed your vote, you will be taken to the next screen which provides the outcome of your vote and other information. At the top of the screen, you will see a recap of the information each member of your group has and the voting rule, which is that all 5 members of your group must vote for box 1 for it to be selected, otherwise box 2 is selected. You will also see a reminder of your vote, which in this case was box 1. You will see the group's decision based on the voting outcome beneath this – which is that box 1 was selected. The group's decision is box 1 because all 5 members of your group voted for box 1. In the actual experiment, if less than 5 people had voted for box 2 instead, the decision would have been box 2. Payoffs: If your group selects the correct box in a round, the payoff to each member will be 100. If you group has selected the wrong box, each member of your group will earn 10. These payoffs only depend on the group decision - not your individual decision. Now that you know the outcome of the initial vote, you will be given the opportunity to communicate with the other members of your group in which case, you will also be allowed to vote again.

Communication. At the bottom of this screen you will see two "Yes" and "No" buttons. Directly above this, you will see the question "Do you wish to communicate?" If all 5 members of your group press the "Yes" button, your group will enter into a chat room. Note that even if you DO choose to communicate, but at least 1 member of your group does not choose to communicate, then your entire group will not enter the chat room. If you group does not choose to communicate, the group's decision from the initial vote will stand as the final decision (which in this case would be box 1).

Costly communication. Please bring your attention to the sentence directly above the question of whether or not you would like to communicate which states that "Communication will cost 20 per minute." If your group decides to communicate you will each be charged a per second fee of .33 (which equals 20 ECUs per minute). The cost of communication will be updated in real-time, so you will pay only for the number of seconds you spent communicating. For example, if you communicate for 30 seconds, you will be charged 10 points. The longer you communicate, the more you are charged. For example, communication for 2 minutes will cost each member of your group 40 points. The communication round will end automatically at 5 minutes (which would be equivalent to a cost of 100 points). As this is a practice session, please press "Yes" to be taken to the communication stage. In the actual experiment you and all other members of your group can press either the yes or no button.

Communication. You have now been directed to the communication screen. On the top of the screen, you are reminded of the composition of box 1 and box 2. You have also been

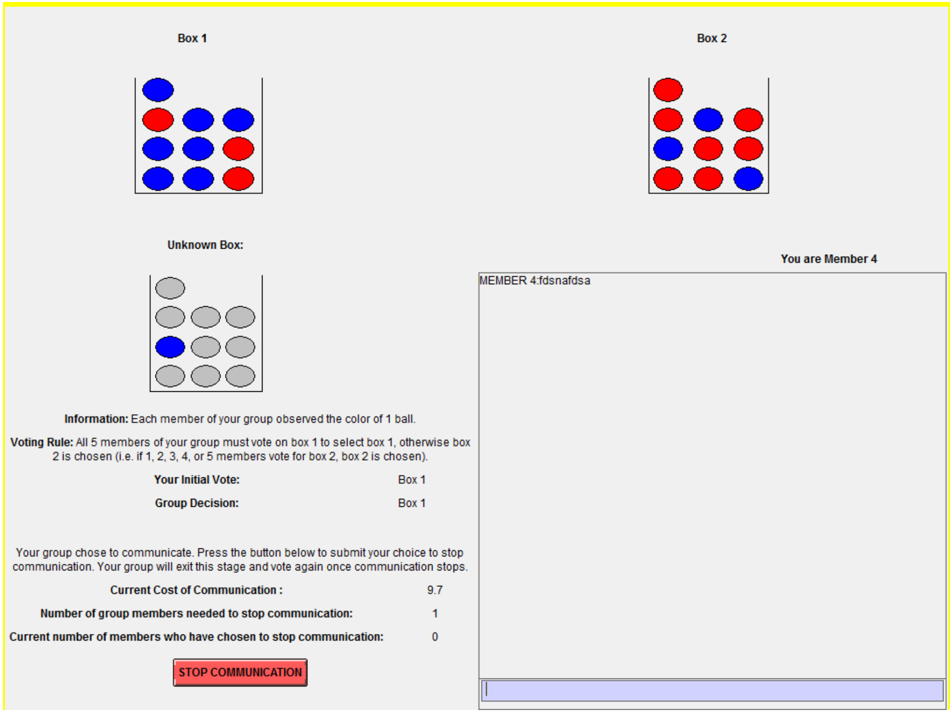

FIGURE 2.7.5. Communication screen.

given the reminder of the unknown box and the ball you chose. Directly below the unknown box, you will see a reminder that each member of your group also viewed one ball and the voting rule. You are also informed of your initial vote and Group Decision – in this example it is box 1 for both. On the right of the screen you see a chat box. This is where you can communicate with the other members of your group. Would please everyone type a message to the members of your practice group, for example "HI," and press enter. You must press enter for your message to be sent. You see now that your message appears on the screen as "Member #." If you are unsure of your member number, look at the top of the chat box. There you will see text that reminds you of your number so that you can track yourself. Member numbers were assigned randomly, and are only used to maintain anonymity in your group. We only have 2 rules that we ask you to follow regarding chat: 1. Please do not use the chat box to reveal identifying information about yourself. 2. Please also refrain from using profanity. On the bottom left side of the screen you will a clock ticking up with the current cost of communication. Communication may last until the clock (cost) reaches 100, unless your decides to exit communication.

Exit communication. To exit this communication, at least one member of your group needs to choose to stop. Once you would like to stop communicating, please click on the red "STOP COMMUNICATION" button. Important: communication will not end until one member presses the STOP COMMUNICATION button. Once one member presses the STOP COMMUNICATION button, all the group members will be immediately directed to the next screen. Please now click on the "Stop Communication" button to be taken to the next voting stage.

