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**Multiplanetary Systems in the Era
of the Roman and PLATO Missions**

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*“Exploration is in our nature.
We began as wanderers, and we are wanderers still.”
— Carl Sagan*

Abstract

The demographic study of multiplanetary systems provides a unique window into the processes of planet formation and evolution. Despite extensive observational efforts, our knowledge of these planetary systems remains incomplete and strongly biased toward close-in planets, leaving their overall architecture and diversity only partially explored. The upcoming *Nancy Grace Roman Space Telescope* and ESA's *PLATO* mission will improve this picture by extending the exploration to complementary regions of parameter space, ranging from cold, distant planets to temperate, Earth-sized worlds orbiting Sun-like stars. This Ph.D. thesis investigates how these two missions will advance the understanding of planetary system architectures. To characterize the multiple-lens microlensing events that will be detectable by *Roman*, a numerical code was developed to compute their light curves. The code employs optimized algorithms capable of accurately resolving triple-lens configurations with enhanced numerical stability and computational efficiency. Using this tool, large-scale simulations of triple-lens microlensing events, representing systems composed of a host star and two planetary companions, were carried out in the high-magnification regime. These simulations enabled a quantitative assessment of *Roman*'s sensitivity to multi-planet configurations and its capability to detect and characterize such systems under realistic observing conditions. In parallel, the analysis of the *PLATO* mission focuses on the long-term dynamical behavior of known multiplanetary systems discovered by *TESS* within the mission's future LOPS2 observing field. The MEGNO chaos indicator was employed to map their dynamical stability and to identify regions where additional Earth-mass planets could exist without destabilizing the configuration. This analysis demonstrates how *PLATO*'s extended time baseline and superior photometric precision will complement observations from *TESS*, enabling the search for smaller, longer-period, and potentially habitable planets in compact nearby systems.

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Introduction

In the autumn of 1995, the discovery of the first exoplanet in orbit around a star similar to the Sun marked the beginning of a new chapter in the history of astrophysics. In the course of just three decades, the science of exoplanets has passed from being relegated to theoretical speculation to a flourishing discipline, contributing to shape our understanding of planetary formation. For centuries astronomers and philosophers have asked themselves if distant stars could host planets, and yet this prospect remained beyond the reach of the instruments at disposal. The technical challenges were multiple; exoplanets emit weak light in comparison to their host star and are at extremely small angular separations. These difficulties have long fed the conviction that the detection of extrasolar planets was an insurmountable challenge. The real turning point came in 1992, with the announcement of the discovery of two planets orbiting the pulsar PSR B1257+12 (Wolszczan et al. 1992). This detection, coming not from a star similar to the Sun but from a stellar remnant, demonstrated our ability to reveal exoplanets. Few years later, the discovery of 51 Pegasi b (Mayor et al. 1995), a gas giant orbiting around a solar-type star, officially marked the beginning of the modern era of exoplanetary physics. The following decades have been witnesses of a great growth in the discoveries that brought from few isolated detections to a catalog in continuous expansion that today has surpassed the number of 6000 confirmed planets (Archive 2025)¹. This progress has been possible thanks to a combination of observational techniques and dedicated instruments always more refined. The first discoveries in the late 1990s and early 2000s were mainly due to ground-based observations. This progress was further accelerated by the advent of space-based observations, which marked a major leap forward in exoplanetary science. High-precision photometry from orbit enabled the detection of planetary transits with

¹<https://exoplanetarchive.ipac.caltech.edu/>

high photometric precision, transforming exoplanet research from a sequence of isolated discoveries into a systematic study of planetary populations. For the first time, it became possible to obtain reliable statistical information on the occurrence and distribution of planets.

Building on these results, a new generation of space missions is being developed to advance exoplanetary research and overcome previous limitations. Among them, the ESA mission *PLATO* (PLANetary Transits and Oscillations of stars) and NASA's *Nancy Grace Roman Space Telescope* will play a central role. *PLATO* will focus on detecting and characterizing Earth-sized planets in the habitable zones of nearby stars through the transit method, while *Roman* will employ gravitational microlensing to explore more distant and colder planetary systems, beyond the reach of transit surveys. Together, these missions will provide a more complete and complementary view of planetary architectures, paving the way toward a deeper understanding of planetary formation and evolution and bringing us closer to the detection of worlds similar to our own.

Equipped with a suite of small telescopes operating simultaneously, *PLATO* will monitor large areas of the sky to identify planetary transits and to measure stellar oscillations with unprecedented temporal coverage. Its long-duration observations will make it possible to detect planets on wider and colder orbits, including potentially habitable Earth-sized worlds. By combining transit detections with ground-based radial velocity measurements, *PLATO* will provide precise estimates of planetary parameters, enabling comparative studies across a wide range of systems.

While *PLATO* will focus on nearby bright stars, *Nancy Grace Roman Space Telescope* will explore a complementary region of the planetary parameter space. Through a dedicated microlensing survey, *Roman* will be sensitive to planetary systems located thousands of parsecs away, including cold planets on wide orbits and objects beyond the snow line. Simulations predict that *Roman* will discover about 1400 new planets, this population will substantially expand the current microlensing sample, which today includes only a few hundred detections limited by ground-based resolution and the lack of parallax constraints, providing a unique statistical view of planetary architectures in distant regions of the Galaxy.

From a handful of planets discovered in the 1990s, we have come to realize that our Galaxy is filled with new worlds (Borucki 2016). One result that has emerged from recent surveys is the abundance of multiplanetary systems, systems hosting more than one planet. Although the first discoveries tended to favor systems with only one giant planet, continuous and high-precision photometric observations have revealed numerous compact systems that contain planets of various sizes (Lissauer et al. 2011; Fabrycky et al. 2014). Overall, more than a thousand of the planets discovered so far are part of multiplanetary systems, and about 40% of the known planetary systems today contain

more than one confirmed planet. The discovery of multiplanetary systems represents a unique opportunity for comparative studies: planets formed within the same protoplanetary disk provide a natural laboratory to test models of planetary formation and evolution. Where once the Solar System was our only reference, the rapid expansion of exoplanetary discoveries has revealed a remarkable diversity of planetary architectures.

This wealth of data now enables us to move beyond individual detections, allowing the identification of recurrent patterns, correlations, and statistical anomalies that offer new insights into the processes of planetary formation and evolution. Understanding these trends is important to place our Solar System in context. In this framework, the new generation of space observatories will play a decisive role. The new instruments will make it possible to characterize multiplanet systems in detail. The purpose of this Ph.D thesis is to investigate how the new missions *Roman* and *PLATO* will contribute to advancing the science of multiplanetary systems.

The thesis is organized as follows. Chapter 1 presents the historical and theoretical context of exoplanetary science, describing the main detection techniques, and describing the current understanding of planetary formation and evolution. Chapter 2 introduces the upcoming *Roman* and *PLATO* missions, highlighting their scientific goals, technical specifications, and relevance for the study of planetary systems. Chapter 3 details the computational methods developed for the analysis of microlensing events, focusing on the three algorithms implemented in the `VBMicrolensing` code and their comparative performance. Chapter 4 presents a systematic simulation study of multiplanet microlensing events, quantifying the detection efficiency of the *Roman* mission and identifying the orbital configurations that maximize the probability of detecting multiple planets. Chapter 5 shifts the focus to transiting systems, analyzing selected multiplanet candidates from *TESS* data, and exploring how future *PLATO* observations could refine their characterization and reveal additional companions. Finally, Chapter 6 summarizes the main results, discusses their broader astrophysical implications, and outlines the perspectives opened by next-generation missions in the study of planetary systems.

1.1 Historic Context

The idea that stars might host other worlds has fascinated humanity for millennia. Ancient philosophers such as Epicurus and Democritus have already hypothesized the existence of a variety of worlds, some similar, others radically different from the Earth. However, this bold vision would have to wait more than two millennia before confirmation was found in direct observation.

The idea of planets beyond our Solar System remained speculation, resurfacing from

time to time in the works of astronomers, from William Herschel to Otto Struve, who in the 1950s advanced the hypothesis of the existence of planets orbiting other stars. Struve showed foresight in suggesting that giant planets with short periods could be identified through the spectroscopic analysis of stellar motion (Struve 1952).

Only in the last years of the twentieth century did technology finally provide astronomers with the instruments necessary to transform such conjectures into observations. Thanks to increasingly sensitive instruments and the development of new observational techniques, it became possible to recognize the presence of planets in an indirect way.

However, the modern history of the search for extrasolar worlds does not begin with a system similar to ours, nor with a star similar to our Sun, but with a pulsar. In 1992 Aleksander Wolszczan and Dale Frail announced that they had identified two planets around PSR B1257+12 (Wolszczan et al. 1992). By analyzing the variations in the period of the radio pulses of the pulsar, they succeeded in identifying the gravitational signatures of the planets orbiting around it, thus revealing the first confirmed multiplanetary system outside the Solar System. It was the beginning of a new branch of astrophysics.

It was necessary to wait another three years for the first exoplanet orbiting a main-sequence star. In 1995 Michel Mayor and Didier Queloz revealed the existence of 51 Pegasi b (Mayor et al. 1995). The discovery was realized using the radial velocity technique, measuring the oscillation of the stellar spectrum due to the motion of the star around the gravitational center.

The discovered planet itself was a surprise, as it presented characteristics never observed in our Solar System: a gas giant in a very close orbit to its star, with a consequent period of only four days. This discovery inaugurated the category of the “Hot Jupiters.”

At the same time, while the radial velocity technique consolidated its effectiveness, other methods, in some cases already conceived in previous decades, began to find fertile ground for development and application. In 1971, Joseph Rosenblatt presented the first rigorous study of the transit method (Rosenblatt 1971), establishing the theoretical bases for what would become one of the most prolific techniques in the discovery of exoplanets. In 1999, David Charbonneau and his collaborators succeeded in observing for the first time the transit of an exoplanet: the star HD 209458 showed a decrease of about 1.7% in luminosity, caused by the passage of a gas giant in front of its disk (Charbonneau et al. 2000). The transit method opened the door to the possibility of measuring the radius of the planet and, combined with radial velocity information, to determining its density.

Another method that awaited observational confirmation was gravitational microlensing, a technique based on general relativity. In 1986 Bohdan Paczyński proposed that compact and dark objects of the Galaxy, the so-called MACHOs (Massive Astrophysical

Compact Halo Objects), could be identified by observing temporary increases in brightness of background stars produced by the gravitational deflection of their light (Paczynski 1986). A few years later, Mao & Paczyński (1991) showed that the same phenomenon could be exploited to reveal binary systems, thanks to small deviations in the light curves, while subsequent studies extended the idea to the search for planets, predicting that a planet orbiting the lens star would produce an additional peak of brightness. (Mao et al. 1991; Gould et al. 1992).

On these premises, in the 1990s the first large dedicated observational projects were born. Among them OGLE in Chile (Optical Gravitational Lensing Experiment), launched in 1992 by Andrzej Udalski, and the MOA collaboration (Microlensing Observations in Astrophysics), operating between New Zealand and Japan. In 1993 OGLE identified the first events in the Galactic center, confirming Paczyński’s vision.

The first planet identified with this method was announced in 2004 by OGLE and MOA, the event OGLE-2003-BLG-235/MOA-2003-BLG-53 (Bond et al. 2004). The model revealed a star of about $0.3 M_{\odot}$ accompanied by a planet of Jovian mass at a distance of a few astronomical units. In the following years, microlensing revealed planets very different from those discovered with other methods: often bodies of mass comparable to or lower than that of Neptune, at orbital distances of several AU, unveiling regions of the planetary parameter space until then unexplored, in particular, the population of planets located beyond the snow line.

In 2008 came the first discovery of a planetary system with the microlensing method. Gaudi et al. announced that the star OGLE-2006-BLG-109, of about $0.5 M_{\odot}$, hosted two planets in an architecture surprisingly similar to that of Jupiter and Saturn, with masses of about 0.7 and $0.3 M_J$ and orbits of a few AU (Gaudi et al. 2008; Bennett et al. 2010). This discovery showed that planetary systems with cold gas giants in resonant configurations were not peculiarities of the Solar System.

While microlensing was showing its potential to discover distant and cold exoplanets, space photometry was taking its first steps. In 2006 the satellite *CoRoT* (ESA–CNES) inaugurated the era of missions dedicated to transits, revealing dozens of planets, among them the first confirmed super-Earth, CoRoT-7b, in 2009 (Léger et al. 2009). But real innovation came that same year with the launch of the *Kepler* telescope, a NASA mission. By monitoring more than 150,000 stars with unprecedented photometric precision, Kepler revealed thousands of transits, confirming more than two thousand planets and identifying numerous multiple systems, many of them extraordinarily compact. One case is Kepler-11, with six planets enclosed within 0.5 AU from the star (Lissauer et al. 2011).

For the first time, a solid statistical basis was obtained: not only planets turned out to be common in the Galaxy, but it was discovered that short-period super-Earths were by far more numerous than hot Jupiters. The analysis of orbital resonances, of

mass distributions and of spatial configurations revealed imprints of the mechanisms of formation, migration, and gravitational interaction, allowing refinement of theories of planet formation.

In the 2010s new ground-based initiatives appeared, such as the networks of robotic telescopes SuperWASP and HATNet, and new space missions. The satellite *TESS* (NASA, 2018) began to map the entire sky in search of planets in transit around bright and nearby stars, optimal objects for follow-up studies (Ricker et al. 2015). *CHEOPS* (ESA, 2019) was designed for high-precision photometric follow-up, allowing the detailed characterization of already known systems and the search for additional planets in multiplanetary architectures (Benz et al. 2021).

Today, the gaze is turned to the near future. The *PLATO* mission (ESA), whose launch is forecasted to occur around 2026, will begin a new phase of research, with the objective of identifying Earth-like planets in the habitable zones of their respective stars, carrying forward the legacy of Kepler (Rauer et al. 2014). *PLATO* will be able to count on unprecedented sensitivity and long observation times, allowing the discovery of planets with orbital periods of years, similar to our Earth.

In parallel, the *Nancy Grace Roman Space Telescope* (NASA, launch foreseen in 2027) will represent a major leap in the capabilities of the microlensing method. Through a dedicated survey in the Galactic bulge, Roman will observe thousands of microlensing events with high photometric precision and cadence, enabling not only the detection of single planets and free-floating objects but also the discovery of multiplanetary systems. Simulations predict that Roman will identify dozens of multiple systems (Chapter4).

The combination of the discoveries of these two missions will allow us to obtain a more complete census of multiplanetary systems and to achieve a deeper understanding of the processes that govern the formation and evolution of planetary systems.

1.2 Detection methods

In the contemporary astronomical panorama, the discovery of exoplanets takes place through a variety of observational techniques. This methodological diversity is necessary in order to explore the full planetary parameter space. The majority of the available techniques are indirect methods: they infer the presence of a planet from the perturbations induced on the central star or on the properties of the radiation crossing the system.

Each of these methods provides a different window in the parameter space. No single method can cover the entire spectrum of planetary configurations; it is through their synergy that we can aspire to a more complete understanding of planetary demographics.

Until now, the methods that have most affected exoplanet detection have been *radial*

velocity method and *transit method*. In the following paragraphs, we describe the principal detection methods, with particular attention to the *microlensing* and *transit* techniques, which constitute the basis of this doctoral thesis.

1.2.1 Transit Method

The photometric transit method is based on the observation of a periodic decrease in the brightness of the target star. When a planet passes in front of the disk of its star, from the observer's point of view, a small fraction of the stellar light is blocked. The decrease in the observed flux is proportional to the area of the planet relative to the area of the stellar disk.

Among the limits of this method we find the main one of geometric nature. In order for a transit to be observable, the planet's orbital plane must be nearly edge-on relative to the observer. The probability that this condition is satisfied corresponds to the fraction of the celestial sphere covered by the planet's cone of shadow. The apparent separation between the center of the planet and that of the star, at the midpoint of the transit, is expressed by the impact parameter b :

$$b = \frac{a \cos i}{R_*}. \quad (1.1)$$

The relation is valid in the case of the circular orbit approximation, where a is the semi-major axis of the orbit. The geometry is illustrated in Figure 1.1.

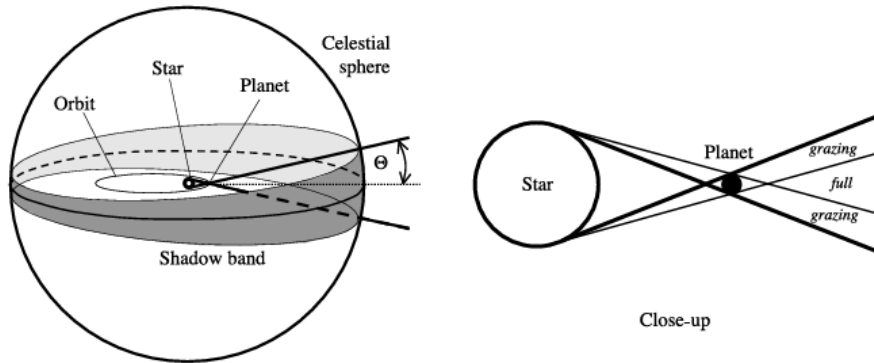


Figure 1.1: Illustration of the geometric configuration determining whether a planetary transit is observable. The left panel shows the star–planet system and the corresponding shadow band projected onto the celestial sphere. The shaded region represents the locus of all possible observer directions for which the planet passes in front of the stellar disk. The opening angle Θ defines the half-angle of the planet's shadow cone. The right panel provides a close-up view of the alignment geometry: depending on the inclination of the orbital plane, an observer may see a full transit or a grazing one. Figure adapted from (Winn 2010)

The probability that a transit occurs is:

$$P \left(|b| < 1 + \frac{R_p}{R_s} \right) = \frac{R_s + R_p}{a}. \quad (1.2)$$

For stellar radii much larger than the planetary ones ($R_s \gg R_p$), this expression reduces to

$$P \simeq 0.5\% \left(\frac{R_s}{R_\odot} \right) \left(\frac{a}{1 \text{ AU}} \right)^{-1}. \quad (1.3)$$

highlighting how transits are much more probable for planets on short orbits (small values of a) and for stars of large radius. For example, a planet like Earth, at 1 AU from the Sun, has a transit probability of just 0.46%. For Jupiter, at 5.2 AU, the probability drops to about 0.09%. In contrast, a 'hot Jupiter' with an orbital period of a few days, located at ~ 0.05 AU from a Sun-like star, has geometric transit probabilities around 10% (Bozza et al. 2016).

An ideal transit (zero impact parameter, absence of limb darkening) produces a trapezoidal-shaped curve, with a flat bottom phase at maximum dimming and symmetric ingress and egress phases. In the light curve four main phases can be distinguished:

- **T1 – First contact:** the moment when the edge of the planet first touches the edge of the star. From this moment on, the observed brightness begins to decrease.
- **T2 – Second contact:** the planet is fully inside the stellar disk. Here begins the 'full' transit phase, where the decrease in brightness stabilizes.
- **T3 – Third contact:** the leading edge of the planet reaches again the opposite edge of the star, marking the start of the egress phase. The brightness begins to rise.
- **T4 – Fourth contact:** the planet leaves the stellar disk completely and the brightness returns to its initial level.

The geometry of these phases and their effect on the light curve are illustrated in Fig. 1.2, which shows the four contacts, the transit depth δ , the total duration T , and the ingress/egress duration τ as defined in the literature.

Assuming no limb darkening effect, the effect by which the star shows a darkening at the edge making the edges of the disk appear less luminous than the center, the transit depth for a planet of radius R_p transiting a star of radius R_* is overestimated and is given by:

$$\delta = \frac{\Delta f}{f} \simeq \left(\frac{R_p}{R_*} \right)^2 \quad (1.4)$$

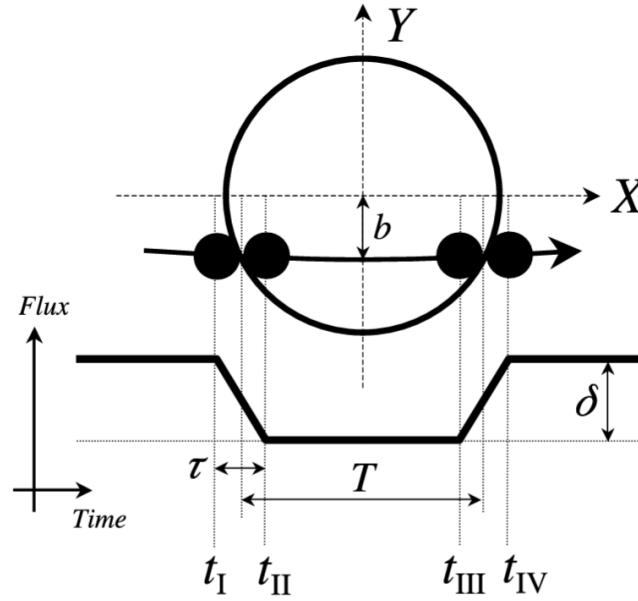


Figure 1.2: Schematic representation of a planetary transit. The upper panel shows the projected orbital geometry, with the impact parameter b indicated. The lower panel shows the idealized light curve with the four contact points (t_I – t_{IV}), the transit depth δ , the total duration T , and the ingress/egress duration τ . Figure adapted from Winn (2010).

A planet with $R_p \approx 0.1R_\odot$ in front of a Sun-like star ($R_* = 1R_\odot$) produces $\delta \sim 1\%$, while an Earth-sized planet ($R_p \approx 0.009R_\odot$) would give $\delta \sim 8 \times 10^{-5}$ (0.008%). The required photometric sensitivity is very high: *Kepler* reached a precision of about 20 ppm (parts per million) over 6 hours of integration for stars with apparent magnitude $V \approx 12$, sufficient to detect transits of Earth-sized planets.

In a real case, the stars do not have a disk with constant brightness. The apparent brightness decreases from the center toward the limb (edge), a phenomenon known as limb darkening.

The intensity profile across the stellar disk is commonly expressed as a function of the angle θ between the line of sight and the stellar radius vector, or equivalently in terms of $\mu = \cos \theta$, where $\mu = 1$ corresponds to the center of the disk and $\mu = 0$ to the stellar limb. For a star with an intensity at the center I_0 , the intensity corresponding to the position of the center of the planetary disk will be given by:

$$\frac{I}{I_0} = 1 - u(1 - \mu), \quad (1.5)$$

where u is the limb darkening coefficient, which is determined by the physical properties of the host star (effective temperature, metallicity, surface gravity) and in the chosen photometric band.

In the case where the planetary disk is completely dark and $R_p \ll R$ the ratio of the

flux is:

$$\frac{\Delta f}{f} = \frac{3(1 - u + u\sqrt{1 - b^2})}{3 - u} \left(\frac{R_p}{R_*} \right)^2. \quad (1.6)$$

The limb darkening considered here corresponds to the linear case, which represents the simplest approximation. More realistic descriptions include quadratic laws or formulations with four non-linear coefficients:

$$\frac{I(\mu)}{I_0} = 1 - \sum_{n=1}^2 u_n (1 - \mu^n) \quad (1.7)$$

$$\frac{I(\mu)}{I_0} = 1 - \sum_{n=1}^4 u_n (1 - \mu^{n/2}). \quad (1.8)$$

In terms of observational consequences, limb darkening alters the shape of the transit light curve.

If the orbit is inclined, with an impact parameter $b > 0$, the duration of the transit decreases, and the curve assumes a more trapezoidal shape with a shorter central phase. Referring to T as the total duration of the transit, one can derive b and the ratio a/R_* (orbital radius in units of the stellar radius).

For a circular orbit approximation:

$$T \approx \frac{P}{\pi} \arcsin \left(\frac{\sqrt{(R_* + R_p)^2 - (bR_*)^2}}{a} \right) \quad (1.9)$$

where P is the orbital period and a the orbital distance. From Kepler's third law,

$$a^3 \approx \frac{GM_* P^2}{4\pi^2} \quad (1.10)$$

we can obtain a given P and M_* (derived, for example, from stellar spectroscopy). Thus, transit observations directly provide R_p (from δ) and a (from P), and constraints on inclination i through Eq. 1.1.

If $b < 1 + R_p/R_*$ we have a transit and from the exact value of b (deduced from the ratio of total duration to partial duration τ) one derives i . Therefore, a single well-measured complete transit reveals the planetary radius, period, and inclination (and even eccentricity if the ingress/egress durations differ from those expected for a circular orbit; in practice, eccentricity is better constrained by combining with radial velocity data or observing the difference between the transit duration and that of a possible secondary eclipse).

Another observable feature is the secondary eclipse: if the planet is sufficiently luminous, when it passes behind the star, an increase in the combined flux is observed when the planet reappears. By measuring the depth of the secondary eclipse, it is possible to derive the planet's temperature.

The transit method is strongly sensitive to large planets in short orbits around small stars. In fact, from the δ equation (Eq. 1.4) we see that for the same planetary radius, it is easier to detect transits on stars with smaller radii. Moreover, if the planet is at a short distance, it will transit more often, offering more opportunities to observe it. For this reason, hot Jupiters are the easiest planets to detect with the transit method and were the targets of the earliest transit search projects.

The transit method, although conceptually simple, is not without challenges. Cases of false positives, that is, transit-like signals that are not due to a planet, are not uncommon. The analyses of Kepler data (Santerne et al. 2013) estimate an average incidence of about 10% false positives; the problem is more severe for hot Jupiter candidates.

The main astrophysical situations that can produce false positives in transit are 4 (Brown 2003):

- **Grazing binaries:** If the target star is part of a binary system, the eclipse of the companion can produce a signal similar to a planetary transit. This is particularly true in the case of partial (grazing) eclipses or when the companion is much smaller and fainter. The main difference is in the shape of the light curve, which takes on a V-shaped shape, losing the classical trapezoidal shape of the transit. A way to have confirmation that this is the physical case is to do follow-up photometry in multiple passbands; if the two objects have different temperatures, the eclipse will be color dependent. A typical example that can be confused with the transit of a giant planet is the case of a brown dwarf periodically eclipsing the primary star.
- **Substellar companion:** A typical example that can be confused with the transit of a giant planet is the case of a low-mass stellar companion or a brown dwarf periodically eclipsing the primary star. Since their radius is similar to that of a giant planet, the eclipses produced can have the same shape as planetary transits. However, if the object emits its own light it can produce a secondary eclipse deeper than that expected from a planet whose light is reflected. Such massive bodies exert strong tidal interactions, deforming it and causing periodic modulations in the brightness.
- **Blended eclipsing binaries (Blended EB):** this occurs when the light curve is contaminated by the eclipses of another binary star in the background that falls within the photometric aperture of the target star. In this case, the light from the

contaminating binary is diluted by the light from the target star, and a stellar eclipse could be mistaken for a planetary transit. The analysis of the centroid of the light during the transit can help to reveal small shifts that indicate the presence of a background object. Batalha et al. (2010) found in the Kepler data that the false positives of this type had the characteristic that the centroid moved toward the brightest and isolated star when the binary eclipse occurred.

- **Hierarchical triple system:** Case similar to the previous one, with the difference that the eclipsing binary system is not in perspective alignment but is part of the same system. The combined light entails a dilution of the signal that can mimic planetary transits. Since the stars of the system are very close, the angular separation is of the order of micro as is difficult to resolve even with instruments of high astrometric precision, such as the ESA Gaia mission. This makes such systems difficult to distinguish from true planetary transits.

In the case of multiplanetary systems, we have the advantage that the false-positive rate decreases drastically for systems with multiple transits (Lissauer et al. 2014); it is indeed very unlikely that two or more transit signals on the same star are due to background eclipsing binaries or other astrophysical phenomena that could mimic a planetary transit curve. This has allowed many planetary systems to be statistically validated without requiring further spectroscopic confirmation, enriching the catalog of planets discovered to date.

1.2.2 Gravitational Microlensing

The microlensing method is based on the effects predicted by the Theory of General Relativity. It occurs when light coming from a background object (the source) is deflected by a massive foreground object (the lens) with respect to the observer's line of sight.

In the weak-field approximation, the deflection angle produced by a lens of mass M at an impact parameter r_0 is given by (Bozza et al. 2016):

$$\alpha = \frac{4GM}{r_0 c^2}, \quad (1.11)$$

where G is the gravitational constant and c is the speed of light. This bending of light causes a temporary increase in the apparent brightness of the source.

In the particular case in which the observer, lens, and source are perfectly aligned, the light is deflected, forming an annular image known as the Einstein Ring, whose angular radius is given by the Einstein angle (θ_E) and corresponds to the angular scale of microlensing:

$$\theta_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}}{D_L D_S}} = \sqrt{kM\pi_{rel}} \quad (1.12)$$

where D_L and D_S are the distances of the lens and the source from the observer, and $D_{LS} = D_S - D_L$ (Gaudi 2012). $\pi_{rel} = \frac{AU D_{LS}}{D_L D_S}$, is the relative lens–source parallax. Typically, for a lens of $0.5M_\odot$ halfway to a source in the galactic bulge ($D_L \sim 4$ kpc, $D_S \sim 8$ kpc), θ_E is of the order of a few hundred microarcseconds (~ 0.7 mas) (Rahvar 2015).

The main problem of microlensing is its observational difficulty. Observing a microlensing event requires precise alignment between lens source and observer. It is assumed that an event is observable when the impact parameter u_0 is smaller than the Einstein angle. The probability of observing a microlensing event is quantified by the *optical depth* τ , which represents the probability that, at any given time, a background source is being magnified by a foreground lens. Considering a lens of mass M , the optical depth is obtained by integrating the contribution of all potential lenses along the line of sight, each weighted by the area of its Einstein ring:

$$\tau = \int_0^{D_S} \frac{4\pi G}{c^2} \rho(D_L) \frac{D_L D_{LS}}{D_S} dD_L, \quad (1.15)$$

where $\rho(D_L)$ is the mass density of the lenses at distance D_L , G is the gravitational constant, and c the speed of light. For typical lines of sight toward the Galactic bulge, τ is of the order of 10^{-6} , while for directions toward the Large Magellanic Cloud it decreases to about 5×10^{-7} .

A related quantity is the microlensing event rate Γ , which describes how frequently microlensing events occur per source star. Assuming that an event takes place when the source crosses a solid angle of width $2\theta_E$ during a time interval dt , the differential event rate can be written as

$$\Gamma dt = \frac{2 dt}{\pi t_E} \int_0^{D_S} n(D_L) D_L^2 \pi \theta_E^2 dD_L = \frac{2\tau}{\pi t_E}, \quad (1.16)$$

where $n(D_L)$ is the number density of lenses and t_E is the Einstein timescale, corresponding to the time required for the source to traverse the Einstein radius. For typical bulge events, with $t_E \approx 20$ – 30 days, this yields an average rate of about $\Gamma \approx \tau / (0.1 \text{ yr})^{-1}$. The short duration and low frequency of microlensing events make observations particularly efficient in regions of high stellar density, such as the Galactic bulge.

Point Lens Microlensing

Knowing the deflection angle, we can write the equations for the simplest case, the case in which the lens is a single point-like object. The geometry of this model is illustrated in Fig. 1.3.

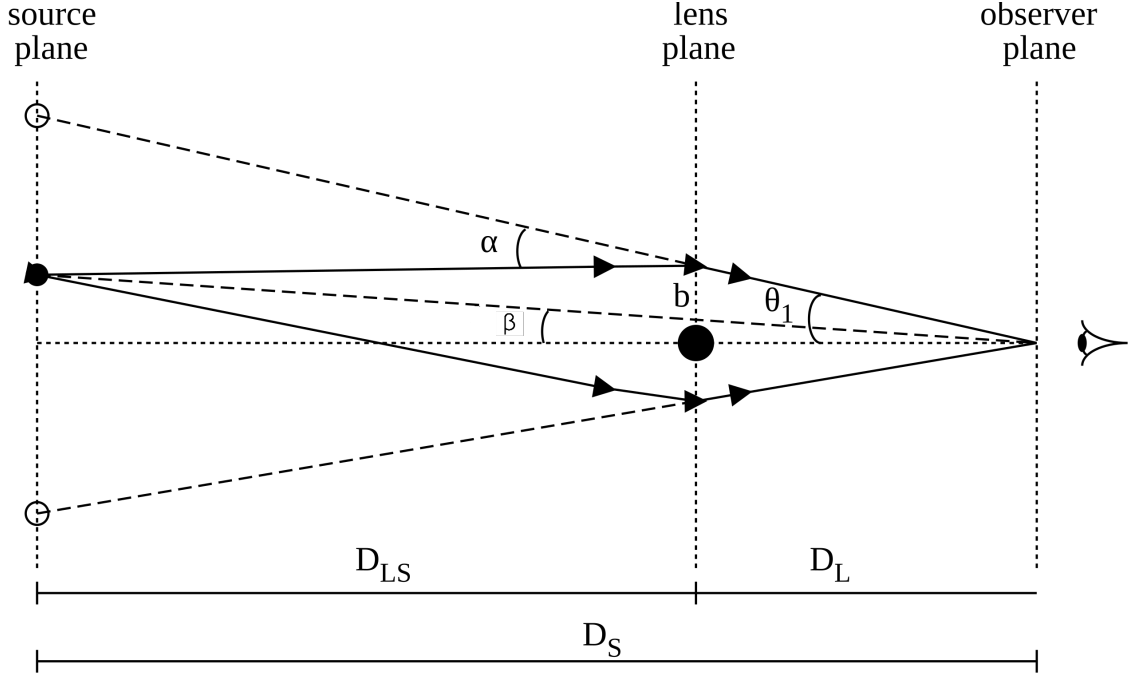


Figure 1.3: Schematic representation of the geometry of gravitational microlensing in the case of a point-mass lens. Image adapted from (Mollerach et al. 2002)

The source S emits light that is deflected by the mass of the lens L , producing apparent images at positions θ_I , different by θ_S , with respect to the line of sight. The fundamental geometric parameters are β that is the angular position of the source in the absence of the lens, $\hat{\alpha}$ is the deflection angle and D_L , D_S , and D_{LS} represent, respectively, the distances of the observer-lens, observer-source, and lens-source distances. This scheme provides the basis for deriving the lens equation.

In the small-angle approximation, justified by the very large distances involved, we can proceed to write the following lens equation.

$$\theta D_{OS} = \beta D_{OS} + \hat{\alpha} D_{LS} \quad (1.13)$$

Defining the parameter α as

$$\alpha = \hat{\alpha} \frac{D_{LS}}{D_{OS}} = \frac{D_{LS}}{D_{OS} D_{OL}} \frac{4GM}{c^2 \theta} \quad (1.14)$$

we obtain the following lens equation:

$$\beta = \theta - \alpha(\theta) \quad (1.15)$$

Substituting with the definition of the Einstein angle:

$$\beta = \theta - \frac{\theta_E^2}{\theta} \rightarrow \theta^2 - \beta\theta - \theta_E^2 = 0 \quad (1.16)$$

For a generic source position, the equation has two distinct solutions, given by (Paczynski 1986):

$$\theta_{\pm} = \frac{\beta \pm \sqrt{\beta^2 + 4\theta_E^2}}{2} \quad (1.17)$$

Among the two solutions, the positive image will be located on the same side as the source, outside the Einstein ring, while the negative image will be located on the opposite side of the source, inside the Einstein angle. We also note that the angular separation between the two images is greater than or equal to twice the Einstein angle $\Delta\theta = \theta_+ - \theta_- = \sqrt{\beta^2 + 4\theta_E^2} \geq 2\theta_E$

Applying a normalization with respect to the Einstein angle, we can rewrite everything in the following way:

$$u_{\pm} = \frac{u \pm \sqrt{u^2 + 4}}{2}; \quad u \equiv \frac{\beta}{\theta_E}, \quad u_{\pm} \equiv \frac{\theta_{\pm}}{\theta_E}. \quad (1.18)$$

Magnification & Paczyński curve

The surface brightness is conserved, which implies that the magnification is given by the ratio of the image area to that of the source. The observed flux increases by a factor A compared to that of the source in the absence of the lens. This factor is given by the ratio between the area of the image in the presence of the lens and that in its absence (Tsapras 2018):

$$A_{\pm} = \left| \frac{\theta_{\pm}}{\beta} \frac{d\theta_{\pm}}{d\beta} \right|. \quad (1.19)$$

Using the values of $\theta_{\pm}$ obtained previously, we get:

$$A_+ = \frac{\theta_+}{2\beta} \left(\frac{\beta}{\sqrt{\beta^2 + 4\theta_E^2}} + 1 \right), \quad A_- = \frac{\theta_-}{2\beta} \left(\frac{\beta}{\sqrt{\beta^2 + 4\theta_E^2}} - 1 \right). \quad (1.20)$$

The total magnification, the sum of the two components, will therefore be given by:

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \quad (1.21)$$

If we consider the lens and the source in general positions, the equation 1.15 becomes vectorial:

$$\vec{\beta} = \vec{\theta} - \vec{\alpha} \quad (1.22)$$

and we notice how the equation can be interpreted as a two-dimensional mapping from the lens plane (image positions) to the source plane.

By introducing the Jacobian, we can then write the magnification in terms of the Jacobian determinant:

$$A = J^{-1} = \left(\det \frac{\partial \beta}{\partial \theta} \right)^{-1} \quad (1.23)$$

For an ideal microlensing event (point–point), the following assumptions are therefore made:

- point-like lens and point-like source;
- rectilinear and uniform relative motion between lens and source;
- absence of contributions to the flux other than the microlensed source (no blending).

Under these conditions, the key variable is $u(t)$, which evolves over time due to relative motion. If the closest approach occurs at time t_0 with minimum impact u_0 , and the transverse velocity is $v_\perp$, we generally have the following.

$$u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{t_E} \right)^2}, \quad (1.24)$$

where $t_E = \frac{r_E}{v_\perp}$ is the so-called *Einstein time*, i.e. the characteristic duration of the microlensing event. The Einstein radius r_E represents the physical size of the Einstein ring in the lens plane, and is defined as $r_E = D_L \theta_E$.

Consequently, the magnification as a function of time is

$$A(t) = \frac{u(t)^2 + 2}{u(t)\sqrt{u(t)^2 + 4}}, \quad (1.25)$$

Indicating with F_* the flux of the star in the absence of the lens, with F_B the flux coming from the background, the flux as a function of time is given by:

$$F(t) = F_* A(t) + F_B = F_* \frac{u(t)^2 + 2}{u(t) \sqrt{u(t)^2 + 4}} + F_B \quad (1.26)$$

This functional form defines the characteristic *Paczynski curve*, which describes the symmetric and achromatic brightening of a microlensing event. As shown in Figure 1.4, the photometric evolution (panel b) directly reflects the underlying lens geometry (panel a): the closer the source trajectory passes to the lens center (smaller u_0), the higher and narrower the peak of the light curve becomes. The shape of the curve depends on five parameters (F_* , F_B , u_0 , t_E , t_0), but only t_E carries physical information about the lens–source system.

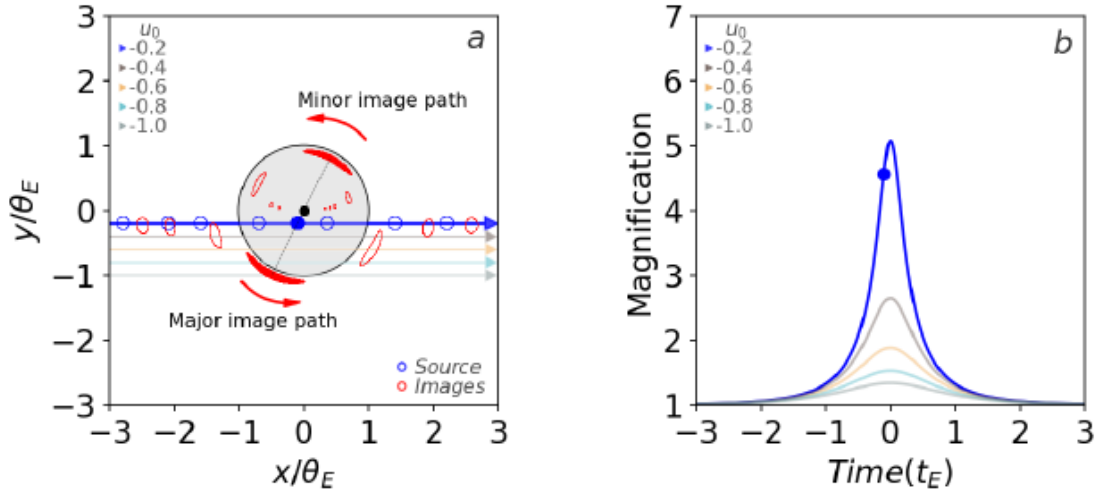


Figure 1.4: Geometry and light curve of a single-lens microlensing event. **(a)** The diagram shows the motion of the source (blue circles) relative to the lens (black dot) in the plane of the sky, expressed in units of the Einstein radius θ_E . As the source approaches the line of sight to the lens, two images are formed: the *major image*, located outside the Einstein ring, and the *minor image*, inside it (red contours). Their apparent paths are indicated by the red arrows. **(b)** The total magnification as a function of time, in units of the Einstein timescale t_E , for different source–lens impact parameters u_0 . Smaller values of u_0 correspond to trajectories passing closer to the lens and thus to higher peak magnifications. The blue point marks the instant shown in panel (a), highlighting the correspondence between geometry and photometric signal. Figure adapted from Tsapras (2018).

Finite source effect

So far we have treated the source as point-like; however, this was an approximation that has limits of validity. If the impact parameter is comparable to the radius of the source, then the point-source approximation is no longer valid. The parameter used to define the size of the source is:

$$\rho_* = \frac{\theta_*}{\theta_E} \quad (1.27)$$

where, given the physical radius of the star R_* , $\theta_* = R_*/D_S$ is the angular radius.

To obtain the magnification in the case of an extended source one must integrate over the area of the source, obtaining:

$$A(y) = \frac{1}{F_*} \int_0^{2\pi} d\phi \int_0^{\rho_*} d\rho \rho I(\rho \hat{R}_E) \mu\left(\sqrt{y^2 + \rho^2 - 2y\rho \cos \phi}\right) \quad (1.28)$$

where $\hat{R}_E = \sqrt{\frac{4GM}{c^2} \frac{D_{LS}D_{OS}}{D_{OL}}}$, $I(r)$ is the surface brightness of the star, μ is the magnification of a point source and $F_* = \int_0^{2\pi} d\phi \int_0^{\rho_*} d\rho \rho I(\rho \hat{R}_E)$ is the flux of the source.

As illustrated in Figure 1.5, the inclusion of the finite-source effect (blue curve) leads to a rounding of the light-curve peak compared to the point-source case (red dashed curve).

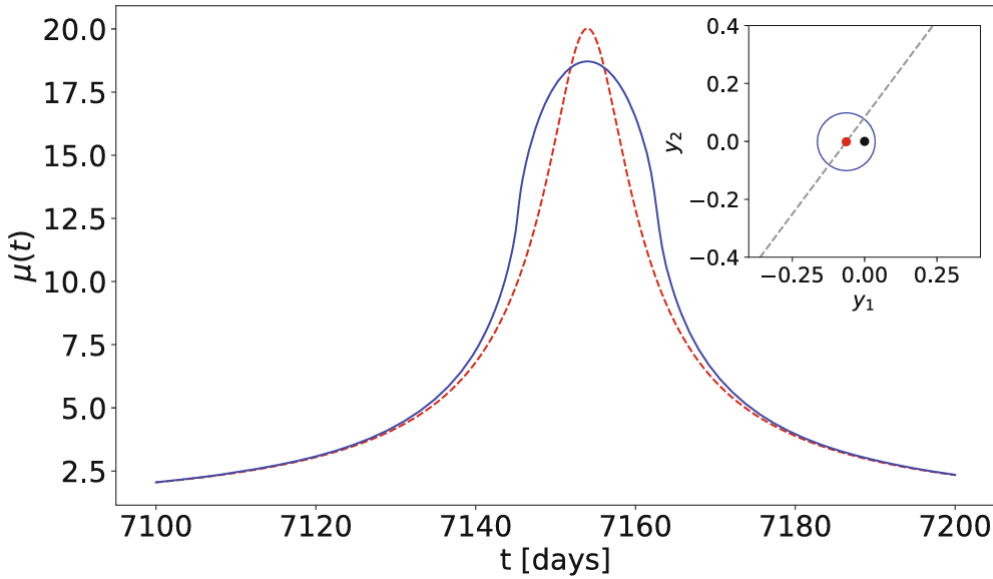


Figure 1.5: Comparison between light curves computed with (solid blue line) and without (red dashed line) the finite-source effect for $\rho_* = 0.1$. The inset shows the lens (black point), the source (red circle), and the trajectory of its center (dashed line). *Image adapted from (Meneghetti 2021).*

Multiple lens

In the case where we consider a system with multiple lenses, we can generalize the lens equation seen for the single point-lens case to a system containing N masses. The

N -body lens equation is:

$$\vec{u} = \vec{y} - \sum_{i=1}^N \frac{\epsilon_i (\vec{y} - \vec{y}_{m,i})}{|\vec{y} - \vec{y}_{m,i}|^2}. \quad (1.29)$$

where $\vec{u} = \vec{\beta}/\theta_E$ is the angular position of the source (in units of Einstein radius θ_E), and $\vec{y} = \vec{\theta}/\theta_E$ is the image position in the lens plane. The terms $\vec{y}_{m,i}$ represent the positions of the individual lensing objects i , and $\epsilon_i = m_i/M_{\text{tot}}$ is the relative mass fraction of each lens compared to the total mass M_{tot} of the lens system.

The next step is to rewrite using the complex notation, introduced by (Witt 1990):

$$\left\{ \begin{array}{l} \vec{u} \rightarrow \zeta = u_1 + iu_2, \\ \vec{y} \rightarrow z = y_1 + iy_2 \end{array} \right\} \implies \zeta = z - \sum_i \frac{\epsilon_i}{\bar{z} - \bar{z}_{m,i}}. \quad (1.30)$$

where z is the complex variable in the lens plane and $\bar{z}$ its complex conjugate.

This equation, if rewritten in complex form, using the complex conjugate of equation 1.30, became a polynomial whose order grows quadratically with the number of lenses, for N lenses the polynomial is of degree $N^2 + 1$:

$$\begin{aligned} & (\zeta - z) \prod_i (\zeta - \bar{a}_i) \prod_j (z - a_j) + \sum_j \left[m_j \prod_{k \neq j} (z - a_k) \right] + \\ & + \left[\prod_j (z - a_j) \right] \sum_i \left[m_i \left(\prod_{p \neq i} (\zeta - \bar{a}_p) \prod_j (z - a_j) + \sum_j \left[m_j \prod_{k \neq j} (z - a_k) \right] \right) \right] = 0 \end{aligned} \quad (1.31)$$

This implies that, in general, a system of N point lenses can produce up to $N^2 + 1$ images of the source. For example, for $N = 1$ (single lens) we have at most 2 images, for $N = 2$ (binary lens) up to 5 images, while for $N = 3$ (triple lens) the maximum number of images rises to 10. However, it should be noted that not all the polynomial solutions correspond to real physical images – some roots turn out to be spurious depending on the source position, the degree of the polynomial only provides an upper limit to the number of images. Many roots can represent images that are not really observable. The general limits for the number of images that a multi-lens system can produce are:

- The minimum number of images is $N + 1$;
- The maximum number of images is $5N - 5$.

The lower limit derives from the theorem on the number of images. When the source is very far from the lens system, each mass acts as an isolated pole on the map, and around

each lens a 'local' image with negative parity is formed. To these images is added a main image, distant with positive parity (Rhie 2001).

