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Advanced magnetic control and reconstruction in fusion devices

by

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Candidate's declaration

I hereby declare that this thesis submitted to obtain the academic degree of Philosophiæ Doctor (Ph.D.) in Information Technology and Electrical Engineering is my own unaided work, that I have not used other than the sources indicated, and that all direct and indirect sources are acknowledged as references.

Parts of this dissertation have been published in international journals and/or conference articles (see list of the author's publications at the end of the thesis).

Napoli, February 6, 2026

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Abstract

The ITER tokamak, currently under construction, is expected to be the first magnetic confinement device capable of producing a net surplus of fusion energy. While awaiting its completion, several satellite projects have been launched to support ITER's mission. Among them are the EU–Japan joint project JT-60SA—presently the world's largest operating fusion device—and the Divertor Tokamak Test (DTT) facility in Italy, which will investigate heat exhaust solutions for next-generation reactors. These satellite tokamaks play a crucial role in developing and validating technologies and control strategies for ITER. In this framework, the initial commissioning phases of tokamak devices pose significant challenges for plasma magnetic control, particularly due to the temporary absence of critical subsystems, such as in-vessel stabilization coils and the full suite of magnetic diagnostics.

This thesis tackles the challenges of early-stage tokamak operations while also proposing advanced control and magnetic reconstruction algorithms for the effective operation of next-generation tokamaks. In particular, the contributions are the following:

- validation of ITER's magnetic reconstruction tools under reduced sensor availability and the development of a fast boundary reconstruction algorithm;
- the design of advanced control strategies for plasma stabilization in early operational scenarios or in the presence of strict actuator constraints.

On the magnetic reconstruction side, this work leverages data from the JT-60SA Integrated Commissioning campaign to preliminarily validate ITER's real-time magnetic reconstruction algorithms, testing the algorithms' reconstruction capability with reduced diagnostics. In fact,

during the JT-60SA IC, only a limited number of magnetic sensors were available. Furthermore, an alternative fast boundary reconstruction algorithm is proposed, based on Extreme Learning Machines (ELMs)—a class of neural networks characterized by fast training and inference times. Its performance is firstly evaluated in the same reduced sensors scenario and then is benchmarked against the XLOC algorithm, that has been used for decades at JET to reconstruct in real-time the plasma boundary, with DTT serving as a case study.

On the control side, the thesis provides a twofold contribution. First, it introduces an ITER-like, model-based magnetic control technique tailored for early operation scenarios lacking in-vessel actuators. This architecture enables the use of external superconducting coils for multiple tasks, including vertical stabilization, and its effectiveness is demonstrated through nonlinear simulations of JT-60SA plasmas.

Second, the thesis develops a Model Predictive Control (MPC)–based scheme for the upgraded RFX-mod2 device. Following the removal of the Inconel vessel and the introduction of a conductive cage, a new electromagnetic model is derived, and consequently an MPC control system is designed. Importantly, the proposed approach explicitly accounts for the hardware constraints on the actuators, such as the use of the already existing one-quadrant converters, thereby ensuring reliable tracking of desired plasma shape and current in RFX-mod2 operations.

Together, these developments contribute to the creation of flexible and resilient magnetic reconstruction and control strategies, crucial for the safe and efficient operation of next-generation tokamak devices.

Keywords: Nuclear fusion, tokamaks, magnetic control, magnetic reconstruction, decoupling control, model predictive control

Sintesi in lingua italiana

Il tokamak ITER, attualmente in costruzione, è destinato a diventare il primo dispositivo a confinamento magnetico in grado di produrre un surplus netto di energia da fusione. In attesa del suo completamento, sono stati avviati diversi progetti satelliti a supporto della missione di ITER. Tra questi figurano il progetto congiunto UE-Giappone JT-60SA — attualmente il più grande dispositivo a fusione operativo al mondo — e il Divertor Tokamak Test (DTT) in Italia, che studierà le strategie per la gestione del carico termico nei reattori futuri. Questi tokamak satelliti rivestono un ruolo fondamentale nello sviluppo e nella validazione di tecnologie e strategie di controllo rilevanti per ITER. In questo contesto, le fasi iniziali di commissioning dei tokamak presentano sfide significative per il controllo magnetico del plasma, dovute in particolare alla temporanea indisponibilità di sottosistemi critici, quali le bobine di stabilizzazione interne e l'intera suite di diagnostiche magnetiche.