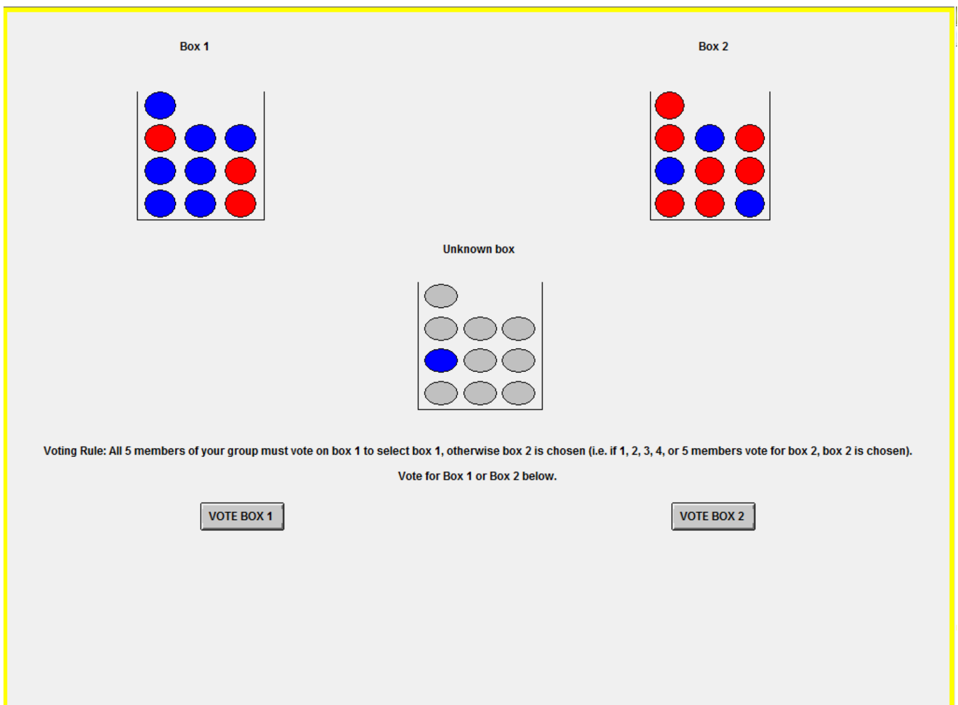

FIGURE 2.7.6. Second voting stage screen.

Second Vote. You should have been directed to another voting screen. As before, you have box 1 and box 2 at the top. The unknown box, which matches either box 1 or box 2, has the original ball that you initially selected at the start of this round. Now, you again have the possibility to vote for either box 1 or box 2. Since you agreed to communicate, your original vote was discarded and only this vote matters for the final decision. If you had chosen not to communicate, the original vote would have been used to calculate your payoff. Please notice that this is the last voting stage – there is no opportunity to communicate again and this will be the final vote that determines the group's decision. The voting rule remains the same: If all 5 members of your group vote for box 1 that will be group's decision. Otherwise, box 2 will be the group's decision. Please now vote for which ever box you would like to vote for.

TABLE 2.7.1. Results screen.

Now that all members of your group have voted for the box that they believe the unknown box matches, you will be taken to the results screen – you will see an example of this screen now. This screen will tell you how many votes were cast for box 1 and how many votes were cast for box 2. Directly below this, you will see your group decision and the below this the true match of the unknown box which is labeled as "Unknown Box Matched." In this example the unknown box matched box 1. We also reveal the final payoff for this round which depends on your group decision and the cost of communication. Recall that the payoff for your decision is the following: 1. If your group chooses the correct box, you will earn 100; 2. If you group chooses the incorrect box, you will earn 10. The cost of communication depends on the amount of time spent in the communication stage before exiting. Directly below the decision payoff, you will have a summary of your group's communication cost. Your round earnings are at the bottom of the screen and will be equal to the decision payoff less the communication cost. Note that it is possible to make negative earnings when your group makes the wrong decision leading to a decision payoff of 10 ECU and the communication costs exceed these earnings, which will only happen if you communicate for longer than 30 seconds. You have been given a start-up of 500 points from which losses will be subtracted and profits will be added. If your overall balance falls below zero you will be initialized with your endowment again and continue playing. 2-time bankrupt participants will be asked to leave the experiment with your show-up fee of 5 CHF. Please click “Continue.” One last screen not directly related to the game. In the event that your group chooses not to communicate, but another group does choose to communicate you will be taken to a tic-tac-toe “time passer” screen similar to the one you see now, where you can play against the computer if you choose so. To see how this works, please click a square. You can play as many games as you like, but you will not earn anything for playing tic-tac-toe, and will immediately be taken to results when the communicating groups finish.

As a recap of what we have discussed in these instructions, we have provided you with a paper “Cheat Sheet” – please feel free to refer to this for basics on payoffs and rules of the experiment. Please click “Continue.”

Are there any questions? The paid rounds will now begin. You will be randomly re-matched into a new group of 5, and a new unknown box will be created randomly. Please press continue to begin. You are now participating at your own pace, please make decisions when you are ready and click buttons when they are available so that they experiment can continue. At the end, please leave the “Cheat Sheet.”

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CHAPTER 3

Organized Crime and Regime Change

3.1. Introduction

Starting from the seminal work of Becker (1968), the economics of crime has attracted the attention of many social scientists. In Becker's work, individuals decide whether to engage in criminal activities or not by rationally comparing the expected returns to crime with the returns of a legitimate business. When the government increases law enforcement, therefore, crime becomes less attractive (Garoupa, 2000). Most of his and subsequent discussion about the economics of crime, however, focuses on the characterization of the optimal law enforcement. To the best of my knowledge, nobody has yet considered the relationship between individuals and criminal organizations, which is the focus of this paper.

Criminal organizations typically operate over different countries and locations. Criminal headquarters, however, have more power where individuals support the criminal organization more than the police force. As reported in Conciliar et al. (2013), "t*he strength of the Italian mafia associations, as well as their increasing influence on the legal economic activity, rest on a diffuse external complicity, namely, special relationships between criminal heads and public officials such as national or local politicians, judges, local administrators and members of the police force*."

Under the assumption that the presence (and persistence) of a criminal organization in a society is attributable to the support of individuals, this paper investigates the strategic interplay between a criminal organization and a large number of citizens and explains why such individuals are more inclined to support a criminal organization rather than reporting its illegal activities to the legal authority. Borrowing from the economic literature on coordination and regime change, I model a criminal organization as an autocratic regime and claim that such criminal organization engages in illegal activities in order to raise citizens support (Edmond, 2013). The criminal organization has private information about its strength and engages in criminal activities. Citizens then observe a noisy signal about the criminal organization strength, which is distorted by the intensity of criminal activities. Having observed their signal, each citizen then decides whether to support or not the criminal organization. Such interpretation further suggests that even those criminal activities which does not seem to have a direct impact on the society - such as retaliation murders between clans - have actually an impact on citizens perception of the organized crime strength.

This view complements the existing interpretation of the organized crime phenomenon, because it highlights the informational effect that criminal activities have on citizens perception of the criminal organization strength. The objective of each citizen is to reduce the loss associated with their decision to support or not the criminal organization. When a critical mass of citizens is not supporting the regime, the criminal organization is overthrown by the legal authority. Citizens who did not support the regime receive a payoff that is normalized to zero, while those who did support the regime bear a cost that is associated with the intensity of law enforcement. In the other scenario where the regime is not overthrown, citizens who supported the criminal organization receive a zero payoff, while those who did not support the regime incur a penalty that is set ex-ante by the criminal organization.