The upper limit, on the other hand, was proposed and demonstrated in two phases. The first formulation dates back to Rhie (2001); Rhie (2003), who explicitly constructed symmetric configurations of lenses capable of producing the maximum number of images and formulated the conjecture that a system of N masses cannot generate more than $5N - 5$ physical images. Subsequently, Khavinson et al. (2004) provided a rigorous mathematical proof of this limit, who reformulated the lens equation as a problem of complex analysis, studying the zeros of harmonic functions. They showed that such functions cannot have more than $5N - 5$ distinct zeros. This result confirmed Rhie's conjecture, establishing the Rhie–Khavinson–Neumann theorem.

Caustics and Critical Curves

In the multiple-lens formalism, we refer to caustics as the loci of points in the source plane where the magnification diverges in the point-source approximation. The corresponding curves in the lens plane are called critical curves; they correspond to points where the determinant of the Jacobian matrix of the lens equation vanishes, $\det J = 0$, which in complex form is given by:

$$\det J = 1 - \frac{\partial \zeta}{\partial \bar{z}} \frac{\partial \bar{\zeta}}{\partial z} \quad (1.32)$$

In the case of multiple lenses, the locus of points characterized by a null Jacobian, the critical curves, is given by:

$$\sum_{i=1}^N \frac{\epsilon_i}{(\bar{z} - \bar{z}_{m,i})^2} = 1, \quad (1.33)$$

The solutions of this equation, when mapped back into the source plane, yield the caustics. Each caustic is composed of continuous lines (called folds) that meet at angular points called cusps. When the source crosses a fold caustic, two images are created or annihilated (image pair creation/annihilation phenomenon), causing sudden variations in magnification.

In particular, the entrance of a point source into a caustic would produce a divergence in magnification; in reality, the stellar source has a finite radius, so the magnitude peak is smoothed to a finite value, which depends on the stellar diameter and its limb darkening. Critical curves and caustics for multiple lenses exhibit complex geometries: for instance, in the binary case, there are typically multiple caustic components, while for $N > 2$ networks of multiple, even more intricate, caustics can exist.

Binary Lens

The binary lens is the most studied case of a multiple lens, and allows us to introduce the complexity of non-trivial caustics. A system composed of two point masses is described by the following parameters: the mass ratio $q = m_2/m_1$ (we assume $m_1 \geq m_2$), the projected separation s (in units of the Einstein radius of the total mass), the angle α that describes the direction of motion of the source relative to the axis connecting the two lenses, in addition to the standard microlensing parameters (time of maximum t_0 , minimum impact parameter u_0 , Einstein time t_E , source flux F_S and background flux F_B).

The geometry of the binary lens configuration is illustrated in Fig. 1.6, which shows the observer, the two lensing masses M_1 and M_2 in the lens plane, and the source in the source plane. The figure highlights the main geometric quantities: the deflection angle α , the angular position of the images θ , and the projected separations in the lens plane.

For the binary case the lens equation is:

$$\zeta = z + \frac{\epsilon_1}{\bar{z}_{m,1} - \bar{z}} + \frac{\epsilon_2}{\bar{z}_{m,2} - \bar{z}} \quad (1.34)$$

that reduces to a fifth-degree polynomial, consequently a system composed of two point masses can produce at most 5 distinct images. If the source is outside the caustics of the binary system, 3 images are formed; if the source falls inside a caustic region, a total of 5 images are generated. When the source crosses a caustic boundary, two of these images appear or disappear, producing a change in the observed brightness. The two new images are characterized by opposite parity and their magnifications diverge as the source approaches the caustic.

Topologies of Binary Caustics

The behavior of caustics in a two-lens system varies as s and q change. There exist three topologically distinct configurations, schematically illustrated in Figure 1:

- **Close configuration:** when the separation is small ($s \ll 1$). In this regime the two masses act almost as a single “equivalent” lens for most of the Einstein region. Three distinct caustics are formed: a central caustic (near the barycenter of the lenses) with four cusps, and two smaller caustics symmetrically placed at the sides (along the lens-lens axis), each with three cusps. The central caustic is of “astroidal” type (rounded rhombus shape), while the peripheral caustics (also called “planetary caustics” in analogy with the case of large mass ratio) are smaller.
- **Wide configuration:** when the separation is large ($s \gg 1$). In this case two

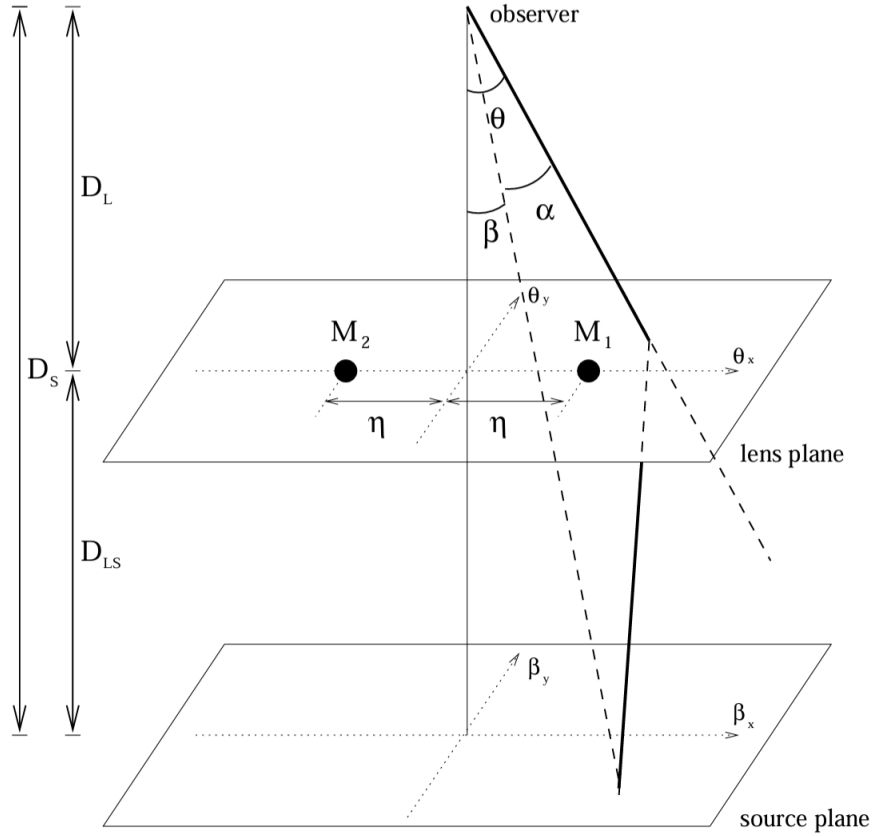


Figure 1.6: Schematic representation of the geometry of a two-point-mass lens system. The observer, the masses M_1 and M_2 in the lens plane, and the source in the source plane are indicated. The angle α represents the deflection, θ the apparent positions of the images, while η describe the projected separation between the lenses in units of the Einstein radius. This schematic forms the basis for the study of the complex caustics generated by binary systems. Figure adapted from (Konno et al. 1999)

separate caustics are also obtained: a small central caustic (near each of the single lenses, but in particular concentrated around the more massive one) and a secondary isolated caustic near the secondary (smaller) lens. For very large s the two lenses act almost independently, generating two nearly separate point-like caustic systems (each lens produces a very small 4-cusp caustic near itself). As s decreases (bringing the lenses closer together), the secondary caustic enlarges and moves away from the secondary lens, reaching maximum size around the so-called lensing zone ($s \sim 1$). In the typical wide configuration ($s > 1$ moderately), the secondary caustic also has 4 cusps (similar to a small astroid) when q is non-negligible.

- **Intermediate (resonant) configuration:** for values of s close to unity ($s \approx 1$). In this limiting case, the central and secondary caustics merge, forming a single large hexagonal caustic (with 6 cusps). This resonant caustic occurs when the secondary lens lies inside or near the primary Einstein ring, producing a single complex caustic structure. Even small variations of s around 1 produce topological transitions: just outside resonance, the hexagonal caustic splits into a central + a secondary (for $s > s_w$, see below) or into a central + two lateral ones (for $s < s_c$).

The transition between these topologies occurs at separation thresholds depending on q . In the limit $q \ll 1$ (planetary case, discussed in detail in the next section), the critical values can be approximated as:

$$s_c \approx 1 - \frac{3}{4}q^{1/3} \quad (\text{below this threshold: close regime with 3 caustics}) \quad (1.35)$$

$$s_w \approx 1 + \frac{3}{2}q^{1/3} \quad (\text{above this threshold: wide regime with 2 caustics}) \quad (1.36)$$

For $s_c < s < s_w$: intermediate regime (single resonant 6-cusp caustic).

These thresholds derive from perturbative analysis for small q . Figure 1.7 shows the diagram of the close, intermediate, and wide regions in the (s, q) plane, with examples of caustics corresponding to each regime.

Degeneracies in Binary Microlensing

A distinctive feature of binary microlensing is the presence of degeneracies—different combinations of physical parameters that give rise to indistinguishable light curves. These degeneracies complicate the interpretation of observational data and the determination of the true physical configuration of the lens system.

The most common case is the wide–close degeneracy. For small mass ratios ($q \ll 1$), configurations with separations s and $1/s$ produce central caustics that are nearly identical

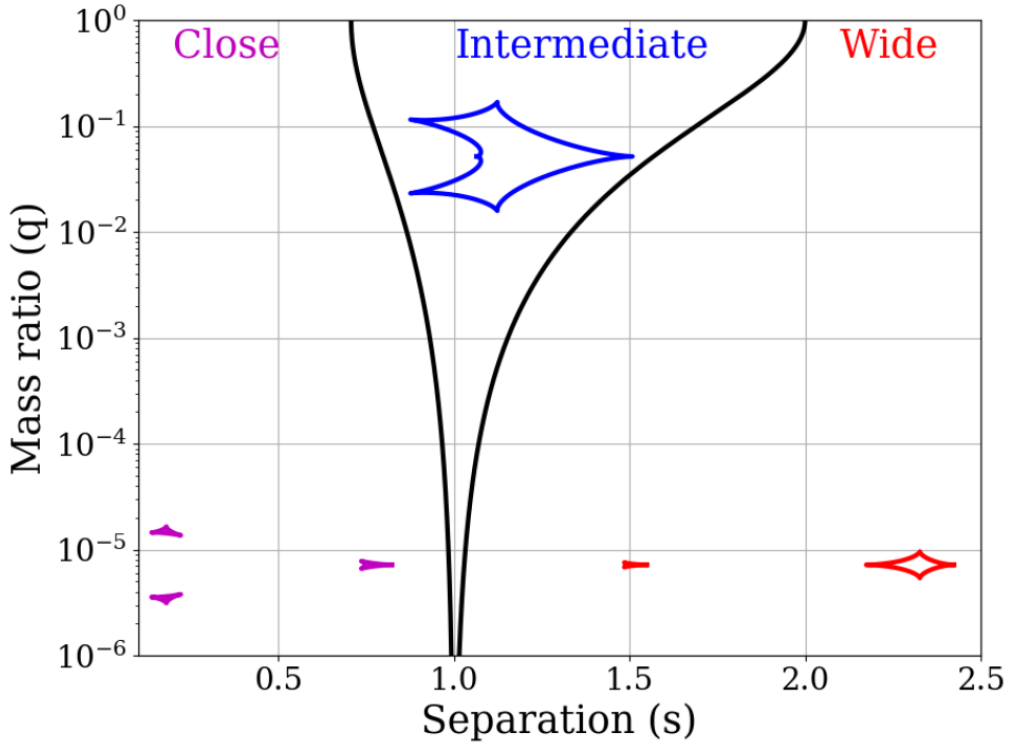


Figure 1.7: Caustic topologies in the (s, q) plane. The three regimes—*close*, *intermediate/resonant*, and *wide*—are separated by the black curves. Example caustics are shown for representative cases in each regime: a close binary ($s = 0.75, q = 10^{-2}$) producing three caustics, a resonant configuration ($s = 1, q = 10^{-2}$) with a single six-cusp caustic, and a wide binary ($s = 1.5, q = 10^{-2}$) showing a central and a secondary caustic, both with four cusps. Image adapted from Tsapras (2018).

in shape and size. Consequently, a planet located outside the Einstein ring ($s > 1$) generates a light-curve anomaly virtually indistinguishable from that produced by a planet with the same mass ratio but lying inside the Einstein ring ($s < 1$). This approximate symmetry, $s \leftrightarrow 1/s$, becomes exact in the limit $q \rightarrow 0$, where the lens equation exhibits a formal invariance under the transformation of the projected separation. In practical terms, this means that, based solely on a central perturbation, it is often impossible to discriminate whether the planetary companion is in a wide or close orbit. Breaking this degeneracy generally requires complementary information, such as microlensing parallax, finite-source effects, or multi-epoch observations with high temporal sampling.

Another common ambiguity is the trajectory-angle degeneracy. Due to the bilateral symmetry of the caustic structure relative to the lens axis, reversing the direction of the source trajectory—i.e., replacing an angle α with its opposite $-\alpha$ —produces an identical light curve if the lens system is static. Similarly, the impact-parameter degeneracy ($u_0 \leftrightarrow -u_0$) arises because trajectories passing “above” or “below” the lens configuration are geometrically mirror images of each other, leading to identical photometric behavior.

By convention, only positive u_0 values are typically reported, but both solutions are physically equivalent when the lens does not exhibit significant orbital motion or parallax distortion.

In more complex systems, further degeneracies emerge from the interplay between different dynamical or observational effects. For instance, microlensing parallax can mimic the signal of orbital motion in a binary lens, producing the so-called parallax–orbit degeneracy. Similarly, the binary–xallarap degeneracy occurs when the orbital motion of a binary source (xallarap effect) reproduces the same asymmetries expected from parallax in a single-source event. These higher-order degeneracies are particularly relevant in long-duration events or those involving bright sources, where the additional parameters increase the model’s flexibility but also its ambiguity.

Understanding and identifying these degeneracies is essential for correctly interpreting observed light curves and inferring the true architecture of microlensing systems. In practice, robust discrimination between degenerate models often requires high-cadence, multi-band photometry, astrometric follow-up, or additional physical constraints from stellar evolution and population synthesis models.

1.2.3 Other Methods

In addition to the transit and microlensing techniques, which constitute the main focus of this thesis, several other observational methods have contributed, and continue to contribute, to the detection and characterization of exoplanets. Each of these techniques probes different physical regimes or observational domains, extending the range of detectable planetary systems beyond the limits of photometric and gravitational approaches.

In this section, we provide a concise overview of some of the most relevant complementary methods, such as *astrometry*, *direct imaging*, and *radial velocity*.

Astrometry

Astrometry is one of the oldest branches of observational astronomy, traditionally dedicated to the measurement of stellar positions, proper motions, and parallax. In the context of exoplanetary astronomy, astrometry takes advantage of the fact that the star and the planet orbit around a common center of mass. Although the motion of the star, due to the high mass ratio with the other components of the system, is very small, it is possible to measure the periodic angular displacement of the photocenter of the star. When measured with sufficient precision, this constitutes the proof of the presence of a substellar companion (Sozzetti 2005).

The amplitude of the stellar motion, projected on the celestial sphere, is expressed by

the following relation:

$$\alpha = \left(\frac{M_p}{M_p + M_\star} \right) \frac{a}{d} \simeq \left(\frac{M_p}{M_\star} \right) \frac{a}{d}, \quad (1.37)$$

where M_p is the planetary mass, $M_\star$ the stellar mass, a the semi-major axis of the planetary orbit, and d the distance of the system from the observer.

To appreciate the observational challenge, consider that a planet of Jovian mass ($M_p \sim 318 M_\oplus$) in orbit at 1 AU from a solar-type star located at 10 pc produces an astrometric signature $\alpha \sim 0.5$ milliarcseconds (mas). For an Earth-mass planet in the same configuration, the signature drops to $\alpha \sim 1.6$ microarcseconds (μas), about three orders of magnitude smaller.

The astrometric position measured for a star at time t results from the superposition of different components; the astrometric model can be expressed as:

$$\Theta(t) = \Theta_0 + \boldsymbol{\mu}(t - t_0) + \pi \mathbf{P}(t) + \Delta_{\text{reflex}}(t) + \boldsymbol{\epsilon}(t), \quad (1.38)$$

where Θ_0 is the reference position at epoch t_0 , $\boldsymbol{\mu}$ represents the proper motion vector, π is the annual parallax with its geometric projection $\mathbf{P}(t)$, and $\Delta_{\text{reflex}}(t)$ is the additional term due to the orbital reflex motion induced by the planet. With astrometry, it is possible to directly constrain the orbital inclination i and the longitude of the ascending node Ω , allowing the determination of the true planetary mass M_p .

The *Gaia* mission of the European Space Agency represents a major contribution to this technique, since it has greatly increased astrometric precision. Launched in 2013, *Gaia* is building a three-dimensional map of the Milky Way, measuring positions, parallaxes, and proper motions for about 1.8 billion stars up to magnitude $G \sim 21$ (Gaia Collaboration et al. 2016; Gaia Collaboration et al. 2021; Gaia Collaboration et al. 2023). For bright stars ($G < 13$), *Gaia* reaches single-measurement precisions of $\sim 20\text{--}30 \mu\text{as}$, and through repeated observations during the mission lifetime, end-of-mission parallax uncertainties reach $\sim 10 \mu\text{as}$ for bright and well-behaved targets. This sensitivity allows the detection of giant planets.

Direct Imaging

The direct imaging method is based on the instrumental capability of separating the faint light emitted or reflected by a planet from the much stronger light of its host star. The main limitation is the small angular separation between the star and its planets, together with the flux contrast. The apparent angular separation θ between the star and the planet is given by the angle subtended by the orbital distance a at the distance d of the system

(both expressed in astronomical units/parallax):

$$\theta \simeq \frac{a}{d} \quad (\text{rad}), \quad (1.39)$$

which, in units of arcseconds, can be written as

$$\theta["] \simeq \frac{a [\text{AU}]}{d [\text{pc}]}. \quad (1.40)$$

For example, a Jupiter-like planet ($a \sim 5$ AU) around a star at 50 pc would yield $\theta \sim 100$ mas, of the order of the angular resolutions achieved today with 8–10 m telescopes equipped with adaptive optics.

The light emitted or reflected by the planet is composed of two main contributions: a reflected component in the visible domain and a thermal component in the infrared. The flux of the reflected component, at a given wavelength λ , can be expressed as:

$$F_{\text{vis},\lambda} = a(\lambda) f(\phi(t)) \left(\frac{R_p}{a} \right)^2 \left(\frac{R_\star^2}{d^2} B_\lambda(T_\star) \right), \quad (2.3)$$

where $a(\lambda)$ is the geometric albedo of the planet, $f(\phi(t))$ is the phase function describing the illuminated fraction visible to the observer as a function of the phase angle $\phi(t)$, R_p is the planetary radius, and a is the semi-major axis of the orbit. $B_\lambda(T_\star)$ denotes the spectral radiance of a blackbody at the stellar effective temperature $T_\star$, $R_\star$ is the stellar radius, and d is the distance between the observer and the system.

The thermal flux component, emitted by the planet itself, can be written analogously as:

$$F_{\text{IR},\lambda} = \left(\frac{R_p}{d} \right)^2 B_\lambda(T_p), \quad (1.41)$$

where T_p is the effective temperature of the planet. The balance between these two contributions determines the observed planetary spectrum, which depends on both the reflective properties of the atmosphere and the intrinsic thermal emission of the planet.

These relations show that, since $R_p \ll R_\star$ and typically $T_p < T_\star$, the flux received from the exoplanet is extremely faint. For example, HIP 65426 b is $\sim 10^4$ times fainter than its star in the near-IR band.

To apply this method, coronagraphs are employed whose goal is to block totally or almost completely the flux from the star, reducing the peak of the PSF. Differential imaging is essential to eliminate the surviving stellar residuals. The most common strategies include: - ADI (angular differential imaging), where the alt-azimuth telescope

keeps the coronagraph fixed while the field rotates, allowing subtraction of the mean PSF and isolation of rotating sources (the planet); - SDI (spectral differential imaging), which exploits spectral differences between planet and star in different bands; - RDI (reference star differential imaging), with calibrator stars of similar speckle patterns; - PDI (polarimetric differential imaging), mostly useful for disks but also applicable to polarized planetary light.

Direct imaging is today confined to a rather restricted category of exoplanets: mostly massive gas giants, young and on wide orbits around nearby stars. The proximity of the system, typically within a hundred parsecs, is necessary to guarantee angular separations large enough to be resolved with current telescopes. The young age of the star, less than a few hundred million years, ensures that the planets still retain part of the gravitational heat from contraction, thus making them much brighter in the infrared than gigayear-old planets.

Among the few known multiplanetary systems detected through direct imaging, the HR 8799 system stands out as a benchmark case (Marois et al. 2010; Marois et al. 2008), with four massive planets directly observed around their host star. HR 8799 is a young (~ 30 Myr) main-sequence A/F-type star ($\sim A5-F0V$), located at a distance of about 133 ly, with a mass of $\sim 1.5 M_{\odot}$ and luminosity of $\sim 4.9 L_{\odot}$. The known planets (HR 8799 b, c, d, e) have masses of several times that of Jupiter ($6-10 M_J$) and wide orbits, with semi-major axes of approximately 14, 26, 41, and 72 AU for e, d, c, and b, respectively.

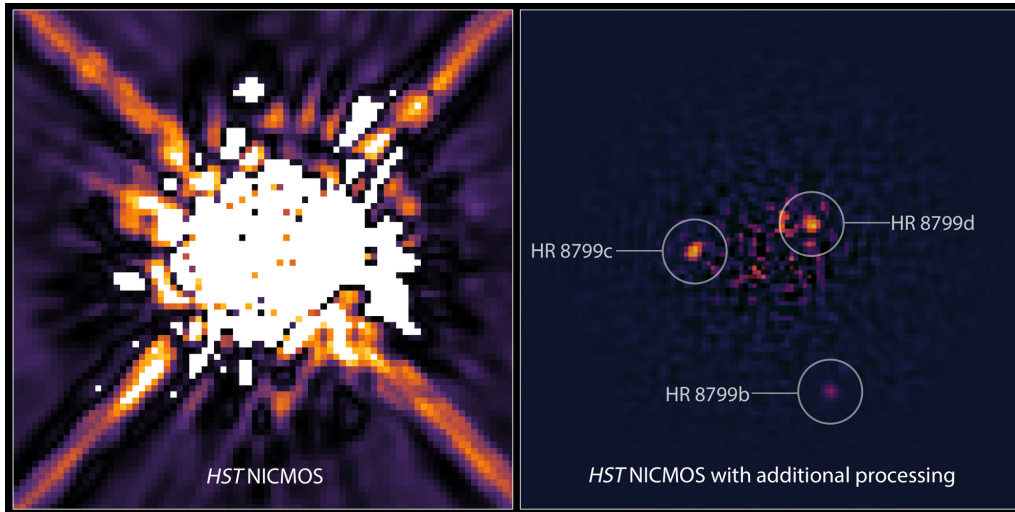


Figure 1.8: Image obtained with the HST space telescope, using the Near Infrared Camera and Multi-Object Spectrometer (NICMOS), of the HR 8799 system. On the left the original coronagraphic image is shown (central star masked), while on the right the same image is displayed after stellar profile subtraction: the four planets HR 8799 b, c, d are visible as bright spots (identified by the labels). Credit image: NASA, ESA, Rémi Soummer (STScI).

Radial Velocities (Doppler Spectroscopy)

The presence of a planet orbiting a star can be detected through the gravitational influence it exerts on its host star. The orbital motion of the star around the barycenter of the system is measured by the Doppler shift of the star's spectral lines. In the presence of a planet, the star will show radial velocities on the order of a few m/s, depending on the mass of the planet and its orbital distance. For example, the largest planet of our solar system, Jupiter, induces a velocity in the Sun of about 13 m/s, while that of the Earth only induces 0.09 m/s. The velocity of the star is given by:

$$V = r \frac{d\nu}{dt} = \frac{2\pi a^2 \sqrt{1-e^2}}{rP} \frac{M_p^2}{(M_s + M_p)^2} \quad (1.42)$$

from which, by projecting, the radial velocity is obtained:

$$V_r = \frac{M_p}{M_s + M_p} \cdot \frac{2\pi a \sin(i)}{P\sqrt{1-e^2}} [\cos(\nu(t) + \omega) + e \cos(\omega)] \quad (1.43)$$

where M_p is the mass of the planet, M_s is the mass of the host star, a is the semi-major axis of the planetary orbit, P is the orbital period, e is the orbital eccentricity, i is the inclination of the orbit with respect to the line of sight (with $i = 90^\circ$ corresponding to an edge-on view), $\nu(t)$ is the true anomaly that describes the position of the planet along its orbit as a function of time, and ω is the argument of periastron.

From the equation 1.43 we obtain the semi-amplitude of the radial velocity:

$$A = \frac{M_p}{M_s + M_p} \frac{2\pi a}{P\sqrt{1-e^2}} \sin i \quad (1.44)$$

which we can rewrite using Kepler's third law:

$$A = \frac{M_p \sin(i)}{(M_s + M_p)^{2/3}} \cdot \frac{(2\pi G/P)^{1/3}}{\sqrt{1-e^2}} \quad (1.45)$$

Equation 1.43 shows how the time dependence of the radial velocity depends not only on the mass and the orbital period, but also on geometric parameters such as the eccentricity e and the argument of periastron ω . The shape of the RV curve varies significantly: for circular orbits ($e = 0$) it is a pure sinusoid, while for eccentric orbits it shows asymmetric and sharper peaks near periastron. Figure 1.9 illustrates these differences.

Typically, a Jovian planet (mass $\sim 1M_J$) with a short period (days) around a solar-type star produces K of the order of tens or hundreds of m/s – easily detectable – while an Earth ($\sim 3 \times 10^{-6}M_\odot$) at 1 AU produces $K \sim 0.1$ m/s, at the extreme limit of

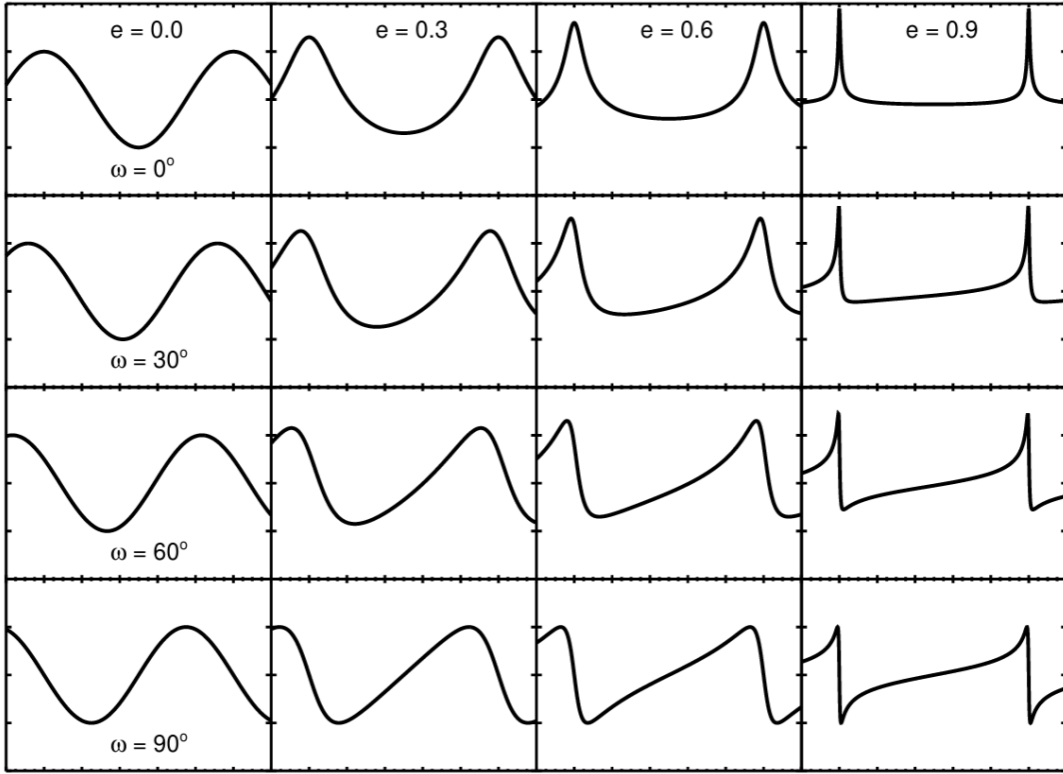


Figure 1.9: Radial velocity curves as a function of time for a planet with the same mass, period, and time of conjunction, but with different eccentricities (e) and arguments of periastron (ω). Figure adapted from Wright et al. (2013).

current instrumental capabilities. Modern high-resolution spectrographs (such as HARPS and successors) reach precisions up to $\sim 0.5\text{--}1\text{ m/s}$, sufficient to detect planets of a few $M_{\oplus}$ on short orbits (Mayor et al. 2003). The Doppler method provides primarily the minimum mass of the planet ($M_p \sin i$), since the orbital inclination i is generally unknown (inclination–mass degeneracy).

The radial velocity method has the advantage of being applicable to relatively nearby and bright stars, stars for which it is possible to obtain a high-quality spectral signal; moreover, the method is sensitive to a wide range of orbital periods, although limited by the duration of observational campaigns. Among the disadvantages, however, we find the impossibility of measuring the planet’s mass with the RV method alone, since it is necessary to know the inclination of the planet, information that can be obtained from the transit method, as well as the difficulty of detecting small and distant planets. Finding Earth analogues with this method requires extreme instrumental sensitivity as well as high precision in the control of stellar activity. The Doppler method, during the first 15 years of exoplanetary science, was the most prolific method in terms of discoveries; it was surpassed only in 2010 by the transit method thanks to the launch of dedicated space missions. Nevertheless, it remains fundamental for confirming the planetary nature of

many of the candidate planets discovered with the transit method.

1.3 Planetary Demography

To date, about 6,000 exoplanets have been confirmed, exhibiting an extraordinary diversity in terms of mass, radius, and host-star properties. This diversity is clearly illustrated in Figures 1.10 and 1.11, which show the distribution of known planets as a function of the orbital period versus the planetary radius and mass, respectively. These diagrams highlight not only the wide range of planetary characteristics, but also the observational biases and discovery domains associated with each detection method.

Such a wealth of data has made it possible to initiate detailed studies of *planetary demography*, that is, statistical analysis of the distributions of physical and orbital parameters across the exoplanet population. Through these analyses, recurring patterns and correlations can be identified, such as the clustering of super-Earths and sub-Neptunes at short orbital periods or the scarcity of massive planets on very tight orbits, offering insights into the physical processes governing planetary formation and migration.

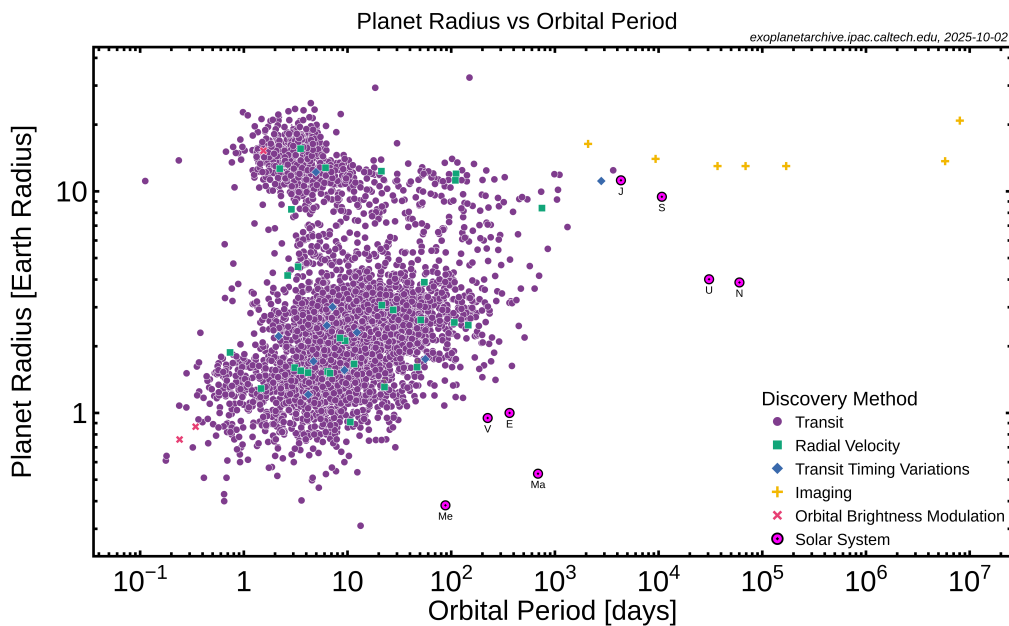


Figure 1.10: Distribution of confirmed planets as a function of planetary radius (in Earth radii) and orbital period (in days). Each point represents a planet discovered with different detection methods, as indicated in the legend. The planets of the Solar System are included for comparison. Data from the *NASA Exoplanet Archive* (<https://exoplanetarchive.ipac.caltech.edu>), updated to October 2, 2025.

Astronomers, on the basis of mass and radius, classify planets as follows:

- **Gas Giants:** Massive planets predominantly composed of hydrogen and helium,

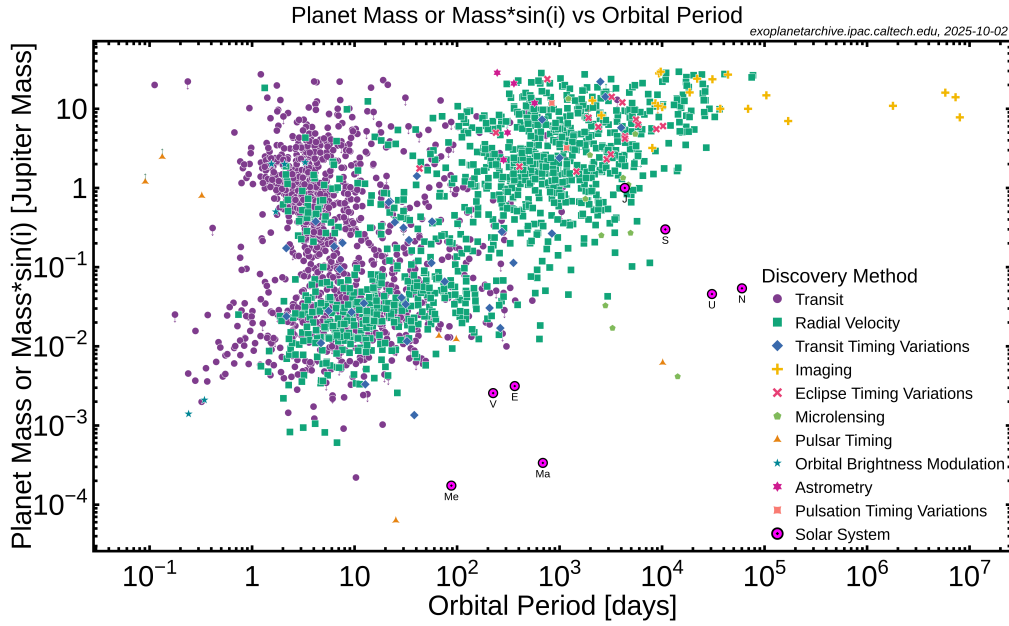


Figure 1.11: Distribution of confirmed planets as a function of planetary mass (or $M \sin i$, in Jupiter masses) and orbital period (in days). Each symbol represents a different discovery method, as indicated in the legend. The planets of the Solar System are shown for comparison. Data from the *NASA Exoplanet Archive* (<https://exoplanetarchive.ipac.caltech.edu>), updated to October 2, 2025.

with relatively small solid cores compared to their total mass. These planets typically have large radii and low mean densities, reflecting their gaseous composition. Gas giants are among the most massive exoplanets detected. Within this class, two main subgroups can be identified: *Hot Jupiters* and *Cold Jupiters*. Hot Jupiters are Jupiter-type planets with very short orbital periods (typically $P < 10$ days). Due to their close proximity to the host star, they are strongly irradiated, often exhibit inflated radii, and can show significant orbital inclinations relative to the stellar equator, an indication of past dynamical interactions or tidal processes after formation. They are frequently found as isolated planets within their systems. Cold Jupiters, on the other hand, are giant planets analogous to Jupiter and Saturn, with longer orbital periods and lower equilibrium temperatures. They tend to form farther from their host stars, often beyond the snow line, and show a higher occurrence rate around metal-rich stars. These planets are frequently found in systems that also host smaller inner planets, suggesting complex formation and migration histories.

- **Neptunian planets:** Planets with sizes and masses comparable to Neptune, typically within the range of 2 to 6 Earth radii ($R_{\oplus}$) and 10 to 50 Earth masses ($M_{\oplus}$). They are common in exoplanetary systems and are frequently found on relatively short orbits. Their compositions are likely dominated by volatile materials and

thick gaseous envelopes over rocky or icy cores.

- **Sub-Neptunian planets:** Objects bridging the gap between Super-Earths and Neptunians, with radii in the range of $1.6\text{--}2.0 R_{\oplus}$. These planets exhibit a wide diversity in densities, indicating a transition region between rocky and gaseous compositions. They are among the most abundant planet types detected by transit surveys.
- **Super-Earths:** Planets with radii between 1.2 and $3 (R_{\oplus})$ and masses between 2 and 10 Earth masses ($M_{\oplus}$). They are abundant around FGK- and M-type stars, predominantly on close-in orbits. Kepler mission statistics show that Super-Earths frequently appear in compact multiple systems. Their existence poses significant challenges for planet formation models, given their absence in our Solar System.
- **Terrestrial planets:** Planets with dimensions comparable to Earth, with radii $\lesssim 1\text{--}1.2 R_{\oplus}$. Only a few of these planets have been confirmed to date, mainly due to detection difficulties. Most of the known exo-Earths orbit closer to their stars than in the habitable zone, though ongoing missions aim to identify true Earth analogs.

Although this classification captures the main structural types of planet, the statistical distributions of planetary masses, radii, and orbital parameters show characteristic features that provide important clues to the mechanisms governing planet formation and evolution.

- **The Radius Valley:** from demographic studies carried out thanks to the Kepler mission data, a lack of planets is evident in the interval of radii between 1.5 and 2 Earth radii, separating two distinct populations. This gap in the radius distribution is also known as the Fulton gap (Fulton et al. 2017). The radius valley is interpreted as the sign of physical processes that remove the atmosphere of small-mass planets that are at reduced orbital distances from their star. Planets that have modest atmospheric layers composed mainly of hydrogen–helium, if subjected to intense stellar radiation or to other mechanisms of atmospheric escape, can become stripped cores and thus smaller. However, the cause of the radius valley remains debated; the main accepted model is that of atmospheric photoevaporation induced by stellar XUV irradiation.
- **The Neptunian Desert:** In demographic studies there is an almost complete absence of Neptune-size planets on short-period orbits. In the radius–period diagram it is possible to notice a “desert” of objects with radii $\sim 4\text{--}6 R_{\oplus}$ and periods of a few days. The Neptunian desert shows how intermediate mass planets

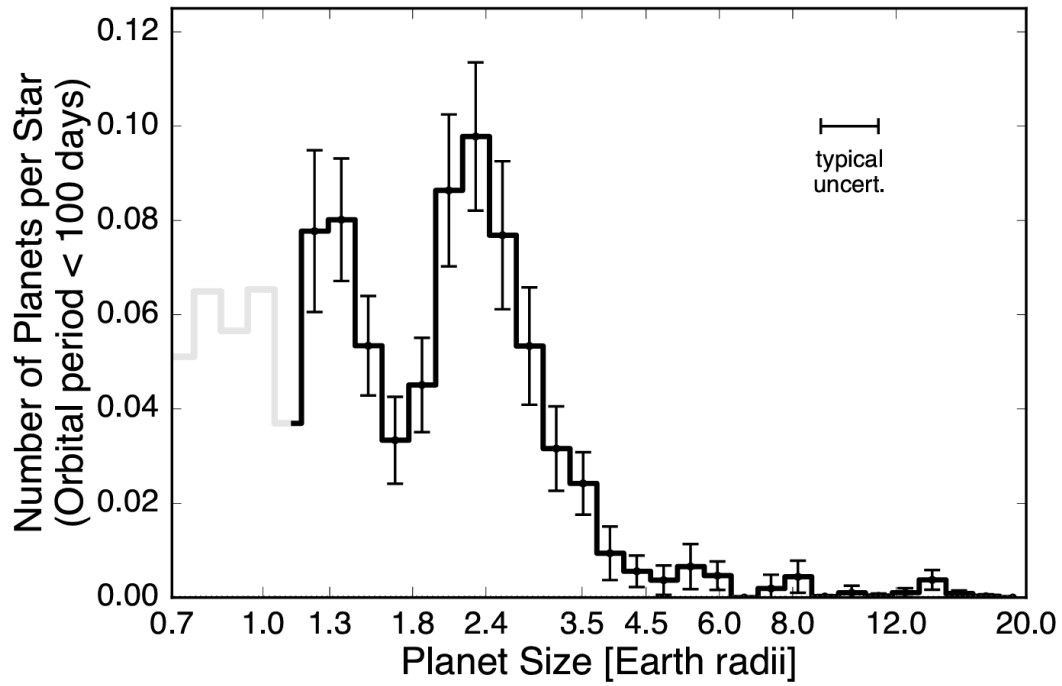


Figure 1.12: Distribution of exoplanet radii (in Earth radii) for planets with orbital periods shorter than 100 days. The observed deficit between 1.5 and 2 $R_{\oplus}$, known as the *Radius Valley* or *Fulton Gap*, separates super-Earths and sub-Neptunes. Figure adapted from (Fulton et al. 2017)

undergo atmospheric ablation processes or tidal instabilities that prevent them from remaining as “hot Neptunes”. An example of a planet discovered at the edge of this region is LTT 9779b, a rare ultra-hot Neptune (Jenkins et al. 2020).

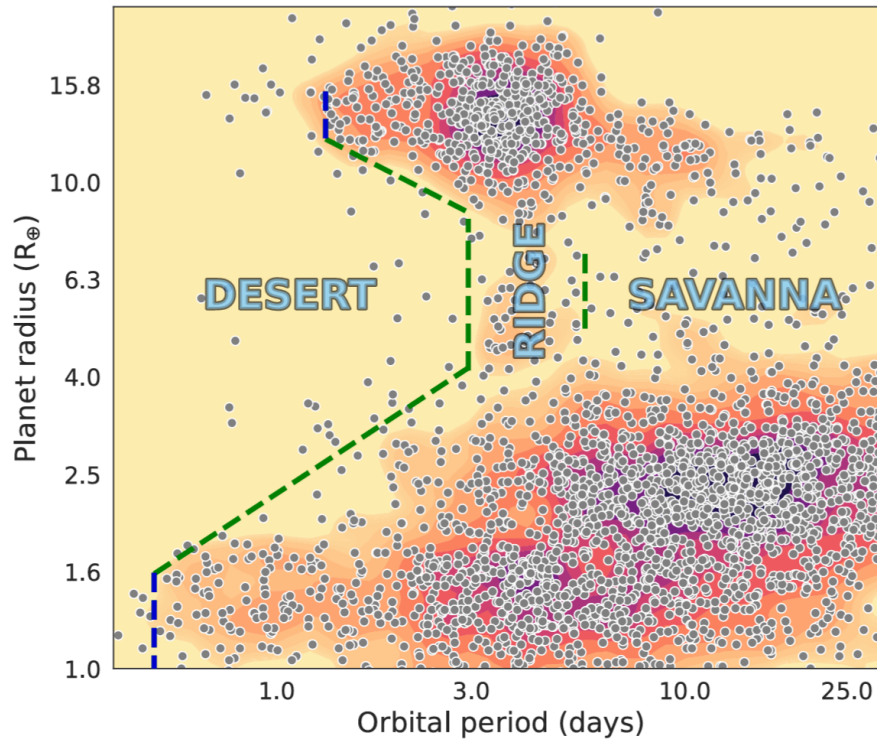


Figure 1.13: Planet radius versus orbital period diagram showing the *Neptunian Desert*, the region nearly devoid of Neptune-size planets at short periods. The plot highlights the “desert”, “ridge” and “savanna” regions, illustrating the transition between hot Jupiters and smaller sub-Neptunes. Figure adapted from (Castro-González et al. 2024).

1.4 Multiple Planetary Systems

To date, around 42% of the exoplanets discovered are located within systems that host more than one planet. Each system presents different conditions, methods, and times of formation; this means that no single physical process alone can explain the entire variety of observed planetary systems. However, the fundamental mechanisms are accretion, migration, disk dissipation, gravitational interactions, and atmospheric loss. In this context, the planetary system that we have characterized best and have been studying for the longest time is our own Solar System (Raymond 2026); the only system observed with such a level of detail as to allow a direct analysis of the physical processes that led to the formation of the planets, the satellites, and the small bodies that compose it. From the orbits and compositions of the inner bodies to the outer gas giants, the Solar System is an excellent tester for theoretical models. The masses and orbits of the inner terrestrial planets show a balance resulting from accretion and subsequent impacts. The positions of the giants and their masses, on the other hand, are indicative of a different accretion mechanism mediated by orbital migration. The ages of meteorites and chondrites provide a clock for the accretion processes of the planetary nuclei. The collection of such data

has been made possible through decades of observations and dedicated space missions.

Comparing the data from our Solar System with those emerging from the discoveries of exoplanets shows that our Solar System is not common. The planets of our Solar System occupy regions of the parameter space that are sparsely populated in current statistics.

Figure 1.14 provides a visual summary. It shows the planets belonging to known multiplanetary systems for which the planetary mass has been constrained. It is evident that most of these systems have been discovered using the transit method, whereas only a few planets in multiple systems have been detected through microlensing.

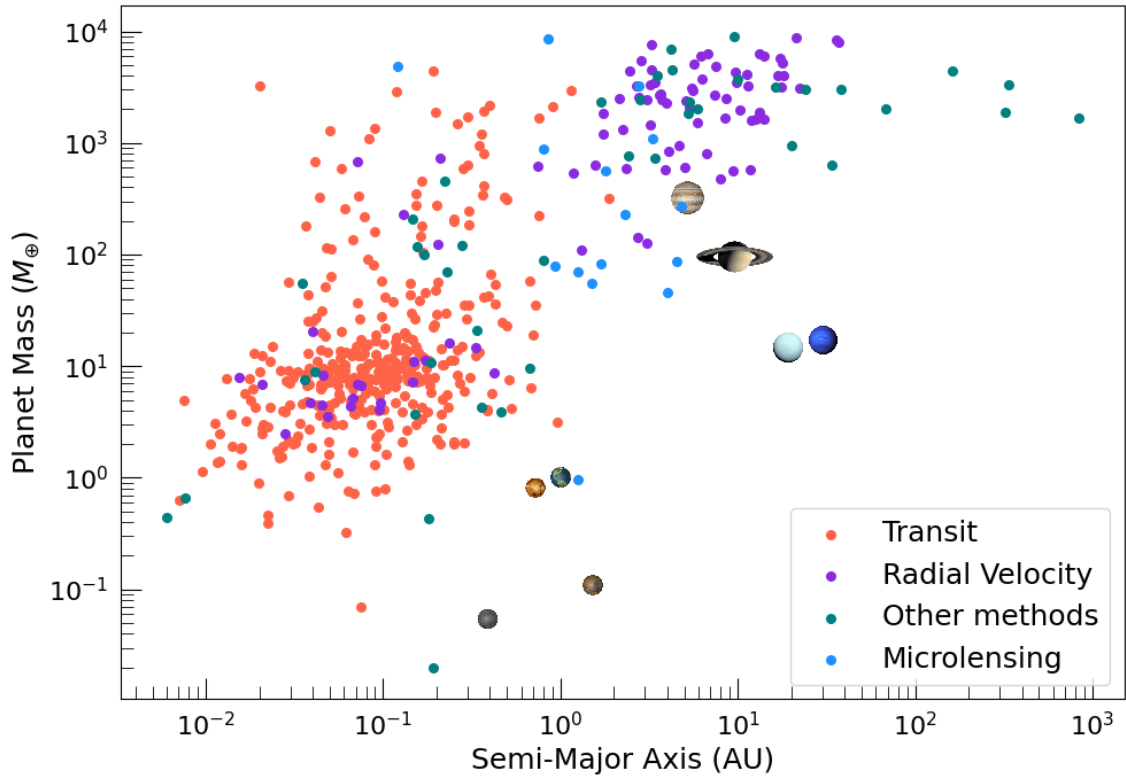


Figure 1.14: Distribution of known exoplanets in multiplanetary systems for which the planetary mass has been constrained. The points are color-coded according to the detection method.