This paper makes two main contributions. It provides a formal characterization of citizen support of the criminal organization, identifying the conditions under which: a) citizens are willing to support the criminal organization; b) the criminal organization resists to the attacks of the legal authority. Specifically, the unique equilibrium is characterized by two thresholds as in the standard global games literature (Morris and Shin 2001, Atkeson 2001). When a citizen receives a signal that is lower than a threshold, he will support the criminal organization, and he will not provide support in any other case. Similarly, when the true type of the criminal organization is below a threshold which is a function of the aggregate mass of citizens supporting the regime, the organization will not be overthrown. Conversely, the organization is overthrown when the mass of citizens supporting the regime is not sufficiently large.

Second, I show that the level of criminal activities is increasing when the probability that the criminal organization will be overthrown in equilibrium becomes higher. This suggests that the higher the probability that the legal authority is able to overthrow the criminal organization regime, the more the criminal organization will engage in criminal activities. Put it differently, the criminal organization performs in more criminal activities when it is actually weak compared to the strength of the legal authority.

In addition to the above contribution, I conjecture that, under some parameter conditions, when we introduce the possibility for a government to endogenously modify the intensity of law enforcement, the optimal response of the legal authority to criminal activities is to reduce the intensity of law enforcement. Assuming that a government wants to maintain the level of criminal activities low, indeed, there are two possibilities to reduce the level of criminal activities. This is because, on one side, the government has the possibility to invest a large amount of resources of law enforcement. Given that the level of criminal activities is increasing with the probability that the criminal organization is overthrown, however, the government may not have enough resources to invest in law enforcement and fight against the regime. When the level of criminal activities is too high, therefore, the government may incur losses whatever the investment in law enforcement. This implies that the optimal reaction of the government would be to reduce the intensity of law enforcement so as to induce the organization to reduce criminal activities.

The paper is organized as follows. Section 3.2 reviews the existing literature. Section 3.3 presents the model, and in section 3.4 I present the main results of the paper. Section 3.5 briefly discusses further extension of the basic model and section 3.6 concludes.

3.2. Related Literature

This paper is related to two main strands of literature. The first refers to the sociological and economic interpretation of the organized crime, where a criminal organization is usually seen as an alternative provider of public services. Gambetta (1993), is one of the most known attempt to give a sociological and economic explanation of organized crime. He views the Sicilian Mafia as an informal provider of public services, alternative to a formal state. When there is lack of formal institutions, organized crime can effectively act as an informal way to implement law enforcement.

A first economic formalization of this view may be attributed to Grossman (1995), in which he sees the Mafia as a competitor to the state in the provision of public services and goods. Protection may indeed be used as an effective substitute good for trust. As reported by Gambetta (1993), *"in every transaction in which at least one party does not trust the other to comply with the rules, protection become desirable, even if it is a poor and costly substitute for trust."* According to his studies, the first private protection organizations in Sicily appear during the post-feudal period.

Varese (1994) follows Gambetta's approach studying the origins of the Russian Mafia. After Gorbachev reforms in the late 80's, Russia was experiencing economic conditions similar to the post-feudal Sicily. The spread of property rights after the post-communist reforms, created in Russia an unregulated market economy with no law enforcement. Because of this lack of formal institutions for property rights and law enforcement, a market for informal protectors.

Following this interpretation, Dixit (2003) points out that any economic activity needs governance, making an explicit link between his theory and Gambetta's work. In a more recent research, Dixit (2007, 2009) focuses on the distinction between governance and government, claiming that there is no reason, from an ex-ante perspective, for which formal institutions should be better than informal institutions. Conventional economic theory takes the existence of the rule of law for granted, assuming that a government has a monopoly over coercion power and law enforcement. In contrast with this view, Dixit claims that criminal organizations can act as alternative modes of governance.

Other authors, such as Becker (1968), Grossman (1995) and Garoupa (1997), focus on the optimal law enforcement. Baccara and Bar-Isaac (2008) focus on the optimal structure of a criminal organization, taking into account the trade-off between internal efficiency versus external vulnerability. Conciliar et al. (2013) study the impact of the leniency program introduced in Italy in 1991. They provide a theoretical framework that links the observed fall in Mafia related murders with the introduction of the new reform.

From a technical perspective, this paper is related to the literature on global games of regime change. Global games are known as coordination games with incomplete information in which players can only observe noisy signals about the underlying states of the world. Carlsson and van Damme (1993) and Morris and Shin (1998) pioneered this literature using this class of games in macroeconomic applications that are characterized by coordination problems and multiplicity of equilibria as in Diamond and Dybvig (1983). Classical examples go from debt pricing, bank runs to currency attacks and are well reviewed in Morris and Shin (2001).

As pointed out by Atkeson (2001), multiplicity of equilibria arises when agents' beliefs about what the other agents' are doing are a function of multiple sources of information. When agents observe only one source of information - typically a noisy signal about the actual state of the world - this multiplicity reduces to a unique equilibrium corresponding that is a function of the actual state of the world.

Global games have also recently been applied to political science. Iaryczower (2006) explains how party leaders discipline their party through the appropriate amount of resource disbursement, in order to guarantee a minimum level of support. Bruno (2008) extends the Hoff-Stiglitz (2004) model of the quest for the rule of law using the global games approach. He claims that the quality of institutions and the information of such quality determine together whether anarchy or the rule of law will prevail in equilibrium.

Finally, this paper is mostly related to Edmond (2013) who provides a model of information manipulation and regime change. An autocratic regime may induce people to riot or not through regime *propaganda*. He proves that as the information becomes very precise, the regime survives more likely. I adopt Edmond's approach here, claiming that a criminal organization behaves as an autocratic regime.

3.3. The Model

3.3.1. Setup.

3.3.1.1. Players.

Criminal Organization. A criminal organization needs the support of citizens to resist to legal authority attacks and maintain the control over its illegal activities. The criminal organization has private information about its strength, summarized by the parameter $\theta \in \mathbb{R}$. One possible interpretation of θ is the relative economic strength of the criminal organization with respect to the legal authority. Lower values of θ are associated to stronger organizations.

The greater the number of people in the society support the criminal organization, the harder the legal authority can overturn the criminal organization. Once the criminal organization knows its type θ, it has the possibility to distort citizens perception of its strength by performing criminal activities, i.e. selecting $a \in \mathbb{R}$.

Citizens. A mass of citizens, each indexed by i, is uniformly distributed over $[0,1]$. They have the common uninformative prior that θ is uniformly distributed over $\mathbb{R}$.[1] Each citizen i receives an i.i.d. signal

$$x_i = \theta - a + \varepsilon_i \tag{29}$$

with $\varepsilon_i \sim \mathcal{N}(0, p^{-1})$, with p indicating the precision of the signal. This heterogeneity may be attributed to the quality of the information each citizen has access to - e.g. the quality of newspapers they read or the news broadcasting channel they watch - or to the personal perception of the criminal power of the organization, which is randomly assigned.

Having observed his signal, each citizen decides whether to support the criminal organization or not, choosing an action $r_i \in \{0,1\}$, where $r_i = 1$ ($r_i = 0$) when a citizen supports (opposes to) the criminal organization.[2]

Let $R := \int_0^1 r_i di$ be the mass of people that supports the criminal organization. When there exists a sufficient mass of people who supports the criminal organization activities - i.e. when $R > \theta$ - the criminal organization survives, while the regime is overthrown otherwise.

[1] Morris and Shin (2001) prove that, as long as conditional expectations are well defined, uninformative priors - i.e. priors with infinite mass of probability - are not an issue.

[2] A citizen is supporting the criminal organization when he does not cooperate with the legal authority. For example, having observed a criminal activity, a citizen supports the criminal organization when he does not report such event to the police authority. This indirect support to the criminal organization, or any general act of non-cooperation with the legal authority is known as *omertà*.

3.3.1.2. Timing. The timing of the game is summarized as follows.

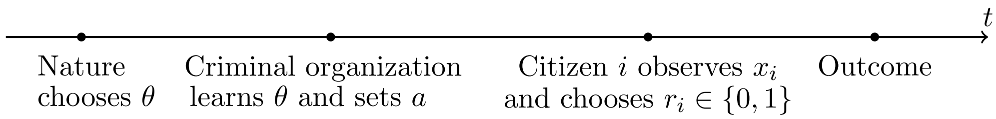

FIGURE 3.3.1. The timing of the game.

Nature draws the parameter $\theta \in \mathbb{R}$. Once the criminal organization learns θ, it selects the level of criminal activities a. Citizens receive idiosyncratic noisy signals, distorted by the level of criminal activities, and decide whether to support or not the criminal organization.

The outcome of the game depends on the strength of citizens support. The criminal organization is not (resp. is) overthrown by the legal authority when there is (resp. there is not) a sufficient mass of citizens supporting its activities, i.e. if $R > \theta$ (resp. $R < \theta$).[3]

3.3.1.3. Payoffs. Assuming that the presence of the criminal organization can only lower the citizens welfare, I represent citizens preferences with a loss function. Because the underlying state of the world is unknown, there are two possible scenarios, depicted as follows.

When the criminal organization is not overthrown, each citizen not supporting the regime receives punishment (i.e. retaliation) $P_M \geq 0$ from the criminal organization, and 0 otherwise. The criminal organization gains $G(R)$ - with R being the aggregate mass of supporters - and bears the cost of engaging criminal activities $C(a)$. Revenues increase with the number of citizens supporting the organization, i.e. $G'(R) > 0 > G''(R)$, while the cost of performing criminal activities is such that $C'(a) > 0$, $C''(a) > 0$ and $C(0) = 0$.

In the other case where the criminal organization is overthrown, each citizen supporting the regime receives a punishment $P_L \geq 0$ from the legal authority, and 0 otherwise. The parameter P_L can be interpreted as the intensity of law enforcement. When the regime is overthrown, the criminal organization only bears the cost of performing criminal activities $C(a)$ and does not gain any revenue.

The following table summarizes the payoffs.

[3] In section 3.5 we analyze and discuss the implications of introducing the legal authority as a third player of the game.

	Regime is overthrown	Regime is not overthrown
Citizen i	$-r_i P_L$	$-(1-r_i)P_M$
Criminal Organization	$-C(a)$	$G(R)-C(a)$

Fig. 3.2: The matrix of payoffs.

The parameters P_L and P_M, both non negative, are monetary-equivalent punishments that either the legal authority or the criminal organization respectively inflict on citizens according to their choice of r_i.[4]

3.3.2. Complete information benchmark. When θ is known by both the citizens and the criminal organization, it is optimal for the criminal organization to set $a = 0$ for any θ. In this case, multiple equilibria arise.

When $\theta \leq 0$, the organization always survives. Because it is a dominant strategy for each citizen to participate, every citizen does so, implying that $R = 1$ and the organization is never overthrown.

When $\theta \geq 1$, it is a dominant strategy for each citizen to select $r_i = 0$, so that $R = 0$ and the organization is always overthrown.

When $\theta \in (0, 1)$, citizens face a standard coordination problem where multiple equilibria are sustainable.[5] For any given value of θ, there exist two equilibria. When each citizen believes all other citizens are (not) supporting the organization, it is optimal for him to do so, so that $r_i = 1$ ($r_i = 0$). Given the symmetry of the game, every citizen supports (does not support) the regime and $R = 1$ ($R = 0$). Which of the two outcomes actually occurs depends on what citizens believe about other citizens behavior.

3.3.3. Incomplete information without criminal activities. When θ is private information of the criminal organization, citizens receive only a noisy signal about θ, as defined by 29. The strategies of the criminal organization and of each citizen are set conditionally on the realization of θ and x_i, respectively.

[4]When either P_L or P_M are chosen endogenously, citizens should also take into consideration this information when computing their posterior beliefs about θ as, for instance, in Angeletos, Hellwig, Pavan (2006). For simplicity, we do not consider here this case.

[5]See, for instance, Diamond, Dybvig (1983).

3.3.3.1. *Strategies.* Having observed θ, the criminal organization chooses the level of criminal activities a. A pure strategy for the criminal organization is a mapping from $\mathbb{R}$, the domain of signals, to $\mathbb{R}_+$, the domain of criminal activities.

A pure strategy for each citizen i maps any signal x_i into a decision to support the criminal organization or not $r_i \in \{0,1\}$. Each citizen i expected utility rewrites, therefore, as

$$V(x_i, r_i) = \int_{\theta < R(\theta)} \left(-(1-r_i)P_M\right) \mu(\theta|x_i) d\theta + \int_{\theta > R(\theta)} \left(-r_i P_L\right) \mu(\theta|x_i) d\theta \tag{30}$$

where $\mu(\theta|x_i)$ is the posterior belief, defined below, of citizen i with signal x_i.