Several dynamical properties appear to be shared between many systems, offering useful points of comparison. A first dynamical classification concerns *coplanarity*: most multi-transiting systems have very small mutual inclinations, typically of the order of a few degrees, i.e. they are nearly coplanar like our own Solar System (Fang et al. 2012). This reflects both formation mechanisms (the protoplanetary disk naturally favors orbital alignment) and observational biases (only nearly coplanar systems can be seen in transit from Earth). Exceptions with larger inclinations exist and are often interpreted as the result of past dynamical instabilities.

Another key distinction separates *resonant* from *non-resonant* systems. In systems with mean-motion resonances (MMRs), the orbital periods of adjacent planets are close to ratios of small integers (e.g., 2:1, 3:2). These resonances imply regular gravitational interactions and often point to convergent migration within the protoplanetary disk. In presence of giant planets, the configurations tend to be found in resonant situations, favoring the 2:1 resonance; for low-mass systems, instead, the periods are slightly different from an exact resonance (Fabrycky et al. 2014). This distribution of orbital period ratios is illustrated in Figure 1.15.

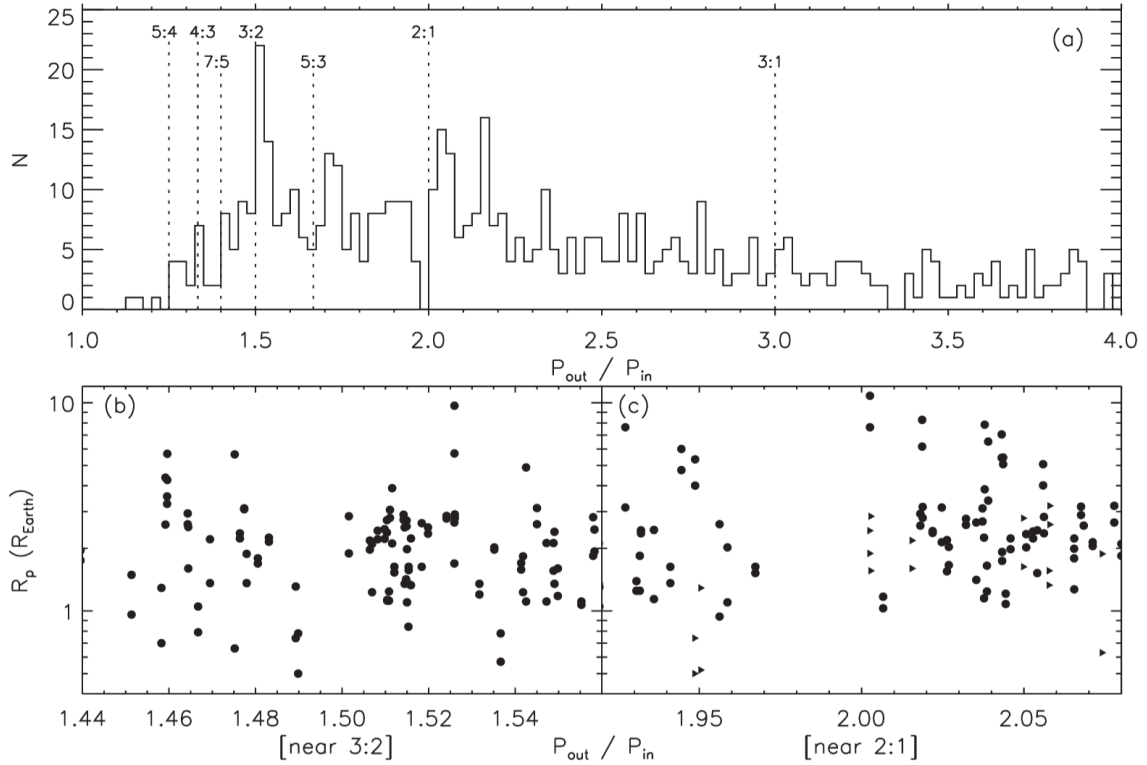


Figure 1.15: Distribution of period ratios $P_{\text{out}}/P_{\text{in}}$ in planetary systems observed by *Kepler*. (a) Histogram of the number of planet pairs (N) as a function of the ratio of orbital periods. The dashed lines indicate the main resonances (5:4, 4:3, 3:2, 2:1, 3:1). (b) and (c) show the planetary radii R_p (in units of Earth's radius, $R_{\oplus}$) as a function of the period ratio for systems near the 3:2 and 2:1 resonances, respectively. Figure adapted from Fabrycky et al. (2014).

For example, recently discovered systems show chains of resonances in which each adjacent planet pair is in or near resonance (Leleu et al. 2021). Ultra-compact systems such as TRAPPIST-1 (seven terrestrial planets) (Gillon et al. 2017) and TOI-178 (six planets in a multi-resonant chain) (Leleu et al. 2021) are notable examples. Conversely, many systems are non-resonant or only *near-resonant*, with period ratios offset from exact commensurabilities.

Thanks to the *Kepler* mission, which has provided us with the most uniform popula-

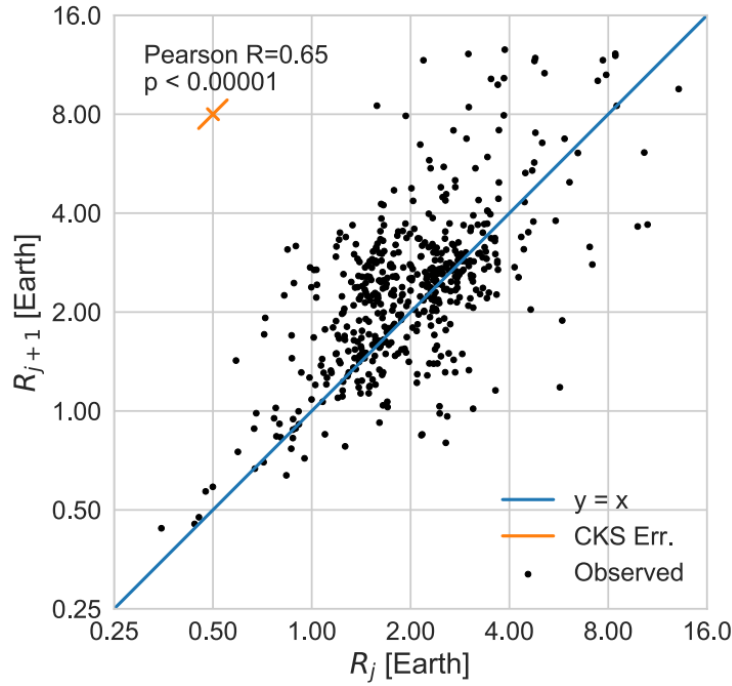


Figure 1.16: Comparison between the radii of adjacent planets in multi-planet systems observed by Kepler. The strong correlation (Pearson $R = 0.65$) indicates that planets within the same system tend to have similar sizes, consistent with the “Peas in a Pod” trend. Figure adapted from (Weiss et al. 2018).

tion of exoplanets we have today, a dominant population of compact planetary systems has been identified. These systems present an architecture with multiple planets with radii between Earth and Neptune ($1R_{\oplus} < R_p < 4R_{\oplus}$), which orbit at distances much smaller than those of Mercury from the Sun. Many of these systems show remarkable internal uniformity; the planets within the same system tend to have similar radii and regular orbital spacings. This feature has been nicknamed “peas in a pod” (Weiss et al. 2018). An example is the already cited TRAPPIST-1 system (Gillon et al. 2017). The homogeneity inside the system suggested by the “peas in a pod” pattern (Figure 1.16) puts constraints on formation models.

1.5 Planetary Evolution

Planet formation takes place within protoplanetary disks of gas and dust orbiting young stars. These accretion disks naturally result from the conservation of angular momentum during the gravitational collapse of a molecular cloud. Their typical mass is a few percent of the stellar mass, and their composition is about 99% gas (mainly H_2 and He) and 1% solid particles such as silicate dust, metal oxides, and ices.

Protoplanetary disks show pronounced radial and vertical gradients in temperature

and density. The vertical scale height $H(r)$ increases with distance from the star due to hydrostatic equilibrium and stellar irradiation. A simple estimate gives $H(r) \approx c_s(r)/\Omega_K(r)$, where c_s is the local sound speed and Ω_K the Keplerian angular velocity (Armitage 2020). As the disk cools with increasing distance from the star, its aspect ratio gradually rises, producing a flared structure characterized by warm, irradiated surface layers and a cold, dense midplane. The resulting radial pressure gradient causes the gas to rotate slightly below the Keplerian speed, an essential factor in the aerodynamic coupling between gas and solids (Weidenschilling 1977).

Condensation fronts or *snow lines* mark the distances from the star beyond which volatile species can exist as ices. The most relevant is the water snow line, located at 2–3 AU for a solar-type star, corresponding to temperatures of 150–170 K (Öberg et al. 2011). Beyond this boundary, the presence of water ice roughly doubles the solid surface density, facilitating faster core formation. Additional snow lines for CO, CO₂, and NH₃ occur farther out, contributing to the chemical and physical stratification of the disk (Qi et al. 2013). At these transitions, solids can accumulate and grow efficiently.

High-resolution observations, especially with ALMA, have provided direct images of these structures. Many disks show concentric rings and gaps in both dust and gas emission (Andrews et al. 2018). These substructures are interpreted as the signatures of pressure variations induced by the formation of planets or changes in disk chemistry near condensation fronts. A well-known example is the HL Tau disk (Figure 1.17), where bright rings and dark gaps likely trace the gravitational influence of nascent planets.

1.5.1 From Dust to Planetesimals

Planet formation begins with the growth of dust grains within the protoplanetary disk. Through gentle collisions, grains stick through van der Waals forces and form porous aggregates that initially grow to millimeter-centimeter sizes ('pebbles') (Blum et al. 2008). Beyond this stage, growth by collisions is limited by two major obstacles: the *fragmentation barrier*, where high-velocity impacts destroy aggregates, and the *radial drift barrier*, where meter-sized solids rapidly spiral into the star due to gas drag (Weidenschilling 1977). These effects prevent a smooth transition from pebbles to kilometer-sized bodies.

One way to overcome these growth barriers is through *streaming instability* (SI), a process in which interactions between drifting solids and the surrounding gas cause particles to gather together (Youdin et al. 2005). When the local dust-to-gas ratio becomes slightly higher than average, pebbles clump into dense filaments that can rapidly collapse under their own gravity, forming planetesimals tens to hundreds of kilometers across.

Planetesimals are expected to form preferentially near pressure maxima or condensation fronts, such as the water snow line, where water ice condenses, increasing the

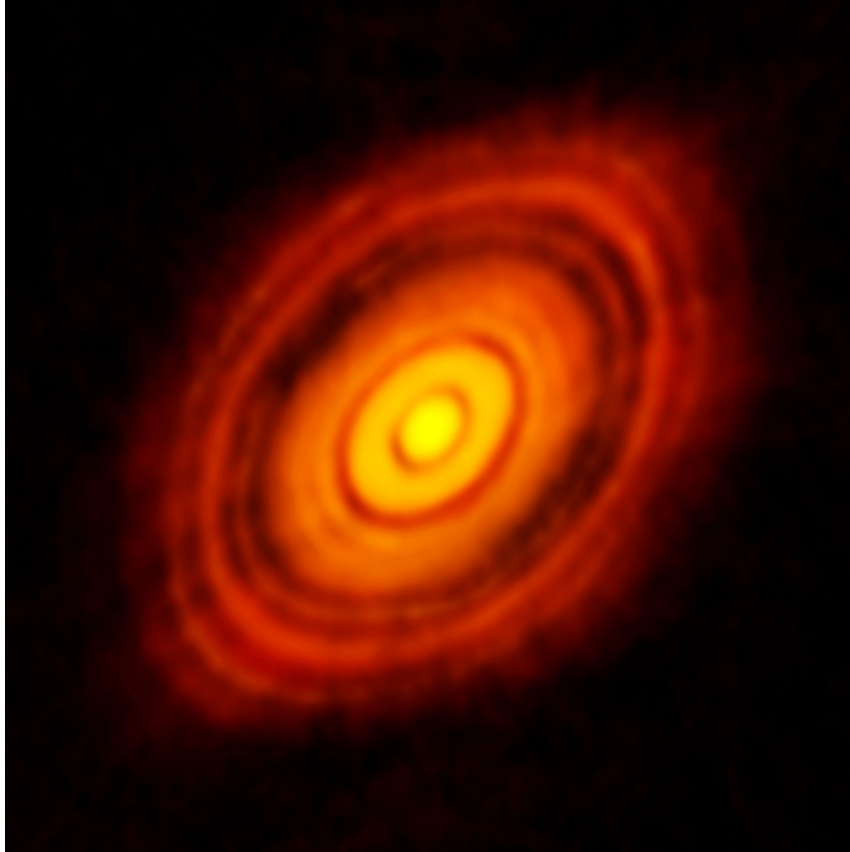


Figure 1.17: ALMA image of the protoplanetary disk around the young star HL Tau, about 450 light-years away. The disk shows a pattern of bright rings and dark gaps, likely shaped by the gravitational influence of forming planets. The disk extends across several tens of astronomical units. Credit: ALMA (ESO/NAOJ/NRAO).

amount of solid material and allowing particles to be more easily trapped (Ros et al. 2013). ALMA observations of ringed disk structures support this scenario, suggesting that planetesimals formed in localized regions rather than uniformly throughout the disk (Andrews et al. 2018). Their composition mirrors the local temperature: rocky and metal rich in the inner disk, mixed rock ice beyond the snow line, and volatile rich in the cold outer regions. This radial diversity explains the compositional contrast between inner asteroids and outer icy bodies and may explain the delivery of water to terrestrial planets.

Figure 1.18 summarizes the main physical processes shaping planetary growth. It distinguishes the divergent evolutionary paths of rocky and giant planets — the former dominated by collisional accretion and giant impacts, and the latter by rapid core formation and gas accretion before disk dispersal.

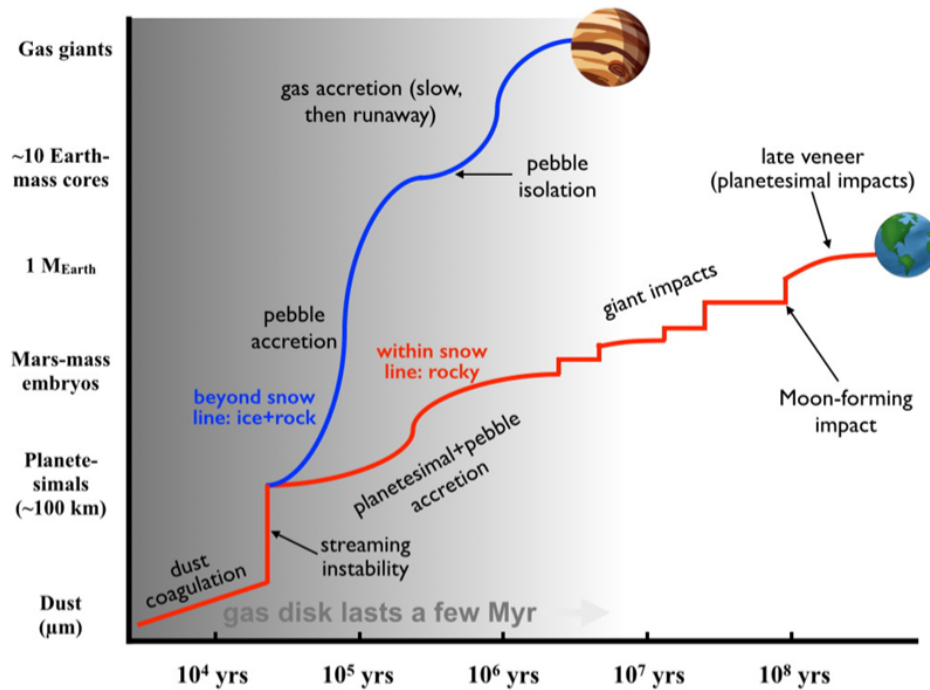


Figure 1.18: Illustration of the main processes leading to the formation of rocky and giant planets. The diagram shows the growth stages from dust grains to fully formed planets and highlights the different evolutionary paths of terrestrial and giant planets, with a focus on pebble accretion, streaming instability, and giant impacts. Adapted from Raymond et al. (2022).

1.6 Planetary Accretion and Disk Instability

Once a first generation of planetesimals ($\sim 10\text{--}100 \text{ km}$) has formed, planetary growth proceeds through two complementary channels: *planetesimal accretion* and *pebble accretion* (Wetherill et al. 1989; Kokubo et al. 1998; Ormel et al. 2010; Lambrechts et al. 2012). In the classical planetary-accretion framework, mutual collisions between kilometer-sized bodies gradually lead to the formation of planetary embryos. At early stages, growth is rapid because gravity increases the likelihood of collisions, but it later slows as impacts become destructive and the growing bodies dynamically perturb the smaller population. In the inner disk, this mechanism can produce lunar to Mars sized embryos ($10^{-2}\text{--}10^{-1} M_{\oplus}$) within $10^5\text{--}10^6$ years (Chambers 2004). Farther out in the disk, slower orbital motion and more energetic dynamical interactions make accretion less efficient, ultimately limiting growth.

Pebble accretion provides a far more efficient pathway. Small (mm–cm) solids drifting inward due to gas drag are accreted onto protoplanetary cores when aerodynamic drag within the Hill sphere dissipates their kinetic energy (Ormel et al. 2010; Lambrechts et al. 2012). This process can rapidly build massive cores ($\sim 10\text{--}20 M_{\oplus}$) in less than a million

years, particularly beyond the water snow line where icy pebbles are larger and more abundant. Once the growing core perturbs the local gas pressure enough to halt the inflow of pebbles—a stage known as the *pebble isolation mass*—solid accretion ceases and gas accumulation begins (Lambrechts et al. 2014). This dichotomy naturally explains the coexistence of small rocky planets in the inner disk and massive gas giants beyond a few astronomical units.

When a solid core exceeds $\sim 10\text{--}15 M_{\oplus}$, it starts accreting gas from the surrounding disk. The *core-accretion* model predicts a slow initial phase controlled by the Kelvin–Helmholtz contraction of the envelope, followed by runaway gas accretion once the envelope mass equals the core mass (Pollack et al. 1996). This process can form Jupiter-mass planets before the gas disk dissipates ($\lesssim 5$ Myr), consistent with the observed correlation between stellar metallicity and the occurrence of gas giants (Fischer et al. 2005; Johnson et al. 2010).

An alternative formation route is the *disk instability* model, where a massive, cold protoplanetary disk becomes gravitationally unstable and fragments directly into self-gravitating clumps (Boss 1997; Mayer et al. 2004). According to the Toomre criterion, instability arises when $Q = c_s \Omega_K / (\pi G \Sigma) \lesssim 1$, a condition met only in very massive disks ($M_{\text{disk}} \gtrsim 0.1 M_*$) at large radii (> 50 AU). Such fragments may contract into giant planets within $10^3\text{--}10^4$ years, but survival requires avoiding inward migration or tidal disruption. Disk instability might explain a few very massive companions on wide orbits, yet the strong planet–metallicity correlation and heavy-element enrichment of most gas giants support core accretion as the dominant formation mechanism.

1.7 Orbital Migration and Dynamical Evolution

During the gaseous phase of the protoplanetary disk, young planets interact gravitationally with the surrounding gas, causing orbital migration that strongly influences the final architecture of planetary systems. Planet–disk interactions can be described by two main regimes: *Type I* and *Type II migration* (Goldreich et al. 1980; Lin et al. 1986; Tanaka et al. 2002). Type I migration affects low- and intermediate-mass planets ($M_p \lesssim 30 M_{\oplus}$) that are unable to open a gap in the disk. In this regime, density waves exert differential torques on the planet, generally driving it inward on timescales of $10^5\text{--}10^6$ years (Ward 1997). However, strong gradients in temperature or density can slow or even reverse this motion through corotation torques (Paardekooper et al. 2010). Type II migration, on the other hand, occurs for massive planets ($M_p \gtrsim 100 M_{\oplus}$) that carve a gap in the disk and then migrate along with its viscous evolution over millions of years (Lin et al. 1993; D’Angelo et al. 2003). Together, these processes offer a natural explanation for the presence of gas giants at several astronomical units as well as the inward migration that

produces “hot Jupiters.”

After gas dispersal (within $\sim 3\text{--}10$ Myr), gravitational interactions among planets and planetesimals dominate. In the Solar System, migration driven by planetesimals in the *Nice model* caused Jupiter to move slightly inward while Saturn, Uranus, and Neptune were pushed outward. This process triggered a dynamical instability that reorganized the outer Solar System and led to the Late Heavy Bombardment (Tsiganis et al. 2005; Gomes et al. 2005). Similar instabilities in extrasolar systems can lead to *planet–planet scattering*, ejecting planets and leaving others on eccentric or inclined orbits (Chatterjee et al. 2008; Jurić et al. 2008). In some cases, tidal dissipation can circularize highly eccentric orbits, leading to the formation of *hot Jupiters*, while in other systems, interactions with distant companions through Kozai–Lidov cycles can drive eccentricity in an alternative way (Wu et al. 2003; Naoz 2016).

Compact systems of super-Earths can stay stable when their planets are locked in resonance chains, as seen in TRAPPIST-1, but in some cases, delayed instabilities may occur, causing planets to collide and merge. More generally, the movements of planets through migration and their gravitational interactions, both while the gas disk is still present and afterward, play a key role in determining the variety of planetary system architectures we observe.

Space missions

2.1 Roman

The Nancy Grace Roman Space Telescope (Roman) (see Fig. 2.1) is NASA's next space observatory dedicated to near-infrared astrophysics (Spergel et al. 2015). This mission was previously known as WFIRST (Wide-Field Infrared Survey Telescope), then renamed in 2020 in honor of Nancy Grace Roman (Akeson et al. 2019). Roman will have the same diameter as Hubble for the primary mirror (2.4 meters) but with a much wider field of view. The 300-megapixel Wide Field Instrument (WFI) will be able to capture in a single exposure an area of the sky about 100 times larger than that framed by Hubble at the same resolution. This makes Roman an ideal telescope for conducting surveys.

2.1.1 Main Scientific Objectives

The *Roman* mission is designed around three core scientific objectives:

1. Dark energy and observational cosmology
2. Exploring exoplanet populations
3. Galactic surveys and general astrophysics

Dark Energy and Observational Cosmology

A key objective of the *Roman* mission is to explore the nature of **dark energy**, which drives the accelerated expansion of the universe. By mapping the history of cosmic expansion and the growth of structures with high precision, *Roman* will determine whether

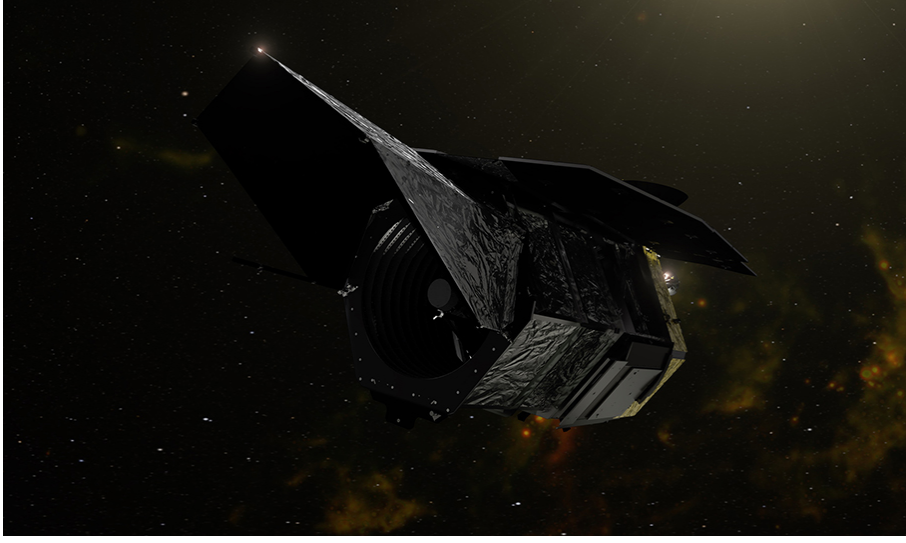


Figure 2.1: Artist's impression of NASA's *Nancy Grace Roman Space Telescope*. Image credit: NASA / GSFC / SVS.

this acceleration comes from a cosmological energy component or from modifications of gravity on large scales (Weinberg et al. 2013; Spergel et al. 2015). This goal will be pursued through the *High Latitude Survey* (HLS), which provides both deep imaging and spectroscopic observations.

The *HLS* will map dark matter through weak gravitational lensing, measuring the *cosmic shear* of about 400 million galaxies up to $z \sim 3$. This will allow a reconstruction of the matter distribution and provide tests of general relativity. The spectroscopic part of the survey will measure redshifts for more than 20 million galaxies (Sanderson et al. 2024), making it possible to detect *baryon acoustic oscillations* (BAO) between $1 \lesssim z \lesssim 3$. These observations will help determine the expansion rate $H(z)$ and place constraints on the growth of cosmic structures.

Observations of thousands of Type Ia supernovae up to $z \approx 1.7$ will extend the cosmic distance ladder using the standard-candle approach. By combining measurements from weak lensing, BAO, and supernovae, *Roman* will place strong constraints on the dark energy equation of state w and its possible evolution over time, offering a comprehensive view of the universe's accelerated expansion.

Exploring exoplanet populations

The second objective of *Roman* is a large-scale census of extrasolar planets using the *gravitational microlensing* method.

We have anticipated that microlensing is a rare phenomenon (probability of $\sim 10^{-6}$ per star), but *Roman* will simultaneously monitor hundreds of millions of stars in the

region of the Galactic *bulge*, maximizing the chances of detecting such events.

The *Galactic Bulge Time Domain Survey* (GBTDS) will be the main campaign for this goal.

Simulations (Penny et al. 2019) predict ~ 1400 detected planets ($M > 0.1 M_{\oplus}$), including ~ 200 Earth-mass or lower-mass planets and numerous free-floating planets. In addition, about $\sim 50,000$ total microlensing events are expected, including roughly 500 produced by isolated black holes and hundreds by neutron stars. Astrometric measurements of the centroids will allow direct estimation of the masses of these compact objects. Figure 2.2 illustrates the predicted *Roman* detection sensitivity compared to that of *Kepler*.

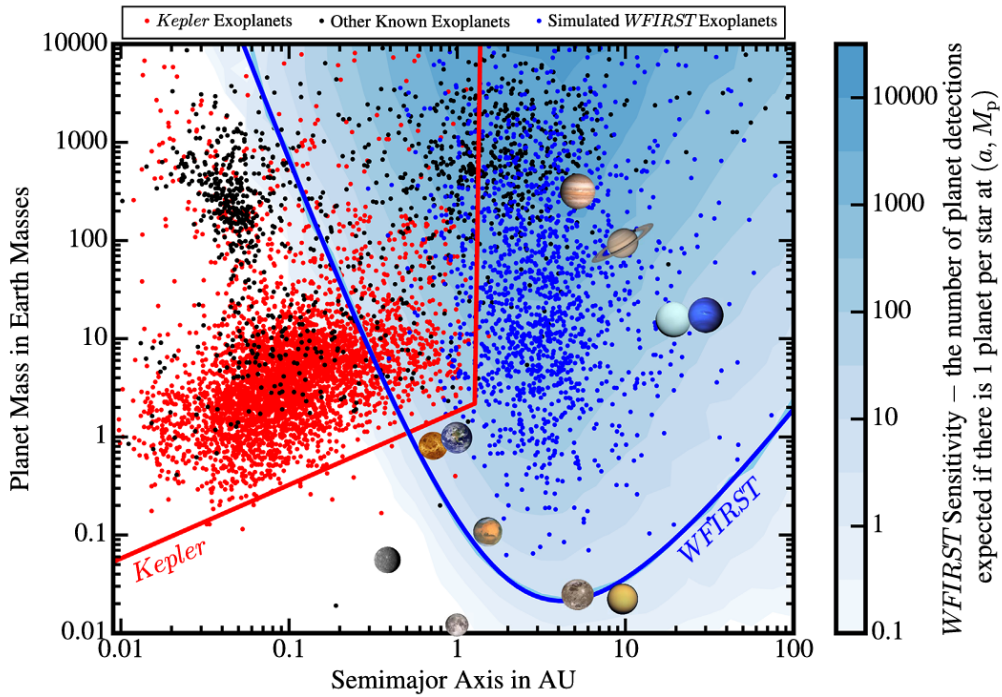


Figure 2.2: Comparison of the *Roman* (formerly *WFIRST*) planet-detection sensitivity with that of *Kepler* in the planet mass–semimajor axis plane. Red dots show *Kepler* detections, black dots other known exoplanets, and blue dots the simulated planets detectable by *Roman*. Blue shading indicates the expected number of detections if each star hosts one planet at a given mass and semimajor axis. *Roman*’s sensitivity extends to lower masses and larger separations, probing a population of cold and free-floating planets inaccessible to previous surveys. Adapted from Penny et al. (2019).

The survey will also yield valuable data for several other research areas, including the detection of variability and pulsations in nearly one million red giant stars of the Galactic bulge, the characterization of populations of compact objects such as isolated black holes and neutron stars, and the discovery of thousands of trans-Neptunian objects (TNOs) in the outer Solar System down to sizes of 10–20 km in diameter.

The *Roman* microlensing program will provide a planetary census that complements

those of *Kepler*, *TESS*, and the upcoming *PLATO* mission. This will help complete the statistical picture of planetary systems in our Galaxy and improve our understanding of both stellar and planetary evolution.

Galactic surveys and general astrophysics

In addition to the two pillar objectives of the mission, cosmology and exoplanets, *Roman* will devote a significant portion of the mission to **general astrophysics** projects, broadening its scientific impact. About 25% of the mission time will be available for community-proposed surveys (*Guest Investigator Program*) (Sanderson et al. 2024).

An example is the *Galactic Plane Survey*, designed to map the Galactic plane down to magnitude ~ 25 in infrared, covering ~ 1000 deg². This will enable studies of the large-scale structure of the Milky Way, stellar populations, star-forming regions, and variable stars.

2.1.2 Mission Payload

The spacecraft is organized into three main modules: (1) the *Optical Telescope Assembly*, featuring a 2.4 m monolithic primary mirror; (2) the instrument module, hosting the *Wide Field Instrument (WFI)* and the *Coronagraph Instrument (CGI)*; and (3) the service module, containing avionics, propulsion, communications, and power systems.

Roman employs a Cassegrain optical design optimized for 0.5–2.3 μm observations. Its Hubble-sized primary mirror, made of low-thermal-expansion fused silica, operates at a controlled temperature of ~ 290 K with passive detector cooling. A long cylindrical *baffle* minimizes stray light from the Sun, Earth, and Moon, enhancing thermal and photometric stability.

The observatory will operate in a *Halo* orbit around the Sun–Earth L2 point, 1.5 million km from Earth, providing a thermally stable environment and continuous sky coverage (about 59% at any given time, 100% over six months). Data will be transmitted via a Ka-band link through NASA’s *Deep Space Network*; each 300 Mpixel WFI exposure produces ~ 0.6 Gbyte of raw data, processed and compressed onboard.

High-precision attitude control ensures pointing stability better than a few tenths of an arcsecond—critical for weak-lensing observations. Roman can slew several degrees in under three minutes and perform fine repointing in seconds, enabling efficient wide-field surveys. The spacecraft measures ~ 12.7 m in length, has a mass of $\sim 8,000$ kg, and will be launched aboard a Falcon Heavy in 2027 for a nominal five-year mission, extendable to ten.

Wide Field Instrument (WFI)

The *Wide Field Instrument* is Roman’s main scientific instrument, designed for wide-field observations in imaging and *slitless* spectroscopy modes. It will provide *Hubble*-like resolution over a field more than one hundred times larger, enabling unprecedented cosmological and galactic surveys.

The WFI covers a field of 0.281 deg^2 ($0.76^\circ \times 0.43^\circ$) through 18 *H4RG-10* detectors (4096×4096 pixels) arranged in a 6×3 array, for a total of about 300 million pixels with a scale of $0.11''/\text{pixel}$. It operates in the $0.48\text{--}2.30 \text{ }\mu\text{m}$ range through six main photometric filters (Z, Y, J, H, F184, W149). The spectroscopic mode uses a *grism* with a resolution of $R \sim 600\text{--}800$ ($1.0\text{--}1.93 \text{ }\mu\text{m}$), capable of separating lines such as $\text{H}\alpha$ and $[\text{OIII}]$ at $z \sim 2$.

The H4RG detectors, cooled to $\sim 95 \text{ K}$, ensure read noise of a few e^- and sensitivity limited by zodiacal background. The PSF is *diffraction-limited* above $1 \text{ }\mu\text{m}$ ($\lambda_{\text{diff}} \sim 0.12''$), and photometry achieves precisions on the order of 1%. Relative astrometry is accurate to a few milliarcseconds, sufficient to measure proper motions and parallaxes over multi-year timescales. The optional *Integral Field Unit (IFU)* channel will provide low-resolution spectra ($R \sim 75$, $0.6\text{--}2.0 \text{ }\mu\text{m}$) for the follow-up of Type Ia supernovae.

Coronagraph Instrument (CGI)

The *Coronagraph Instrument* (CGI) is a technological demonstrator for the direct imaging of exoplanets and circumstellar disks at high contrast. It is the first space coronagraph equipped with *deformable optics* and active wavefront control, capable of suppressing stellar light by factors up to $10^8\text{--}10^9$, allowing direct observation of giant planets around stars within 30 light-years.

The coronagraph will operate for about 6–12 months during the primary mission, observing known giant planets (e.g., $\beta \text{ Pic b}$, 47 UMa c) and bright disks such as those of Fomalhaut and HR 8799.

2.1.3 Observing strategy and main surveys

Roman will devote most of its observing time to three main programs (*Core Community Surveys*) that reflect its fundamental scientific goals: (1) the *High Latitude Wide Area Survey* for cosmology and dark energy, (2) the *High Latitude Time Domain Survey* for the study of Type Ia supernovae, and (3) the *Galactic Bulge Time Domain Survey* for the search for planets through gravitational microlensing. The remaining time will be reserved for general observation programs (GO) and additional community-proposed surveys.

Figure 2.3 shows the current design footprint of the *Roman Core Community Surveys*, illustrating the complementary sky coverage of the three main programs.

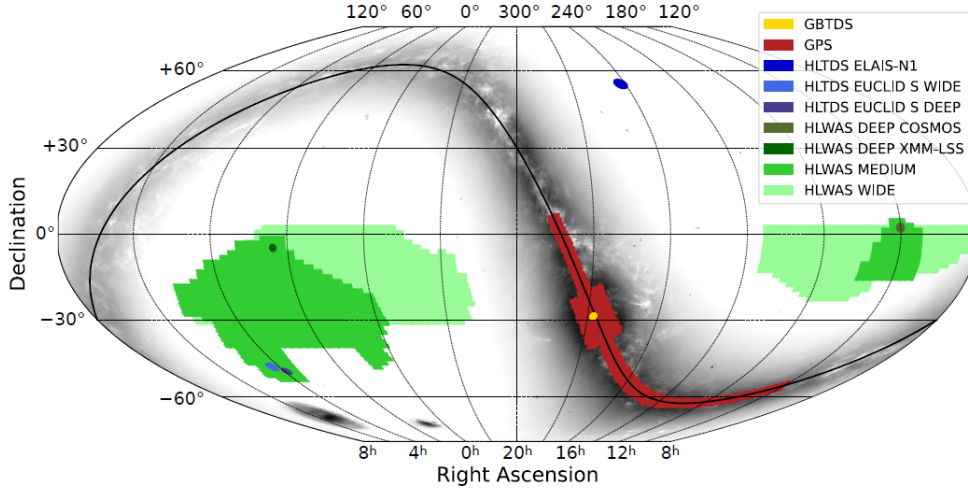


Figure 2.3: Equatorial projection of the planned *Roman Core Community Survey* (CCS) footprints. The High Latitude Wide Area Survey (HLS) fields are shown in shades of green, the High Latitude Time Domain Survey (HLTDS) in blue, and the Galactic Bulge Time Domain Survey (GBTDS) in yellow. The red region marks the preliminary Galactic Plane Survey (GPS). The background grayscale map shows stellar density from *Gaia*. Figure adapted from (Zasowski et al. 2025)

High Latitude Wide Area Survey

The *HLS* will cover about 2000 deg^2 of extragalactic sky at high latitudes, combining deep imaging in four filters (Y, J, H, F184) with *slitless* spectroscopy in the near infrared. The imaging exposures (depth $\sim 26.5\text{--}27$ AB) will provide shapes and multicolor photometry for more than 4×10^8 galaxies, while the spectroscopic component ($R \sim 600$) will produce over 20 million redshifts up to $z \sim 3$, tracing emission lines such as $H\alpha$ and [OIII]. These data will enable measurements of *weak lensing*, *baryon acoustic oscillations* (BAO), and peculiar motion distortions (RSD), constraining the dark energy equation of state with unprecedented precision. The HLS will also provide a huge archive of galaxies, quasars, and gravitational lenses for studies of cosmic evolution and large-scale structure.

High Latitude Time Domain Survey

The *High Latitude Time Domain Survey* (HLTDS) will observe selected fields with regular cadence (every ~ 5 days) to discover and monitor thousands of Type Ia supernovae up to $z \approx 1.7$. The program will adopt three levels of depth: *Wide*, *Medium*, and *Deep*, balancing area and sensitivity. Infrared observation will reduce the effects of dust extinction, while

the use of a single instrument will ensure photometric uniformity and minimization of systematic errors. Multi-band light curves and IFU spectra for a subsample will calibrate the magnitude–redshift relation, improving constraints on the cosmological parameters w_0 and w_a . It is estimated that, overall, Roman will collect about 2700 high-quality SNe Ia, integrating the results of the HLS in a multi-probe cosmological approach.

Galactic Bulge Time Domain Survey

The *Galactic Bulge Time Domain Survey* is the program entirely dedicated to exoplanetology. Roman will continuously monitor, over several seasons, a region of $\sim 2 - 3 \text{ deg}^2$ in the Galactic *bulge* (centered at $|b| \sim 1.5^\circ$, $l \sim 0^\circ - 5^\circ$), where the stellar density and the probability of gravitational alignments are highest.

The observations will take place in six campaigns of ~ 72 days each, distributed over five years, when the bulge is visible from the L2 point. During each season, Roman will take images with the wide infrared filter W149 every ~ 15 minutes, obtaining almost continuous coverage of the light curves of tens of millions of stars. Occasional observations in the Z087 filter (every ~ 12 hours) will be inserted to determine colors and correct for extinction. Each 52 s exposure will reach magnitudes of ~ 23.5 (S/N ~ 5), with millimagnitude photometric precision for the brightest stars.

This strategy will allow the identification of $\sim 54,000$ microlensing events, including over a thousand planets revealed as deviations in the light curves. The sensitivity will cover the orbital range 0.3–30 AU, complementary to transit surveys such as *Kepler*. In addition, thanks to the distinct vantage point from L2, Roman will measure microlensing parallax effects in combination with ground-based observations (e.g., OGLE, LSST), enabling direct determination of the masses and distances of the lenses.

Roman’s high resolution will also allow measurements of astrometric microlensing (image shifts of a few mas), identifying isolated black holes and brown dwarf lenses, and creating the first statistical census of compact remnants in the Galaxy.

2.2 PLATO

The PLATO mission (PLANetary Transits and Oscillations of stars) is an M3-class mission of the European Space Agency (ESA), selected in 2014 within the framework of the Cosmic Vision 2015–2025 program (Rauer et al. 2024) (see Fig. 2.4). Its main objective is the detection and characterization of extrasolar planetary systems, with particular attention to Earth-type planets located in the habitable zone of stars of the same class as our Sun.

PLATO combines two observational approaches: high-precision photometry for the detection of planetary transits and asteroseismology for the accurate determination of stellar parameters. The combined analysis of these data will allow extremely precise measurements of the radius, mass, and mean density of planets, as well as the age, mass, and radius of the host stars (Rauer et al. 2024).

The payload of PLATO consists of an ESA satellite equipped with 26 cameras (24 “normal” white-light cameras and 2 “fast” cameras with color filters), together with the necessary data processing and power supply systems. The launch is planned for the end of 2026.

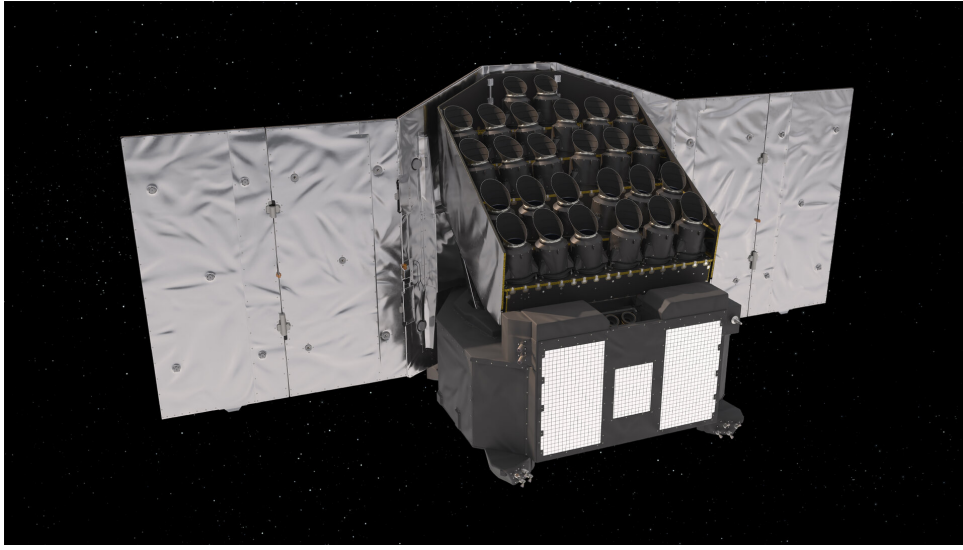


Figure 2.4: Artist’s impression of ESA’s *PLATO* (PLAnetary Transits and Oscillations of stars) spacecraft. Image credit: ESA / ATG Medialab.

2.2.1 Main Scientific Objectives

The scientific program of the **PLATO** mission (*PLAnetary Transits and Oscillations of stars*) addresses key questions in exoplanetary science and stellar astrophysics, aiming to understand the formation, evolution, and diversity of planetary systems within the context of Galactic evolution (Rauer et al. 2024).

PLATO seeks to determine how planets and their systems form and evolve, to assess whether the Solar System represents a typical or exceptional case, and to measure the occurrence rate of potentially habitable, Earth-sized planets ($\eta_{\oplus}$) in the Milky Way. Its discoveries will provide fundamental observational constraints for theories of planet formation and evolution, linking stellar properties to planetary architectures and habitability.

Characterization of exoplanets

PLATO is designed to determine the fundamental parameters of planets - radius, mass, mean density, and age - in a wide variety of systems, including in particular terrestrial planets in the *Habitable Zone* (HZ) of solar-type stars. For Earth-sized planets ($R_p \sim 1 R_\oplus$) orbiting G0V-type stars with visual magnitude $V \approx 10$, photometry is expected to achieve precisions of about 3–5% on the planetary radius and 10% on the stellar age; for slightly fainter stars ($V \approx 11$), the uncertainty on the radius is reduced to about 5% (Rauer et al. 2024). The high brightness of the target stars will also allow effective ground-based spectroscopic *follow-up*, enabling planetary mass measurements with precisions on the order of 10% for terrestrial planets ($R_p \leq 2 R_\oplus$, $M_p \leq 10 M_\oplus$).

The combination of PLATO photometry and high-resolution radial velocities will make it possible to derive mean densities and internal planetary structures, constraining composition (iron, silicon, and ice content) and differentiation processes. Particular attention will be devoted to rocky planets within the HZ, defined by the “*recent Venus*” and “*early Mars*” limits (Kasting et al. 1993), which represent the inner and outer habitability boundaries for terrestrial-type planets, respectively. Estimation of the fraction of Earth-like planets ($\eta_\oplus$) per host star represents one of the main indicators of the scientific success of the mission.

Thanks to its long-term observational strategy, PLATO will also explore poorly sampled regions of parameter space, expanding the knowledge of planets with orbital periods longer than 80 days, thus complementing the results of the *Kepler* and *TESS* missions.

Evolution of planetary systems

The age of planetary systems is a key parameter for understanding how the physical properties of planets – such as structure, composition, and atmosphere – evolve over time. PLATO will provide precise and homogeneous stellar ages through asteroseismology, allowing the correlation of planetary characteristics with the age of their host stars and the validation of formation and dynamical evolution models over extended time scales. This information will be crucial, for example, to analyze the solidification times of terrestrial planets and the atmospheric loss of gaseous planets in the early evolutionary stages.

In the case of stars without detectable oscillations, the age will be estimated through photometric measurements of the rotation period and the application of gyrochronology and magneto-gyrochronology relations (Barnes 2003). The determination of age will also be particularly useful for resolving the observed degeneracy in hot or temperate giant planets (*warm Jupiters*), in which similar mass and radius can correspond to different

evolutionary configurations: young and metal-poor planets or older but heavy-element-enriched ones.

Architectures of planetary systems

PLATO will make it possible to investigate the architectures of planetary systems, studying the distribution of masses, types (rocky or gaseous), and orbital parameters (semi-major axes, eccentricities, inclinations) of the discovered planets. Such data will be fundamental to understanding whether the Solar System represents a particular case or a common configuration. The high precision of the planetary parameters will also make it possible to test emerging correlations between planetary and stellar characteristics, such as the relation between metallicity and frequency of gas giants.

A relevant case is the *radius valley* (Fulton et al. 2017). Thanks to its extended temporal coverage and larger sample of planets on wider orbits, *PLATO* will allow testing whether this trend is universal or confined to the compact systems observed by *Kepler*. Another key objective concerns the *Hot Neptune Desert*. Observations from *PLATO* will help clarify the physical mechanisms responsible for this gap, such as atmospheric photoevaporation and disk-driven planetary migration.

Correlation between stellar properties and planetary frequency

The detailed characterization of the stars observed by *PLATO* will make it possible to study the relationships between stellar parameters (metallicity, chemical composition, magnetic activity, spectral type, age) and planetary properties (frequency, mass, composition, radius). Previous investigations, based on missions such as *Kepler* and RV surveys, have indicated preliminary correlations but limited by the uncertainty in stellar parameters (Howard et al. 2012; Fressin et al. 2013). The photometric and seismic catalogs of *PLATO* will provide a high-quality reference for calibrating planetary occurrence rates and testing the relations between stellar composition and planetary composition (Adibekyan et al. 2021).

Stellar seismology

Another objectives of *PLATO* is the improvement of the knowledge of the internal structure and evolution of stars. The solar-like oscillations measured in the photometric data will be modeled through advanced asteroseismic analyses, providing constraints on processes of angular momentum transport, convection, and differential rotation. The integration between seismic data and spectroscopic observations will make it possible to derive stellar parameters (mass, radius, age) with high precision, improving the cali-

bration of stellar evolution models. For stars without detectable seismic signals, PLATO will measure photometric rotation periods, allowing the estimation of ages through gyrochronological methods and extending the characterization also to later-type stars.

2.2.2 Observing strategy

The observing strategy of the **PLATO** mission has been designed to maximize the scientific yield in the detection and characterization of exoplanets, while at the same time optimizing the photometric quality and the thermal stability of the spacecraft. The observatory, located in orbit around the L2 Lagrangian point of the Sun–Earth system, will have an extremely wide field of view ($\sim 2132 \text{ deg}^2$) that will allow the simultaneous monitoring of thousands of bright stars. The nominal mission foresees an operational duration of four years, with the possibility of extension up to about 8.5 years, depending on fuel reserves and the stability of the pointing systems (Rauer et al. 2024).

Observation phases and general scheme

PLATO will be launched at the end of 2026 aboard an Ariane 62 rocket and placed in a stable orbit around the Sun–Earth L2 Lagrangian point, about 1.5 million km from Earth. The nominal mission is expected to last at least four years, with possible extensions up to about eight years thanks to generous consumables and system margins.

The observing strategy is organized around two main *Long Observation Phases* (LOPs), each lasting about two years. One phase targets a field in the southern hemisphere (LOPS2), while the other focuses on a field in the northern hemisphere (LOPN1). During these periods, the spacecraft will remain pointed at the selected fields for extended intervals, with rotations of 90° carried out every few months to ensure thermal stability and efficient power generation. An additional observing mode based on shorter *step-and-stare* pointings is planned in the event of a mission extension. This approach would allow coverage of a wider area of the sky and increase the number of detected planets.

The choice of the PLATO sky fields reflects a balance between scientific goals and technical constraints, such as illumination conditions, stray-light control, and communication requirements. The layout of these long-duration fields, along with their overlap with regions observed by earlier exoplanet missions including *Kepler*, *K2*, *CoRoT*, and *TESS*, is shown in Figure 2.5.

To support this selection, Montalto et al. (2021) assembled the all-sky *PLATO Input Catalogue* (PIC), which identifies stars well suited for high-precision photometric observations. Building on this work, Nascimbeni et al. (2022); Nascimbeni et al. (2025) identified the most favorable regions for the LOP fields by considering visibility constraints, stellar

crowding, and photometric signal-to-noise requirements.

Each LOP field will be monitored almost continuously, achieving a duty cycle exceeding 93% thanks to the stable observing conditions at the Sun–Earth L2 point. Observations will be carried out using 26 telescopes, consisting of 24 *Normal Cameras* and 2 *Fast Cameras*, which together cover a total field of view of approximately 2150 deg^2 . The Normal Cameras are organized into four groups of six, with slight offsets that create overlapping areas on the sky. A central region of about 325 deg^2 will be observed simultaneously by all 24 cameras, providing the highest photometric precision, while the surrounding areas will be covered by between 6 and 12 cameras. This pattern of overlap results in regions with different levels of photometric quality, corresponding to the high-priority target categories P1 and P2.

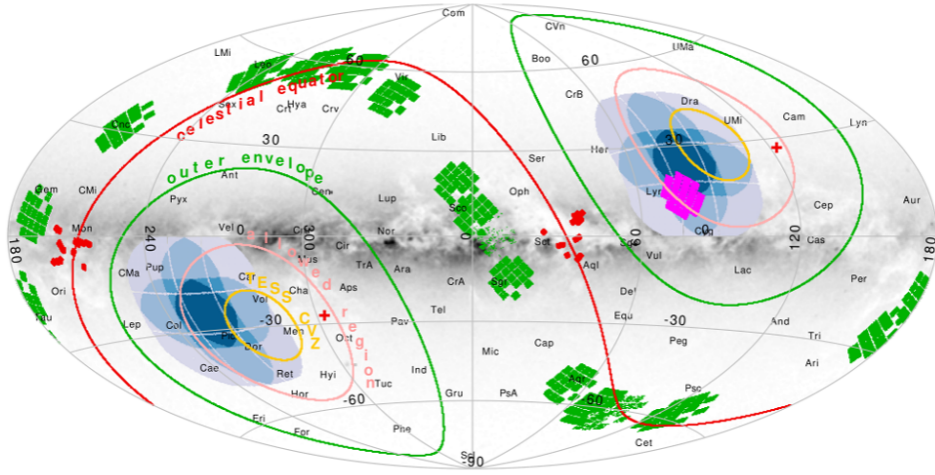


Figure 2.5: Aitoff map in galactic coordinates of the long-duration observation fields of the *PLATO* mission. The southern field **LOPS2** and the northern field **LOPN1** are shown. Shades of blue indicate the number of cameras (*Normal Cameras*) observing the same region of the sky, while pink areas represent the regions allowed for long-duration observations ($|\beta| > 63^\circ$). Also highlighted are the regions observed by previous missions (*CoRoT*, *Kepler*, *K2*, *TESS*). Figure adapted from Nascimbeni et al. (2025).

Stellar observational samples

The observing strategy groups the monitored stars into five main samples (P1–P5), defined by magnitude, spectral type, and the expected level of photometric noise (Rauer et al. 2024). This scheme is designed to meet the requirements of both asteroseismology and planet detection, while making efficient use of onboard data processing resources.

- **Samples P1 and P2 (bright sample)** – They include the stars of greatest scientific interest: about 15,000 F5–K7 type stars with $V \leq 11$ (P1) and a subset of more than

Sample	Target V mag	Noise (ppm in 1 h)	Type	# Targets	Sampling (s)
P1	≤ 11	< 50	F5–K7	$\geq 15,000$	25
P2	≤ 8.5	< 50	F5–K7	$\geq 1,000$	25
P4	≤ 16	–	M	$\geq 5,000$	25
P5	≤ 13	–	F5–K7	$\geq 245,000$	600

Table 2.1: Overview of the stellar sample requirements for *PLATO* dwarf and subgiant targets, adapted from Rauer et al. (2024).

1,000 extremely bright stars ($V \leq 8.5$, P2). These stars are observed with rapid cadence (25 s) through dedicated imagerettes, with photometric noise lower than 50 ppm/h. They represent the core of the asteroseismology program and the basis for the precise characterization of rocky planets in the habitable zone.

- **Sample P4 (M dwarfs)** – Includes at least 5,000 red dwarfs and late-type stars (K–M) up to $V \leq 16$, also observed with rapid 25 s imagerettes. This sample takes advantage of the high frequency of planets around M dwarfs to explore the population of terrestrial planets in low-mass and low-luminosity stellar environments.
- **Sample P5 (statistical)** – Includes about 245,000 F5–K7 type stars up to $V \leq 13$, observed with a sampling of 600 s. The light curves are processed onboard and transmitted in compressed form. Despite lower precision compared to P1/P2, this sample will provide essential statistical information on planetary occurrence rates and on the correlations between stellar and planetary parameters.

2.2.3 The LOPS2 field

The first long-duration observational field, LOPS2 (Long Observation Phase South 2), represents the most significant phase of *PLATO*’s scientific program. It is centered at the equatorial coordinates $\alpha = 06^{\text{h}}21^{\text{m}}14.5^{\text{s}}$, $\delta = -47^{\circ}53'13''$ (ICRS), corresponding to $l = 255.94^{\circ}$, $b = -24.62^{\circ}$ in galactic coordinates and $\lambda = 101.06^{\circ}$, $\beta = -71.12^{\circ}$ in ecliptic coordinates. The high ecliptic latitude ($|\beta| \sim 71^{\circ}$) guarantees a continuous operational cycle greater than 93%, thanks to an optimal geometry for the illumination of the solar panels and the thermal stability of the satellite (Nascimbeni et al. 2025).

The LOPS2 field covers an area of about $2,149 \text{ deg}^2$, within which each star is observed by a variable number of telescopes (from 6 to 24), with greater overlap in the central region ($\sim 325 \text{ deg}^2$). It contains about 8,200 P1 stars, 700 P2, 12,400 P4, and more than 167,000 P5, according to the *PLATO* Input Catalog v2.0.0 (EOL22 scenario). This region contains a wide range of important targets, including known planetary systems with more than 100 confirmed exoplanets and over 500 candidates identified by *TESS*. It also hosts several bright stars, such as TOI-700 and HD 23472, nearby open clusters including

NGC 2516, IC 2391, and Alessi 34, as well as the Large Magellanic Cloud (LMC). The field lies within the southern *TESS Continuous Viewing Zone*, allowing close observational collaboration and making it easier to carry out spectroscopic follow-up with instruments such as HARPS, ESPRESSO, and SPIRou.

From a technical point of view, the LOPS2 observations will use variable cadences: 2.5 s for the *Fast Cameras*, 25 s for the P1–P4 samples, and 600 s for the P5 sample. Photometric extraction will be performed onboard, with transmission to Earth of reduced light curves and selected imagerettes. Data will be released on a quarterly basis, with public publication expected about 12–18 months after the acquisition period.

2.2.4 Expected mission yield

The overall scientific yield of *PLATO* is assessed in terms of the number of planets that can be detected and characterized, as a function of detection efficiency, observing strategy, and stellar sample properties.

Mission simulations and predictive models developed by Rauer et al. (2024) and Matuszewski et al. (2023) suggest that, under the nominal “2+2” observing scenario (two years on LOPS2 and two years on LOPN1, with 24 NCAMs in operation), *PLATO* should detect between about 4000 and 5000 confirmed planets. Of these, several hundred are expected to be Earth-sized or super-Earth planets located within the habitable zones of their host stars. In extended observing configurations, such as a 3+2+1 year scenario, the total number of detected planets could rise above 10000, substantially improving the statistical coverage across different stellar populations and orbital properties.

The predicted planet yield depends on a range of factors, including the observing strategy, the size and brightness distribution of the target sample, intrinsic planet occurrence rates, geometric transit probabilities, and the overall detection efficiency of the mission. The latter primarily depends on the transit depth and on the photometric noise level of the light curves, expressed through the noise-to-signal ratio (NSR). For the P1 stellar sample ($V \leq 11$, $\text{NSR} < 34 \text{ ppm h}^{-1}$), Earth-sized planets ($R_p \approx 1 R_{\oplus}$) are expected to be detected with nearly 100% efficiency. For the P5 sample ($V \leq 13$), detection efficiency remains close to 100% for Neptune-sized and giant planets, while it decreases to about 50% for planets smaller than $2 R_{\oplus}$ when $\text{NSR} < 80 \text{ ppm h}^{-1}$, and approaches zero for noisier conditions.

During the step-and-stare phases, *PLATO* will be sensitive to planets with orbital periods up to ~ 30 days, maintaining similar efficiency criteria as in the long-duration observing fields. While the detection of small planets becomes more difficult in the presence of higher noise levels, the mission’s unique combination of long time baselines, continuous high-precision photometry, and uniform stellar characterization is expected

to greatly expand the sample of known transiting exoplanets.

The planet occurrence rates used in the yield calculations are largely drawn from Fressin et al. (2013), with specific adjustments applied to the habitable zone. In this regime, current estimates remain highly uncertain and span a broad range, from 2% to 100%, for small planets orbiting bright stars ($V \leq 11$).

2.3 TESS

The *Transiting Exoplanet Survey Satellite* (TESS) is a NASA space mission dedicated to the detection of extrasolar planets through the transit method (Ricker et al. 2015) (see Fig. 2.6). Launched on 18 April 2018, TESS orbits the Earth in a highly elliptical 13.7-day orbit in a 2:1 resonance with the Moon, ensuring a stable thermal environment and continuous data downlink. The spacecraft carries four identical refracting telescopes ($f/1.4$, 10.5 cm aperture), each covering a $24^\circ \times 24^\circ$ field of view. Combined, they form a total instantaneous field of $24^\circ \times 96^\circ$ per observing sector (Ricker et al. 2015). TESS performs a nearly all-sky survey, monitoring each sector for approximately 27 days before moving to the next. The mission's primary scientific goal is to conduct a comprehensive census of transiting exoplanets around bright, nearby stars, identifying small planets orbiting both solar-type stars and M dwarfs suitable for follow-up and atmospheric characterization.

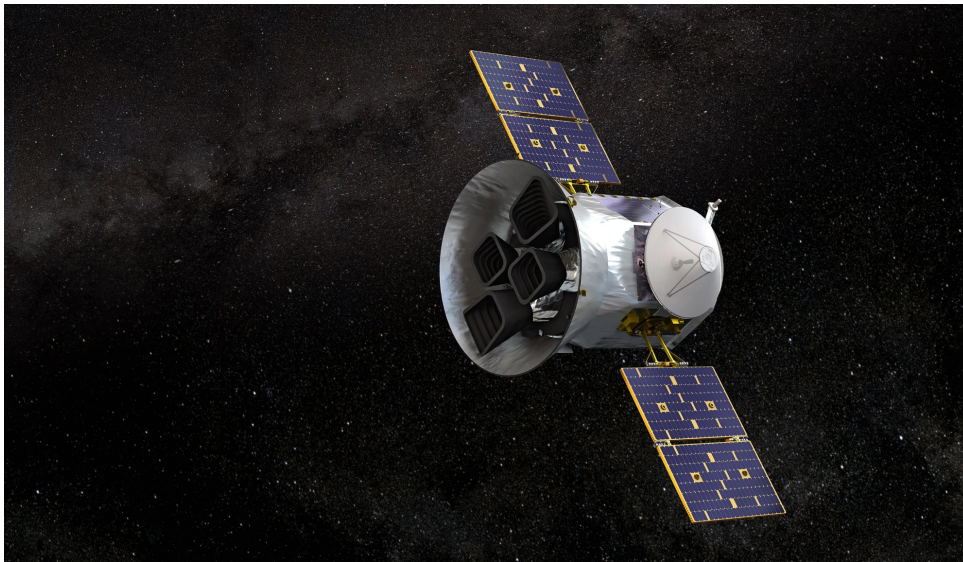


Figure 2.6: Artist's impression of NASA's *Transiting Exoplanet Survey Satellite* (TESS). Image credit: NASA / MIT.

2.3.1 Technical Overview

Each of the four TESS cameras consists of a lens assembly with an $f/1.4$ aperture and a focal plane hosting four MIT/Lincoln CCID-80 CCDs, each with 2048×2048 pixels. This configuration yields a total detector size of 4096×4096 pixels per camera, with a pixel scale of $21'' \text{ pixel}^{-1}$ and a half-encircled energy within one pixel (Ricker et al. 2015). The cameras operate over a broad spectral band from 600 nm to 1000 nm, roughly corresponding to the Cousins I band, maximizing sensitivity to cool and low-mass stars while minimizing photon noise.

During observations, each camera continuously acquires 2-second exposures in a shutterless mode. These short frames are co-added on board into longer integrations: originally, 2-minute cadences for pre-selected targets and 30-minute Full Frame Images (FFIs). In the first extended mission, FFI cadence was reduced to 10 minutes, and in the second extension to 200 seconds. This improvement enhances temporal resolution and transit detectability for short-duration events. Photometric precision reaches $\sim 200 \text{ ppm hr}^{-1}$ for $T \approx 10 \text{ mag}$ stars, sufficient to detect terrestrial-sized planets transiting small stars.

2.3.2 Observing Strategy and Sky Coverage

TESS observes the sky by dividing it into 26 sectors, each covering $24^\circ \times 96^\circ$ and monitored for ~ 27 days (two spacecraft orbits) before rotating to the next field (Ricker et al. 2015). During the primary two-year mission (2018–2020), TESS surveyed 13 sectors in the southern ecliptic hemisphere in the first year and 13 in the northern hemisphere in the second, nearly completing all-sky coverage. This strategy produces two Continuous Viewing Zones (CVZs) near the ecliptic poles, where adjacent sectors overlap and targets are observed continuously for up to one year per hemisphere.

The mission has since undergone several extensions. The first extended mission (**EM1**, July 2020–June 2022) repeated the hemisphere coverage and introduced new observing cadences: 20 s for selected targets, 2 min for standard postage stamps, and 10 min for FFIs. The second extension (**EM2**, September 2022–October 2024) further shortened the FFI cadence to 200 s while maintaining the other integration modes. A third extension (**EM3**) is planned for 2025–2028, pending NASA Senior Review approval, with continued sky scanning that revisits both hemispheres and the ecliptic plane.

2.3.3 Scientific Output and Mission Extensions

TESS was designed to detect thousands of transiting exoplanets around bright stars ($I < 12$), serving as a bridge between the deep-field statistical survey of *Kepler* and

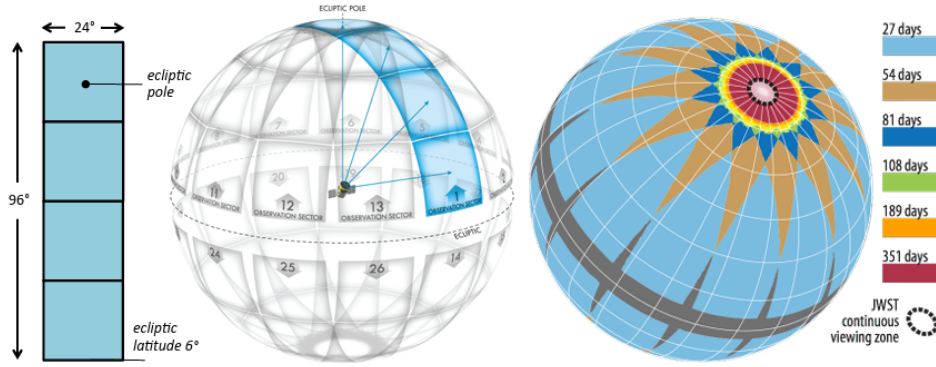


Figure 2.7: Schematic illustration of the *TESS* sector tiling strategy. Each blue stripe represents a $24^\circ \times 96^\circ$ observing sector, progressing around the sky to achieve full coverage. Near the ecliptic poles, the overlapping sectors define the Continuous Viewing Zones (CVZs), where targets are observed for up to a full year. Figure adapted from Ricker et al. (2015).

the upcoming focused characterization by *PLATO* and *JWST*. To date, the mission has identified over 6000 planet candidates (TESS Objects of Interest, TOIs), with several hundred confirmed through ground-based spectroscopy and photometry (Hesse 2025). The extended missions have expanded the sample of long-period planets and improved the ephemerides of previously discovered systems, while also enabling variable-star studies, eclipsing binaries, and transient events across the sky.

2.3.4 Synergy with PLATO

The *PLATO* mission will observe two long-duration fields (LOPN1 and LOPS2) for several years, and these regions have already been extensively covered by *TESS*. According to Eschen et al. (2024), each star within the southern *PLATO* field (LOPS2) will have been observed by *TESS* for an average of ~ 330 days by 2025, with central regions near the southern ecliptic pole monitored for over 1000 days. In these regions, *TESS* has already detected more than 500 planetary candidates. Consequently, *TESS* will have discovered nearly all transiting giant planets with orbital periods up to ~ 30 days (and up to ~ 100 days in the CVZs) within the *PLATO* fields. *PLATO*, in turn, will reach superior sensitivity to smaller ($1-4 R_\oplus$), longer-period planets thanks to its extended continuous monitoring and improved photometric precision. Thus, *TESS* provides a comprehensive catalog of pre-identified targets, while *PLATO* will refine and extend these results, enabling precise characterization and detection of new terrestrial planets. Together, the two missions offer complementary and synergistic views of planetary system architectures across the sky.

Three Algorithms for Multiple-Lens Microlensing Events

This chapter is based on:

**“VBMicroLensing: Three algorithms for multiple lensing
with contour integration”**

V. Bozza, V. Saggese, G. Covone, P. Rota, and J. Zhang

Astronomy & Astrophysics, 694, A219 (2025)

(Bozza et al. 2025)

3.1 The challenge of Multiple-Lens Microlensing

Thanks to the increase in the photometric precision and in the temporal coverage of microlensing surveys, and thanks to the future contribution of space surveys, the number of analyzed events is destined to grow rapidly (Mao 2012; Gaudi 2012; Tsapras 2018; Mroz et al. 2023; Covone et al. 2000). Future datasets will increasingly yield complex systems which will involve multiple compact objects as lens, with geometric configurations and mass ratios that make the lens equation more difficult to solve. The modelling therefore requires increasingly accurate and stable numerical approaches, capable of guaranteeing an efficient sampling of the light curves and of the astrometric variations.

The main challenges derive from the highly non-linear character of the lens equation; for a number of lenses N , the equation can be rewritten as a complex polynomial of degree N^2+1 (Witt 1990). However, this formulation, although formally exact, introduces serious problems of numerical stability and a high computational cost, which grows rapidly with N . When one or more masses have extremely small mass ratios or very wide separations, rounding errors and numerical instabilities become dominant, compromising the accuracy of the solutions. In such conditions, traditional methods manage to maintain numerical stability only by resorting to representations in extended algebra, with an inevitable increase of the computational cost and a significant slowdown of the simulations.

Over the years, several approaches have been developed to face these problems. Among the most widespread methods are the inverse ray-shooting (Kayser et al. 1986; Dong et al. 2006), which projects a large number of rays in the image plane to reconstruct the amplification regions, and the contour integration, initially introduced for binary lenses (Dominik 1995; Dominik 1998; Gould et al. 1997) and later implemented in public codes such as VBBinaryLensing (Bozza 2010; Bozza et al. 2018; Bozza et al. 2021). The latter has represented a remarkable step forward in the modelling of binary events, thanks to an optimized management of the errors and to an adaptive sampling of the contour of the source, capable of balancing efficiency and accuracy. However, the direct extension of such methods to multiple systems has encountered significant limits: the numerical complexity increases in a non-linear way, and the numerical noise related to the polynomial resolution tends to grow rapidly already for $N=3$. In parallel, methods such as the image-centered ray-shooting (Bennett et al. 1996; Bennett 2010) have shown great flexibility and have been widely employed in real cases, but they also require an accurate resolution of the lens equation for the center of the source, an operation that becomes burdensome in multiple or dynamic systems. When the configurations vary due to the orbital motion of the lens masses, the generation and reuse of the amplification maps turn out to be ineffective, making the calculations prohibitively slow for extended statistical analyses.

In light of these difficulties, it becomes necessary to develop new algorithms capable of combining the robustness and the precision of the contour integration with a better efficiency in the treatment of systems with many lenses. The goal is not only to solve the lens equation for specific configurations, but to make possible the automated and large-scale analysis of complex events, such as those expected from future high-precision surveys. This assumes an even greater value in view of the launch of the Nancy Grace Roman Space Telescope (Observations Time Allocation Committee et al. 2025), which promises to revolutionize the field of microlensing thanks to a continuous temporal coverage and to an unprecedented sensitivity (100 ppm). It is therefore expected that the Roman will reveal a large number of signals so far unobserved, due not only to very low-mass planets, but also to distant stellar companions or, in some cases, even to natural satellites. These effects, although of small amplitude, could manifest themselves in planetary microlensing events. It becomes evident the necessity to develop new algorithms capable of combining the robustness and the precision of the contour integration with a better efficiency in the treatment of systems with multiple components.

In this work, a set of three innovative algorithms is presented, designed to improve the numerical resolution and the stability of the calculations in the presence of multiple lens masses. The first approach, called *Singlepoly*, directly extends the classical polynomial formulation to multiple systems, maintaining the structure and philosophy of the *VBBinaryLensing* code. The second, *Multipoly*, introduces an original idea of multi-polynomial approach: instead of solving a single global polynomial, the equation is rewritten in local reference systems centered on each lens, minimizing the numerical errors due to the scale differences among the masses. Finally, the algorithm *Nopoly* represents a paradigm shift, applying a modified Newton-Raphson method directly on the original lens equation, without resorting to its polynomial transformation.

3.2 Algorithms for Computing the Magnification

3.2.1 Inverse Ray Shooting

The *Inverse Ray Shooting* (IRS) technique directly exploits the lens equation. It is a method in which a large number of light rays are projected from the image plane back to the source plane. In practice, rays are launched uniformly over the image plane, with the construction of a grid, and traced through the gravitational potential of the lenses, employing directly the lens equation without the need for manipulations. The relative density of rays that land inside the source area provides a measure of the magnification. This technique was first described by Kayser et al. (1986) and has since been widely applied e.g., (Dong et al. 2006; Gaudi et al. 2008).

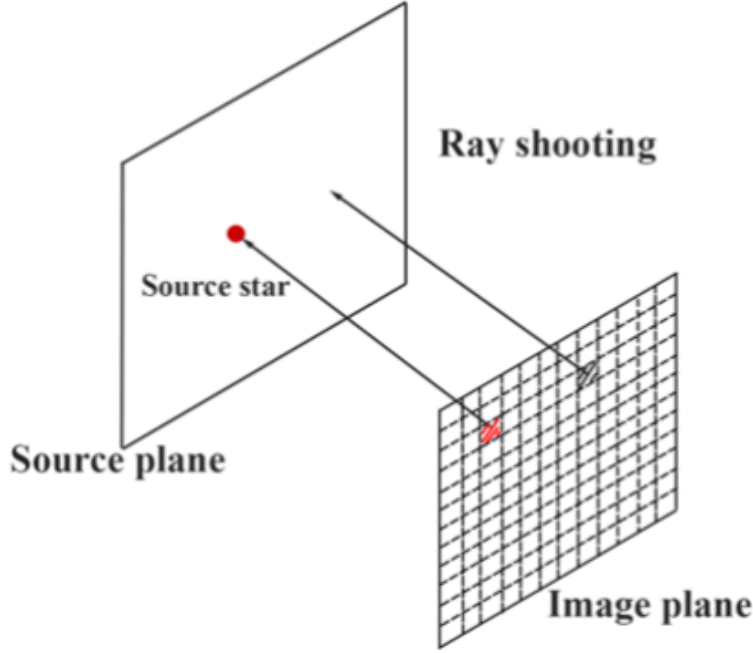


Figure 3.1: Schematic representation of the Inverse Ray Shooting (IRS) method. A uniform grid of rays is projected from the image plane toward the source plane, following the deflections imposed by the gravitational potential of the lenses. The density of rays collected in each region of the source plane provides a direct estimate of the local magnification.

As shown in Fig. 3.1, rays originating from a regular grid in the image plane are traced back to the source plane, where they illuminate the source region according to the local magnification pattern.

The main advantages of IRS are conceptual simplicity and robustness: it naturally handles systems with many lenses and highly complex light curves, and it allows the inclusion of dynamical effects (e.g., orbital motion of the lenses) without solving high-degree polynomials. It also incorporates limb darkening in a straightforward way. Its main drawback is computational cost: accurate results require very large numbers of rays, especially near caustics where magnification gradients are steep.

In the IRS formalism, to build a magnification map one considers a large region of the lens plane and generates a uniform distribution of rays in that space. Each ray, starting from a point $\vec{\theta}$ on the image plane, is deflected by the N lens masses according to the lens equation and arrives at a position $\vec{\beta}$ on the source plane. The landing positions are accumulated on a discrete source-plane grid (pixels). The point-source magnification of a pixel (i, j) is estimated as

$$\mu_{ij} = \frac{N_{ij}}{N_{\text{rays}}}, \quad (3.1)$$

where N_{ij} is the number of rays falling into pixel (i, j) and N_{rays} is the expected number of rays in the absence of lensing (i.e., a uniform distribution). Regions where rays cluster more densely than average correspond to higher magnification. This discrete definition follows from Liouville’s theorem (surface brightness conservation) and the standard definition of magnification as the ratio of image to source areas.

Because the method is Monte Carlo in nature, it introduces statistical (Poisson) noise in the magnification map. Each pixel receives N_{ij} rays with fluctuations of order $\sqrt{N_{ij}}$. To reduce this noise below observational thresholds, a very high number of rays per pixel is required (often thousands). Modern simulations employ up to 10^8 – 10^{11} rays in total, making the statistical error negligible compared to observational photometric uncertainties. It is also common to apply smoothing or filtering techniques to mitigate residual shot noise, or to use sub-pixel interpolation in regions of strong magnification gradients.

The IRS method is straightforward but computationally expensive, as it requires computing deflections for enormous numbers of rays and collecting statistics on the source plane. For this reason, it has become practical only with the advent of parallel hardware (multi-core CPU clusters or GPU acceleration). GPU implementations can compute billions of deflections per second, generating large magnification maps (e.g. 4096^2 pixels) in reasonable timescales. Zhang et al. (2022), for instance, showed that by replacing the traditional circular lens-plane setup with a rectangular adaptive grid and interpolating deflections, one can achieve a speedup of $\sim 100\times$ without significant loss of accuracy.

As illustrated in Fig. 3.2, IRS provides a direct and visual way to explore the structure of magnification patterns in multi-lens systems. In the case of a single stellar lens (top panel), the map exhibits circular symmetry, with magnification peaking at the Einstein radius. When a low-mass companion is introduced (bottom panel), this symmetry is broken: a complex network of caustics and magnification gradients appears, reflecting the non-linear interference between the lens potentials. The resulting structure encodes information about the companion’s mass ratio and separation, making IRS simulations an indispensable diagnostic tool for planetary microlensing studies.

Alternative approaches have been developed. Tree codes (Wambsganss 1999) reduce computational cost by grouping distant lenses, at the expense of some approximation. Image-centered ray shooting (Bennett 2010), which launches rays near known image positions.

In summary, IRS remains the most general and versatile method, with no restrictions

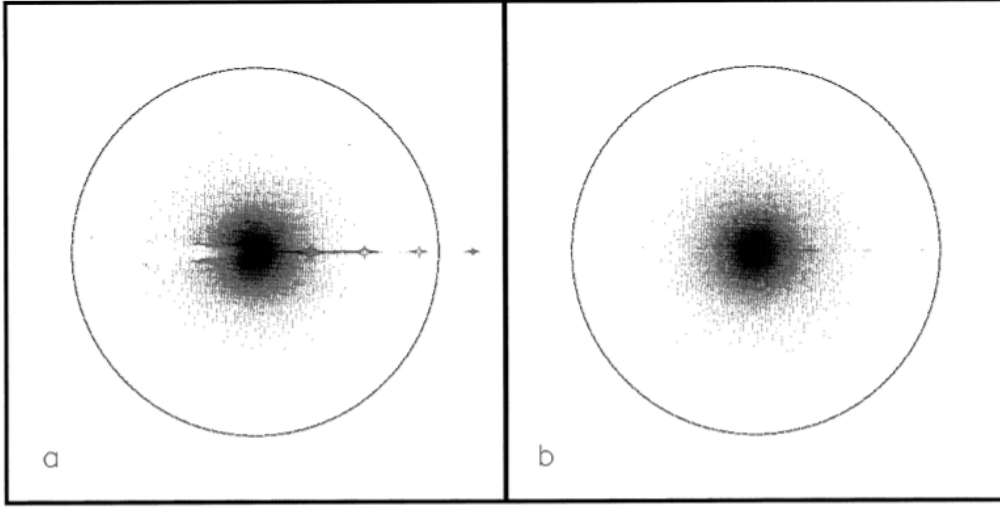


Figure 3.2: Example of magnification patterns generated through the Inverse Ray Shooting technique (adapted from Wambsganss (1997)).

on the number of lenses and a natural treatment of finite-source effects, though it comes at a high computational cost. Ongoing improvements in hardware and algorithms are steadily increasing its efficiency. Once a magnification map is generated, light curves can be produced quickly for any source trajectory, which is essential for high-magnification events where small shifts in the source plane lead to rapid changes in amplification. Despite being more computationally demanding than semi-analytic alternatives, IRS continues to be a cornerstone of modern microlensing simulations.

3.3 Contour Integration Method in Microlensing

A second methodology that is placed side by side with the inverse ray shooting is the contour integration based on Green's theorem (Schramm et al. 1987; Gould et al. 1997; Dominik 1998). This methodology is based on the conversion of a surface integral into a line integral performed on the contour of the image. In this section we describe this methodology, which in many cases turns out to be the most efficient technique.

3.3.1 Green's Theorem and Area Integration

In the formalism that we have introduced of microlensing, the lens equation acts as a map that associates each point of the image plane ($\vec{x}$) to a position of the source plane:

$$\vec{y} = \vec{f}(\vec{x}) \quad (3.2)$$

The conservation of surface brightness, consequence of Liouville's theorem, makes so that the gravitational lens does not alter the specific intensity along each light ray. The direction and the spatial distribution of the rays are modified, but not the number of photons per unit of area, time, frequency and solid angle. It follows that the observed intensity on the image plane $I_l(\vec{x})$ is equal to the source intensity evaluated in the corresponding position of the source plane:

$$I_l(\vec{x}) = I_s(\vec{y}(\vec{x})). \quad (3.3)$$

Let A_s be the region of the source plane occupied by the extended source and A_l the region of the image plane covered by its images. The *total magnification* μ is defined as the ratio between the total observed flux (integral of the brightness on the image plane) and the emitted flux (integral of the brightness on the source plane):

$$\mu = \frac{\int_{A_l} I_l(\vec{x}) d^2\vec{x}}{\int_{A_s} I_s(\vec{y}) d^2\vec{y}}. \quad (3.4)$$

This definition represents the most general form of the magnification in microlensing and is valid for any brightness distribution $I_s(\vec{y})$ (Dominik 1998).

For each image i generated by the lens system, the area A_i of the image region can be expressed using Green's theorem as a line integral over its contour C_i :

$$A_i = \frac{1}{2} \oint_{C_i} (x dy - y dx). \quad (3.5)$$

In complex notation $z = x + iy$, this can be rewritten compactly using the *wedge product* defined by $z_1 \wedge z_2 = \Im(\bar{z}_1 z_2) = x_1 y_2 - y_1 x_2$. The total signed image area is then

$$A_{\text{tot}} = \sum_i \Pi_i A_i = \frac{1}{2} \sum_i \Pi_i \oint_{C_i} (x dy - y dx), \quad (3.6)$$

where $\Pi_i = \pm 1$ is the *parity* of the i -th image (negative-parity images contribute with opposite sign due to orientation reversal). The final magnification is obtained as $\mu = A_{\text{tot}}/A_s$, with A_s the unlensed source area.

For a circular source of radius ρ , the source boundary can be parametrized as $\zeta(\phi) = \rho e^{i\phi}$ with $\phi \in [0, 2\pi)$. For each angle ϕ_j , one solves the lens equation to obtain the image positions $z_{i,j}$. The line integrals along each image contour are then approximated by summing contributions along small arcs between consecutive points. In compact form, the basic trapezoidal rule yields contributions proportional to $(z_{i,j+1} \wedge z_{i,j})$, and summing

over all arcs reconstructs the total area.

3.4 Singlepoly

In the Singlepoly method we exploit the polynomial associated with the lens equation (1.31). The method is a generalization of the one used in VBBinaryLensing. The roots of the polynomial, which correspond to the positions of the physical images or spurious solutions (ghost images) in the lens plane, can be found through numerical algorithms for polynomial root search. Singlepoly, as well as VBBinaryLensing, adopts the algorithm of Skowron et al. (2012). The direct resolution of the polynomial quickly becomes burdensome as the number of lenses increases because of the high polynomial degree and the numerical problems associated. The accuracy of the roots degenerates as one root is found, since to find the next one it is necessary to divide the polynomial by the previous root and this operation introduces numerical errors. A way to reduce the numerical errors is to center the reference system with respect to which the polynomial is constructed on the smallest mass. In the case of a single planet ($N = 2$) it is sufficient to center the origin on the planet to have a significant reduction of numerical noise; however, in the case of a planetary system with more than one planet, this stratagem starts to be less effective and the roots near the other bodies can result inaccurate, in particular when the mass ratio of both objects accompanying the primary are very low. From Figure 3.3 it can be seen that the algorithm keeps its effectiveness when the second mass, numbering them in decreasing order with respect to the mass, has a mass smaller than $10^{-3}m_1$. In this situation the algorithm does not fail even in cases where the third object has a mass smaller than $10^{-7}m_1$.

From the implementation point of view, the algorithm is a direct extension of the numerical techniques developed within the code VBBinaryLensing (Bozza 2010; Bozza et al. 2018; Bozza et al. 2021). In the Singlepoly code, the discretization of the contour is managed through an adaptive grid of angular sampling, which is dynamically refined as a function of the proximity to the caustics. To improve convergence and reduce the error, the algorithm implements the parabolic corrections (Bozza 2010). These corrections consist in including higher-order terms in the estimate of the elementary area associated with each contour arc, allowing to reach a numerical error of the order of $O(\Delta\phi^5)$, compared to the order $O(\Delta\phi^3)$ of the standard trapezoidal method.

A further relevant element is represented by the extension of the local error estimators, which in the original context of VBBinaryLensing were based on the behavior of the ghost images. The ghost images are used to diagnose the distance of the source from a fold caustic, providing a quantitative indicator of the level of accuracy required in that region of the domain. In the binary case the ghost images, if they exist, are always only

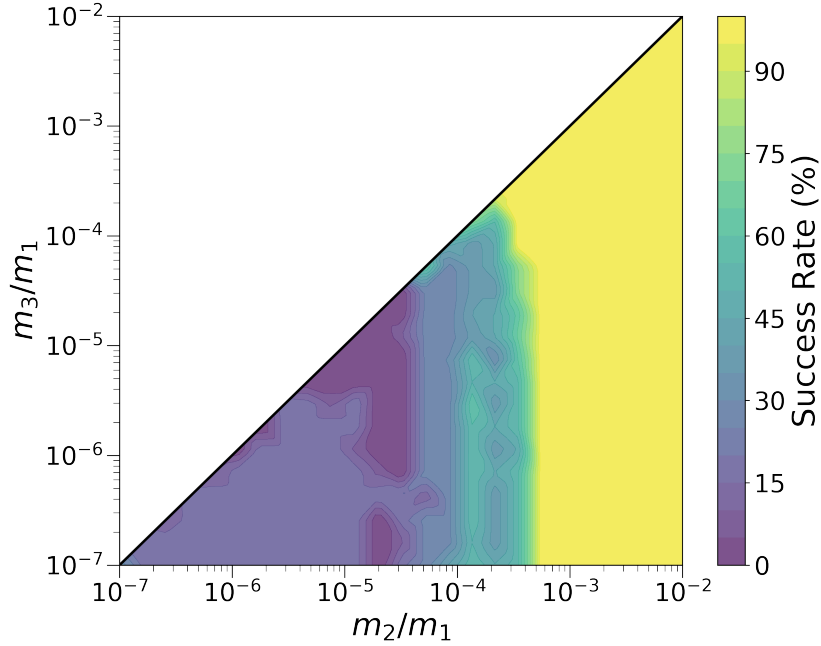


Figure 3.3: The figure illustrates the limitations of the Singlepoly algorithm. For this analysis, five lens configurations were selected, with separations between 0.2 and 1.2 and the mass ratio parameters were varied keeping $m_3 < m_2 < m_1$. For each configuration, five source trajectories were tested, and the success rate of the Singlepoly algorithm was evaluated. The figure clearly shows that the Singlepoly algorithm fails when both mass ratios are on the order of 10^{-4} or smaller. Figure adapted from (Bozza et al. 2025)

two, since the polynomial has degree 5 and 3 images are surely real, from the image theorem. In the case of multi-lens systems, however, the situation is more complex, because the total number of solutions (real and spurious) grows rapidly with the number of lenses involved. Singlepoly implements a selection logic based on the criterion of minimum distance between ghost images: among all the pairs of spurious solutions, the one with the smallest mutual separation is identified, which typically corresponds to the two images that will merge during a fold caustic crossing. By monitoring this distance as a function of time or of the source position, the code predicts the appearance of new images and consequently adjusts the sampling density and the integration tolerance.

3.5 Multipoly

The Multipoly method represents a numerically more stable extension of the Singlepoly method. As introduced in the previous section, its introduction arises from the need to reduce the inaccuracy in the search for the roots of the polynomial associated with the lens equation. With the Multipoly method, the global problem of finding the roots of

the polynomial is decomposed into the resolution of different polynomials, each with its respective reference system centered on one of the lenses composing the system. This approach allows exploiting the greater stability of the roots found in proximity to the center of reference of the system.

The `Multipoly` algorithm chooses as the first reference system the one centered on the lens with the *smallest mass* among the N components of the system, indicated with ℓ_1 . In the local frame associated with that lens, the coordinates in the image plane are expressed as

$$z' = z - z_{\ell_1}, \quad (3.7)$$

where z_{ℓ_1} is the complex position of the considered lens. Similarly, the position of the source is translated into the same reference system:

$$\zeta' = \zeta - z_{\ell_1}. \quad (3.8)$$

The lens equation in the local frame thus takes the form

$$\zeta' = z' - \sum_{j=1}^N \frac{m_j}{z' - (z_j - z_{\ell_1})}, \quad (3.9)$$

from which, eliminating the conjugate term $\overline{z'}$, one obtains the complex polynomial

$$P_1(z') = 0, \quad (3.10)$$

Unlike the `Singlepoly` method, `Multipoly` does not completely solve the polynomial $P_1(z')$. The resolution proceeds incrementally; Whenever a new root $z_k^{(1)}$ is found, the algorithm evaluates its position with respect to the different lenses of the system by calculating the distances

$$d_j(z_k^{(1)}) = |z_k^{(1)} - z_j|, \quad j = 1, \dots, N. \quad (3.11)$$

The nearest lens is then identified as

$$\hat{\ell} = \min_{j \in \{1, \dots, N\}} d_j(z_k^{(1)}). \quad (3.12)$$

If the root found is closer to the lens around which the frame is centered ($\hat{\ell} = \ell_1$), it is accepted as belonging to the domain of that lens. If instead $\hat{\ell} \neq \ell_1$, it means that the root is closer to another lens: in that case, the process of solving the polynomial $P_1(z')$ is interrupted, and the algorithm moves to the frame centered on the lens $\ell_2 \equiv \hat{\ell}$.

The procedure then continues in an analogous way, moving from lens to lens in order

of increasing mass.

Once the last frame is reached, centered on the lens of greater mass ℓ_N , the method already has a set of roots found in the previous frames, each referring to a local domain. All the roots found are first translated into the new reference system and used to deflate the final polynomial:

$$P_N(z) = \frac{P_{N-1}(z)}{\prod_{k=1}^{n_{N-1}} (z - z_k^{(N-1)})}. \quad (3.13)$$

The polynomial thus obtained has reduced degree, of the order of the remaining images to be determined, and can be solved completely, ensuring that all roots are found.

These last roots will be those most subject to numerical error.

Figure 3.4 illustrates an example of the `Multipoly` algorithm applied to a three-lens system, with the lenses indicated in gray and numbered in order of increasing mass. The opaque roots represent the real images, while the semi-transparent ones correspond to the spurious (*ghost*) images generated during the resolution of the intermediate polynomials.

Compared to the `Singlepoly` method, the `Multipoly` approach focuses on the accuracy of the solutions rather than on computational speed. The resolution of several local polynomials, instead of a single global polynomial of degree $N^2 + 1$, reduces the propagation of rounding errors and the loss of numerical significance in cases where the mass ratios are extreme ($m_i/m_j \ll 1$) or the separations between the lenses are very small.

In particular, the planetary images formed near a small-mass lens are calculated with accuracy comparable to that of a binary lens centered on the planet, avoiding the instability observed in `Singlepoly`. The computational complexity grows linearly with the number of lenses, but the gain in numerical robustness is significant.

The method also retains an important advantage: it preserves the information related to the spurious (*ghost*) images produced at each intermediate step, an advantage not present in the method that we will see in the next section, `Nopoly`. These are used to estimate the proximity of the source to the caustics, as done in `Singlepoly`.

3.6 Nopoly

The `Nopoly` method represents a different approach compared to the previous `Singlepoly` and `Multipoly` algorithms. The `Singlepoly` and `Multipoly` methods use the polynomial formulation of the lens equation but this transformation introduces several sources of numerical errors: the conversion to polynomial form itself can amplify arithmetic inaccuracies, and the successive polynomial divisions performed after each root is found may further degrade numerical stability. From this consideration arises the idea behind

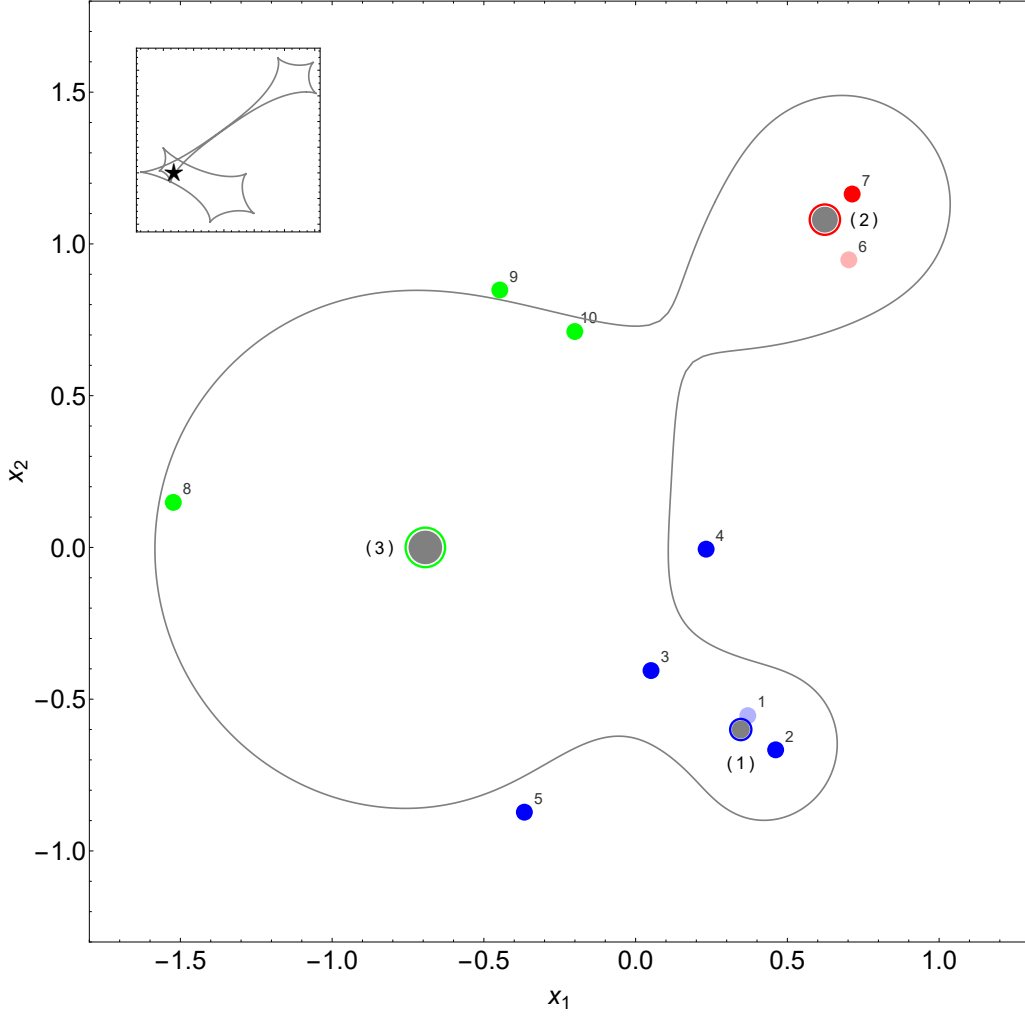


Figure 3.4: Schematic representation of the functioning of the `Multipoly` algorithm in a three-lens system. The lenses are indicated in gray and numbered (1), (2), (3) in order of increasing mass, corresponding to the order of construction of the local polynomials. The critical curves associated with each lens are shown, and the roots found are colored according to the polynomial from which they derive. The opaque roots represent real images, while the transparent roots represent spurious (*ghost*) solutions. Figure adapted from (Bozza et al. 2025)

the `Nopoly` method: to solve the lens equation directly from its original form through a Newton–Raphson type method. An additional advantage of applying directly the lens equation is its more favorable scaling behavior. When the polynomial formulation is used, the degree of the polynomial increases as $N^2 + 1$, where N is the number of lenses. As a result, the number of roots to be found scales quadratically with N , including non-physical solutions. It should be noted, however, that the *ghost images* are not entirely a drawback, as we have explained in the previous sections. In contrast, according to the image theorem, the number of physical images increases only linearly with N , which makes the `Nopoly` method more efficient as N grows.