3.3.3.2. *Equilibrium concept.* We look for a symmetric Perfect Bayesian Equilibrium (PBE), defined by the collection

$$\left(a^*(\theta), (r_i^*(x_i))_{i\in[0,1]}, R(\theta, a), \mu(\theta|x_i)\right) \tag{31}$$

with

$$\forall\theta,\ a^*(\theta) \in \arg\max_{a\in\mathbb{R}} \left\{G(R(\theta,a))\, \mathbb{I}_{\{R(\theta,a)>\theta\}} - C(a)\right\} \tag{32}$$

$$\forall i, \forall x_i,\ r_i^*(x_i) \in \arg\max_{r_i\in\{0,1\}} V(x_i, r_i) \tag{33}$$

$$R(\theta, a) := \int_0^1 \int_{-\infty}^{+\infty} r_i^*(x_i)\, \varphi(x_i|\theta, a) dx_i di \tag{34}$$

$$\mu(\theta|x_i) := \frac{\Pr(x_i - \theta + a(\theta)) \Pr(\theta)}{\int_{-\infty}^{+\infty} \Pr(x_i - \theta' + a(\theta')) \Pr(\theta') d\theta'} \tag{35}$$

where, specifically: $a * (\theta)$ maximizes the criminal organization's utility for any θ, $r_i^*(x_i)$ maximizes each citizen i expected utility for any given signal x_i; $R(\theta, a)$ is the expected mass of citizens supporting the criminal organization for a given θ and a given level of criminal activities a, with $\varphi(\cdot)$ being a normal p.d.f.; $\mu(\theta|x_i)$ is the posterior belief of each citizen i for a given signal x_i. Finally, the indicator function $\mathbb{I}_{\{.\}}$ is equal to one when $R(\theta, a) > \theta$ and equal to zero otherwise.

3.4. Results

Let us first focus on the case of no endogenous information distortion, that is the case where $a(\theta) = 0$, for any θ. Let $\Pr(\theta < R(\theta)|x_i) := \int_{\theta<R(\theta)} \mu(\theta|x_i)d\theta$ be the citizen i belief that the criminal organization will not be overthrown, and $\Pr(\theta > R(\theta)|x_i) = 1 - \Pr(\theta < R(\theta)|x_i)$ be the complementary probability. Therefore,

LEMMA 3.1. *Citizen i chooses $r_i = 1$ if and only if*

$$\Pr(\theta < R(\theta)|x_i) \geq \frac{P_L}{P_L + P_M} \tag{36}$$

PROOF. See Appendix. □

From (36), we see that each citizen decision to support the criminal organization depends on the ratio between the level of law enforcement P_L and the retaliation of the criminal organization P_M. It is easy to show that, when the level of P_M increases, each citizen i will support the criminal organization more often. On the other hand, when P_L increases, less citizens are willing to support the criminal organization, implying that (36) is satisfied less often.

3.4.1. Morris-Shin benchmark.

I now explicitly characterize the equilibrium, assuming that each citizen i adopts a cutoff strategy, that is:

$$r_i(x_i) = \begin{cases} 1 & \text{if} \quad x_i < \widehat{x}_i \\ 0 & \text{if} \quad x_i > \widehat{x}_i \end{cases} \tag{37}$$

Using definition (34), the aggregate mass of supporters $R(\theta)$ is continuous and strictly decreasing in θ, so that $R(0) > 0$ and $R(1) < 1$.[6] Since $R(\theta)$ is a real-valued mapping from $\mathbb{R}$ to $[0,1]$, then there exists a unique threshold $\theta^* \in [0,1]$ such that

$$\begin{cases} R(\theta) > \theta^* \text{ if } \theta < \theta^* \\ R(\theta) = \theta^* \text{ if } \theta = \theta^* \\ R(\theta) < \theta^* \text{ if } \theta > \theta^* \end{cases} . \tag{38}$$

which allows us to rewrite $\mathbb{P}(\theta < R(\theta^*)|x_i)$ as $\mathbb{P}(\theta < \theta^*|x_i)$. The following figure provides a graphical interpretation of (10).

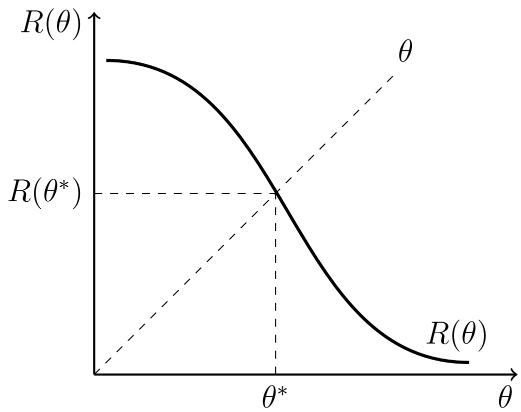

FIGURE 3.4.1. The aggregate mass of citizens supporting the criminal organization decreases with θ.

We are now able to prove that there exists a unique equilibrium à la Morris-Shin (2001), defined by threshold rules. Each citizen i supports criminal activities when he receives a signal x_i lower than a threshold x^* and the criminal organization resists resist to attacks if θ is lower than a threshold θ^*.

[6] This argument follows from the normality assumption. It can be easily checked, indeed, that $\mathbb{P}(x_i|\theta) = e^{\cdot - \frac{(x_i-\theta)^2}{2}p}$ is strictly decreasing in θ for any $\theta \geq x_i$.

PROPOSITION 3.2. *[Morris-Shin benchmark] The game has a unique equilibrium defined by thresholds* (x^*, θ^*) *which simultaneously solve the following system*

$$\begin{aligned} &i)\ \mathbb{P}\left(x_i \leq x^* | \theta = \theta^*\right) = \theta^* \\ &ii)\ \mathbb{P}\left(\theta \leq \theta^* | x_i = x^*\right) = \tfrac{P_L}{P_L + P_M} \end{aligned} \tag{39}$$

PROOF. See Appendix. □

3.4.2. Comparative Statics. Using an explicit characterization of the equilibrium thresholds, we can derive policy implications under this simple framework.

Let $\Phi\left(\sqrt{p}\left(x^* - \theta^*\right)\right)$ replace $\mathbb{P}\left(x_i \leq x^* | \theta^*\right)$, where $\Phi\left(\cdot\right)$ is a standard normal cdf, with precision p. Condition *i)*, then, rewrites as

$$\Phi\left(\sqrt{p}\left(x^* - \theta^*\right)\right) = \theta^* \iff x^* = \theta^* + \frac{1}{\sqrt{p}}\Phi^{-1}\left(\theta^*\right) \tag{40}$$

which makes x^* an increasing function of θ^*. This implies that, as it becomes harder to overthrow the criminal organization (i.e., as θ^* increases), the probability that each citizen i supports the criminal organization decreases.

Condition *ii)* of (39) rewrites, instead, as

$$\mathbb{P}\left(\theta \leq \theta^* | x_i = x^*\right) = \tfrac{P_L}{P_L + P_M}$$

$$\iff \frac{\int_{-\infty}^{\theta^*} \sqrt{p}\varphi\left(\sqrt{p}(x^* - \theta)\right) d\theta}{\int_{-\infty}^{+\infty} \sqrt{p}\varphi\left(\sqrt{p}(x^* - \theta)\right) d\theta} = \tfrac{P_L}{P_L + P_M}$$

$$\iff 1 - \Phi\left(\sqrt{p}\left(x^* - \theta^*\right)\right) = \tfrac{P_L}{P_L + P_M} \tag{41}$$

where $\varphi\left(.\right) = \Phi'\left(.\right)$. Using (40), we can substitute x^* in (41) and, rearranging the terms, get

$$\theta^* = \frac{P_M}{P_L + P_M}. \tag{42}$$

and

$$x^* = \frac{P_M}{P_L + P_M} + \frac{1}{\sqrt{p}}\Phi^{-1}\left(\frac{P_M}{P_L + P_M}\right) \tag{43}$$

As intuition suggests, both thresholds are increasing in the level of retaliation, P_M, and decreasing in law enforcement, P_L. When θ^* increases, indeed, the probability that the criminal organization resists to attacks rises; when x^* increases, instead, more people are willing to support the organization. The policy implication is straightforward: higher levels of law enforcement increase the probability that the criminal organization may be overthrown.

The precision of the signal also plays an important role. Indeed, as precision tends to infinity - i.e. as the variance goes to zero - incomplete information vanishes, so that the signal that makes a citizen indifferent between supporting or not becomes equal to θ^*. Because in this case citizens would be able to coordinate perfectly, this suggests that increasing the precision in the signal is also a welfare-increasing policy.

3.4.3. Criminal Activities as Endogenous Information Distortion. When we consider the case of endogenous information manipulation, the criminal organization distorts the perception of the citizens according to the value of θ. Because the distortion shifts the distribution of the signals that the citizens are receiving, the thresholds that determine whether the regime is overthrown or not will be different from that derived without signal distortion.

In this respect, let θ_a^* be the threshold value that determines whether the organization is overthrown or not in case of endogenous distortion, i.e. when $a^*(\theta) > 0$ for some θ. Because the organization survives whenever $\theta < \theta_a^*$, the distortion is effective when $\theta_a^* > \theta^*$. In other words, because the citizens willingness to support the regime is increasing with θ_a^*, an effective distortion increases the probability that the regime is not overthrown.

Let us focus on the case of endogenous distortion. Given that a is unobservable, each citizen does not have additional information about θ other than the signal x_i; this ensures the equilibrium to be unique (see Edmond (2012), Angeletos et al. (2007)). The criminal organization solves

$$\max_{a\in\mathbb{R}_+} G\left(R\left(\theta,a\right)\right)\mathbb{I}_{\{R(\theta,a)>\theta\}}-C\left(a\right) \tag{44}$$

Since each citizen i is not able to disentangle $\theta-a$ from the observed signal x_i, $R\left(\theta,a\right)$ just depends on the difference $\theta-a$. This implies that (44) can be rewritten as

$$\begin{aligned} &\max_{a\in\mathbb{R}_+} G\left(R\left(\theta-a\right)\right)-C\left(a\right) \\ &\text{s.t. } R\left(\theta-a\right)\geq\theta \end{aligned} \tag{45}$$

which has two possible solution, according to the true value of θ.

Proposition 3.3. *Assume* $\frac{\partial R}{\partial a}>0$ *and* $\frac{\partial^2 R}{\partial a^2}\leq 0$. *When the criminal organization solves* (45)*, then*

Case 1. *If* $\theta\geq\theta_a^*$*, the optimal level of criminal activities is* $a^*\left(\theta\right)=0$.

Case 2. *If* $\theta<\theta_a^*$*, the optimal level of criminal activities* $a^*\left(\theta\right)$ *is increasing in* θ*.*

Proof. See Appendix. □

The intensity of the distortion and, therefore, the intensity of criminal activities, depend on the actual value of θ. When θ is close to 0 - the case of a powerful organization - there is no reason to perform criminal activities. This is because the organization knows that, on average, signals are very low and the probability that each citizen will support the regime is very high. The optimal distortion, in this case, is such that $a^*\left(\theta\right)=0$. As θ rises, the organization is more willing to use criminal activities to distort citizens perception of the regime. Therefore, there exists some range of $\theta\in(0,\theta_a^*)$, in which the criminal organization sets $a^*\left(\theta\right)>0$.

When θ reaches the threshold value θ_a^*, the criminal organization is indifferent between performing criminal actions or not. When $\theta\geq\theta_a^*$, the criminal organization has again no reason to perform illegal actions, since they are costly and they do not add any improvement to the criminal organization's welfare.

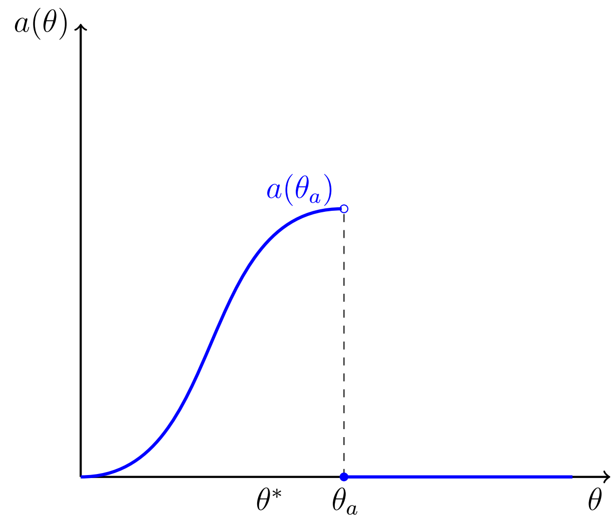

FIGURE 3.4.2. Criminal activities as a function of θ.

Focusing on the last and more interesting case, the mass of supporters is no longer a continuous function. As long as $\theta < \theta_a^*$, the signal is artificially pushed down by criminal activities, which are still positive. Therefore, the aggregate mass of supporters is greater with respect to case without signal-jamming. When θ is just above θ_a^*, the criminal organization does not have incentive to perform costly criminal actions, since it would be overthrown anyway. In this case, the mass of supporters dramatically falls and declines even more as θ becomes bigger. Picture 3.5 summarizes.