3.6.1 Newton–Raphson Algorithm

To simplify the notation and make the following derivations more compact, we adopt the auxiliary complex functions introduced by Bozza et al. (2025):

$$S_q(z) = \sum_{i=1}^N \frac{m_i}{(z - a_i)^q}. \quad (3.14)$$

$S_q(z)$ provides a compact notation for the contribution of each lens. From this definition it follows that the residual of the lens equation can be written as

$$L(z) = \bar{\zeta} - \bar{z} + S_1(z), \quad (3.15)$$

which vanishes when z corresponds to an image of the source and directly measures how far a trial point lies from solving the lens equation.

The local properties of the mapping are described by the Jacobian determinant,

$$J(z) = 1 - S_2(z) \overline{S_2(z)}, \quad (3.16)$$

whose zeros trace the critical curves in the image plane, marking the boundary between regions of opposite image parity.

Starting from an initial guess z_k , a refined approximation of the image position can be obtained by applying an iterative correction of the form $z_{k+1} = z_k + \epsilon$, where ϵ is a small complex increase. In this formulation, the iteration aims to minimize the complex residual of the lens equation $L(z)$, driving it to zero through successive linearizations of the mapping around z_k .

Expanding the lens equation to first order around the z_k in ϵ leads to a linearized system for the complex conjugate pair $(\epsilon, \bar{\epsilon})$:

$$\begin{cases} \bar{L}(z_k) = \epsilon + \overline{S_2(z_k)} \bar{\epsilon}, \\ L(z_k) = \bar{\epsilon} + S_2(z_k) \epsilon. \end{cases} \quad (3.17)$$

The linearized system can then be solved for the correction ϵ .

Solving Eq. (3.17) yields the Newton–Raphson correction step:

$$\epsilon = \frac{\bar{L}(z_k) - \overline{S_2(z_k)} L(z_k)}{J(z_k)}, \quad (3.18)$$

This relation shows that the correction term ϵ depends on both the residual of the lens equation $L(z_k)$ and the local derivatives of the mapping described by $S_2(z_k)$.

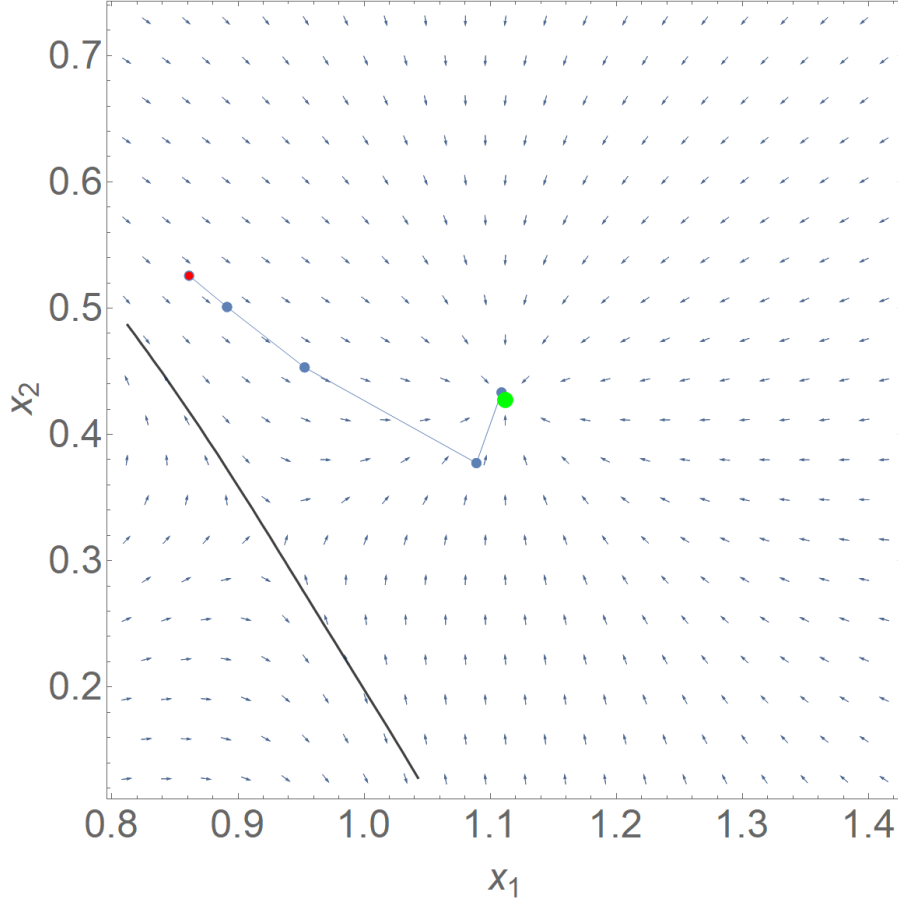


Figure 3.5: Vector field of the Newton–Raphson correction ϵ in the image plane. The arrows represent the direction of the correction. Starting from an initial guess (red dot), the iteration converges toward a true image position (green dot) in regular regions, while close to the critical curve (gray line, where $J(z) \rightarrow 0$) the vectors become distorted and the convergence may fail. Figure adapted from (Bozza et al. 2025)

The updated image position is then obtained as

$$z_{k+1} = z_k + \epsilon, \quad (3.19)$$

which represents one iteration of the Newton–Raphson method.

When the initial guess z_0 lies sufficiently close to a true image, the sequence z_k converges quadratically toward the solution, making the method extremely efficient. Near the critical curves, where $J(z) \rightarrow 0$, the denominator in Eq. (3.18) amplifies the correction term, possibly resulting in overshooting or divergence. In regular regions, the vectors smoothly guide the iteration toward the image positions and we can see an example in the Figure 3.5

The essential advantage of this approach is that no polynomial is constructed: each iteration acts directly on the function $L(z)$, preserving numerical precision even for

systems with strong gravitational gradients or highly unbalanced masses. However, this method presents a major limitation: the need to vary the initial conditions many times in order to find all the roots, also putting us in the situation where we can never be sure of having identified them all.

3.6.2 Numerical Corrections to the Newton–Raphson Method

To guarantee the robustness of the iteration, the Nopoly method applies a series of numerical corrections that modify the step ϵ whenever it tends to diverge or oscillate. The goal is to prevent the Newton-Raphson process from leaving the domain of convergence. By dynamically regulating ϵ , the method preserves the stability of the iteration even in regions where the Jacobian determinant $J(z)$ approaches zero.

a) Oscillation control. After each iteration, the position z_{k+1} is compared with the two previous ones. If $|z_{k+1} - z_{k-1}| < |\epsilon|$, the case is interpreted as an oscillation around a quasi-convergent region. To break the chain, the length of the step ϵ is reduced by multiplying it by a random factor.

b) Avoid explosive steps. To prevent the correction ϵ from growing excessively when the denominator, the Jacobian, becomes too small, the following control factor is introduced:

$$f = \frac{|\epsilon|}{|z_k - z_{k-1}| [1 + \min(1, |J_k|/|J_{k-1}|)]}. \quad (3.20)$$

Whenever $f > 1$, the step is rescaled as $\epsilon \mapsto \epsilon/f$ to enforce the condition $f \leq 1$. In practice, if the absolute value of the Jacobian $|J_k|$ is smaller than that of the previous iteration, the step is shortened to avoid entering unstable regions near the critical curve. Conversely, when $|J_k|$ increases, it is allowed to proceed faster in regular regions of the mapping.

c) Avoiding crossings of critical curves. The Newton-Raphson iteration may drive the trajectory across a critical curve, thus switching between regions of opposite parity. This may occur when the step ϵ is evaluated near a point where the Jacobian $J(z)$ approaches zero, with a consequent sign inversion in the determinant and a change in the local mapping properties. These transitions can compromise the local consistency of the search; to maintain the efficiency and organization of the exploration process, the algorithm therefore introduces a correction that prevents individual branches from crossing the critical curves.

Whenever $\text{sign}(J_{k+1}) \neq \text{sign}(J_k)$, it is assumed that the iteration has crossed a critical

curve. A correction δ is introduced to bring the new point z_{k+1} back to the same parity region as z_k .

The local estimate of the distance from the critical curve, derived from the single-lens configuration, is given by

$$\delta_0 = \frac{S_2}{S_3} [(1 - J)^{1/4} - 1], \quad (3.21)$$

which, for small values of J , becomes $\delta_0 \simeq -J S_2/S_3$. The correction actually applied at each iteration is then

$$\delta = \delta_0 \left[1 - \frac{J(z_k)}{J(z_k + \epsilon)} \right], \quad (3.22)$$

In applying this correction, we check that the new position $z_{k+1} = z_k + \epsilon + \delta$ falls within the same parity region as z_k ; if this does not happen, we further increase the correction. Figure 3.6 shows an example where this correction comes into play.

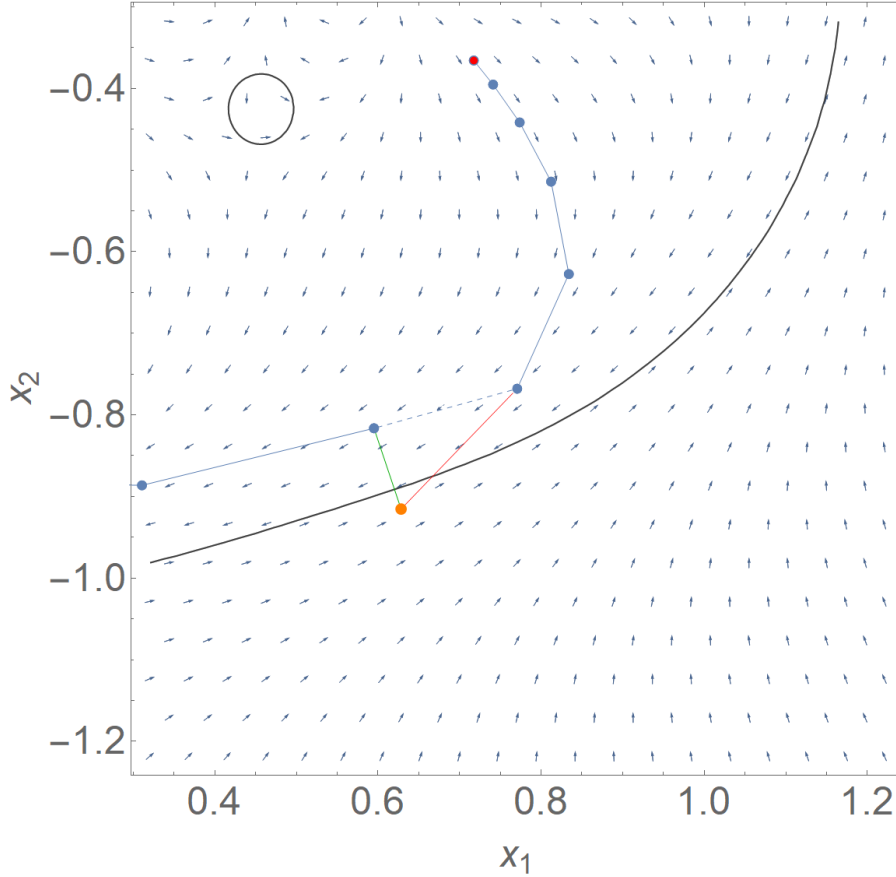


Figure 3.6: Example of the correction δ (Eq. 3.22) that prevents crossing a critical curve. The Newton step would bring the iteration beyond the critical curve (gray), but the correction (green line) returns it to the initial region, preserving the sign of the Jacobian. Figure adapted from (Bozza et al. 2025)

d) Backtracking correction. If after the update the lens equation is less satisfied ($|L(z_{k+1})| > |L(z_k)|$), it means that the direction of the step is wrong or too long. In this case, an additive correction κ is applied, based on the local gradients of J and $|L|^2$:

$$dJ = 2S_3 \overline{S_2}, \quad (3.23)$$

$$dL^2 = -\overline{L} + \overline{S_2} L, \quad (3.24)$$

$$\kappa = -\frac{|L|^2}{\text{Re}[dJ]\text{Re}[dL^2] + \text{Im}[dJ]\text{Im}[dL^2]} dJ. \quad (3.25)$$

The updated iteration $z_{k+1} = z_k + \epsilon + \delta + \kappa$ thus becomes corrected and closer to the physical solution.

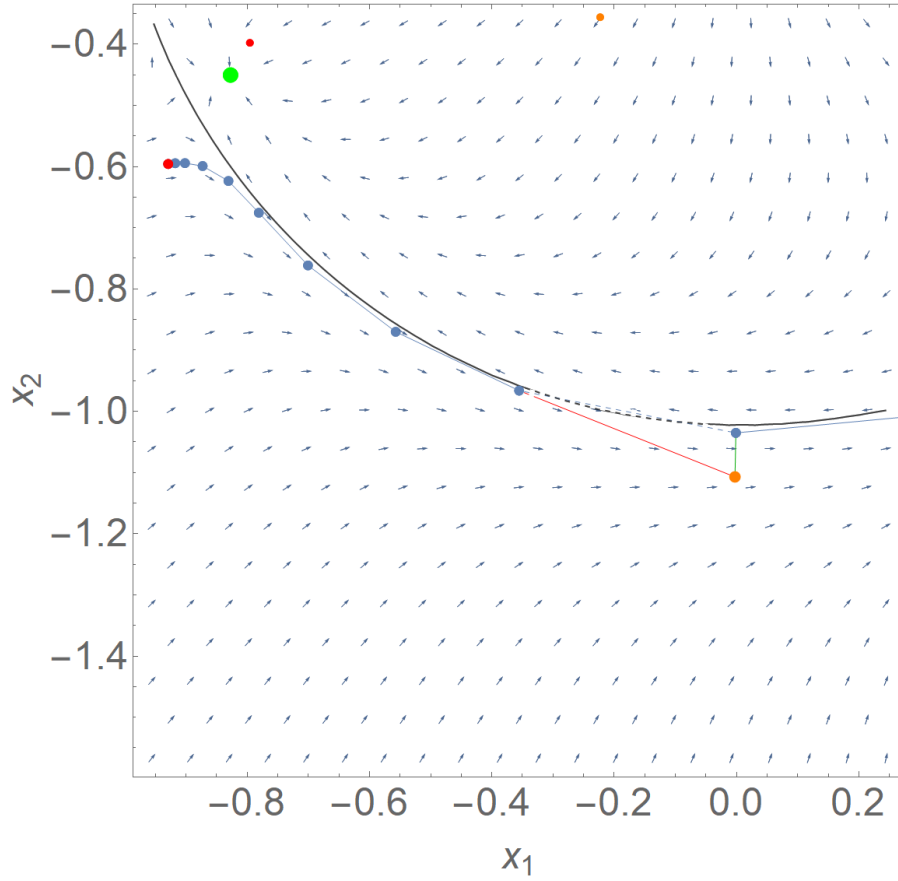


Figure 3.7: Example of the correction κ (Eq. 3.25) applied when the last Newton step worsens the adherence to the lens equation. The orange arrow represents the uncorrected step, while the green one shows the shortened and reoriented displacement that brings the iteration closer to the critical curve and improves convergence. Figure adapted from (Bozza et al. 2025)

3.6.3 Stopping Criteria

To ensure both accuracy and computational efficiency, each iterative chain in the Nopoly algorithm is evaluated until a set of conditions is reached. These stopping conditions determine whether the sequence of iterations has reached a physically meaningful image position. The conditions to be satisfied are:

- the raw Newton-Raphson correction falls below the accuracy threshold, $|\epsilon| < 3 \times 10^{-11}$;
- after the application of possible corrective terms (to mitigate oscillations or divergences), the step becomes comparable to machine precision, $|\epsilon| < 10^{-14}$;
- the lens equation itself is fulfilled to within machine accuracy, $|L(z_k)| < 10^{-15}$.

These criteria ensure that the algorithm stops once the estimated image position is stable to a precision beyond a physically relevant scale.

Conversely, a chain is terminated and rejected when convergence clearly fails, that is, if:

- more than nine consecutive corrections δ are required to restore stability, indicating that the iteration is trapped near a critical curve;
- the total number of iterations exceeds 100, implying that no satisfactory convergence has been achieved within a reasonable computational effort.

These stopping criteria allow for a balance between reliability and efficiency: they prevent the algorithm from wasting time on divergent or cyclic sequences, while preserving its sensitivity to real image solutions, even in complex multi-lens configurations.

3.6.4 Error Estimate and Acceptance of Solutions

For each image identified by the Nopoly algorithm, a numerical uncertainty is assigned to evaluate the reliability of the final value. The error estimator combines two distinct contributions:

$$\text{Err} = |\epsilon| + \frac{3 \times 10^{-15}}{|J|}. \quad (3.26)$$

The first term, $|\epsilon|$, is the amplitude of the last Newton-Raphson correction, and provides a measure of the deviation from the final converged position. It quantifies the precision reached by the process.

The second term originates from the finite precision of the floating-point arithmetic. Assuming that all quantities in the lens equation are evaluated within the machine precision (about 10^{-15}), the error on the computed position is magnified by the inverse of the Jacobian determinant, $1/|J|$. This amplification reflects the local sensitivity of the lens mapping: near critical curves, where $|J| \rightarrow 0$, even infinitesimal numerical perturbations in the equation can produce relatively large variations in the image position.

The combined estimator in Eq. (3.26) provides a conservative measure of the numerical accuracy of each solution.

3.6.5 Duplicate Removal

After each Newton–Raphson chain has converged, the new solution z is checked against all previously identified images. The comparison is restricted to solutions with the same parity, same sign of the Jacobian determinant J , since images of opposite parity are in distinct regions of the mapping.

A new solution is accepted only if the distance from every other image z_i satisfies the condition

$$\frac{|z - z_i|^2}{\text{Err}^2 + \text{Err}_i^2 + 4 \times 10^{-20}} > 10, \quad (3.27)$$

which ensures that the separation between the two solutions is significantly larger than their combined numerical uncertainties. If this inequality is not satisfied for any z_i , the two solutions are flagged as duplicates; only the one with the smaller estimated error is kept in the final set. In this way, redundant solutions are removed.

3.6.6 Initial Conditions

As we previously mentioned, one of the weak points of this method is the need to choose the initial conditions appropriately in order to find all the images; therefore, the selection of z_0 becomes important.

The Nopoly algorithm adopts a hierarchical four-level strategy (illustrated in Figure 3.8).

Level 1 – Base images.

Outside any caustic, $N + 1$ images are expected (one of positive parity and N negative).

The positive-parity image is initialized as

$$z_+ = \zeta + \sum_i \frac{\zeta - a_i}{2} \left(\sqrt{1 + \frac{4m_i}{|\zeta - a_i|^2}} - 1 \right), \quad (3.28)$$

which is a first-order correction to the unperturbed source position ζ , obtained by adding the combined deflection from all lenses.

For each lens i , the corresponding negative-parity image is given by:

$$z_{-i} = a_i - \frac{\zeta - a_i}{2} \left(\sqrt{1 + \frac{4m_i}{|\zeta - a_i|^2}} - 1 \right), \quad (3.29)$$

Level 2 – Central images.

If two lenses i, j are very close ($|a_i - a_j| < m_i + m_j$), their potential generates an additional image z_{ij} near the center of mass of the pair, of negative parity ($J(z_{ij}) < 0$). To capture these solutions, a specific seed is introduced:

$$z_{ij} = \frac{m_i a_j + m_j a_i}{m_i + m_j}, \quad (3.30)$$

useful for capturing weak images in the center of the system.

Level 3 – Partner images.

For each image z already found, two additional *seeds* are generated on opposite sides of the nearest critical curve, according to

$$z_a^\pm = z + \delta_0(2 \pm i), \quad (3.31)$$

where δ_0 is the estimated distance from the image position to the closest critical curve, as derived from Eq. (3.21).

This choice is motivated by the local behavior of the lens mapping near caustics: when the source approaches or crosses a caustic, the number of images changes by two, and a pair of images with opposite parity appears on the two sides of the associated critical curve.

Each seed is refined through the Newton–Raphson iteration (Eq. (3.19)) to verify whether it converges to a new image or corresponds to a previously known one. In the second case, the solution with the smaller error is retained (Sect. 3.6.5). This process continues until no further new images are found, ensuring a complete sampling of the image plane for the given source position.

Level 4 – Final random search. After the previous steps, a parity check is performed according to the image theorem:

$$n_- - n_+ = N - 1, \quad (3.32)$$

If this relation is not satisfied, it means that at least one image is still missing. In this case, a final exploratory search is activated to recover any undetected solutions. This step performs a localized random sampling of the image plane around each lens position, generating trial seeds according to

$$z_{i,k,\lambda} = a_i + \lambda, \sqrt{m_i}, e^{ik\pi/6}, \quad k = 0, \dots, 11, \quad (3.33)$$

where λ is randomly drawn within a small interval around unity (typically of order a few tenths).

Each generated seed is processed through the standard Newton–Raphson iteration (Eq. (3.19)), and the resulting solutions are accepted or discarded following the duplicate and error criteria discussed in Sect. 3.6.5.

In summary, `Nopoly` combines the generality and precision of the Newton–Raphson method with a set of corrective rules that guarantee its stability even in critical regions of the image plane. Although slower than polynomial methods for small N , it becomes progressively more efficient and scalable as the number of lenses increases, establishing itself as a reference for the study of complex configurations in gravitational microlensing.

3.7 Comparison of Performance and Accuracy of the Methods

We have conducted a systematic analysis of the computational performance and accuracy of the three proposed algorithms: `Singlepoly`, `Multipoly`, and `Nopoly`.

The goal of this comparison is to delineate the operational domains in which each algorithm performs optimally, identifying the regimes of magnification and multiplicity, that favor one approach over the others.

Figure 3.9, adapted from Bozza et al. (2024), illustrates the average computation times of the three algorithms as a function of the magnification for a system composed of three lenses. The analysis was carried out for four representative geometrical configurations:

- (i) a linear alignment of the three masses;
- (ii) an equilateral triangle;
- (iii) an isosceles triangle;
- (iv) a planetary-type configuration analogous to the Sun–Jupiter–Earth system: a primary lens accompanied by two secondary bodies with mass ratios typical of giant and terrestrial planets.

For each configuration, an ensemble of source positions was simulated and grouped into bins of comparable magnification. Within each bin, the mean computational time required to obtain all image solutions was evaluated for each algorithm, providing a measure of their efficiency as a function of the magnification regime.

The data in Figure 3.9 show a consistent trend in all cases: in the case of a triple-lens system, `Singlepoly` is on average the fastest algorithm, followed by `Multipoly`, while `Nopoly` is slower by about an order of magnitude. This efficiency hierarchy (`Singlepoly` > `Multipoly` > `Nopoly`) remains unchanged across all four selected configurations. However, as discussed in the reference section, `Singlepoly`, for certain configurations where both secondary objects have large mass ratios, shows a degradation in numerical precision due to the intrinsic instability of solving high-degree polynomials. In such cases, `Multipoly`, while sacrificing some speed, maintains higher stability thanks to the dynamic re-centering of local frames and the resolution of local polynomials. The `Nopoly` method, on the other hand, in the case of triple lenses, turns out to be the least efficient, with computation times roughly five times longer than those of `Multipoly`.

Figure 3.10 shows the average computation times of the three algorithms as a function of N , for a moderate magnification regime ($\mu \simeq 20$). For $N = 2$, the reference code `VBBinaryLensing` (black circle) is also included, confirming its superior efficiency for this specific case thanks to dedicated optimizations. For $N = 3$, `Singlepoly` maintains the lead in speed, followed by `Multipoly`, while `Nopoly` remains slower by about a factor of five. However, the dashed curve associated with `Singlepoly` shows that for $N \geq 4$, the algorithm loses reliability: solving polynomials of degree $N^2 + 1$ (for example, degree 17 for four lenses) results in poor numerical stability. Consequently, for $N > 4$, the use of `Singlepoly` is not recommended.

From the scalability point of view, `Multipoly` sequentially solves $\sim N$ local polynomials of degree $N^2 + 1$, with an overall complexity $\mathcal{O}(N^2)$; `Singlepoly` also scales as $\mathcal{O}(N^2)$ but through the resolution of a single global polynomial. `Nopoly`, instead, operates through independent iterative chains on each image and scales as $\mathcal{O}(N)$, proportional to the number of physical images of the system. This difference in behavior explains the intersection point between the `Multipoly` and `Nopoly` curves observed in Figure 3.10: starting from $N \approx 5$, the iterative `Nopoly` becomes not only more stable but also faster. This is a highly significant result, opening the way to simulations of complex multiple systems that were previously prohibitive with polynomial-based methods.

The figure also includes a comparison with external codes: for $N = 3$, the `triplelens` code by Kuang et al. (2021) shows computation times higher by a factor of $\gtrsim 5$ –10 compared to `Multipoly` and `Nopoly`, confirming the effectiveness of the optimizations implemented in `VBMicrolensing`, such as parabolic corrections, error estimators, and intelligent management of ghost images. For $N \leq 4$, `Multipoly` thus represents the

optimal choice; for $N \geq 5$, `Nopoly` emerges as the reference method for both efficiency and numerical stability.

3.8 Conclusions

In conclusion, none of the three algorithms introduced in this work can be considered a general solution applicable in every case, but each excels in a different regime:

- **Singlepoly** offers maximum speed for simple systems ($N \leq 3$), but suffers from conditioning problems for higher N or in the presence of triple systems where the secondary objects have a high mass ratio with respect to the main lens, such as planetary systems.
- **Multipoly** represents an excellent compromise between accuracy and speed for systems of medium complexity ($3 \leq N \leq 4$), maintaining high precision even in numerically difficult scenarios.
- **Nopoly**, although slower in simple cases, scales favorably and becomes the preferred method for complex systems ($N \geq 5$), being the only one to maintain manageable computation times and guaranteed stability.

The synergistic combination of the three algorithms within `VBMicroLensing` constitutes a versatile tool for computing magnification in microlensing events involving multiple lenses.

These results have direct implications for future high-precision microlensing observation campaigns, such as those planned with the *Nancy Grace Roman* Space Telescope. In this context, the ability to model events with multiple deflectors and complex caustic structures will be crucial for the identification of multiple planets and faint companions in binary or multiple stellar systems. Thanks to `VBMicroLensing`, it will be possible to tackle these problems with a combination of speed, accuracy, and scalability that was previously unattainable.

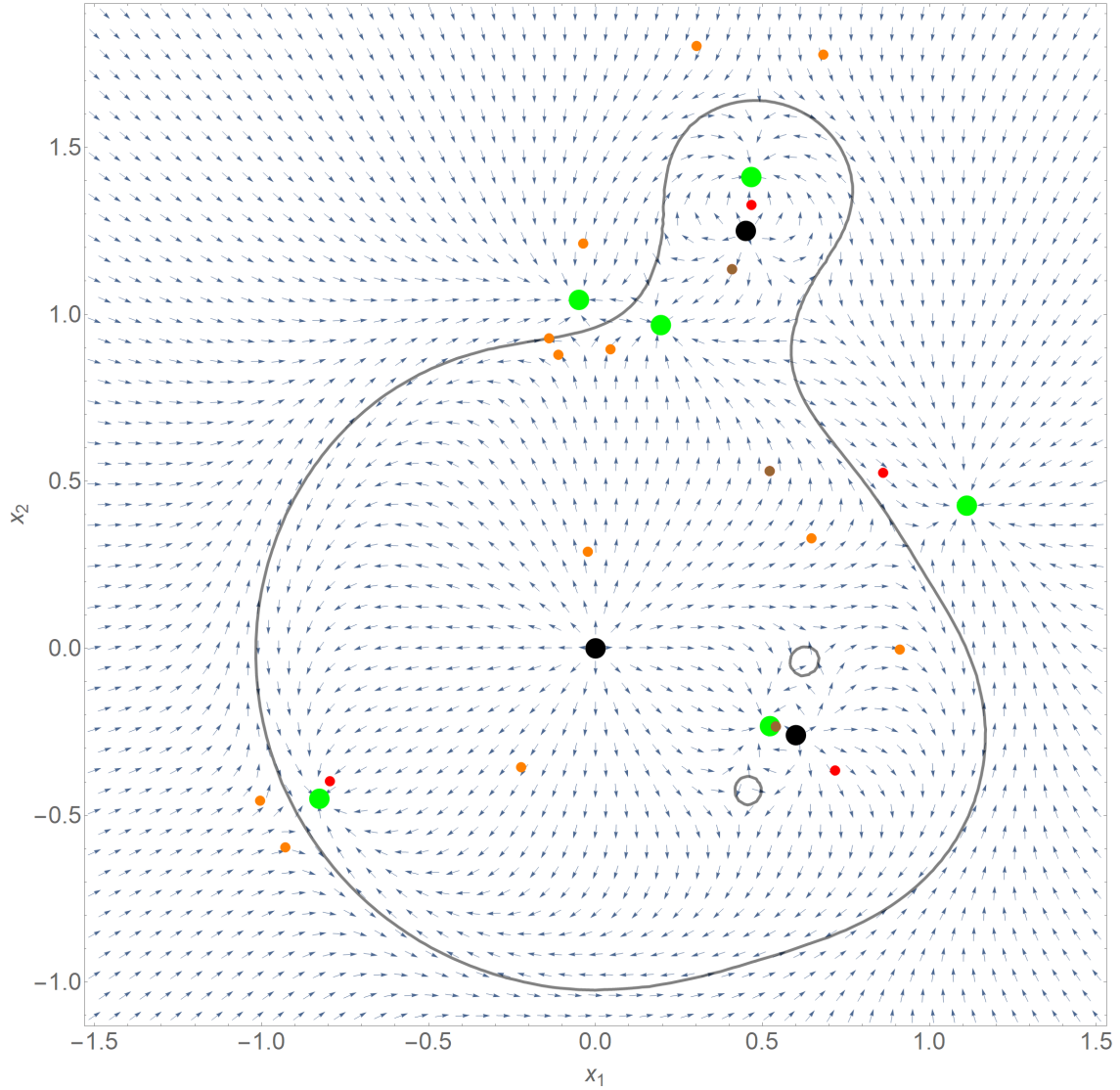


Figure 3.8: Comprehensive illustration of the image search procedure implemented in the `Nopo1y` algorithm. The red points indicate the first-level initial seeds based on perturbative guesses, while the brown points represent the secondary central seeds introduced to capture images near the system barycenter. The green points correspond to the successfully identified image positions, and the orange points denote the partner seeds generated beyond the critical curves (in gray). Progressing through successive levels, the algorithm explores the image plane, avoiding instabilities and redundant solutions, and recover all the physical images of the multi-lens system. Figure adapted from (Bozza et al. 2025)

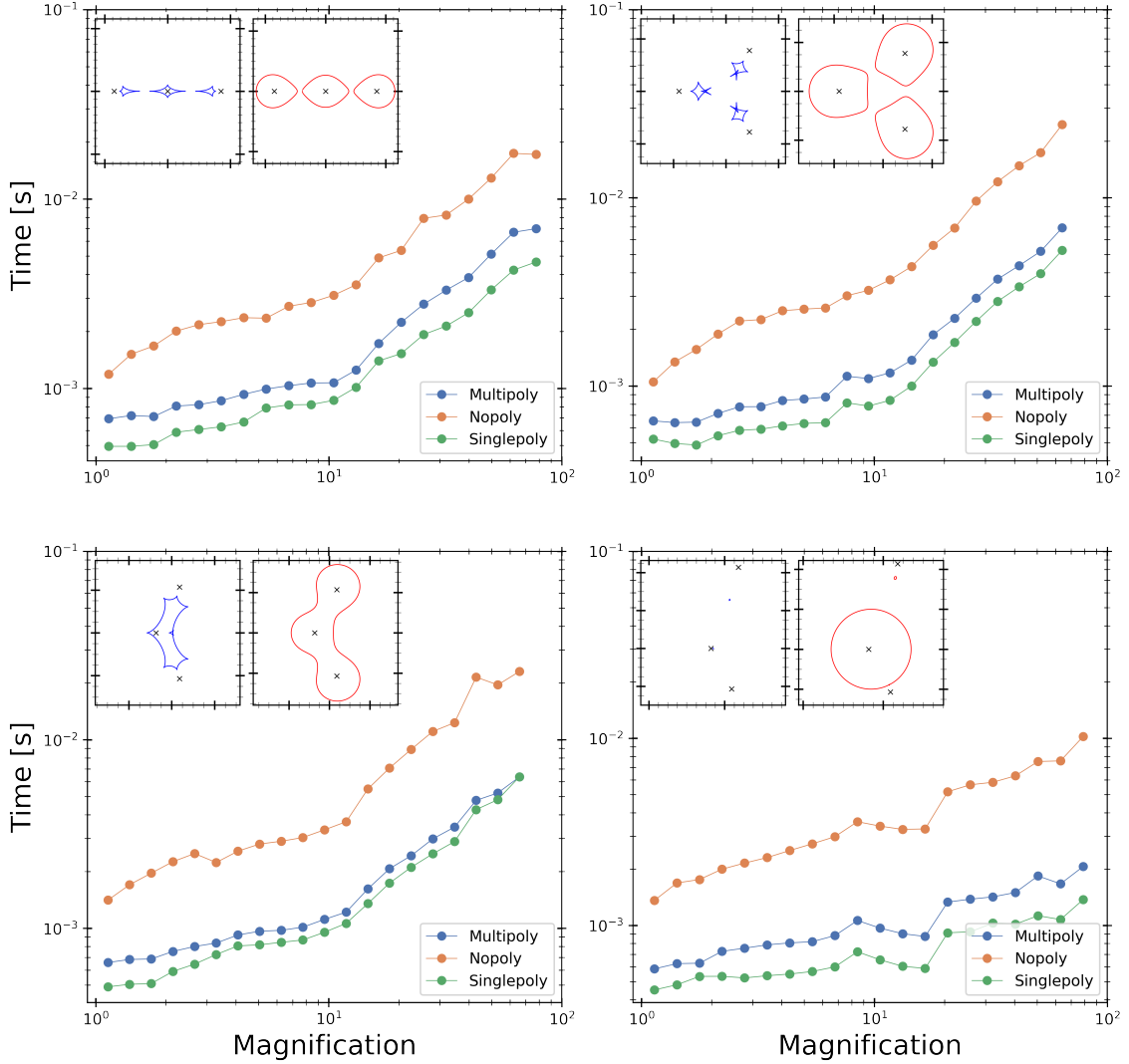


Figure 3.9: Average computation times for the three algorithms Singlepoly (red circles), Multipoly (green squares), and Nopoly (blue triangles), as a function of the total magnification for a system composed of three lenses. The analysis includes four representative geometrical configurations: (a) linear alignment of the three masses, with parameters $(q_2, q_3, s_2, s_3, \theta, \rho) = (1, 1, 1.7, 1.7, \pi, 10^{-3})$; (b) equilateral triangle, $(1, 1, 1.5, 1.5, \pi/3, 10^{-3})$; (c) isosceles triangle, $(1, 1, 1.2, 1.2, 0.7\pi, 10^{-3})$; (d) planetary-type configuration, $(3.3 \times 10^{-6}, 10^{-3}, 1, 2, 0.7\pi, 10^{-3})$, corresponding to a Sun–Jupiter–Earth analog with one primary lens and two secondaries of planetary masses. Each point represents the average computation time for all sources within a given magnification bin. The insets display the caustics (black) and lens positions (red) for each configuration. The hierarchy of efficiency (Singlepoly > Multipoly > Nopoly) is maintained in all cases, with Singlepoly being the fastest but less stable at very high magnifications where numerical instabilities appear, while Multipoly and Nopoly preserve reliable convergence. Figure adapted from (Bozza et al. 2025).

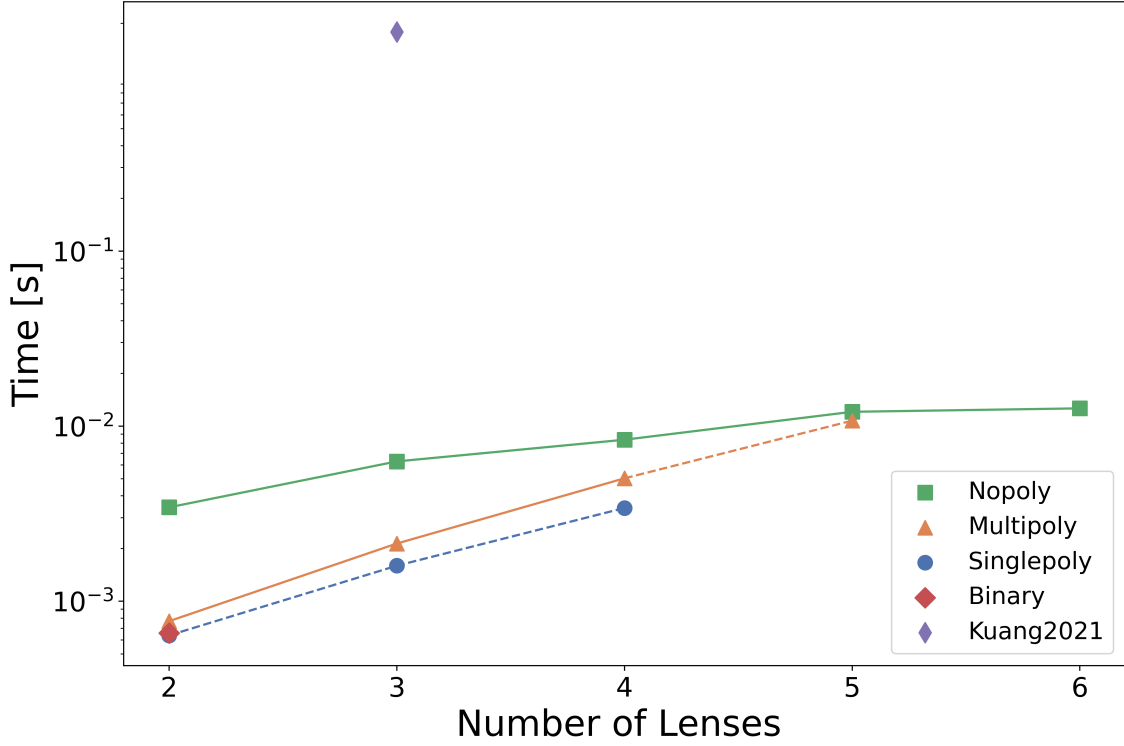


Figure 3.10: Average computation times of the three algorithms `Singlepoly` (red circles), `Multipoly` (green squares), and `Nopoly` (blue triangles), as a function of the number of lenses N . For comparison, the figure also reports the performance of `VBBinaryLensing` (valid for $N = 2$) and of the `triplelens` code by Kuang et al. (2021). The results refer to events with an average total magnification of $A \simeq 20 \pm 5$, computed across four representative geometrical configurations and averaged over all source positions. For $N \lesssim 3$, `Singlepoly` remains the fastest algorithm but gradually loses numerical accuracy beyond $N \simeq 4$ (dashed line). `Multipoly` preserves excellent numerical stability up to intermediate multiplicities. Conversely, `Nopoly`, though initially slower, exhibits a less strong dependence $\mathcal{O}(N)$, becoming the most efficient approach for systems with $N \geq 5$. Figure adapted from (Bozza et al. 2025)

Detection Rates of Multiplanetary Microlensing Events with Roman

This chapter is based on:

“Predictions of the Nancy Grace Roman Space Telescope Galactic Exoplanet Survey. V. Detection Rates of Multiplanetary Systems in High-Magnification Microlensing Events”

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4.1 Introduction

The present work is part of the *Roman Galactic Exoplanet Survey (RGES)* yield-forecast series, which aims to quantify the mission’s expected discovery potential under realistic survey conditions. Earlier studies have focused on single bound planets (Penny et al. 2019), free-floating planets (Johnson et al. 2020), exomoons (Lastovka et al. 2025) and the precision of host-lens mass and distance measurements (Terry et al. 2025). Here we extend the framework to investigate triple-lens events, in particular planetary systems with two planets.

In the previous sections, we have seen that among the planets discovered to date, only about $\sim 40\%$ of them reside in multiplanetary systems, and the majority of these systems have been revealed through the transit method (Struve 1952; Seager et al. 2003) or via radial velocity measurements. However, the microlensing method also plays a major role in revealing planetary systems distant from our Solar System or planets not detectable by other techniques.

Roman will carry out an extensive dedicated survey toward the Galactic bulge, continuously monitoring millions of stars with high cadence and photometric precision (Penny et al. 2019). Thanks to its large infrared field of view, *Roman* will significantly expand the sensitivity to cold and distant planets. This will not only enable the discovery of new planets through microlensing but also the detection of new multiple systems, which are currently rare among microlensing discoveries.

The combination of continuous high-cadence monitoring and photometric precision will make it possible to identify weak signals produced by low-mass companions around lensing stars much more easily than is currently possible with ground-based observations (or with *Gaia*). Moreover, the greater sensitivity of *Roman* could even allow the detection of exomoons if they produce anomalous perturbations in light curves.

This chapter investigates the capability of *Roman* to identify microlensing events arising from triple lens configurations (planetary systems). Such configurations, represent the simplest extension beyond the binary (star-planet) case.

We focus on this class of systems because the likelihood of detecting unambiguous signatures from more than two planets within a single microlensing event is small. As the number of lenses increases, the geometry of caustics becomes progressively more complex, leading to the superposition of planetary perturbations and to brief anomalies that are often difficult to distinguish from noise or degeneracies in the modeling (Shvartzvald et al. 2012). Even under ideal conditions, the identification of distinct planetary contributions becomes difficult when the caustic regions strongly overlap.

A number of previous studies have examined the detectability of multi-planet systems

through numerical simulations. The pioneering works of Gaudi et al. (1998) and Han et al. (2001) first demonstrated that *high-magnification* microlensing events—those in which the source trajectory passes very close to the lens center—offer the most favorable conditions for detecting additional planets, owing to their strong sensitivity to perturbations in the central caustic region. Subsequent investigations explored the theoretical complexity of three-body lensing, with Chung et al. (2005) and Han (2006) providing systematic analyses of the morphology and mutual interactions of caustics in triple-lens geometries.

In recent years, more detailed simulation-based studies have refined this picture. Kuang et al. (2023) modeled planetary systems resembling a scaled version of our Solar System, composed of a host star with Jupiter- and Saturn-like companions, and showed that the two planetary signatures can remain individually recognizable when the corresponding caustics are either well separated or only partially overlapping. In parallel, Fatheddin et al. (2023) extended the parameter space by exploring a broad range of planetary masses and separations, analyzing both isolated and interacting caustic configurations. Their results emphasized how the degree of caustic overlap affects detectability, although their analysis did not specifically target high-magnification events.

The present work builds upon these foundations but focuses deliberately on the high-magnification regime ($A_{\max} \gtrsim 100$). Such events occur when the source star passes extremely close to the line of sight to the lens center, traversing the region dominated by the central caustic. This geometry maximizes the sensitivity to subtle deviations produced by additional planets, including low-mass companions that might otherwise remain undetectable. From an observational standpoint, this focus is particularly relevant because the *Roman* mission, with its continuous, high-cadence infrared monitoring and exceptional photometric precision, is expected to record a substantial number of high-magnification events. These conditions make it uniquely suited to reveal the complex light-curve features characteristic of triple-lens systems.

4.1.1 Known Multi-Planet Microlensing Systems

Although rare, several confirmed microlensing events have revealed systems with more than one planet. Below is a brief overview of these discoveries, which collectively illustrate the diversity of architectures detectable through microlensing.

To date, 11 planets have been discovered within binary systems, while fewer than a dozen microlensing events have revealed systems with more than one planet:

OGLE-2006-BLG-109L. This was the first confirmed multiplanetary system detected via microlensing (Gaudi et al. 2008; Bennett et al. 2010). The lens consists of a solar-type star orbited by two gas giants, with a configuration reminiscent of Jupiter and Saturn in

our own Solar System. The detection of the orbital motion of the outer planet during the event allowed partial constraints on its orbit, marking a milestone in the capability of microlensing to probe planetary architectures.

OGLE-2012-BLG-0026L. Another well-studied system, composed of a G-type main-sequence star in the Galactic disk and two cold Jovian planets (Han et al. 2013; Beaulieu et al. 2016). The planetary masses were determined with $\sim 5\%$ precision, showing that when parallax and finite-source effects are measurable, microlensing can yield highly accurate physical parameters.

OGLE-2014-BLG-1722L. A probable two-planet system orbiting a low-mass star, containing two Saturn-mass planets (Suzuki et al. 2018). It was the first multiple-planet microlensing discovery obtained in a low-magnification event. Even in the absence of parallax or finite-source effects, Bayesian modeling enabled reliable estimates of the planets' masses and semi-major axes.

OGLE-2018-BLG-1011L. A microlensing event showing a distinctive double-peaked anomaly in its light curve, interpreted as two Jovian planets orbiting a low-mass star (Han et al. 2019). Modeling placed the system at a distance of about 7 kpc in the Galactic bulge.

OGLE-2019-BLG-0468L. A system composed of a G-type host star ($\sim 0.9 M_{\odot}$) with two cold giant planets (Han et al. 2022). Its light curve exhibited three distinct anomalies that could only be explained by a triple-lens model, extending the known population of cold gas giants beyond the snow line.

KMT-2020-BLG-0414L. A particularly complex case involving a low-mass star, an Earth-mass planet ($q_2 \sim 10^{-5}$), and a brown-dwarf companion (Zang et al. 2021). Although only one component is truly planetary, this very high-magnification event ($A_{\max} \sim 1450$) demonstrated the extreme sensitivity of microlensing to intricate multi-body configurations.

KMT-2021-BLG-1077L. The most distant known multiplanetary system detected through microlensing (Han et al. 2022). It consists of a late M-dwarf with two cold giant planets located at about 8.2 kpc. This event confirmed that modern survey networks can detect multiple planetary signals even in the densely crowded fields of the Galactic bulge.

4.1.2 Parameterization

We present here the geometric convention adopted in this work to describe a triple-lens system, illustrated in Figure 4.1. The figure shows a configuration with a star and two planets. Each minor body is characterized by two parameters: the mass ratio with respect to the star, $q = M_p/M_*$, and the projected separation s from the star (in units of θ_E). The parameters q_3 and s_3 denote the mass ratio and projected separation of the second planet relative to the star. The parameter ψ , defines the relative angular configuration of the two planets: it represents the angle (measured counterclockwise) between the axis connecting the star to the first planet and the axis connecting the star to the second planet.