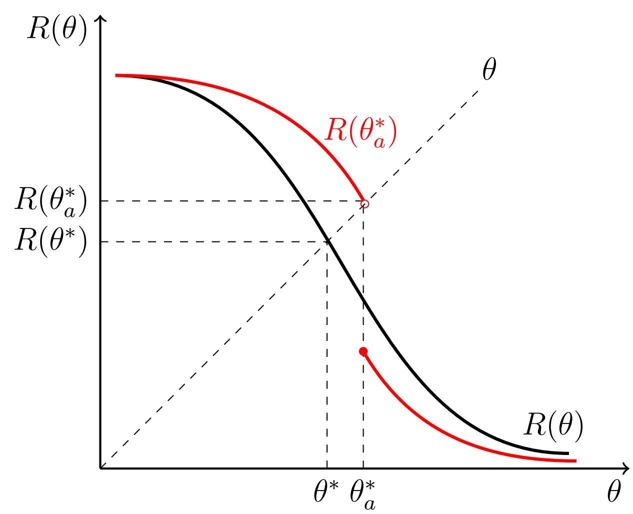

FIGURE 3.4.3. The aggregate mass of supporters with signal distortion.

The position of the new curve, depicted in blue, is of course arbitrary. We do not know whether the constraint is binding or not at the threshold θ_a^*.

3.5. Discussion and Extension

The following section is a preliminary attempt to extend the model analyzed above. Let us consider the case where the legal authority is able to select endogenously the intensity of law enforcement P_L and see how this affect the equilibrium of the game.

I assume that a government has access to a technology that allows to observe criminal activities a. The state parameter θ, however, is still private information of the criminal organization: even if the government can observe $a^*(\theta)$, it cannot observe the actual θ. After the criminal organization sets the level of criminal activities, the government chooses the intensity of law enforcement based on the observed value of $a^*(\theta)$. Citizens, therefore, observe their signal and decide whether to support or not the criminal organization. Again, the criminal organization resists to attacks if there is a sufficient mass of people that supports the organization. Figure 3.6 summarizes the new timing.

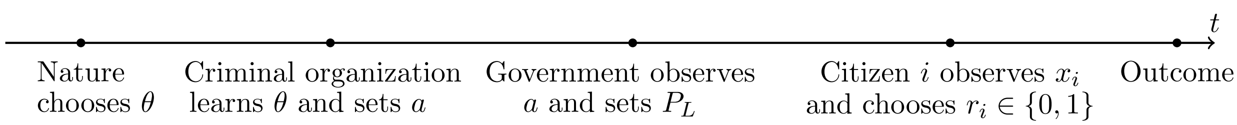

FIGURE 3.5.1. The timing of the game with endogenous law enforcement.

For the sake of simplicity, I focus on the case where the choice of P_L does not convey any information to citizens.[7] Each citizen i, therefore, selects his optimal action according to the sole source of information x_i. The equilibrium thresholds change, however, because they are affected by the intensity of law enforcement P_L.

3.5.0.1. Government's problem. The government observes the level of criminal activities a, then decides the intensity of law enforcement. The government's strategy, therefore, maps the intensity of law enforcement P_L from $\mathbb{R}_+$ to $\mathbb{R}_+$. I assume that the government seeks to solve, for any a,

[7]This assumption prevents the consideration of multiple equilibria as in Angeletos et al. (2006).

$$\max_{P_L \in \mathbb{R}_+} \quad W(P_L, a) - C(P_L)$$
$$\text{s.t.} \qquad P_L \leq E$$

where the welfare function $W(P_L, a)$ is decreasing in a, $C(P_L)$ is the cost of implementing law enforcement and is such that $C'(\cdot) < 0$, and E is the maximum amount of resources that the government can allocate to law enforcement. One reason that justifies why the welfare function of the government is decreasing in the level of criminal activities is that criminal activities scare citizens and increases the perception of living in an unsafe environment. Because citizens may take this perception into account when they are asked to vote and elect a new government, a high level of criminal activities loads the incumbent government bears with the implicit cost of a reduced probability of re-election.

When the government wants to reduce as much as possible criminal activities, we may obtain a counter-intuitive result. When the level of criminal activities is too high, the government incurs losses whatever the investment in law enforcement. This is because the level of criminal activities increases with the probability of overthrowing the criminal organization. After observing a large amount of criminal activities, therefore, the optimal reaction of the government would be to reduce the intensity of law enforcement, so as to induce the criminal organization to reduce the level of criminal activities.

When fighting crime is too costly, we may obtain a paradoxical equilibrium outcome where the government reduces law enforcement to reduce the level of criminal activities.

3.6. Conclusion

This model provides an alternative interpretation of criminal activities performed by organized crime. When we consider criminal organizations as autocratic regimes, criminal activities can be interpreted as a distortion of citizens perception of the criminal organization's strength.

A good empirical example may be the Mafia murders in the early 90's in Italy. In that period, the Sicilian Mafia was under the pressure of very intense legal authority's attack. In particular, the two prosecuting magistrates Giovanni Falcone and Paolo Borsellino obtained important successes in the fight against Mafia. Between May and July 1992, however, the two judges were killed by the Sicilian Mafia, an assassination ordered by the Corleonesi's family. Following other similar and dramatic events attributable to the Sicilian Mafia, in 1993, the Italian Minister of Justice temporarily reduced law enforcement for 140 jailed mafia leaders "*in order to avoid further murders,*" as he said.[8]

This model pictures two possible scenarios without criminal activities. Either the criminal organization is so powerful that it has no need to distort the citizens perception of its strength or, in the other case, the criminal organization is actually overthrown by the legal authority. This interpretation shed new light on the rise and fall of organized crime murders and terrorist attacks observed in Italy before and after 1992.

3.7. Appendix

Proof of Lemma 3.1. Consider the expected utility of each citizen i, $V(x_i, r_i)$ as defined in (30). Having observed x_i, when citizen i chooses $r_i = 1$, he gets $-P_L \mathbb{P}(\theta > R(\theta)|x_i)$. Choosing $r_i = 0$, instead, citizen i gets $-P_M[1 - \mathbb{P}(\theta > R(\theta)|x_i)]$. Citizen i, therefore, chooses to support the criminal organization when

$$-P_L \mathbb{P}(\theta > R(\theta)|x_i) \geq -P_M[1 - \mathbb{P}(\theta > R(\theta)|x_i)]$$

which is equivalent to (36). *Q.E.D.*

Proof of Proposition 3.2. We prove that there exists unique thresholds (x^*, θ^*) which solve the system of equations defined in (39). Because citizens play according to a threshold strategy, as defined by (37), I proceed in steps through iterated elimination of weakly dominated strategies.