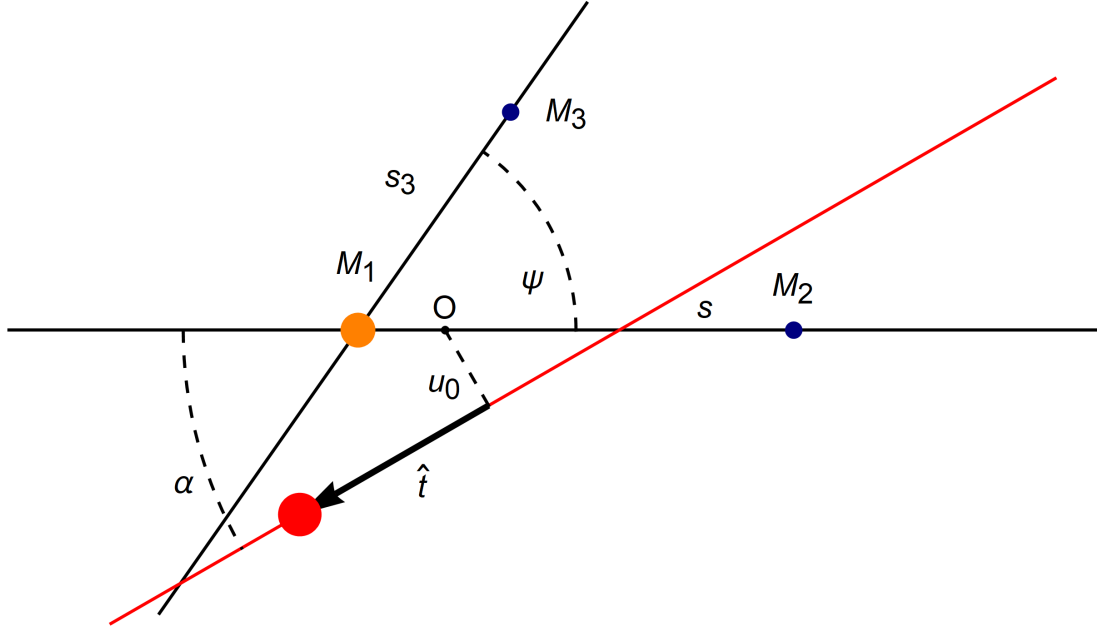


Figure 4.1: Schematic convention used for the geometry of triple-lens configurations. Figure adapted from (Saggese et al. 2025)

In addition to the parameters describing the lens configuration, it is necessary to specify the geometric parameters of the source trajectory with respect to the lens. We define $\hat{t}$ as the unit vector tangent to the source trajectory, indicating the direction of the relative source–lens motion in the lens plane. The orientation of the trajectory is identified by the angle α between $\hat{t}$ and the star–planet 1 axis (the axis along which the first planet lies). The parameter u_0 represents the minimum projected distance between the source and the system’s center of mass, expressed in units of θ_E , and thus defines the impact parameter of the event. The time at which this closest approach occurs is denoted by t_0 , while the Einstein timescale t_E corresponds to the time required for the source to traverse an angular distance equal to one Einstein radius. Finally, $\rho = \theta_*/\theta_E$ indicates the

normalized angular radius of the source star, which becomes relevant when finite-source effects are significant, for instance when the source crosses or approaches a caustic.

It is worth noting that, compared to the binary-lens case—where the parameterization requires only one mass ratio q , one separation s , and one trajectory angle α —the triple-lens case introduces three additional degrees of freedom: a second mass ratio q_3 , a second projected separation s_3 , and the relative position angle ψ between the two planets. Including also the source trajectory parameters (u_0 , t_0 , t_E , and ρ), the full description of a triple-lens microlensing event is thus provided by the set $(q_2, s_2, q_3, s_3, \psi, u_0, \alpha, t_0, t_E, \rho)$, which fully defines both the lens configuration and the source motion.

4.2 Simulations

This section describes how the sample used for the simulations was constructed and the method employed to study *Roman*’s capability to detect all the planets in the system.

4.2.1 Sample

To simulate realistic microlensing events with a triple lens, we used as a starting point the Galactic microlensing population models developed by Penny et al. (2019), based on a physically consistent representation of the spatial, kinematic, and demographic distributions of lens and source stars in the Milky Way. In particular, we employed an updated and statistically representative sample of microlensing events generated for the binary-lens case (star + planet), kindly provided by the authors of Penny et al. (2019). This sample serves as the basis for our extension to systems with two planets, ensuring that our simulations are grounded in astrophysically plausible conditions and reflect the characteristics of the lens and source populations expected in the Galactic bulge field observed by *Roman*.

The statistical distributions of the main physical parameters adopted in our simulated sample are shown in Figure 4.2. They illustrate the diversity of the underlying planetary population inherited from Penny et al. (2019) and extended to include a second companion, highlighting the broad dynamical range of planetary separations and mass ratios considered in this work.

Each event in the sample is described by a complete set of physical and observational parameters defining its geometry and dynamics. Among these are the physical parameters: the *planet-to-star mass ratio* $q = M_p/M_*$, the *projected separation* s (expressed in units of the Einstein radius θ_E), and the kinematic parameters, including the *Einstein timescale* t_E and the *normalized source radius* $\rho = \theta_*/\theta_E$.

Starting from this set of single-planet events, we constructed double-planet configurations by pairing together events whose lens stars have similar masses. More precisely, for each pair of combined events, we required that the relative mass difference between the two host stars be less than 5% ($|\Delta M_\star|/M_\star < 0.05$). This ensures that the two selected events have comparable length and time scales (since θ_E and t_E depend on the lens mass), allowing us to merge them into a single system with two planets without introducing significant discrepancies due to differing stellar masses. The first event in the pair provides the parameters of the “first planet” (q, s), while the second provides those of the “second planet” (q_3, s_3). The relative orientation ψ between the two planets (the angle between their projected orbital planes, as defined in Sec. 2.4) is assigned randomly: for each pair, we build 12 distinct configurations, assigning ψ values uniformly distributed in the interval $[0^\circ, 360^\circ)$. This sampling over 12 angles ensures broad coverage of the possible relative configurations of the two planets on the sky. Additionally, in each two-planet configuration, we label the “first planet” as the more massive one (i.e., with larger q) and the “second planet” as the less massive one, such that by construction $q > q_3$. Although this pairing method does not guarantee that the resulting planetary orbits are dynamically stable over long timescales, it nonetheless provides an effective set of configurations for investigating the observable signatures of triple lenses in high-magnification microlensing events. Indeed, our focus is on the instantaneous imprints of the planets in the light curves rather than the long-term orbital evolution of the system.

To focus on the perturbations associated with the central caustic (which dominate the signal in high-magnification cases), we set the source trajectories to have extremely small impact parameters: $u_0 = 0.01, 0.001$, and 0.0001 . These values ensure that the source passes very close to the optical axis of alignment, producing events with peak magnification $A_{\max} \gtrsim 100$ (i.e., high-magnification events). For each triple-lens configuration (defined by the planetary parameters and angle ψ), we consider two possible trajectory angles α (orientation of the trajectory relative to the star–planet 1 axis, as defined in Sec. 2.4). The two α values assigned are taken from the corresponding original binary events used for that configuration: in practice, we inherit the trajectory angles already present in the two single-planet events that were combined. This approach ensures that, for each configuration, at least two different trajectory inclinations relative to the caustics are explored. In all configurations, the planets are ordered such that $q > q_3$. Throughout the simulations, we retain the Einstein timescale t_E and normalized source radius ρ from the original binary event used to build each configuration: this means that the transition from binary to triple lens does not alter the event’s timescale or the source size relative to θ_E , focusing the analysis exclusively on the additional effects introduced by the second planet.

Applying the procedure described above to the entire initial sample of binary events,

we obtained a total of approximately 1.298 million synthetic light curves (1.298×10^6) covering a wide range of triple-lens geometries and observational conditions. This extensive simulated dataset forms the statistical foundation for the detectability analyses described in the following sections.

4.2.2 Method

To obtain the *observed* light curves (i.e., including photometric noise), we combined the theoretical magnitude at the telescope with the photon flux expected from the source, any blending component, and the sky background, using the flux values provided by the simulated sample of Penny et al. (2019) and corresponding to the broad W149 filter of *Roman*. Each event was simulated over a time window of 72 days with a sampling interval of 15 minutes, a duration consistent with the planned observing campaigns of the mission (Penny et al. 2019). *Roman* will conduct six observing cycles toward the Galactic Bulge, each lasting 72 days, with high cadence; thus, our choice to simulate a single 72-day season for each event ensures that the light curve duration fully covers the event, even in cases with very long Einstein timescales.

To generate the expected light curves, we used the `VBMicrolensing` code (see Chapter 3 of the thesis), adopting in particular the *Multipoly* algorithm, which—as previously discussed—is the most suitable choice for a triple-lens event.

Each simulated configuration is defined by the parameters (q, s) and (q_3, s_3) of the two planets, the relative orientation angle ψ , the impact parameter u_0 , the Einstein timescale t_E , and the normalized source radius ρ .

Although the instrumental characteristics of *Roman* are well known, we adopted a simplified noise model in order to isolate the detectability effects specifically due to the signatures of triple lenses. In particular, we applied Poisson-type noise to the simulated fluxes, drawing at each time step an observed photon count from a Poisson distribution with a mean equal to the theoretical expected flux. This model ignores additional noise sources such as readout noise or thermal background but remains adequate to capture the primary photometric limitations relevant to evaluating the detectability of triple-lens signals in high-magnification events.

To objectively determine whether the presence of the second planet (i.e., the triple nature of the lens) can be recognized from the light curve, each simulated curve was fitted using a simpler binary-lens model (star + one planet). The fitting was performed using a non-linear least-squares minimization algorithm (Levenberg–Marquardt). The binary fit was carried out employing initial guesses for the main planet parameters (q, s, α) , based on the actual triple configuration but excluding the second, less massive planet. From the fit we obtained the best-fit parameters and the corresponding χ^2_{binary} value. This was then

compared with the corresponding χ^2_{triple} computed by evaluating the same light curve (including noise) under the original triple model (without additional fitting, since the triple model is exactly the one used to generate the curve). We define $\Delta\chi^2 = \chi^2_{\text{binary}} - \chi^2_{\text{triple}}$ as the improvement in data-fitting quality when moving from the binary-lens model to the triple-lens model. We consider a triple lens to be *successfully detected* when this improvement exceeds a predefined threshold, which we set to $\Delta\chi^2 = 160$. This threshold is consistent with the detection criteria adopted in previous studies on planetary detection via microlensing (Gould et al. 2010; Suzuki et al. 2018; Penny et al. 2019).

Our numerical framework provides systematic coverage of the parameter space relevant to planetary triple-lens microlensing events in the high-magnification regime, exploring a broad spectrum of astrophysical and geometric configurations. Through this parametric exploration, we sample a large diversity of light-curve morphologies expected in multi-planetary systems. This approach allows us to assess how detection sensitivity depends on the actual lens architecture and to identify the specific parameter regimes in which secondary planetary perturbations generate observationally resolvable and statistically significant deviations compared to the binary-lens model. The resulting simulation dataset allows us to evaluate *Roman*'s detectability performance for planetary systems with two planets and provides a direct numerical complement to analytical studies and previous predictive works on the detectability of multi-planet microlensing events (Gould et al. 2010; Suzuki et al. 2018; Fatheddin et al. 2023).

4.3 Results

In this section, we present the main results emerging from the analysis of the simulations introduced in the previous section. We recall that our goal is to quantify how efficiently the *Roman* Space Telescope will be able to identify microlensing events produced by systems containing two planetary companions in the high-magnification regime, and to understand how this detectability varies with different orbital and physical parameters. The analysis proceeds in several steps. We first examine the global detection efficiency of the simulated sample and its dependence on the orbital configuration of the two planets. We then explore in detail the morphological diversity of the resulting light curves, showing how the interplay between the two planetary caustics shapes the observable anomalies. Finally, we investigate the physical origin of the efficiency trends, identifying the key parameters that control the detectability of triple-lens signatures.

4.3.1 Global detection efficiency and dependence on orbital regime

Analyzing the entire simulated sample, consisting of 1.3×10^6 light curves, we found that 66.3% meet the detection criterion, that is, they show a fit improvement of $\Delta\chi^2 > 160$ when moving from the binary-lens to the triple-lens model. About two-thirds of all simulated two-planet events exhibit signals strong enough to be identified by *Roman* according to our threshold. This highlights the high sensitivity of the *Roman* survey to deviations introduced by multiple planetary companions, especially when the caustic structures generated by the two planets interact and produce pronounced anomalies in the light curve.

To analyze how detectability depends on the orbital configuration of the planets, each simulated event was classified based on the projected separations s and s_3 of the two planets relative to the Einstein radius, assigning each planet to one of the three regimes defined in Sec. 2.3: close, resonant, or wide. Each event therefore falls into one of nine possible combinations of orbital regimes (*close/close*, *close/resonant*, *close/wide*, *resonant/close*, *resonant/resonant*, *resonant/wide*, *wide/close*, *wide/resonant*, *wide/wide*).

This subdivision allows us to carry out a detailed investigation of which configurations produce the most detectable signals. Table 4.1 presents the detection statistics grouped according to this classification.

The results show that, under high-magnification conditions, the detectability of a triple-lens event is primarily dictated by the presence of a planet in the resonant regime, that is, when the projected separation s (or s_3) is close to unity. In these cases — such as the *resonant/resonant*, *close/resonant*, and *wide/resonant* configurations — the success rates reach approximately 92.8%, 91.4%, and 89.6%, respectively. The physical reason behind this behavior is that planets located near the Einstein ring generate large, centrally connected caustics that overlap with the central magnification region. As the source passes through this area, even small perturbations in the lens potential translate into strong, easily detectable anomalies in the light curve, ensuring a high probability of identifying the system’s multiple-planet nature.

In contrast, configurations in which *both* planets are outside the resonant zone lead to substantially lower efficiencies. In these non-resonant regimes, the planetary caustics are smaller, more isolated, and generally positioned far from the central caustic, resulting in a perturbation on the central caustic of more modest amplitude. As a result, the corresponding light-curve deviations are weaker. In the *wide/wide* case, with both planets at large separations, the detection efficiency drops to around 54–55%, reflecting the limited dynamical influence of such distant companions on the central magnification pattern. Intermediate results are obtained for mixed but non-resonant combinations, such as *resonant/close* (about 80.8%) or *close/close* (about 70.2%), where smaller caustics

Orbital regime	Simulated events	Success (%)	Sim. fraction	Detection prob.	Expected events
resonant / resonant	14,518	92.82	0.01	9.32×10^{-4}	0.76
close / resonant	72,472	91.37	0.06	4.58×10^{-3}	3.73
wide / resonant	77,891	89.56	0.06	4.83×10^{-3}	3.93
resonant / close	49,281	80.81	0.04	2.75×10^{-3}	2.24
close / close	219,303	70.23	0.17	1.06×10^{-2}	8.68
wide / close	263,649	69.02	0.20	1.26×10^{-2}	10.25
resonant / wide	46,240	63.91	0.04	2.05×10^{-3}	1.67
close / wide	258,603	55.44	0.20	9.91×10^{-3}	8.08
wide / wide	294,748	54.65	0.23	1.11×10^{-2}	9.08

Table 4.1: Detection statistics by orbital regime. For each combination of regimes ($s, /, s_3$) we report: the number of simulated events, the raw success rate (fraction of events with $\Delta\chi^2 > 160$), the fraction of total simulations, the effective detection probability (accounting for the expected fraction of planetary events in the survey), and the number of expected detections over the full duration of the *Roman* survey. Table adapted from (Saggese et al. 2025)

still occasionally interact with the central region but do not produce dominant features.

4.3.2 Expected detection rate for triple-lens events

We estimated the expected number of detectable triple-lens microlensing events that the *Roman* survey could observe. Starting from the yield model of Penny et al. (2019), we considered the population of planetary microlensing events and applied successive multiplicative factors derived from our simulations.

We begin with an estimated total of 1,400 planetary microlensing events expected during the nominal *Roman* mission (out of $\sim 54,000$ total microlensing events detections).

The relevant probabilities considered in our estimate are:

- **High-magnification fraction:** About 6.9% of planetary events have a high-magnification geometry ($u_0 < 0.01$).
- **Average detection efficiency for triple lenses:** For these favorable high-magnification events, our simulations yield an average detection efficiency of 66% for recognizing the presence of a second planet.

From these two probabilities, we can compute the fraction and number of detectable triple-lens systems:

From these two probabilities, we can compute the overall fraction and number of detectable triple-lens systems. Combining the above factors gives an effective detection

fraction of

$$0.069 \times 0.66 = 0.045,$$

which means that about 4.5% of all planetary microlensing events are expected to be recognizable as triple-lens systems. Multiplying this fraction by the total number of planetary events (1,400) gives approximately

$$0.045 \times 1400 = 63,$$

detectable triple-lens systems throughout the entire *Roman* microlensing survey.

The last column of Table 4.1 shows how these expected detections are distributed among the various orbital regimes. For each configuration, the detection probability was computed as the product of:

1. the probability that a planetary event has a high-magnification geometry (6.9%),
2. the specific detection efficiency from our simulations for that configuration, and
3. a normalization factor accounting for the relative frequency of each orbital regime in the simulated sample.

Overall, our analysis indicates that the *Roman* mission is expected to detect dozens of multi-planet systems through microlensing. In this context, “detection” means that the presence of a second planet can be identified with high statistical significance ($\Delta\chi^2 > 160$) in the observed light curve.

4.3.3 Light-curve morphologies across orbital regimes

Figures 4.3–4.11 show representative examples of simulated light curves for the various orbital combinations considered (see Table 4.1). Although not statistically exhaustive, these examples illustrate how the geometric configuration of a two-planet system influences the morphology and strength of the anomalies expected in the light curve.

When at least one planet lies near the Einstein ring (resonant regime), the light curve typically shows very strong, often complex deviations centered around the peak magnification. These anomalies arise from a large, structured central caustic and often appear as blended, multi-component features where the contributions from both planets overlap (see, for example, Figure 4.3). In such cases, the individual signatures of each planet may be difficult to disentangle, as the combined effect produces a single, dominant anomaly near the peak.

In contrast, systems in which both planets lie in non-resonant regimes (e.g., *close/close* or *wide/wide* configurations) generate simpler and more isolated perturbations. These

appear in the light curve as short, localized features (small bumps or dips) often far from the main peak, since the smaller, more distant planetary caustics are intersected by the source only in the event’s wings (see Figure 4.11). Consequently, signals from two non-resonant planets tend to be weaker and less conspicuous, reducing the likelihood of being noticed above normal microlensing residuals.

In mixed configurations (e.g., *close/resonant* or *wide/resonant*), the light curve generally exhibits two temporally separated features: a strong main anomaly near the peak caused by the resonant planet, and a secondary, weaker deviation due to the non-resonant companion (Figure 4.6). The temporal separation and differing morphology of the two signals increase the chances of identifying each planet individually, as each anomaly can be associated with a distinct lensing body.

In summary, the presence of a planet in the resonant regime tends to dominate the dynamics of a triple-lens light curve, while the lower-mass companion (typically the second planet, with $q_3 < q$) mainly contributes localized perturbations or slight asymmetries. These trends, illustrated qualitatively in the figures, help explain the different detectability levels observed across orbital regimes (as discussed in the previous section) and provide an intuitive framework for interpreting potential real events involving multiplanetary systems.

4.3.4 Physical implications of detection efficiency trends

We first assess the effect of the source impact parameter u_0 on triple-lens detectability. In our simulations, we considered three decreasing u_0 values (0.01, 0.001, and 0.0001) for each configuration, keeping the lens geometry fixed. The results, shown in Table 4.2, indicate that the fraction of events where the second planet is detected remains nearly constant with varying u_0 . The percentage of events exceeding the $\Delta\chi^2 > 160$ threshold hovers around 65%, with no significant systematic trend (only a slight increase in efficiency for $u_0 < 0.01$ compared to $u_0 = 0.01$). Even with a stricter threshold ($\Delta\chi^2 > 1000$), values remain comparable (about 50% of events in all cases). This suggests that, for the high-magnification events considered, trajectory geometry (as long as it lies in the strong central amplification regime) plays a secondary role in the ability to detect multiple planets.

To explore how the physical and geometric parameters influence detectability, we examined the distribution of several key quantities in our simulations (Figure 4.12). As expected, the detectability of a second planet is primarily governed by its mass ratio, while the relative angular orientation between planets, ψ , has a negligible effect. The histograms in Figure 4.12 illustrate this trend: increasing the mass ratio of the outer planet, q_3 , increase the detection efficiency, whereas variations in ψ produce no systematic

Table 4.2: Fraction of simulated triple-lens events exceeding two $\Delta\chi^2$ thresholds (fit improvement over a single-planet model), as a function of the impact parameter u_0 . Table adapted from (Saggese et al. 2025)

u_0	Events with $\Delta\chi^2 > 160$ (%)	Events with $\Delta\chi^2 > 1000$ (%)
0.0001	66.23	50.35
0.0010	69.17	53.06
0.0100	63.44	46.59

change.

Larger mass ratios (q or $q_3 \gtrsim 10^{-3}$, corresponding to a Jupiter-mass planet orbiting a solar-mass star) generate stronger perturbations in the lensing potential. These, in turn, lead to more extended caustic structures and greater deviations in the light curve, resulting in a higher probability of detection. In contrast, low-mass companions ($q \sim 10^{-4}$ or below) produce only subtle perturbations in the central caustic that are often comparable to, or smaller than, the photometric noise level, making them difficult to identify as statistically significant signals.

The flat behavior with respect to ψ indicates that rotating the planet configuration (with all other parameters fixed) barely changes the total “anomaly cross-section” presented to possible source trajectories, since the total caustic area—and thus the chance of intersection—remains nearly invariant under rotation.

Regarding the source trajectory angle α , the detection efficiency appears roughly constant across the full range of orientations, reflecting the near-isotropy of magnification patterns in high-magnification events. Since in all cases the source passes very close to the lens center, the direction of motion has little effect on the probability of encountering anomalies (though it may affect their detailed shape). Note, however, that because our sample was built from already-detectable binary events, the base caustic geometry tends to favor trajectories α nearly orthogonal to the star–planet axis (as also discussed by Bozza et al. (2025)). This intrinsic symmetry may lead to a bimodal distribution of α among events showing detectable deviations. Nevertheless, the overall probability of detecting a second planet does not vary significantly with α across the high-magnification range considered.

Another key factor influencing detectability is the event’s *Einstein timescale*, t_E . Our simulations indicate that the detection efficiency increases with longer t_E , especially when moving from short- to long-duration events. Events with short timescales are more likely to be missed or insufficiently sampled, as planetary perturbations may occur over very brief intervals with only a few data points. Conversely, longer events provide extended temporal coverage, allowing potential anomalies to be better resolved and increasing the likelihood that at least one significant deviation is adequately sampled.

A similar behavior is observed with the ratio u_0/ρ between the impact parameter and the normalized source radius. Detection efficiency rises with increasing u_0/ρ , as higher values correspond (for fixed u_0) to smaller source radii, reducing finite-source smoothing effects. Smaller sources yield sharper caustic-crossing signals, making anomalies more evident and easier to detect. Conversely, large angular sources tend to smooth caustic features, decreasing the likelihood of sharp, high-amplitude variations.

Figure 4.13 shows a set of hexbin density maps summarizing the fraction of detectable triple-lens events across several two-parameter projections of the simulated parameter space. Each panel bins the simulations on a regular hexagonal grid, with color indicating the detection fraction within that bin (number of detections divided by total events in the bin). The four panels correspond respectively to the (q, q_3) , (s, s_3) , (q, s) , and (q_3, s_3) planes. Detection efficiency increases with the planets' mass ratios and when at least one planet lies near the resonant separation ($s \sim 1$), and decreases for low-mass companions or wide/close non-resonant configurations, in agreement with the trends discussed above.

Figure 4.14 shows how the detection efficiency of a triple-lens system varies with the secondary planet's mass ratio (q_3), for three different values of the main planet's mass ratio ($q = 10^{-2}$, 10^{-3} , and 10^{-4}). When the secondary planet is massive ($q_3 = 10^{-3}$), detection efficiency remains high regardless of the main planet's mass. However, as q_3 decreases, efficiency declines, more sharply when the first planet is very massive. For instance, at $q = 10^{-2}$, the efficiency at $q_3 = 10^{-5}$ is much lower than for smaller q . This occurs because a large primary caustic (from a planet with $q \sim 10^{-2}$) can overshadow the weak perturbations produced by the second planet, reducing their detectability even though it is physically present.

The detection of triple-lens systems is driven primarily by high planetary mass ratios and favorable caustic geometries. The highest efficiencies occur when at least one planet lies near the Einstein ring and has $q \gtrsim 10^{-3}$. Conversely, very low-mass companions, large-source stars (high ρ), or short-duration events (small t_E) tend to produce weak or smoothed signals that often fall below the detection threshold. Overall, the dominant factors determining successful detection of a second planet are the planets' masses, the source's angular size, the event duration, and a resonant caustic configuration, while other parameters—such as trajectory angle α or relative orientation ψ —have only minor impact on detection probability.

4.4 Conclusions

This work demonstrates the strong potential of the *Nancy Grace Roman Space Telescope* to detect and characterize microlensing events produced by complex planetary archi-

tectures. We focused on *triple-lens* configurations—systems in which two planets orbit the same host star—and explored the conditions under which such events can be distinguished from simpler single-planet (binary-lens) models. Using a large suite of simulated high-magnification light curves that include realistic photometric noise and parameters consistent with the planned *Roman* survey, we find that roughly two thirds ($\sim 66\%$) of two-planet events satisfy the adopted detection criterion ($\Delta\chi^2 > 160$), indicating that *Roman* will be highly sensitive to the characteristic signatures of multiple planets. The efficiency rises above 90% when both planets are relatively massive ($q \gtrsim 10^{-3}$), while systems containing only low-mass companions ($q \lesssim 10^{-4}$) are rarely detectable. These results mark a substantial improvement over current ground-based microlensing campaigns, which have so far identified only a handful of such systems.

The detectability of triple-lens events is determined not only by the planets’ masses but also by the event geometry. Strong anomalies occur when the source trajectory passes through or near the merged central caustic produced by the combined gravitational influence of the two planets, yielding light-curve deviations that cannot be reproduced by any single-planet model. Resonant configurations, where one or both planets lie close to the Einstein ring ($s \sim 1$), are particularly favorable because they generate large, overlapping caustics that dominate the high-magnification region. In contrast, systems in the *close–close* or *wide–wide* regimes exhibit small, displaced caustics that are seldom intersected by the source, resulting in lower detection probabilities. These behaviors are consistent with the parameter-space trends discussed earlier, reinforcing the dominant role of resonance and mass ratio in shaping the observable signatures of multiple planets.

By combining our detection efficiencies with the expected rate of high-magnification events, we estimate that approximately $\sim 0.1\%$ of all microlensing events observed by *Roman* will correspond to triple-lens configurations, or about sixty detections over the full mission lifetime. Although this fraction is small, it represents a dramatic improvement over current capabilities and will provide the first statistically meaningful sample of multi-planet microlensing detections beyond the snow line. Our approach differs from that of Kuang et al. (2023), who modeled Solar-analog systems (Sun–Jupiter–Saturn), yet both studies converge on the same conclusion: under favorable geometries, particularly during high-magnification events, microlensing offers a powerful route to detecting systems with multiple planets that are inaccessible to other techniques.

Overall, the *Roman* microlensing survey will not only expand the census of exoplanets at wide separations but will also open a new observational window on the diversity of planetary architectures, bridging the gap between inner systems probed by transits and radial velocities and the outer regions accessible through microlensing. Future extensions of this work will explore a broader parameter space, including different source radii, orbital inclinations, and orientations, as well as higher-order effects such as parallax

and lens orbital motion, to refine detectability estimates and guide the interpretation of upcoming *Roman* discoveries.

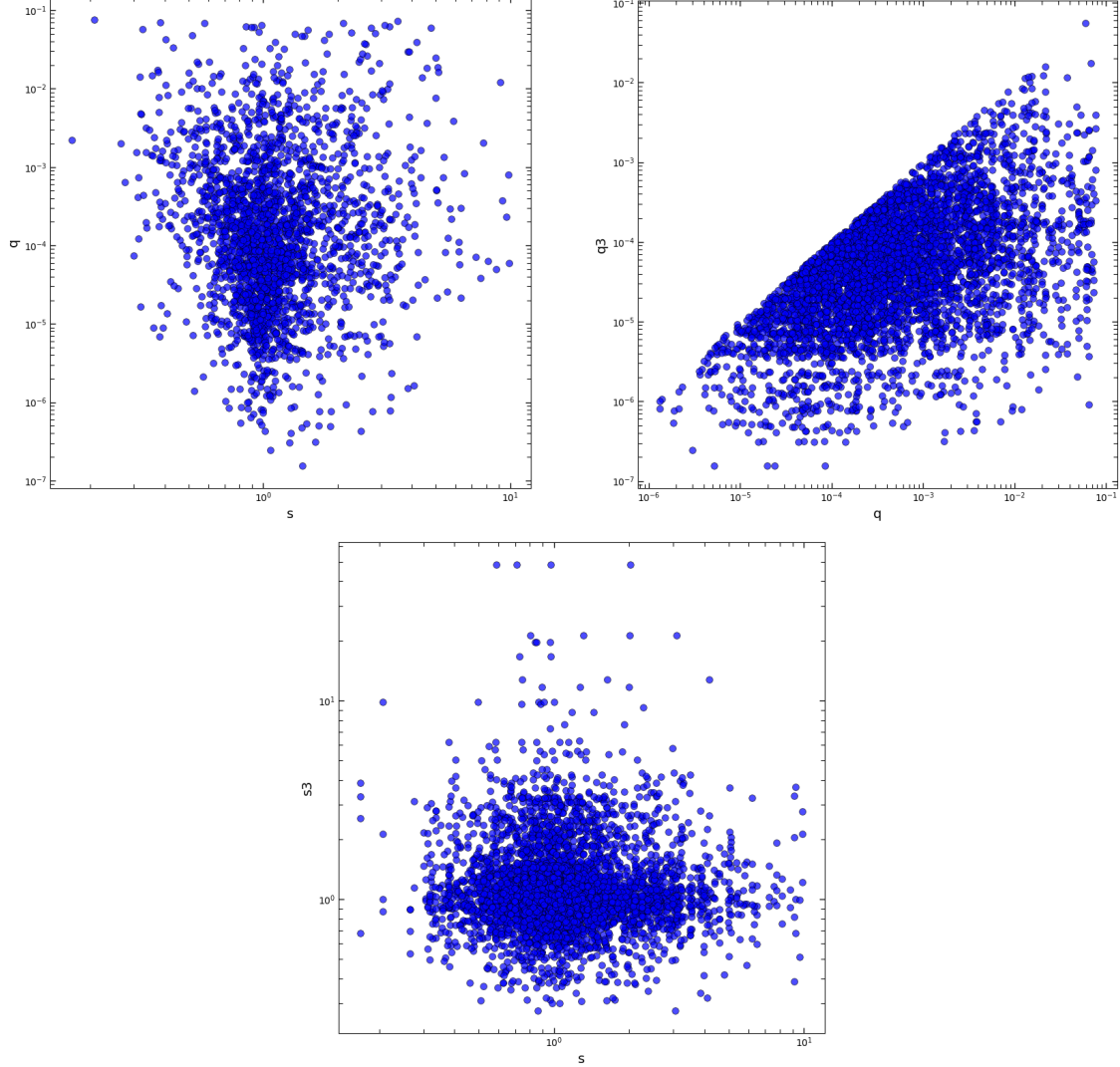


Figure 4.2: Distributions of the main physical parameters in the simulated sample of triple-lens events. Top left: planet-to-star mass ratio q versus projected separation s for the first planet. Top right: correlation between the mass ratios of the two planets, q_2 and q_3 . Bottom: projected separations of the two planets, s_2 and s_3 . Each point corresponds to a single simulated system drawn from the population synthesis of Penny et al. (2019), extended to include a second planet. The distributions are expressed in units of the Einstein radius θ_E , covering the typical range of planetary microlensing events ($10^{-7} \lesssim q \lesssim 10^{-1}$, $0.1 \lesssim s \lesssim 10$).

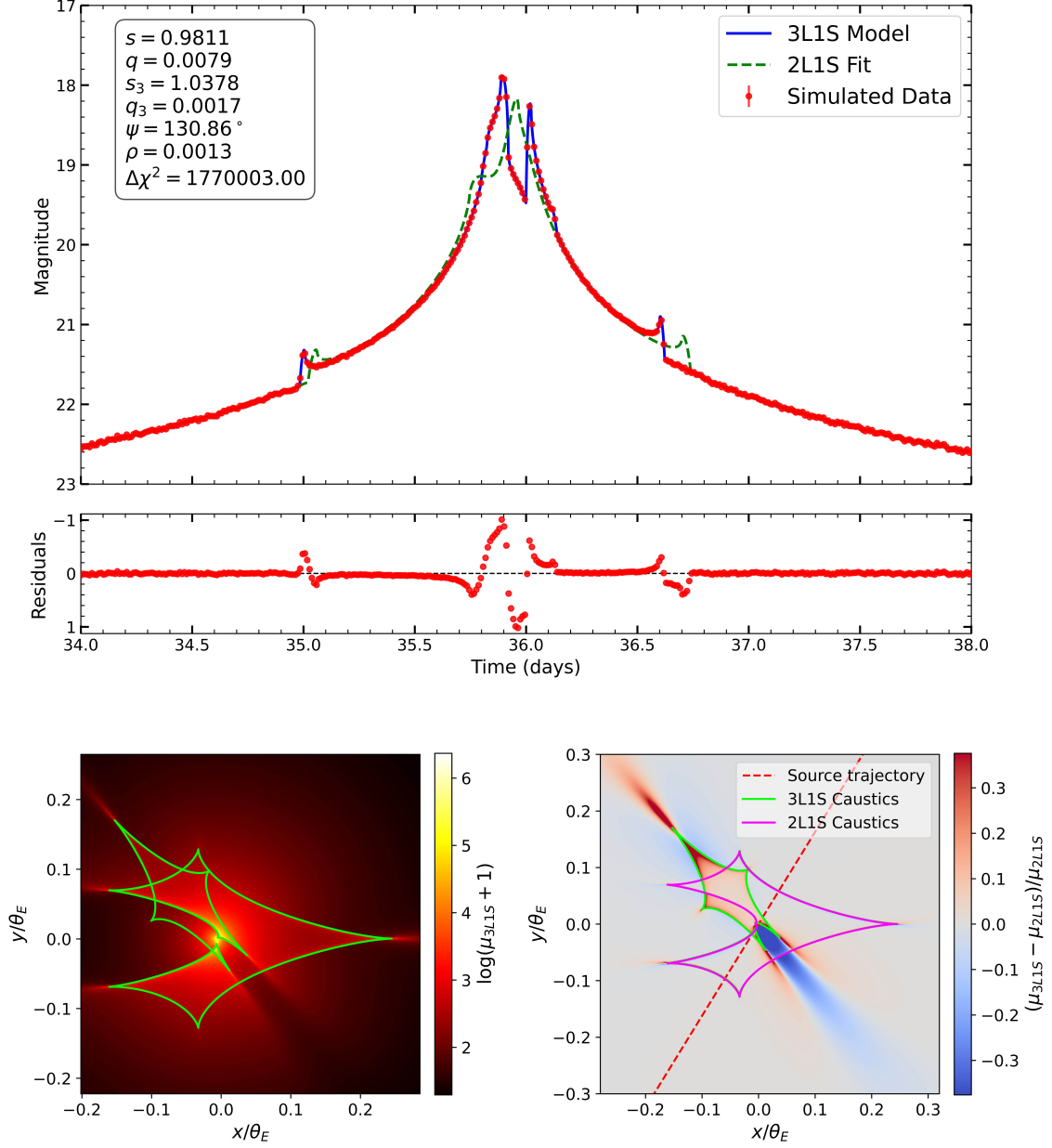


Figure 4.3: Simulated light curve for a triple-lens system with both planets in the resonant regime. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

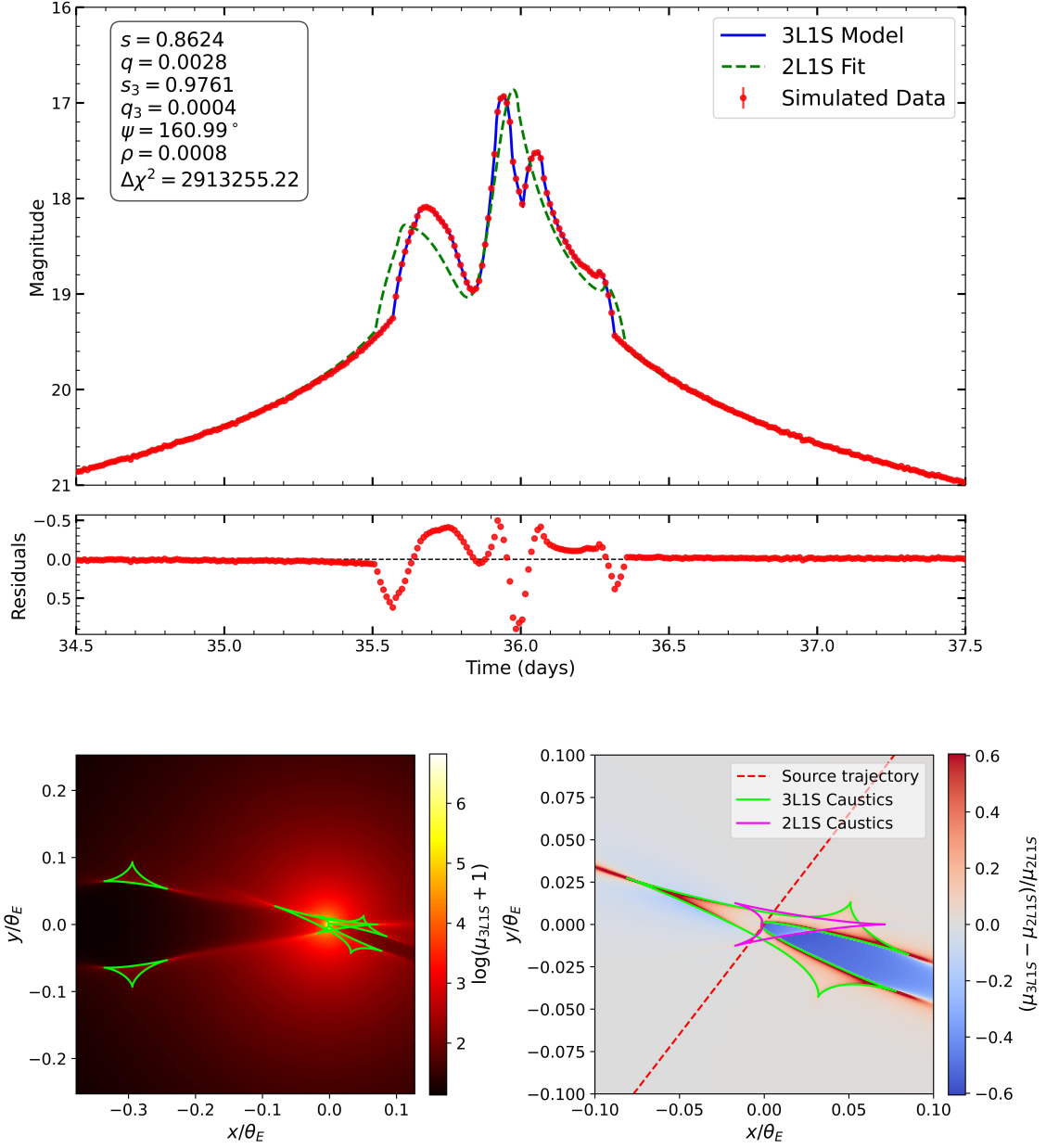


Figure 4.4: Simulated light curve for a triple-lens system with a close and a resonant planet. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

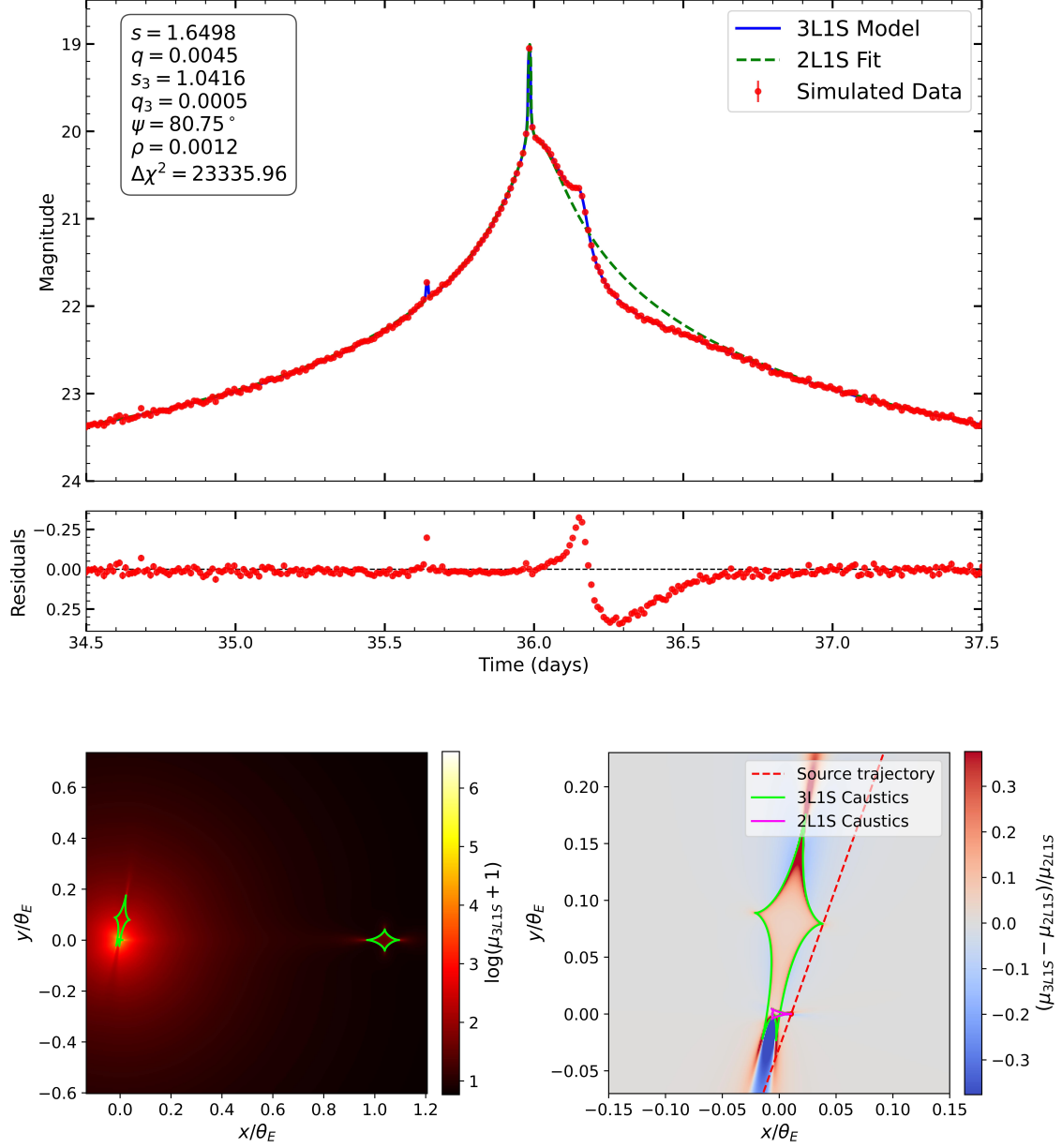


Figure 4.5: Simulated light curve for a triple-lens system with a wide and a resonant planet. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

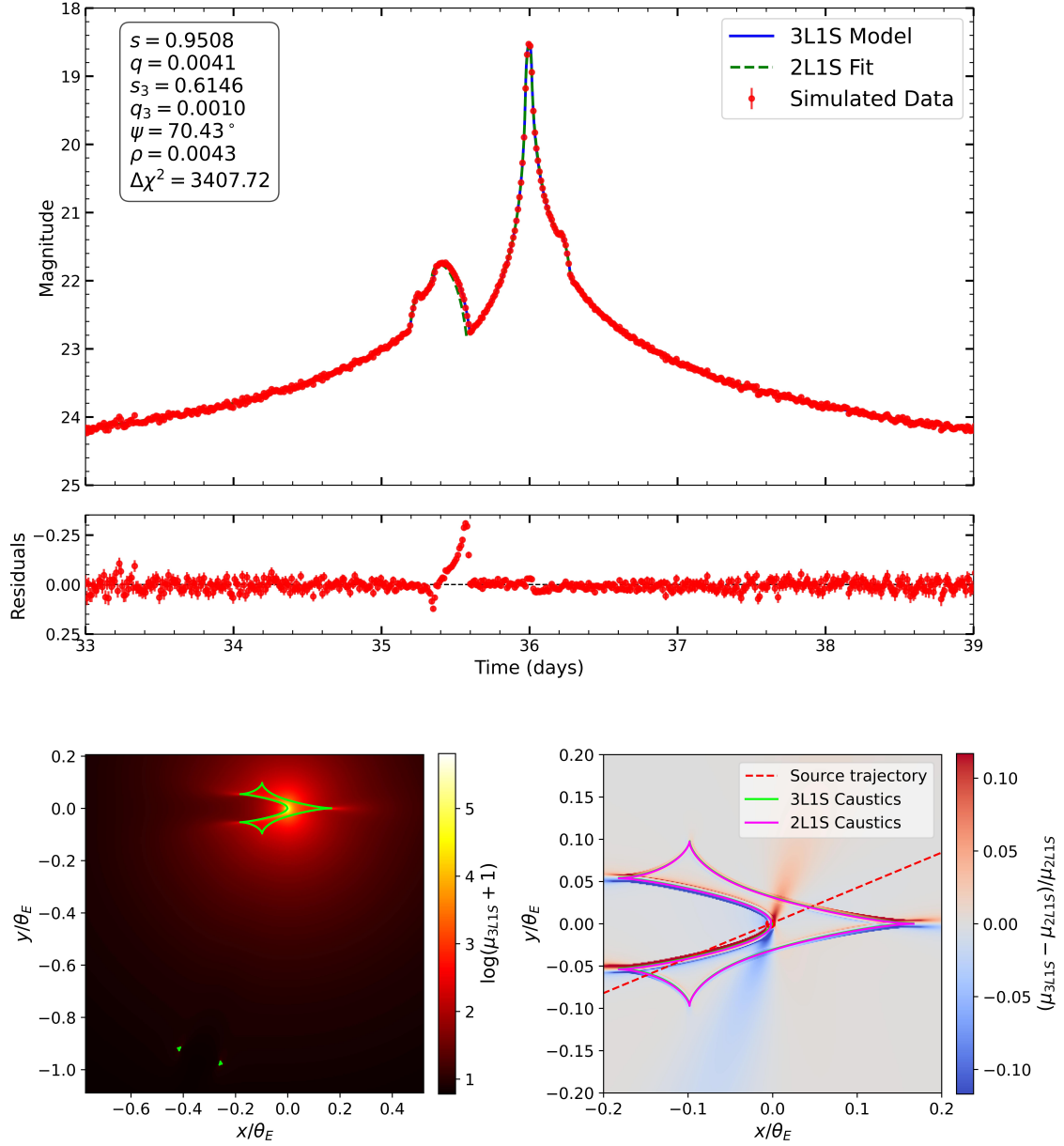


Figure 4.6: Simulated light curve for a resonant-close configuration. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

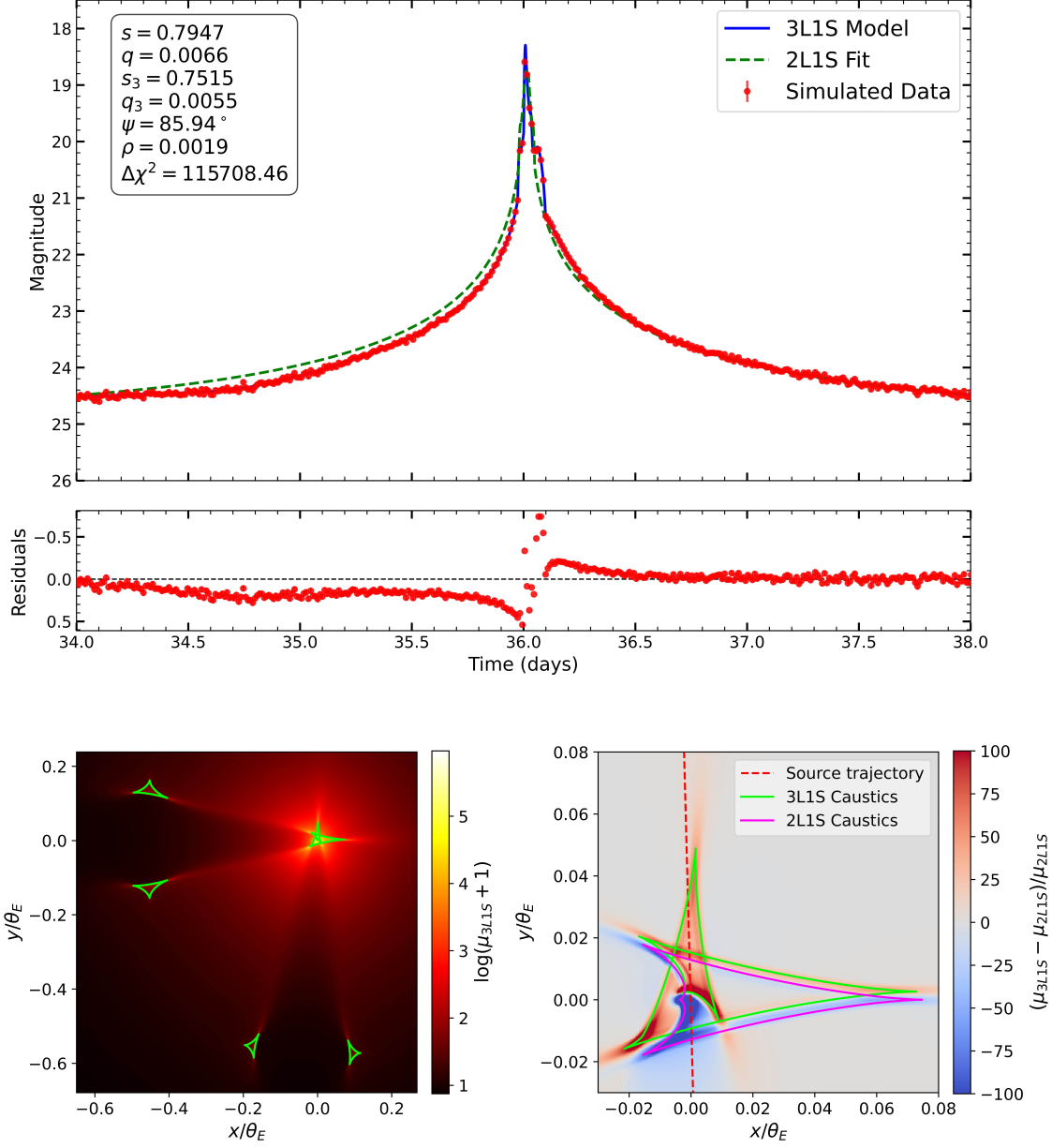


Figure 4.7: Simulated light curve for a system with two close-in planets. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

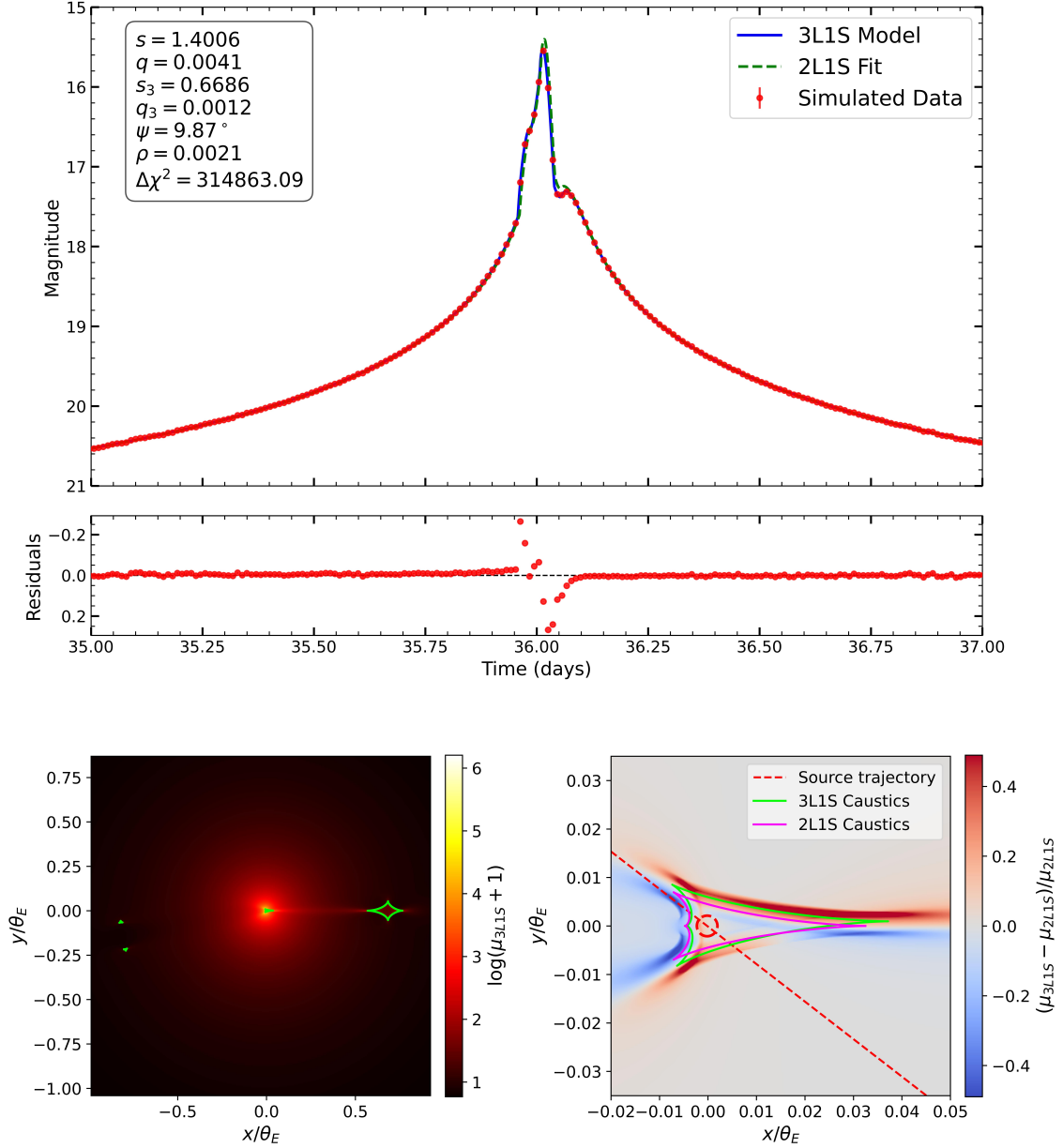


Figure 4.8: Simulated light curve for a wide-close configuration. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

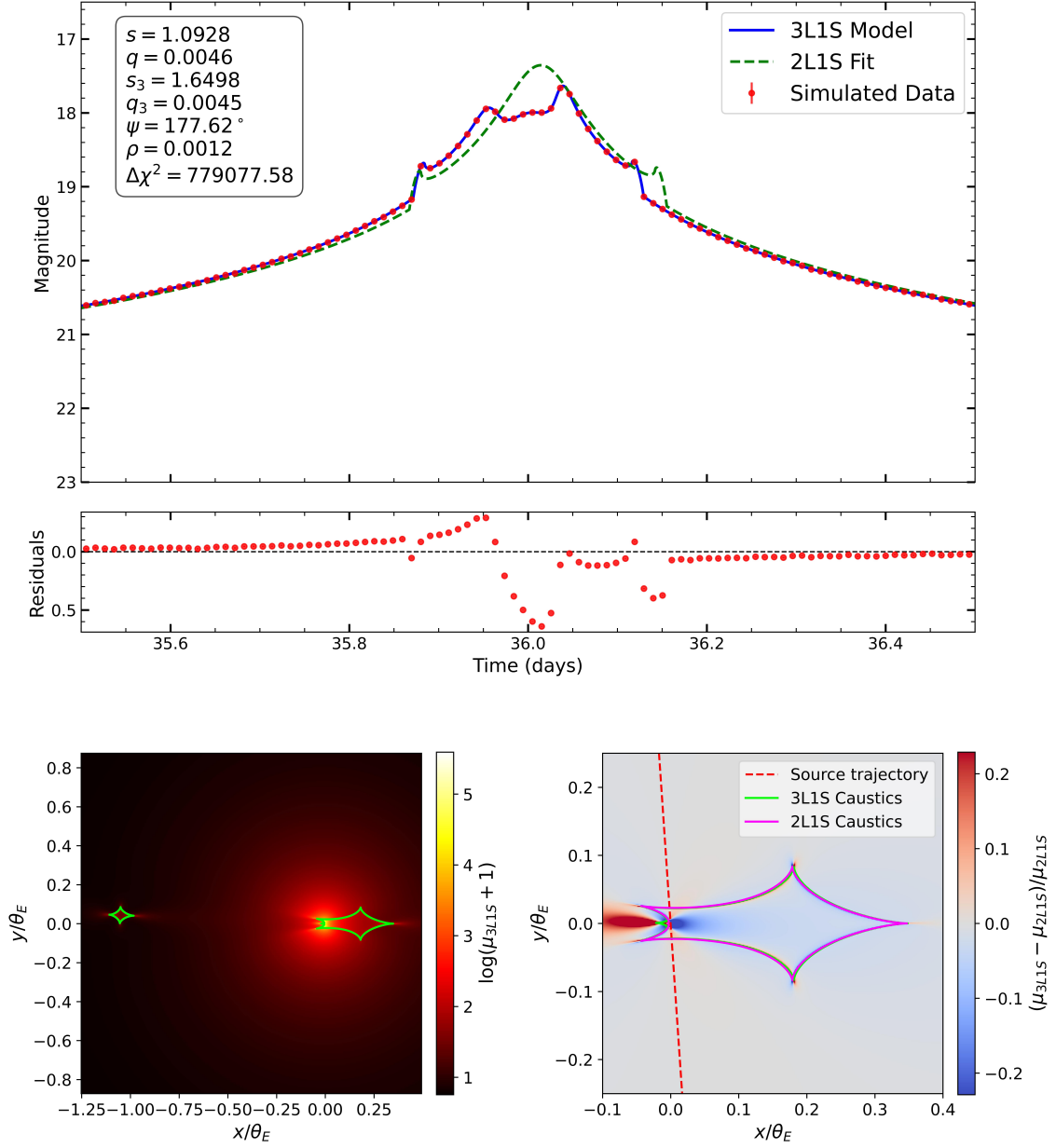


Figure 4.9: Simulated light curve for a resonant-wide configuration. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

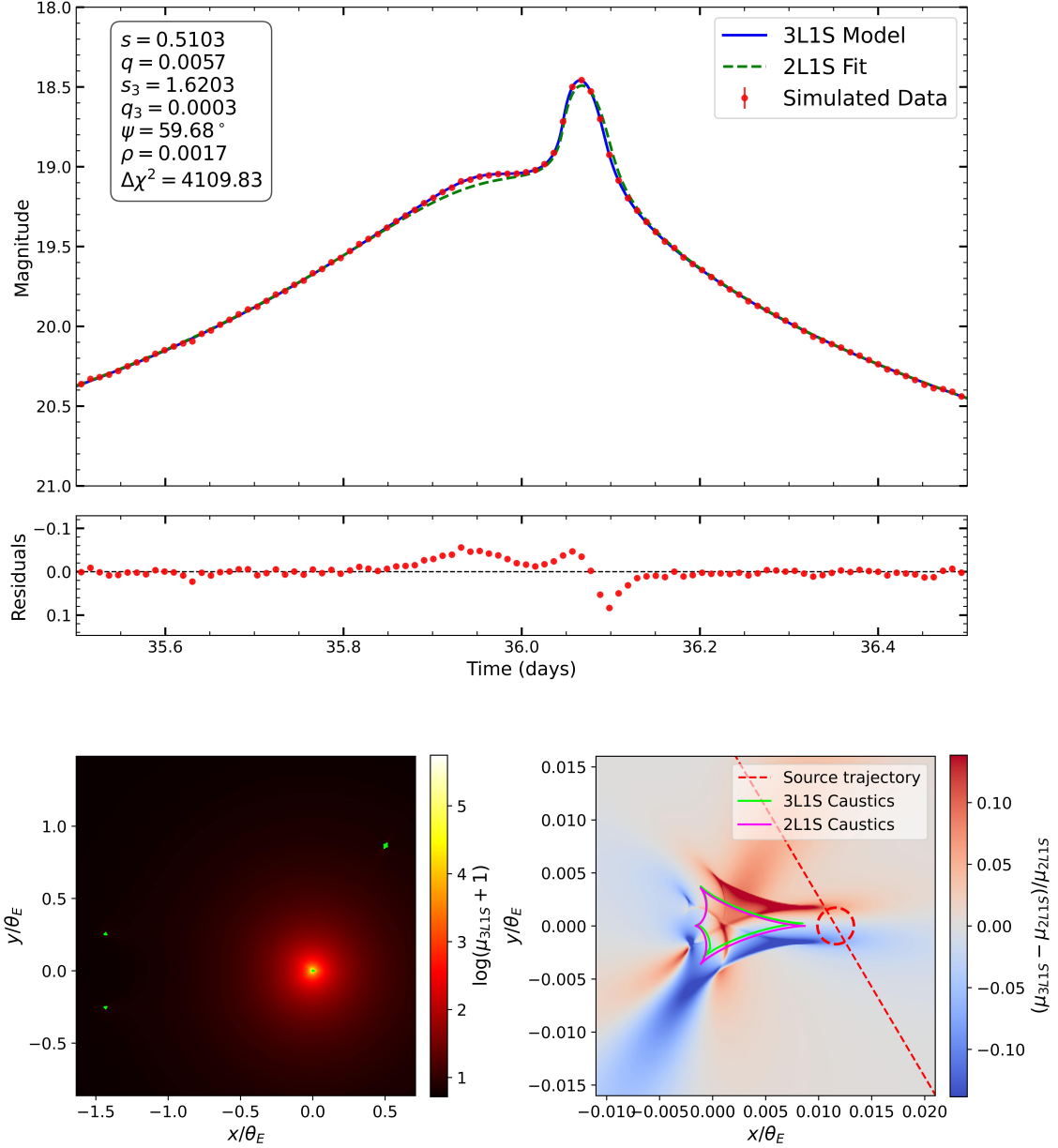


Figure 4.10: Simulated light curve for a close-wide configuration. The bottom-left panel shows the magnification map for the triple-lens system, while the bottom-right panel displays the map of the relative differences between the triple- and binary-lens models near the central caustic. Figure adapted from (Saggese et al. 2025)

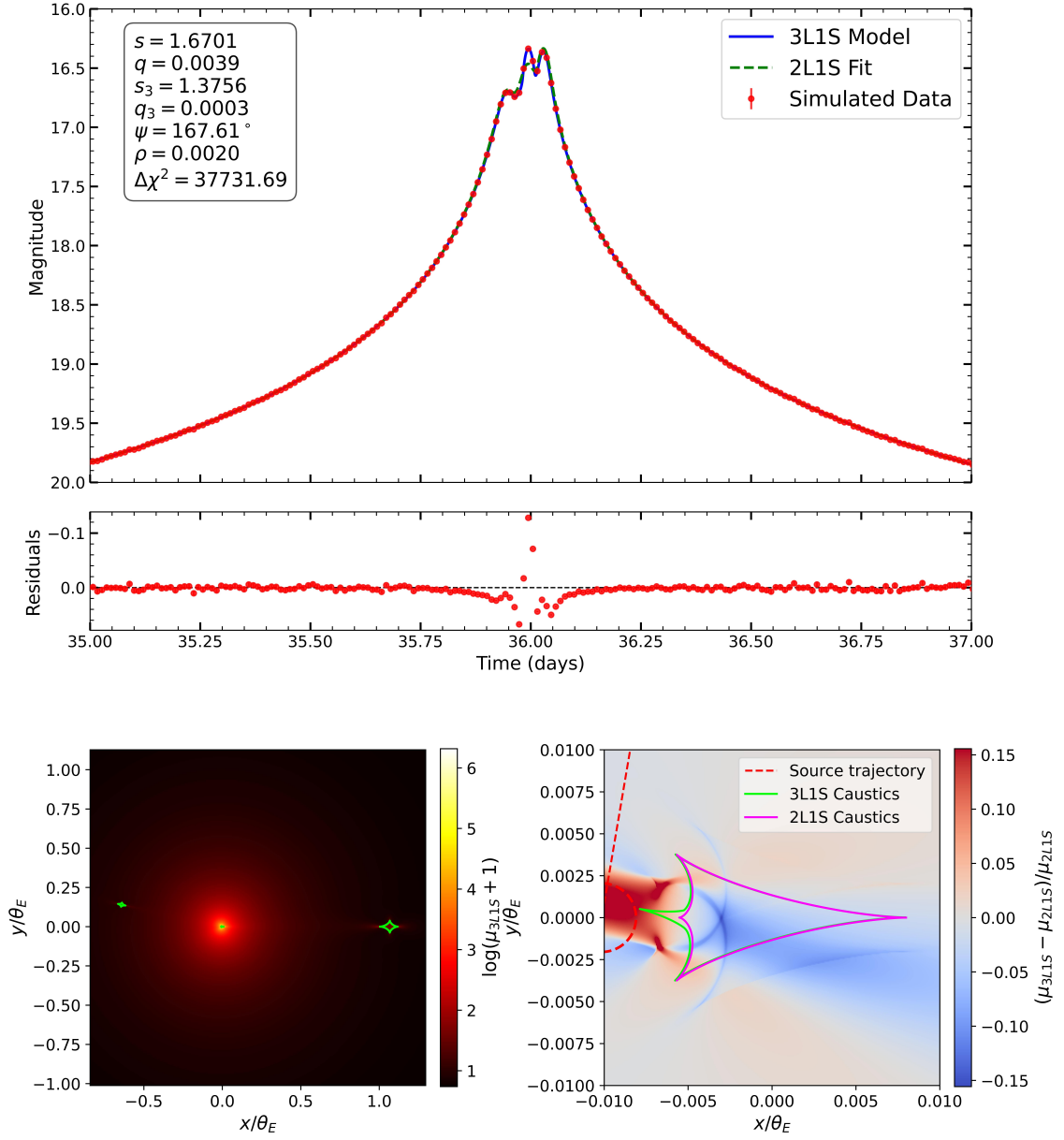


Figure 4.11: Simulated light curve for a system with two wide-separation planets. Figure adapted from (Saggese et al. 2025)

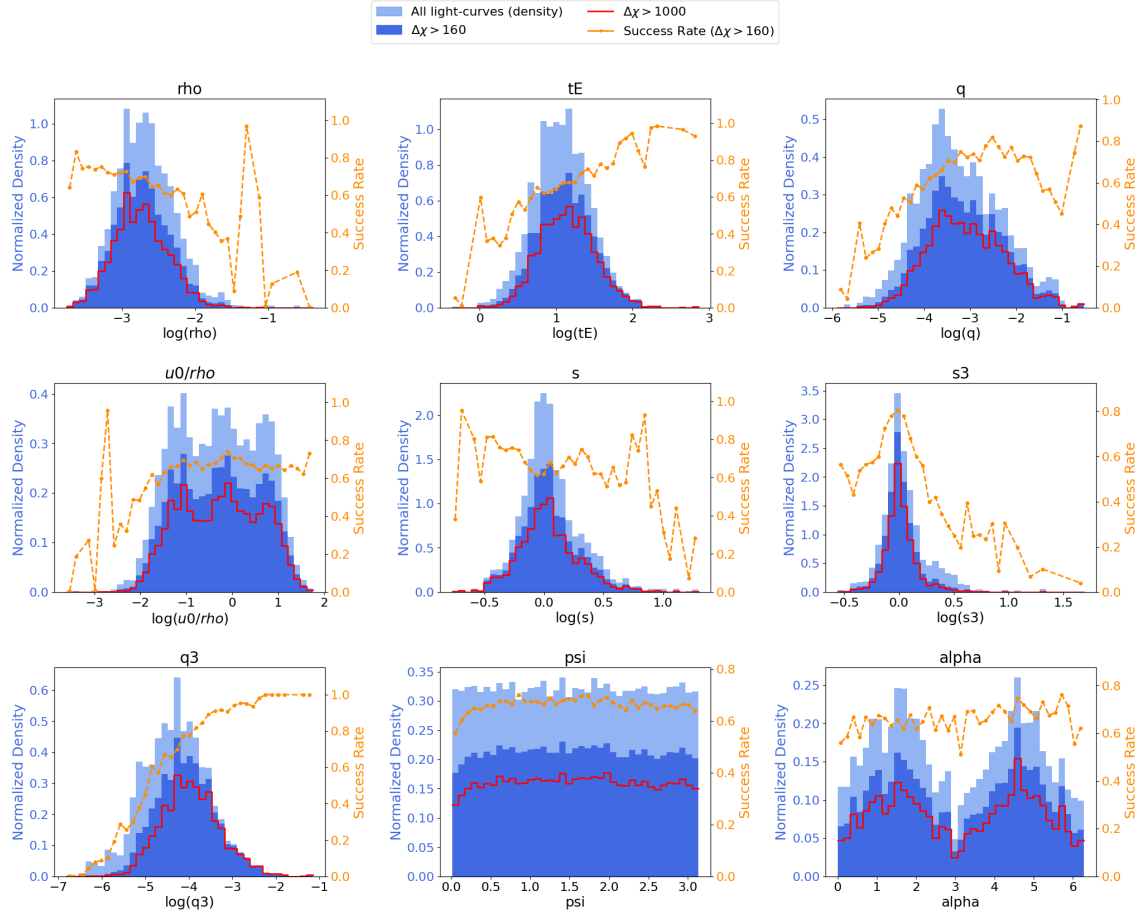


Figure 4.12: Detection efficiency of triple-lens events as a function of physical and geometric parameters. Each panel shows the normalized distribution of all simulated events (light blue), those with significant signals ($\Delta\chi^2 > 160$, dark blue), those with very strong signals ($\Delta\chi^2 > 1000$, red line), and the corresponding success rate (orange curve). Efficiency increases with the outer planet's mass ratio q_3 and separation s_3 , while remaining nearly insensitive to the relative angular separation ψ between planets. Figure adapted from (Saggese et al. 2025)

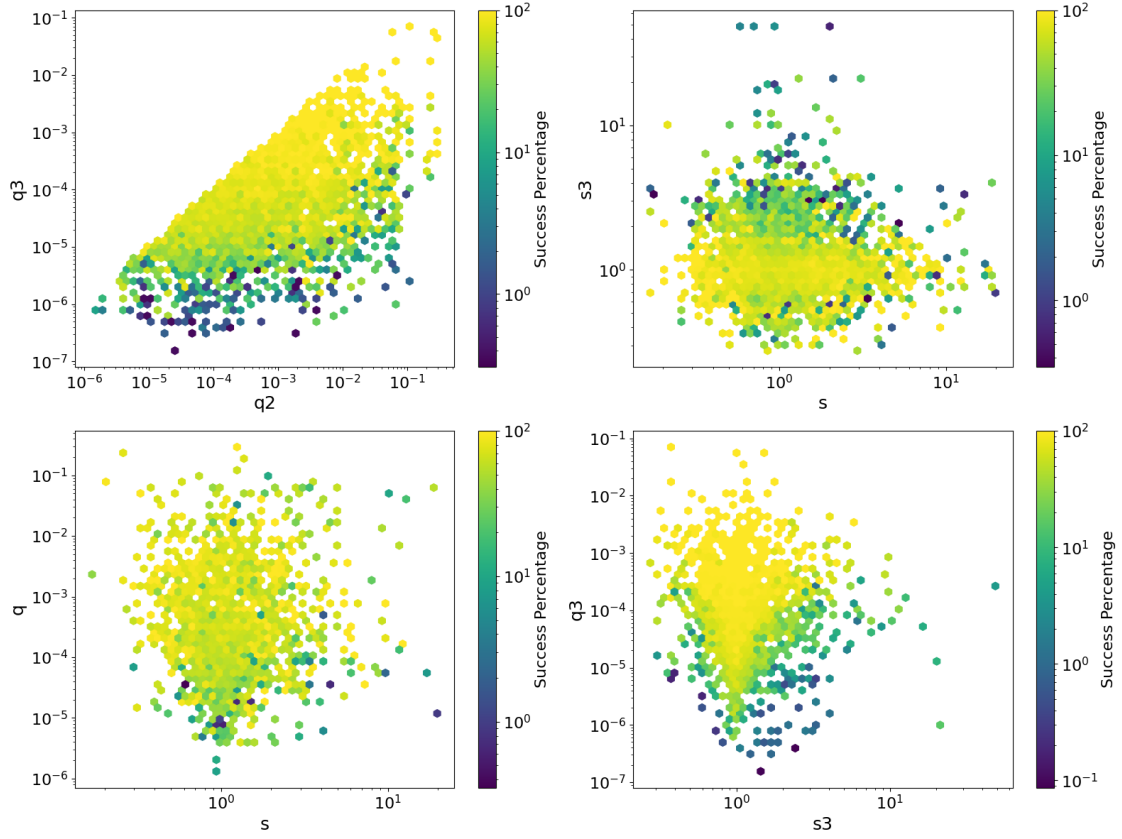


Figure 4.13: Detection efficiency across parameter spaces. Each panel shows a hexbin map in which color encodes the fraction of detectable triple-lens events within each bin of the simulated parameter space: *top-left* (q , q_3), *top-right* (s , s_3), *bottom-left* (q , s), and *bottom-right* (q_3 , s_3). Detection efficiency rises with planetary mass ratio and is enhanced when at least one planet lies near the Einstein ring ($s \sim 1$), consistent with the increased caustic size and anomaly amplitude. Figure adapted from (Saggese et al. 2025)

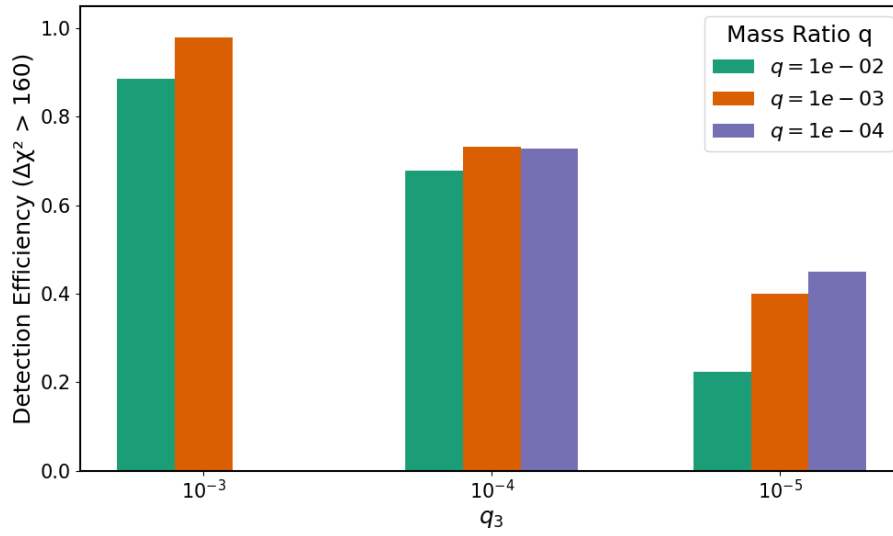


Figure 4.14: **Detection efficiency versus q_3 for different q values.** Detection efficiency of triple-lens systems as a function of the secondary planet's mass ratio (q_3), for three different main-planet mass ratios ($q = 10^{-2}$, 10^{-3} , and 10^{-4}). When the secondary planet is massive ($q_3 = 10^{-3}$), detection efficiency is high regardless of q . As q_3 decreases, efficiency drops in all cases, most steeply when the main planet is more massive. Figure adapted from (Saggese et al. 2025)

Dynamical Analysis of TESS Planetary Systems in PLATO's LOPS2 Field

5.1 Introduction

This chapter is based on a work currently in preparation.

In the introductory Chapter 1 we noted that the *Kepler* mission revealed recurring patterns in the architecture of planetary systems. Statistical studies showed that planets of the same system tend to have similar sizes and regularly spaced orbits, with frequent orbital ratios close to resonant zones (Lissauer et al. 2011; Fabrycky et al. 2014; Lissauer et al. 2014; Weiss et al. 2018; Lissauer et al. 2023). Among these trends, the ‘peas-in-a-pod’ pattern suggests that planets form and evolve in an organized way, retaining signatures of their migration history and interactions within the protoplanetary disk.

Such empirical patterns could suggest that many planetary systems that we observe might be dynamically ‘packed’, which means that their configuration saturates all long-term stable orbital zones (Naoz et al. 2013).

However, recent numerical and observational analyses have shown that this picture is not universal. Some multiplanetary systems exhibit residual stable zones, ‘gaps’ between adjacent planets, where additional bodies could exist without disrupting the stability of the system (Hsu et al. 2019; Horner et al. 2025). These regions often coincide with low-order mean-motion resonances or secularly protected zones, where long-term orbital stability can persist.

The *Transiting Exoplanet Survey Satellite* (TESS) has expanded our view of multiplanetary systems, identifying thousands of new candidates across nearly the entire sky. Its

sensitivity, however, declines for small planets ($R_p \lesssim 1 R_\oplus$) and for longer orbital periods ($P \gtrsim 20$ days), due to the limited photometric precision and shorter observing baselines compared to *Kepler* (Sullivan et al. 2015; Guerrero et al. 2021).

Recent studies (Horner et al. 2025) suggest that many planetary systems discovered by TESS are already packed, but not all, some systems could retain stable orbital niches that could harbor undetected terrestrial planets. TESS is less sensitive to Earth-sized or sub-Earth-sized planets

The upcoming *PLATO* mission by the European Space Agency, already introduced in Chapter 2, is designed to overcome the observational limitations of current transit surveys. Its high photometric precision and extended temporal coverage will allow the detection of Earth-sized planets around bright, Sun-like stars, precisely the regime where *TESS* sensitivity rapidly declines.

Among the mission’s planned observing fields, the first long-duration pointing, LOPS2 (Long Observation Phase South 2), presents an excellent opportunity. As discussed in Chapter 2, this part of the southern sky contains numerous multiplanetary systems already identified by *TESS* and complementary ground-based surveys. These systems provide ideal natural laboratories for studying the dynamical structure of compact planetary architectures and for investigating the possible presence of additional low-mass planets hidden in dynamically stable gaps.

In this work, we focus on a subset of well-characterized multiplanetary systems detected by *TESS* and located within the *PLATO* LOPS2 field. By analyzing their dynamical stability, we aim to identify regions of phase space where additional Earth-mass planets could exist without significantly perturbing the known configurations. Here, we focus on how *PLATO* and *TESS* can work together, not to find long-period planets, but to improve the detection and characterization of multiplanetary systems.

5.2 Systems analyzed

Within the LOPS2 region, the *TESS* mission has identified dozens of confirmed exoplanets, including several multiplanet systems that are valuable for dynamical studies and future high-precision follow-up.

According to Eschen et al. (2024), fifteen confirmed multiplanetary systems lie within the LOPS2 field, but not all of them were discovered using the transit method. The spatial distribution of known transiting and non-transiting planetary systems within this region is shown in Figure 5.1.

Restricting the sample to the systems originally discovered by *TESS* through transits, and for which all known planets were detected photometrically, yields the following

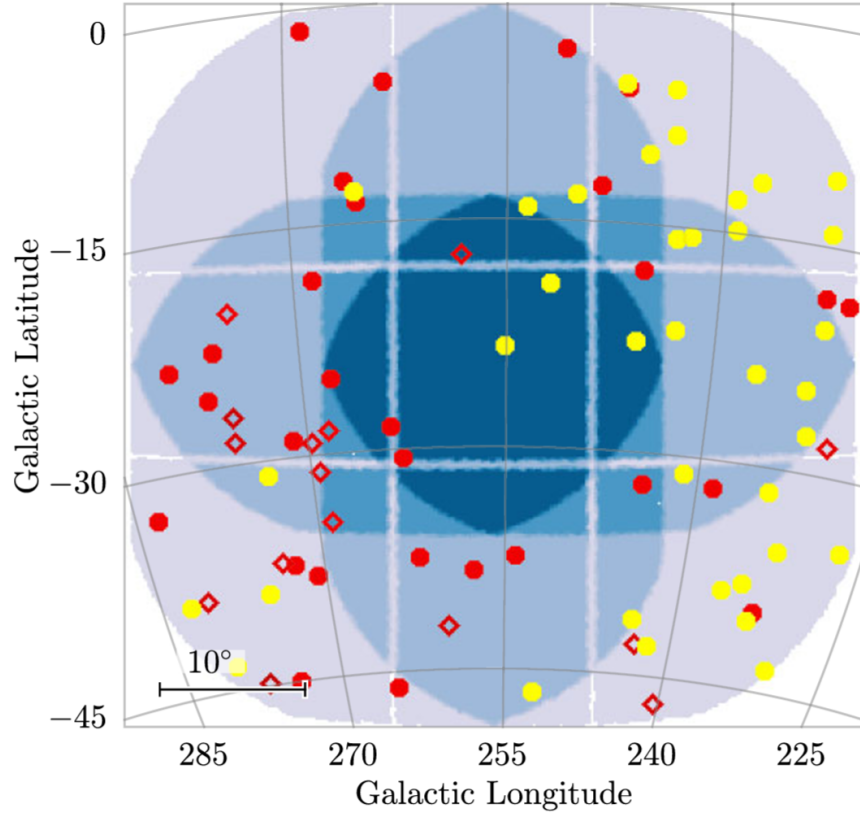


Figure 5.1: Map of the **LOPS2** region in Galactic coordinates, showing the location of stars hosting at least one known transiting planet. Systems discovered by *TESS* are shown in red, while those found by ground-based surveys appear in yellow. Filled symbols indicate single-planet systems, and open diamonds represent multiplanet systems. The background shading illustrates the coverage of *PLATO*’s Normal Cameras, with darker areas corresponding to higher observational overlap. Figure adapted from Eschen et al. (2024).

subset: HD 23472, HD 28109, LHS 1678, TOI-216, TOI-2525, TOI-270, TOI-286, TOI-451, TOI-700, and TOI-712.

These systems span a wide range of stellar types (from late M dwarfs to early G stars) and planetary architectures, providing a representative sample of compact, dynamically interacting systems observable by both *TESS* and *PLATO*. They include compact sub-Neptune systems such as TOI-270 and TOI-700, resonant or near-resonant gas giant pairs such as TOI-216 and TOI-2525, and dynamically rich architectures such as TOI-712 and TOI-431. All are located within the LOPS2 field, ensuring optimal coverage and photometric precision for long-term *PLATO* observations.

In this work, we analyze the dynamical stability of the most promising systems (see Section 5.3 for the methodology with which they were chosen). Our goal is to map the stability of their orbital configurations and identify regions of parameter space where additional Earth-mass planets could exist without disrupting the known systems. The

stellar and planetary parameters adopted, of the systems chosen for the analysis, are summarized in Table 5.1 and Figure 5.2, made in the same style as the figure present in (Eschen et al. 2024), provides a visual overview of their orbital architectures.

Host Star	Vmag	$R_{\star} (R_{\odot})$	$M_{\star} (M_{\odot})$	$T_{\text{eff}} \text{ (K)}$	Max Period (days)
HD 28109	$9.40^{+0.02}_{-0.02}$	$1.45^{+0.04}_{-0.04}$	$1.26^{+0.08}_{-0.08}$	6120^{+50}_{-50}	84.26
TOI-270	$12.62^{+0.03}_{-0.03}$	$0.380^{+0.020}_{-0.020}$	$0.400^{+0.020}_{-0.020}$	3386^{+137}_{-131}	11.38
TOI-286	$9.87^{+0.10}_{-0.10}$	$0.800^{+0.020}_{-0.020}$	$0.800^{+0.020}_{-0.020}$	5132^{+50}_{-50}	39.36
TOI-451	$9.12^{+0.02}_{-0.02}$	$0.90^{+0.02}_{-0.02}$	$0.90^{+0.05}_{-0.05}$	5530^{+50}_{-50}	16.36
TOI-700	$13.15^{+0.07}_{-0.07}$	$0.420^{+0.020}_{-0.020}$	$0.410^{+0.020}_{-0.020}$	3482^{+70}_{-70}	37.42
TOI-712	$10.90^{+0.02}_{-0.02}$	$0.674^{+0.017}_{-0.017}$	$0.73^{+0.03}_{-0.03}$	4622^{+61}_{-61}	84.84

Table 5.1: Stellar parameters of multiplanetary systems in the *PLATO* LOPS2 field, discovered by *TESS* using the transit method. Listed are the visual magnitude, stellar radius, mass, effective temperature, and the longest orbital period among confirmed planets in each system.

5.3 Method

To assess the dynamical stability of the selected multiplanetary systems and to identify regions where additional Earth-mass planets could exist without destabilizing the configuration, we employed a two-stage numerical framework based on the MEGNO (Mean Exponential Growth factor of Nearby Orbits; Cincotta et al. (2000); Cincotta et al. (2003)) chaos indicator — a well-established diagnostic for identifying chaotic versus quasi-periodic motion in planetary systems (e.g. Hinse et al. (2010); Jenkins et al. (2019); Delrez et al. (2022)).

The MEGNO computations were carried out with the *REBOUND* N-body package (Rein et al. 2012), using the *WHFast* symplectic Wisdom–Holman algorithm (Rein et al. 2015), which ensures long-term energy conservation and computational efficiency. The time-averaged value, $\langle Y(t) \rangle$, amplifies subtle stochastic variations in the orbital evolution, enabling a clear separation between regular and chaotic dynamics: for quasi-periodic motion, $\langle Y(t \rightarrow \infty) \rangle \approx 2$, whereas for chaotic trajectories $\langle Y(t \rightarrow \infty) \rangle$ diverges approximately linearly with time, reflecting the local Lyapunov exponent.

For systems lacking direct mass determinations of their planets, we estimated planetary masses from their measured radii using the empirical mass–radius relation of Chen et al. (2017), which provides a probabilistic scaling across the terrestrial, neptunian, and jovian regimes based on a large observational sample.

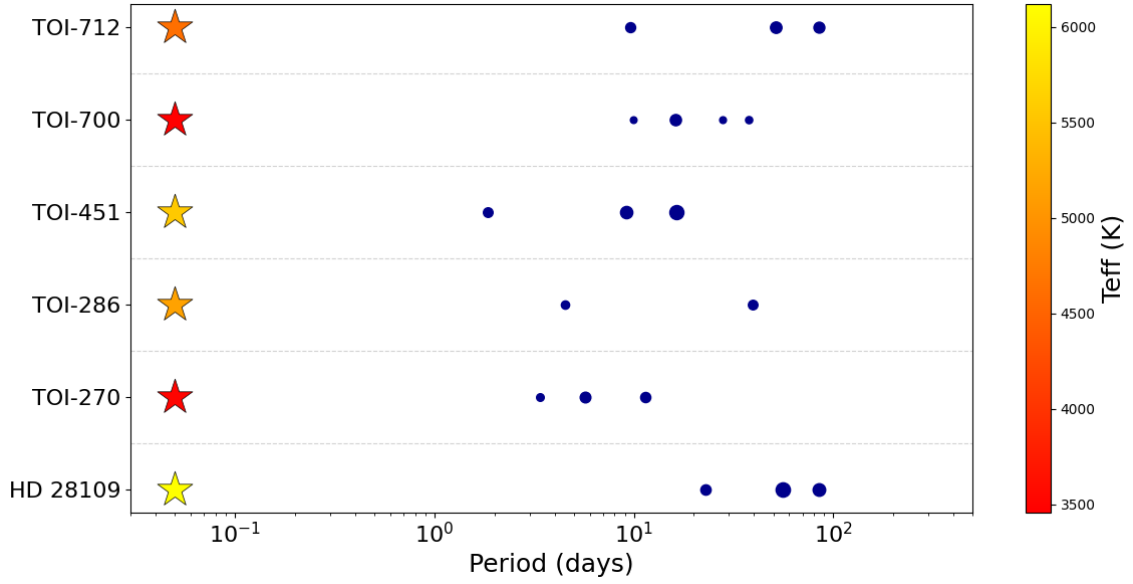


Figure 5.2: Orbital period distribution in multiple planetary systems, with host stars color-coded by their effective temperature (T_{eff}). Planets are shown as dark blue circles, with sizes scaled according to their planetary radius (R_p). Each system is separated by a horizontal black line, and stars are arranged in order of increasing stellar temperature.

For each system, we constructed a two-dimensional grid of initial orbital conditions covering a broad range in semi-major axis (a) and eccentricity (e). The semi-major axis was sampled with 256 uniformly spaced points between the orbits of the innermost and outermost known planets, while the eccentricity was divided into 30 intervals from circular configurations ($e = 0$) up to highly eccentric ones ($e = 0.9$).

At each grid node, a test planet of one Earth mass ($1, M_{\oplus}$) was inserted, with its mean anomaly (M) and argument of pericenter (ω) randomly drawn from uniform distributions within $[0, 2\pi]$. A single integration was then performed for each (a, e) pair, yielding the corresponding MEGNO value. This approach produces two-dimensional MEGNO maps that capture the global dynamical structure of each system without averaging over orbital angles.

In total, each simulation consisted of $256 \times 30 = 7680$ individual integrations per system.

The orbital integrations were performed using a symplectic scheme with a fixed time step of $\Delta t \approx P_{\text{inner}}/30$, covering 10^6 orbital periods of the innermost known planet, depending on the system’s compactness and dynamical timescales.

This grid resolution was chosen as a compromise between dynamical resolution and computational feasibility. Future developments will include higher-resolution grids and extended integration times.

To focus on the dynamically relevant inner regions, we excluded the innermost zone

close to the host star—where tidal and relativistic effects dominate—and the far outer regions beyond the last known planet. This approach ensured that the analysis probed primarily the intermediate zones where additional terrestrial planets could plausibly exist.

The resulting MEGNO stability maps in the (a, e) plane provided a global view of the dynamical architecture of each system, highlighting long-term stable zones, chaotic regions, and dynamically forbidden areas between the known planets. These maps form the basis for evaluating the potential habitability and the existence of additional planets within each system.

Before performing the MEGNO analysis, we excluded from our sample those systems that already appeared dynamically compact according to the Hill stability criterion.

Hill stability provides a first-order estimate of whether adjacent planets can avoid orbit crossing over long timescales (Gladman 1993). For two planets with semi-major axes $a_1 < a_2$ and total mass $(m_1 + m_2) \ll M_*$, the system is considered Hill stable if their separation exceeds a few mutual Hill radii,

$$R_{H,12} = \left(\frac{m_1 + m_2}{3M_*} \right)^{1/3} \frac{a_1 + a_2}{2}, \quad (5.1)$$

which typically requires $\Delta = (a_2 - a_1)/R_{H,12} \gtrsim 3.5$. In multiplanetary systems, empirical studies indicate that long-term stability generally demands $\Delta \gtrsim 10$ –12 (Chambers et al. 1996; Fang et al. 2013).

For this reason, only systems with sufficiently wide orbital separations—where additional planets could potentially exist without violating basic Hill stability—were selected for the dynamical analysis. The systems considered here are: TOI-286, TOI-270, TOI-451, TOI-700, HD 28109, and TOI-712.

5.4 Results and Discussion

The MEGNO analysis provides an overview of the dynamical structure of each selected multiplanet system. The resulting stability maps in the (a, e) plane reveal stable regions, chaotic zones, and resonance boundaries between the known planets, helping to identify areas where additional Earth-mass planets could potentially exist. In general, systems with wider orbital separations show broad stable regions, whereas compact or near-resonant systems exhibit narrower and more fragmented zones of stability. In the following subsections, we examine each system in detail, discussing the extent of stable regions and what they suggest about the long-term stability of the system and the possible presence of additional Earth-like planets.

5.4.1 TOI-286

Overview

TOI-286 is a system composed of a K3V-type dwarf star and two confirmed exoplanets: TOI-286 b, classified as a super-Earth, and TOI-286 c, a sub-Neptune (Hobson et al. 2024). The two planets are located on opposite sides of the so-called “radius valley,” discussed in Chapter 1, a feature generally attributed to atmospheric photoevaporation or to differences in core accretion efficiency during planet formation.

TOI-286 b, the inner planet, has a radius of $1.42 R_{\oplus}$ and completes an orbit in about 4.5 days. Its high density suggests a predominantly rocky composition, likely with a substantial iron-rich core, and its equilibrium temperature is estimated to be around 979 K. In contrast, TOI-286 c, the outer planet, has a larger radius of about $1.86 R_{\oplus}$ and orbits the star in approximately 39.4 days. Its lower density indicates the possible presence of a significant water layer or a thin hydrogen–helium atmosphere (Hobson et al. 2024). The architecture of the TOI-286 system is illustrated in Figure 5.3.

The system was characterized through observations by the *TESS* space telescope and follow-up high-resolution spectroscopy with ESPRESSO (Pepe et al. 2021). These complementary datasets provided accurate planetary radii, masses, and orbital parameters, confirming the transiting nature of both planets and ruling out the presence of close stellar companions. Kinematic analysis further indicates that the host star is a member of the Milky Way’s young disk population, although it does not appear to be associated with any known stellar moving group (Hobson et al. 2024).

Dynamical Analysis

The dynamical analysis of TOI-286 was carried out following the procedure described in Section 5.3, using the REBOUND N -body integrator to compute the MEGNO indicator and evaluate the long-term dynamical stability of the system.

The MEGNO stability map for TOI-286 (Figure 5.4) reveals a well-defined stability dome located between the two known planets, extending roughly from $a \simeq 0.06$ to 0.20 AU. Within this region, orbits with low to moderate eccentricities ($e \lesssim 0.4$) maintain MEGNO values close to $\langle Y \rangle = 2$, indicating quasi-periodic, dynamically regular motion. The dome’s boundaries become increasingly chaotic at higher eccentricities, where resonant overlaps drive orbital diffusion. The center of the dome, corresponding to an orbital period of about 18.8 days, lies near the 4:1 mean-motion resonance with the inner planet TOI-286 b, suggesting that this resonance plays a stabilizing role in defining the structure and extent of the stable zone.