Step 1. Let us fix a very low candidate threshold $\widehat{x}_{i0} \in (0, 1)$ such that $\mathbb{E}[V(\cdot, 0)|\widehat{x}_{i0}] < 0$. This implies that, for any $x_i \leq \widehat{x}_{i0}$, because citizen i is so confident that θ is actually low, his best response is to set $r_i = 1$ regardless of what the other players are doing. Consider now a

[8] See http://www.ilfattoquotidiano.it/2010/11/11/conso-nel-93-non-rinnovai-il-41-bis-per-lucciardone-e-evitai-altre-stragi/76421/ (in Italian).

very high threshold, $\widehat{x}_i^0 \in (0,1)$ such that $\mathbb{E}\left[V(\cdot,1)\,|\widehat{x}_i^0\right] < 0$. Similarly, when citizen i observes any signal $x_i \geq \widehat{x}_i^0$, his best response is to set $r_i = 0$ regardless of what the other players are doing. Thresholds $\widehat{x}_{i0}$ and $\widehat{x}_i^0$ define two regions where each player i has a weakly dominant strategy.

Step 2. Starting from $\widehat{x}_{i0}$ (resp. $\widehat{x}_i^0$), let us define a slightly higher (resp. lower) threshold $\widehat{x}_{i1}$ (resp. $\widehat{x}_i^1$) so that $\mathbb{E}\left[V(\cdot,0)\,|\widehat{x}_{i1}\right] < 0$ (resp. $\mathbb{E}\left[V(\cdot,1)\,|\widehat{x}_i^1\right] < 0$) for any $x_i \leq \widehat{x}_{i1}$ (resp. $x_i \geq \widehat{x}_i^1$). This implies that, for any $x_i \leq \widehat{x}_{i1}$ (resp. $x_i \geq \widehat{x}_i^1$), each citizen i has a weakly dominant strategy by playing $r_i = 1$ (resp. $r_i = 0$).

Step 3. Repeating iteratively step 2, let us define two (bounded) series of monotone increasing (resp. decreasing) thresholds $\{\widehat{x}_{ik}\}_{k=0}^K$ (resp. $\left\{\widehat{x}_i^k\right\}_{k=0}^K$), with $k \in \mathbb{N}$. Because the series are both monotone and bounded, let $\widehat{x}_{i\infty} := \lim\limits_{K\to+\infty} \{\widehat{x}_{ik}\}_{k=0}^K$ (resp. $\widehat{x}^\infty := \lim\limits_{K\to+\infty} \left\{\widehat{x}_i^k\right\}_{k=0}^K$) be such that $\mathbb{E}\left[V(\cdot,0)\,|\widehat{x}_{i\infty}\right] \leq 0$ (resp. $\mathbb{E}\left[V(\cdot,1)\,|\widehat{x}_i^\infty\right] \leq 0$) . Citizens threshold strategies rewrite then as

$$r(x_i) = \begin{cases} 0 & \text{if} \quad x_i > \widehat{x}_i^\infty \\ 1 & \text{if} \quad x_i < \widehat{x}_{i\infty} \end{cases}$$

and because citizens are ex-ante symmetric, the threshold strategy reduces to

$$r(x_i) = \begin{cases} 0 & \text{if} \quad x_i > \widehat{x}^\infty \\ 1 & \text{if} \quad x_i < \widehat{x}_\infty \end{cases}$$

for every citizen i.

Step 4. We now need to show that $\widehat{x}_\infty = \widehat{x}^\infty = x^*$. This reduces to show that

$$\begin{cases} \mathbb{E}\left(V(\cdot,0)\,|\widehat{x}_\infty\right) = 0 \\ \mathbb{E}\left(V(\cdot,1)\,|\widehat{x}^\infty\right) = 0 \end{cases}$$

which is true if and only if citizen i is indifferent between $r_i = 0$ and $r_i = 1$. Because of the normality assumption, the probability that a citizen supports the criminal organization is continuous and strictly decreasing in x_i, implying that for each threshold candidate θ^*, there exists a unique threshold $x^*$for which a citizen is indifferent between the two actions. Therefore $\widehat{x}_\infty = \widehat{x}^\infty = x^*$.

Step 5. We last need to prove that the threshold θ^* exists and is unique. A criminal organization is not overthrown if $\theta \leq \Phi\left(\sqrt{p}\left(x^* - \theta\right)\right)$; because the probability on the right hand side is continuous and strictly increasing in θ, for any given value x^*, there is a unique value of θ such that $\theta^* = \Phi\left(\sqrt{p}\left(x^* - \theta^*\right)\right)$. *Q.E.D.*

Proof of Proposition 3.3. I proceed in two steps. Consider *Case 1.* When $\theta \geq \theta_a^*$, the utility of the criminal organization is decreasing in a because performing criminal actions only increase costs. This is because, for any $\theta \geq \theta_a^*$, $G\left(R\left(\theta - a\right)\right) - C\left(a\right) = 0$. Therefore it is optimal for the Mafia to set $a^*\left(\theta\right) = 0$.

Consider now *Case 2.* The optimal level of criminal activities $a^*\left(\theta\right)$ is characterized by the first order necessary condition that solves the criminal organization problem. Specifically, for any $\theta < \theta_a^*$,

$$G'\left(R\left(\theta - a\right)\right) \frac{\partial R\left(\theta - a\right)}{\partial a} = C'\left(a\right) \tag{46}$$

The second order condition is also satisfied since, for any $\theta < \theta_a^*$:

$$G''\left(R\left(\theta - a\right)\right)\left(\frac{\partial R\left(\theta - a\right)}{\partial a}\right)^2 + G'\left(R\left(\theta - a\right)\right)\frac{\partial^2 R\left(\theta - a\right)}{\partial a^2} - C''\left(.\right) < 0 \tag{47}$$

Conditions (46) and (47) implicitly define the optimal level of criminal activities that the criminal organization is willing to perform for a given value of θ. To show that the level of such criminal activities is increasing in θ, we just need to prove that the cross derivative of the objective function is positive. In other words, $a^*\left(\theta\right)$ is increasing in θ because

$$\frac{\partial}{\partial \theta}\left\{G\left(R\left(\theta - a\right)\right) - C\left(a\right)\right\} = G'\left(R\left(\theta - a\right)\right)\frac{\partial R\left(\theta - a\right)}{\partial \theta}$$

and

$$\frac{\partial^2}{\partial \theta \partial a}\left\{G\left(R\left(\theta - a\right)\right) - C\left(a\right)\right\} = -G''\left(R\left(\theta - a\right)\right)\left(\frac{\partial R\left(\theta - a\right)}{\partial \theta}\right)^2 - \frac{\partial^2 R\left(\theta - a\right)}{\partial \theta \partial a}G' > 0 \tag{48}$$

where $\frac{\partial^2 R(\theta - a)}{\partial \theta \partial a} < 0$ because of decreasing returns to criminal activities. *Q.E.D.*

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