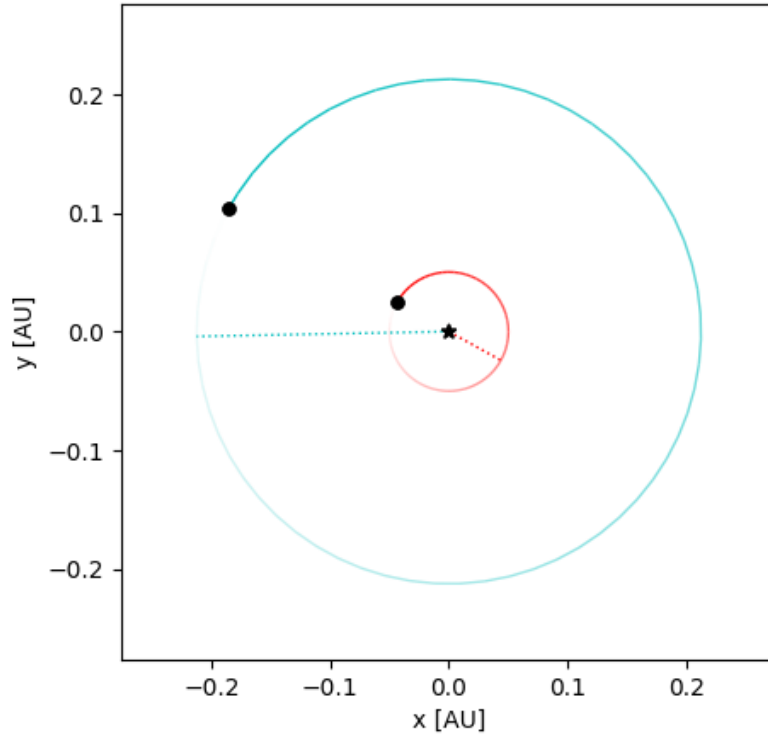


Figure 5.3: Orbital configuration of the TOI-286 system. The red and cyan lines represent the orbits of TOI-286 b and TOI-286 c, respectively, around their host star (black star symbol). The positions correspond to a representative orbital phase. Figure obtained with REBOUND

The dynamical picture that emerges is that of a system that is *not dynamically full*. A wide region between TOI-286 b and c remains available for the long-term survival of additional Earth-mass planets.

From a physical perspective, TOI-286 therefore appears to host a broad and dynamically stable 'corridor' between its two known planets. Unlike the tightly packed, resonant chains of compact multiplanet systems, the architecture of TOI-286 appears dynamically relaxed, offering sufficient space for an additional body to persist without perturbing the known configuration.

5.4.2 TOI-270

Overview

TOI-270 is a nearby M3V-type star hosting three transiting exoplanets detected by the *TESS* mission: TOI-270 b, TOI-270 c, and TOI-270 d (Günther et al. 2019; Van Eylen et al. 2021; Kaye et al. 2022; Mikal-Evans et al. 2023; Coulombe et al. 2025). The system is particularly compact, with all planets residing within 0.1 AU from the host star (Figure 5.5).

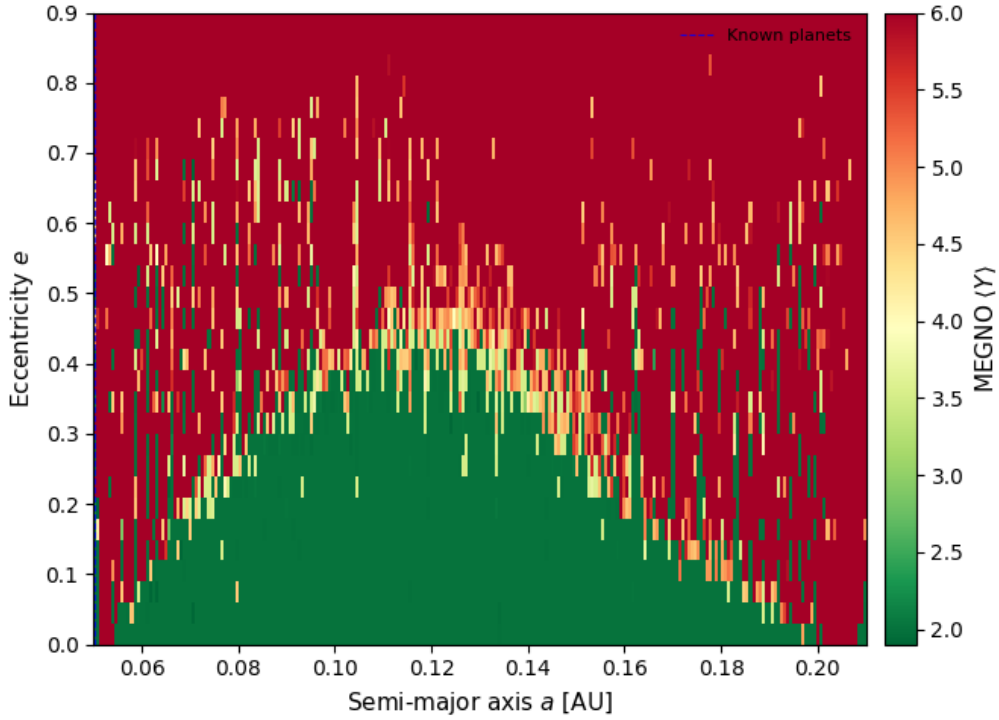


Figure 5.4: MEGNO stability map for the TOI-286 system in the semi-major axis (a)–eccentricity (e) plane. The color scale indicates the MEGNO parameter $\langle Y \rangle$, where values near 2 (green) correspond to quasi-periodic, long-term stable orbits, while higher values (red) indicate chaotic or unstable configurations.

The planets have orbital periods of 3.36, 5.66, and 11.38 days, and radii of 1.25, 2.42, and 2.13 $R_{\oplus}$, respectively (Günther et al. 2019). Mass measurements from transit-timing variations and radial velocities yield $M_b \simeq 1.5, M_{\oplus}$, $M_c \simeq 6.2, M_{\oplus}$, and $M_d \simeq 4.2, M_{\oplus}$ (Kaye et al. 2022), implying a rocky composition for the inner planet and volatile-rich envelopes for the two outer sub-Neptunes. The system spans the so-called “radius valley,” making it an excellent target for investigating atmospheric escape and compositional diversity among low-mass planets around M dwarfs (Kaye et al. 2022; Coulombe et al. 2025; Mikal-Evans et al. 2023).

The orbital architecture of TOI-270 is remarkably regular and compact. The period ratios between adjacent planets are close to 5:3 and 2:1, forming a near-resonant 5:3:2 chain that likely originated through smooth disk migration followed by partial tidal damping (Kaye et al. 2022). Such resonant spacing helps maintain long-term stability despite the system’s tight configuration and may have prevented chaotic divergence during the disk-dissipation phase.

From a dynamical standpoint, TOI-270 satisfies the Hill stability criterion but lies near the threshold of compactness. The mutual separations between adjacent planets are $\Delta_{bc} \approx 9.3$ and $\Delta_{cd} \approx 13.8$ mutual Hill radii, indicating that the region between planets b

and c is dynamically full, whereas a modestly wider gap exists between planets c and d. This outer interval could host limited islands of stability.

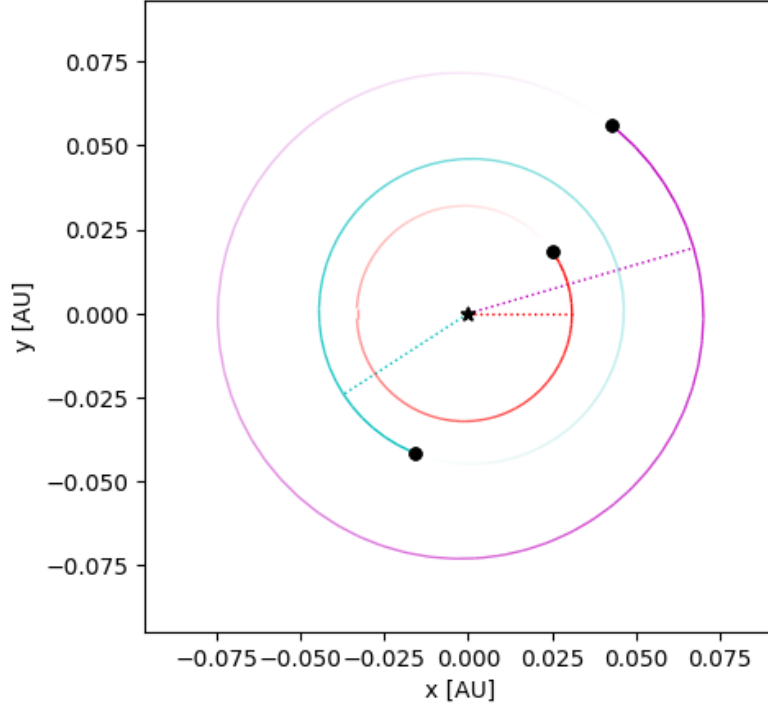


Figure 5.5: Orbital configuration of the TOI-270 system. The red, cyan, and magenta lines represent the orbits of TOI-270 b, c, and d, respectively, around their host star (black star symbol). The positions correspond to a representative orbital phase. Figure obtained with REBOUND.

Dynamical Analysis

The MEGNO stability map for the TOI-270 system (Figure 5.6) reveals a highly compact and dynamically constrained configuration. The phase space between the three known planets is almost entirely dominated by chaotic behavior, with $\langle Y \rangle > 4$ for most values of semi-major axis and eccentricity. No dynamically stable “corridor” is evident between the planets, confirming that TOI-270 is dynamically full in its current configuration.

The absence of any significant stable zone between the planets is consistent with the near-resonant architecture, where mutual gravitational interactions and resonance overlap produce a tightly coupled dynamical chain.

Its compact, near-resonant architecture and low eccentricities suggest that the system may have experienced convergent migration within the protoplanetary disk, followed by partial tidal circularization that stabilized the present configuration. The resulting configuration is dynamically quiet but tightly constrained, leaving no room for additional stable companions in the inner system.

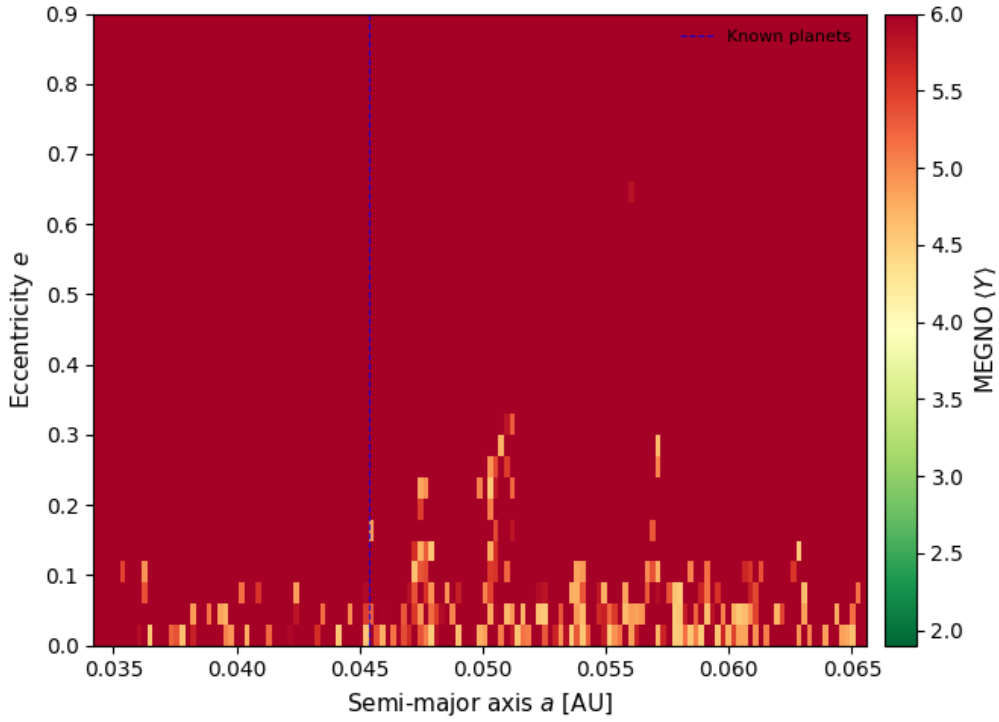


Figure 5.6: MEGNO stability map for the TOI-270 system in the semi-major axis (a)–eccentricity (e) plane. The range in a extends from the orbit of the innermost planet (TOI-270 b) to that of the outermost one (TOI-270 d). Values close to $\langle Y \rangle = 2$ (green) correspond to quasi-periodic, long-term stable orbits, whereas higher values (yellow–red) indicate chaotic motion. The dashed blue vertical lines mark the positions of the intermediate planets (TOI-270 c).

5.4.3 TOI-451

Overview

TOI-451 (TIC 257605131) is a young, late-type dwarf star (spectral type G9–K0V) located at a distance of about 124 pc. It is a confirmed member of the Pisces–Eridanus stellar stream, a kinematically coherent association with an estimated age of 120 ± 8 Myr (Newton et al. 2021). The youth of the star is supported by lithium absorption, rapid rotation ($P_{\text{rot}} = 5.1 \pm 0.1$ days), and modest UV excess, typical of systems younger than 150 Myr. Stellar parameters derived from spectroscopic and photometric analyses indicate $M_{\star} = 0.95 \pm 0.02 M_{\odot}$, $R_{\star} = 0.88 \pm 0.03 R_{\odot}$, and $T_{\text{eff}} = 5550 \pm 60$ K, consistent with a G9–K0 dwarf. The star also has a distant comoving M-dwarf binary companion at ~ 4700 AU, making TOI-451 a wide multiple system (Newton et al. 2021).

The *TESS* mission detected three transiting planets—TOI-451 b, TOI-451 c, and TOI-451 d—forming a compact system within 0.11 AU of the star. Their orbital periods are 1.85, 9.19, and 16.36 days, corresponding to semimajor axes of 0.027, 0.081, and 0.106 AU, respectively. The planets have radii of 1.9, 3.1, and 4.1, $R_{\oplus}$, placing the inner planet near the rocky–volatile transition and the outer two in the sub-Neptune regime. The architecture is approximately near a 5:2:3 sequence of period ratios, though not locked in resonance, suggesting that an initially resonant configuration may have been disrupted by tidal or disk-induced damping during the system’s early evolution. The orbital configuration of the system is illustrated in Figure 5.7. Follow-up observations with *Spitzer* and ground-based photometry confirmed the transits and refined the orbital parameters, while high-contrast imaging ruled out nearby stellar contaminants.

Given its youth, TOI-451 provides a rare glimpse into the early stages of planetary evolution. At ~ 120 Myr, its planets are expected to still be contracting and cooling, while intense high-energy radiation from the host star drives active photoevaporation (Newton et al. 2021). TOI-451 b, the innermost planet, likely represents a dense super-Earth that has lost most of its primordial atmosphere under extreme irradiation ($T_{\text{eq}} \sim 1100$ K), whereas TOI-451 c and d may have retained significant volatile envelopes or low-mass H/He atmospheres ($T_{\text{eq}} \sim 900\text{--}700$ K). The detection of an infrared excess around the star further suggests the presence of a debris disk, potentially analogous to the early Solar System’s asteroid or Kuiper belt (Newton et al. 2021).

Dynamical Analysis

The MEGNO stability map of TOI-451 (Figure 5.8) reveals a rich dynamical structure within its compact orbital architecture. Between TOI-451 b and c, a broad dome-shaped region of regular motion extends from about $a \simeq 0.045$ to 0.075 AU, with the maximum

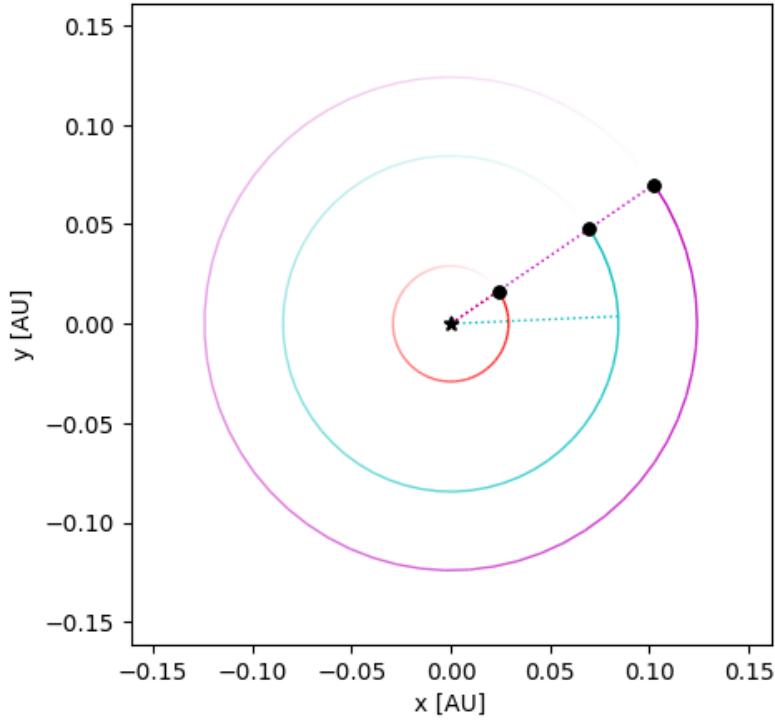


Figure 5.7: Orbital configuration of the TOI-451 system. The red, cyan, and magenta lines represent the orbits of TOI-451 b, c, and d, respectively, around their host star (black star symbol). Figure generated with REBOUND

stability located near $a \simeq 0.050$ AU, corresponding to an orbital period of ~ 4.5 days. This position matches closely the 5:2 mean-motion resonance with TOI-451 b ($P_b = 1.85$ days), suggesting that the region’s stability is maintained by resonant protection. Within this dome, orbits with eccentricities up to $e \lesssim 0.25$ display MEGNO values near $\langle Y \rangle \simeq 2$, indicative of quasi-periodic, long-term regular motion, while higher eccentricities quickly lead to chaotic diffusion. The shape of the dome and its resonance-centered peak point to a dynamically protected corridor where additional low-mass planets could potentially reside without destabilizing the known configuration.

A second, narrower island of stability appears between TOI-451 c and d, centered at $a \simeq 0.11$ AU ($P \simeq 13.7$ days), which corresponds to the 3:2 resonance with TOI-451 c ($P_c = 9.19$ days). This confined zone hosts stable orbits at low eccentricity ($e \lesssim 0.15$), while adjacent regions show rapidly increasing MEGNO values, reflecting strong resonant overlap and chaotic behavior. Interior to TOI-451 b ($a < 0.035$ AU), low-eccentricity orbits remain stable in principle, though any ultra-short-period planet there would experience intense tidal and radiative effects, making long-term survival unlikely. Beyond the orbit of TOI-451 d ($a > 0.12$ AU), a wider region of regular motion emerges, suggesting that outer companions or residual debris could persist undisturbed.

Overall, the MEGNO analysis portrays TOI-451 as a young but dynamically mature

system, with distinct resonant corridors separating each pair of planets. The broad stability dome near the 5:2 resonance and the smaller 3:2 island farther out indicate that the system, while compact, is not completely dynamically full. Its present configuration likely reflects the aftermath of disk-driven migration and tidal damping, with resonance sculpting preserving long-term orbital order. These findings suggest that TOI-451 has already settled into a quasi-stable dynamical state, offering a valuable snapshot of how compact multi-planet systems stabilize within the first few hundred million years of evolution.

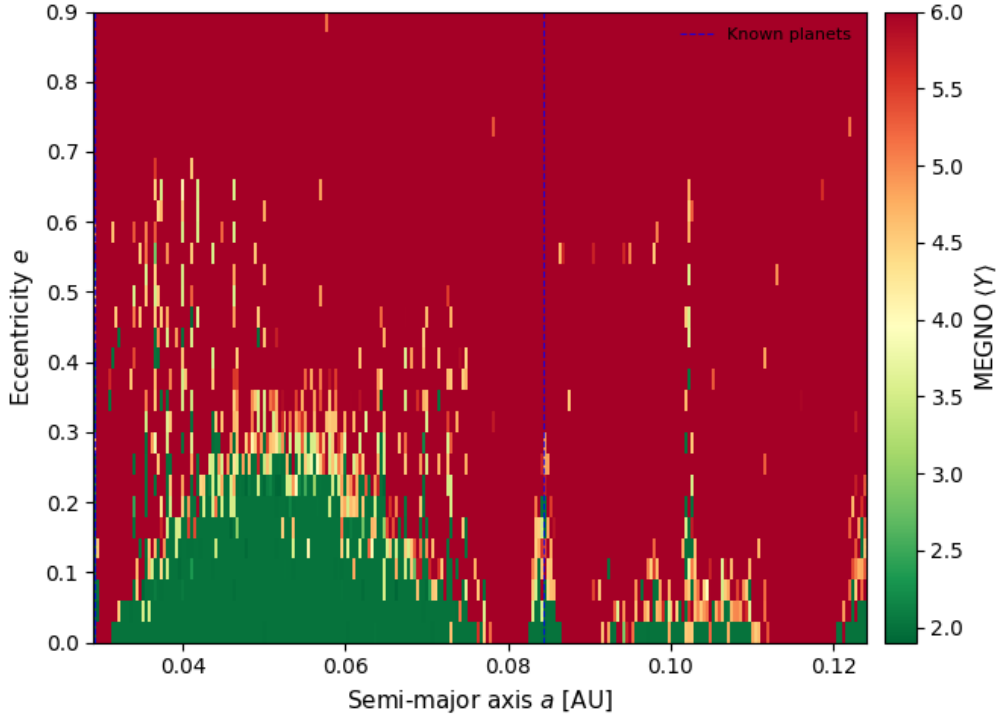


Figure 5.8: MEGNO stability map of the TOI-451 system in the semi-major axis (a)–eccentricity (e) plane. The explored range in a extends from the orbit of the innermost planet (TOI-451 b) to just beyond that of the outermost planet (TOI-451 d). The color scale represents the MEGNO parameter $\langle Y \rangle$. Vertical blue dashed lines mark the locations of the known planet TOI-451 c.

5.4.4 TOI-700

Overview

TOI-700 is a nearby M2.5V-type main-sequence star located at a distance of about 31 pc (Rodríguez et al. 2020). The system hosts four transiting exoplanets—TOI-700 b, c, d, and e—discovered and confirmed through *TESS* photometry and follow-up observations (Gilbert et al. 2020; Gilbert et al. 2023). The planets have orbital periods of approximately

9.98, 16.05, 27.8, and 37.42 days, and radii of 1.07, 2.68, 0.95, and 1.05, $R_{\oplus}$, respectively. The stellar luminosity ($L_{\star} \simeq 0.023, L_{\odot}$) places planets d and e near or within the classical habitable zone, making TOI-700 one of the most compelling M-dwarf systems for studies of terrestrial planet formation and potential habitability (Gilbert et al. 2023).

The system exhibits a well-ordered but compact orbital architecture (Figure 5.9). The two inner planets (b and c) are closely spaced but dynamically separated, while the outer planets (e and d) occupy wider orbits at the edge of the compact inner region. The low eccentricities ($e < 0.1$) and small mutual inclinations ($< 1^{\circ}$) inferred from transit analyses indicate a dynamically cold, coplanar configuration, likely the product of strong disk damping followed by long-term tidal evolution (Rodriguez et al. 2020). The period ratios ($P_c/P_b \simeq 1.61$, $P_e/P_c \simeq 1.73$, and $P_d/P_e \simeq 1.35$) show no evidence of exact resonant locking but suggest a near-commensurate chain consistent with smooth convergent migration during the early stages of formation.

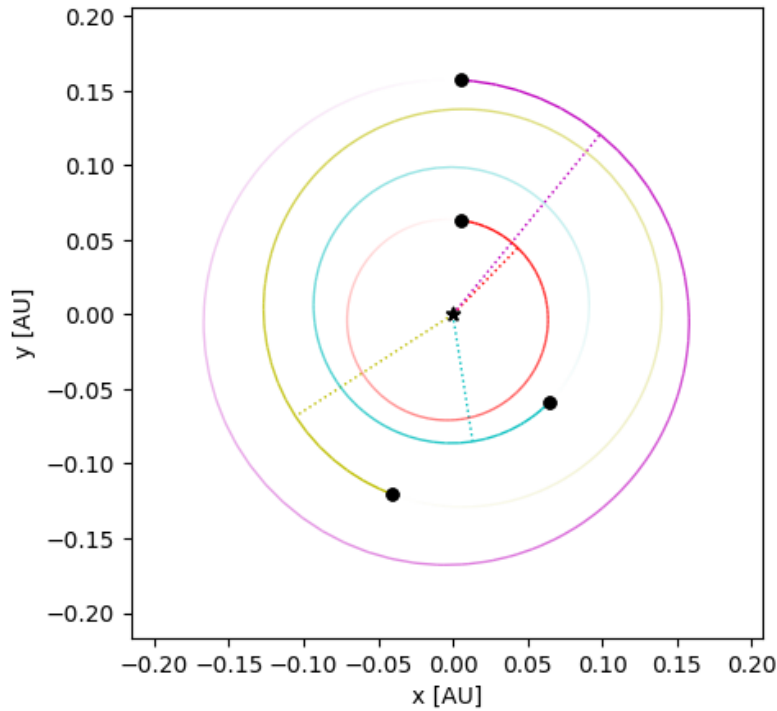


Figure 5.9: Orbital configuration of the TOI-700 system. The red, cyan, yellow, and magenta lines represent the orbits of TOI-700 b, c, e, and d, respectively, around their host star (black star symbol). Figure generated with REBOUND.

Dynamical Analysis

The MEGNO stability map for TOI-700 (Figure 5.10) reveals that the system is dynamically saturated, with no extended regions of long-term stability between the known planets.

The absence of low-MEGNO regions implies that the phase space between TOI-700 b, c, e, and d is already dynamically full: any additional body inserted in these intervals would rapidly experience chaotic diffusion or orbital crossing. This is consistent with the extremely compact orbital spacing of the system, where mutual separations are only marginally above the empirical threshold for long-term stability ($\Delta \sim 10\text{--}12$ mutual Hill radii).

TOI-700 appears as a fully packed, dynamically complete system, in which all stable zones permitted by Hill stability are already occupied by the four known planets. Any additional Earth-mass body placed between them would likely be unstable on timescales shorter than 10^6 orbits. This outcome reinforces the picture of TOI-700 as a long-term stable but tightly constrained M-dwarf system, consistent with a formation history dominated by disk migration that led to a compact configuration.

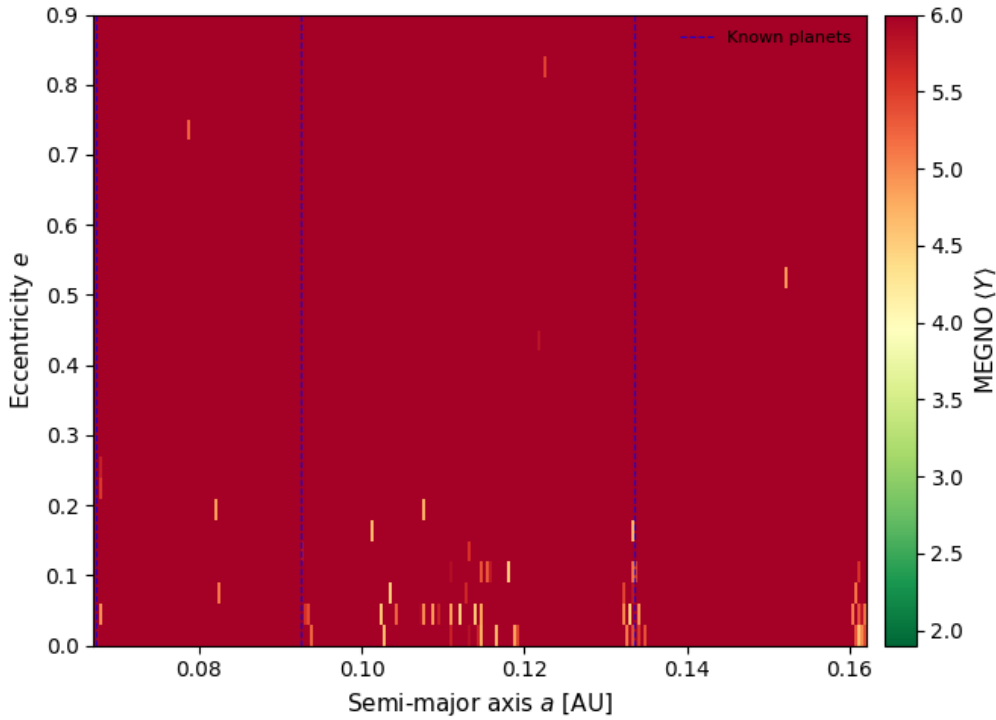


Figure 5.10: MEGNO stability map for the TOI-700 system in the semi-major axis (a)–eccentricity (e) plane. Green regions ($\langle Y \rangle \approx 2$) correspond to quasi-periodic, long-term stable motion, while higher MEGNO values (yellow to red) indicate chaotic or unstable trajectories. The blue dashed lines mark the locations of the known planets included within the simulated range of a , whereas the innermost and outermost planets lie at the boundaries of the plot.

5.4.5 HD 28109

Overview

HD 28109 (TOI-282) is a bright F8/G0V main-sequence star located at a distance of about 140 pc (Dransfield et al. 2022). It hosts three transiting sub-Neptunian planets—HD 28109 b, c, and d—discovered by the *TESS* mission and confirmed through extensive ground-based follow-up with ASTEP, LCO, and high-resolution spectroscopy from *ESPRESSO* and *HARPS*. The planets have orbital periods of 22.89, 56.01, and 84.26 days and radii of $2.20^{+0.10}_{-0.10}$, 4.23 ± 0.11 , and $3.25 \pm 0.11 R_{\oplus}$, respectively. Mass estimates derived from a joint analysis of transit timing variations and radial velocities yield $M_b = 18.5^{+9.1}_{-7.6}$, $M_c = 7.9^{+4.2}_{-3.0}$, and $M_d = 5.7^{+2.7}_{-2.1} M_{\oplus}$, implying a dense inner planet and two outer planets of lower density, consistent with volatile-rich sub-Neptunes.

The architecture of HD 28109 (Figure 5.11) is defined by two outer planets (*c* and *d*) near the first-order 3:2 mean-motion resonance ($P_d/P_c \simeq 1.50$), producing transit timing variations up to about one hour. The inner planet *b* orbits well inside the pair, at $a \simeq 0.136$ AU, with a period ratio $P_c/P_b \simeq 2.45$, indicating that it is dynamically decoupled from the resonant subsystem. The low eccentricities ($e < 0.2$) and mutual inclinations ($< 1^\circ$) point to a dynamically cold configuration, likely shaped by smooth disk migration and modest tidal damping. Given its brightness ($V = 9.4$) and location within the *TESS* continuous viewing zones, HD 28109 stands out as an excellent laboratory for studying the structure, evolution, and atmospheric properties (with JWST) of warm Neptune-class planets in compact multiplanet systems.

Dynamical Analysis

The MEGNO stability map for HD 28109 (Figure 5.12) shows that the system is dynamically saturated, with no significant long-term stable regions between its three known planets.

Between HD 28109 b and c, the phase space is dominated by chaotic diffusion, with $\langle Y \rangle > 4$ over nearly the entire range of a and e . The absence of a stable corridor suggests that the two planets are dynamically packed, leaving no viable intermediate orbits. A similar pattern is seen between planets c and d.

HD 28109 appears to be a dynamically packed system, where the mutual spacing of the planets leaves little to no room for additional bodies between them.

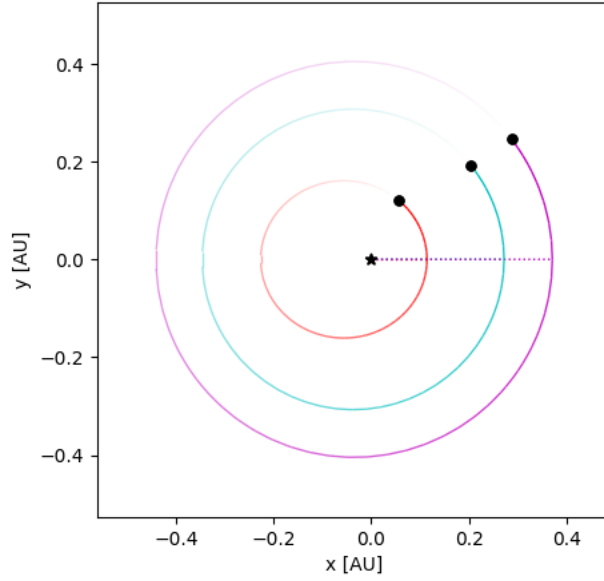


Figure 5.11: Orbital configuration of the HD 28109 system. The red, cyan, and magenta lines represent the orbits of HD 28109 b, c, and d, respectively, around their host star (black star symbol). Figure generated with REBOUND.

5.4.6 TOI-712

Overview

TOI-712 is a young, nearby K4.5V star located at a distance of approximately 50 pc and with an estimated age between 0.5 and 1.1 Gyr (Vach et al. 2022). The system hosts three validated transiting mini-Neptunes—TOI-712 b, c, and d—discovered by the *TESS* mission, with orbital periods of 9.53, 51.7, and 84.8 days, and radii of 2.05 , 2.70 , and $2.47 R_{\oplus}$, respectively.

The host star is moderately active, with a rotation period of ~ 12.5 days derived from *TESS* photometry, consistent with its young age and enhanced spot modulation. The stellar luminosity ($L_{\star} \simeq 0.16, L_{\odot}$) places the conservative habitable zone between 0.34 and 0.84 AU, meaning that TOI-712 d lies near its inner edge. Given their relatively cool equilibrium temperatures and volatile-rich compositions, the two outer planets represent rare examples of temperate mini-Neptunes orbiting a young star—ideal laboratories to study the early evolution of planetary atmospheres in regimes where photoevaporation and core-powered mass loss are expected to be weak.

The architecture of the system (Figure 5.13) shows a moderately compact configuration, with the three known planets spaced by period ratios of $P_c/P_b \approx 5.4$ and $P_d/P_c \approx 1.64$. This non-resonant but ordered structure, together with dynamical simulations indicating long-term stability, suggests that TOI-712 experienced a relatively calm

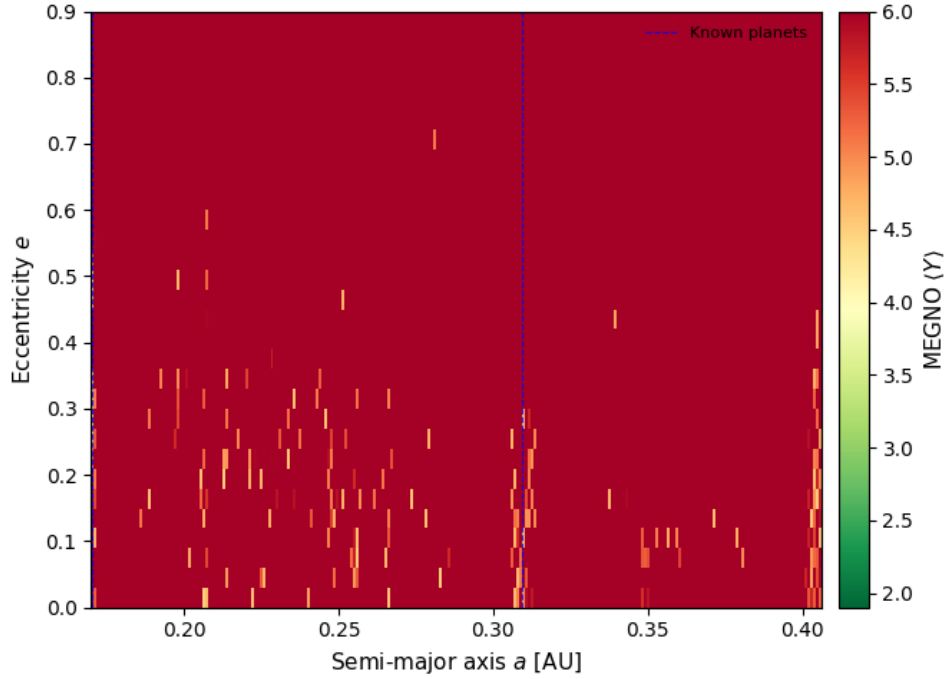


Figure 5.12: MEGNO stability map for the HD 28109 system in the semi-major axis (a)–eccentricity (e) plane. The vertical dashed lines mark the positions of the known planets within the simulated range of a , with the innermost and outermost planets located near the plot boundaries.

formation and migration history. The stable region beyond ~ 0.7 AU could host additional low-mass planets, including potentially habitable ones, while future *TESS* observations may confirm the presence of the inner rocky candidate.

Dynamical Analysis

The MEGNO stability map for TOI-712 (Figure 5.14) reveals a predominantly chaotic dynamical structure between the two innermost planets, with $\langle Y \rangle$ values exceeding 4 across most of the explored (a, e) space. Only a narrow corridor appears around $a \simeq 0.17$ AU, corresponding to a small island of stability centered near the 3:1 mean-motion resonance with TOI-712 b (orbital period ~ 29.5 days).

This stable region is confined to low eccentricities ($e \lesssim 0.15$) and spans less than a few hundredths of an astronomical unit in semi-major axis, indicating that any additional planet in this region would have to occupy an extremely fine-tuned orbit to remain dynamically stable. Such a configuration is unlikely to host an undetected terrestrial planet, as even small perturbations, arising from mutual interactions or tidal effects, would likely disrupt the delicate resonance balance over Gyr timescales.

Beyond this feature, the remaining phase space between TOI-712 c and d does not

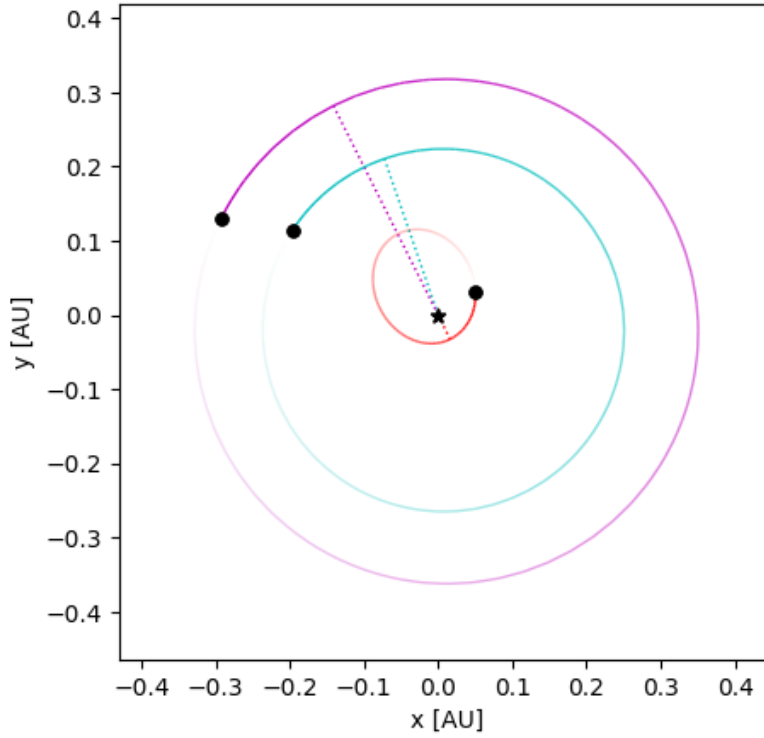


Figure 5.13: Orbital configuration of the TOI 712 system. The red, cyan, and magenta lines represent the orbits of TOI 712 b, c, and d, respectively, around their host star (black star symbol). Figure generated with REBOUND.

exhibit any extended stable domains. The current configuration therefore appears close to dynamical packing, although a small degree of additional stability cannot be completely ruled out in specific resonant configurations.

5.5 Conclusions

The dynamical analysis of the multi-planet systems located within the *PLATO* LOPS2 field reveals a broad diversity of orbital architectures and stability regimes. Using MEGNO-based N -body simulations, we mapped the stability landscapes of each system to identify potential regions where additional Earth-mass planets could exist without destabilizing the known configurations. The resulting maps show that while some systems still preserve narrow corridors of long-term stability, most of the architectures discovered by *TESS* are already close to dynamical saturation.

Among the analyzed systems, TOI-286 and TOI-451 stand out as the most promising candidates for hosting undetected planets. Both display extended low-eccentricity regions of quasi-periodic motion between their known planets, consistent with dynamically stable corridors where terrestrial-mass bodies could survive over Gyr timescales. TOI-712 also

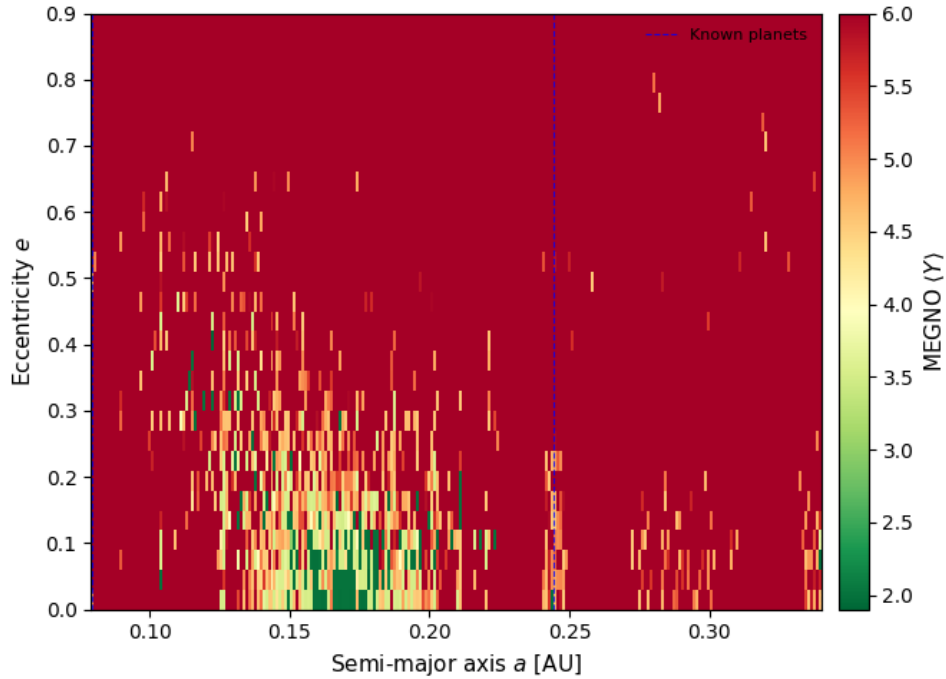


Figure 5.14: MEGNO stability map for the TOI 712 system in the semi-major axis (a)–eccentricity (e) plane. The vertical dashed lines mark the positions of the known planets within the simulated range of a , with the innermost and outermost planets located near the plot boundaries.

exhibits a small but distinct island of stability, located near the 3:1 mean-motion resonance with its innermost planet. However, this region is extremely narrow in semi-major axis and limited to very low eccentricities, suggesting that the existence of an additional planet in such a finely tuned orbit is possible but unlikely. In contrast, systems such as TOI-270, TOI-700, and HD 28109 exhibit strongly chaotic interplanetary zones dominated by resonance overlap, leaving virtually no stable regions within the inner architectures.

Overall, these results suggest that most *TESS*-discovered multiplanetary systems are dynamically compact. Their configurations are typically well ordered, near the threshold of long-term stability. The few systems that retain limited but significant stability niches—often near low-order mean-motion resonances—represent the most promising targets for future searches.

In this context, *PLATO*, with its superior photometric precision, extended temporal baseline, and continuous monitoring of southern targets, will significantly broaden the parameter space explored by *TESS*. The mission will be capable of detecting transits at much longer orbital periods, where *TESS* completeness drops rapidly due to its limited time coverage, while simultaneously exploring the dynamically allowed but observationally elusive regions in the inner parts of these systems. In particular, *PLATO*’s enhanced sensitivity will enable the detection of small Earth-sized or sub-Earth-sized planets that

remain below the current detection threshold of *TESS*.

The synergy between *TESS* and *PLATO* will therefore be essential to fully characterize the dynamical architectures of nearby planetary systems. While *TESS* provides a broad statistical census of compact transiting systems across the sky, *PLATO* will expand this view by reaching smaller planets, longer orbital periods, and resonance-protected orbits, offering a more complete picture of compact multiplanetary systems.

Conclusions

In this Ph.D. thesis, the contribution of the space missions *Roman* and *PLATO* to the study of multiple planetary systems has been investigated. They will employ two different but complementary detection methods for the types of planets detectable: gravitational microlensing and transit photometry.

In Chapters 3, and 4, the potential of the *Nancy Grace Roman Space Telescope* in the field of microlensing has been analyzed. The calculation of the light curve of microlensing events produced by multiple lenses presents numerical challenges. To overcome these obstacles, three new algorithms were developed and implemented in the `VBMicrolensing` code (Chapter 3):

- **Singlepoly**, a direct extension of the polynomial method, fast but numerically unstable for $N > 3$ or extreme mass ratios.
- **Multipoly**, a hybrid multi-polynomial approach that decomposes the problem into local reference systems, proving to be the optimal compromise for $N = 3$ and $N = 4$.
- **Nopoly**, an iterative Newton-Raphson-based method operating directly on the lens equation, slower for $N \leq 4$ but significantly more robust and scalable for $N \geq 5$.

The performance analysis demonstrated the complementarity of the three methods, providing a versatile tool for modeling complex systems. The method that proved most competitive in the case study of triple multiplanetary events (star + 2 planets) was the Multipoly, and therefore it was employed to conduct a large-scale simulation to understand *Roman*'s capability to detect two-planet systems (Chapter 4) in the case where the source trajectory is close to the central caustic ($u_0 < 0.01$), high-magnification regime.

The simulations showed that *Roman* will be able to identify the triple nature of the system in about **66%** of cases. The detection efficiency was found to be maximal ($> 90\%$) for systems with at least one planet in resonant configuration and when the mass ratios of the secondary objects relative to the primary were high ($q > 10^{-3}$). Based on these efficiencies and the expected event rates, we estimate that *Roman* will discover about 63 multi-planetary systems through microlensing.

In Chapter 5, we focused on the dynamical characterization of a selected sample of multi-planet systems discovered by *TESS* and located within the future long-duration observation field LOPS2 of *PLATO*. This analysis aimed to identify dynamically stable regions that could potentially host additional, as-yet undetected Earth-mass planets, and to explore the synergy between *TESS* and *PLATO* in extending the completeness of transit detections toward longer periods and lower masses.

Using MEGNO-based N -body simulations, we investigated the stability of the following systems: TOI-270, TOI-451, TOI-700, TOI-712, TOI-286, and HD 28109. The results revealed a clear diversity of dynamical architectures across these systems:

- **TOI-270, TOI-700, and HD 28109** exhibit strongly chaotic interplanetary zones dominated by resonance overlap, leaving virtually no stable regions between the known planets. These systems appear dynamically compact and close to the threshold of long-term stability.
- **TOI-286 and TOI-451** stand out as the most promising candidates for hosting additional planets. Both display extended low-eccentricity regions of quasi-periodic motion between their known planets, consistent with dynamically stable corridors where terrestrial-mass bodies could survive for Gyr timescales.
- **TOI-712** presents a narrow but distinct island of stability near the 3:1 mean-motion resonance with its innermost planet. Although this region is extremely limited in semi-major axis and eccentricity, it suggests the possible—albeit fine-tuned—existence of another small companion.

Overall, the dynamical maps indicate that most *TESS*-discovered multi-planet systems in the LOPS2 field are already compact, with little room for additional planets within their architectures. However, the few systems that retain small but significant stability niches—typically associated with low-order mean-motion resonances—represent high-priority targets for future searches.

The results of this thesis highlight the important role that upcoming space missions will play in advancing our understanding of planetary system architectures. By combining complementary detection techniques—gravitational microlensing and transit photometry—*Roman* and *PLATO* will jointly probe a vast range of planetary environments, from

cold, wide-orbit planets beyond the snow line to compact, dynamically interacting multi-planet systems in the inner regions; paving the way toward a more complete and unified picture of planetary system formation and evolution across the Galaxy.

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