Questa tesi affronta le sfide delle operazioni nelle fasi iniziali dei tokamak, proponendo anche algoritmi avanzati di controllo e ricostruzione magnetica per un'efficace gestione dei tokamak di nuova generazione. In particolare, i contributi sono i seguenti:

- la validazione degli strumenti di ricostruzione magnetica di ITER in condizioni di disponibilità ridotta dei sensori e lo sviluppo di un algoritmo veloce di ricostruzione del bordo del plasma;
- la progettazione di strategie di controllo avanzate per la stabilizzazione del plasma in scenari operativi iniziali e in presenza di vincoli stringenti sugli attuatori.

Per quanto riguarda la ricostruzione magnetica, questo lavoro utilizza i dati della campagna di Integrated Commissioning di JT-60SA per una validazione preliminare degli algoritmi di ricostruzione magnetica real-time

di ITER, valutandone la capacità di ricostruzione in condizioni di diagnostiche limitate. Durante la fase IC di JT-60SA, infatti, solo un numero ridotto di sensori magnetici era disponibile. Inoltre, viene proposto un algoritmo alternativo di ricostruzione rapida del bordo, basato sulle *Extreme Learning Machines* (ELMs) — una classe di reti neurali caratterizzate da tempi di addestramento e inferenza molto rapidi. Le sue prestazioni sono valutate inizialmente nello stesso scenario a sensori ridotti e successivamente confrontate con l'algoritmo XLOC, utilizzato da decenni al JET per la ricostruzione real-time del bordo plasma, considerando DTT come caso di studio.

Per quanto riguarda il controllo, la Tesi fornisce un duplice contributo. In primo luogo, introduce una tecnica di controllo magnetico di tipo ITER-like, basata su modelli fisici, pensata per scenari operativi iniziali privi di bobine interne. Tale architettura consente l'impiego delle bobine superconduttrici esterne per molteplici funzioni, inclusa la stabilizzazione verticale, e la sua efficacia è dimostrata tramite simulazioni nonlineari di plasmi JT-60SA.

In secondo luogo, la Tesi sviluppa uno schema di controllo basato su *Model Predictive Control* (MPC) per il dispositivo RFX-mod2 aggiornato. A seguito della rimozione del vessel in Inconel e dell'introduzione di una gabbia conduttiva, è stato progettato un nuovo modello elettromagnetico e un sistema di controllo MPC. In particolare, l'approccio proposto tiene conto esplicitamente dei vincoli hardware sugli attuatori, come l'utilizzo degli attuali convertitori monodirezionali nelle fasi operative iniziali, garantendo così un controllo affidabile della forma del plasma e della corrente durante l'avvio di RFX-mod2.

Nel complesso, questi sviluppi contribuiscono alla definizione di strategie di ricostruzione e controllo magnetico flessibili e robuste, fondamentali per un funzionamento sicuro ed efficiente dei tokamak di nuova generazione.

Parole chiave: Fusione nucleare, tokamak, controllo magnetico, ricostruzione magnetica, controllo decoupled, controllo predittivo (MPC)

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List of Acronyms

The following acronyms are used throughout the thesis.

AVA	Adaptive Voltage Allocation
CCS	Cauchy Condition Surface
CS	Central Solenoid
DTT	Divertor Tokamak Test
ELM	Extreme Learning Machine
EF	Equilibrium Field
FS	Field Shaping
FT	Flat-Top
GS	Grad Shafranov
HDE	Horizontal Displacement Event
IC	Integrated Commissioning
LCFS	Last Closed Flux Surface
LLNL	Lawrence Livermore National Laborator

LTV	Linear Time Varying
MHD	Magneto-Hydro-Dynamic
MIMO	Multi-Input Multi-Output
MW	Magnetizing Windings
MPC	Model Predictive Control
NBI	Neutral Beam Injector
NN	Neural Network
OP1	First Operation
PF	Poloidal Field
PFCC	Poloidal Field Current Controller
PTL	Positive Triangularity L-mode
PTH	Positive Triangularity H-mode
QP	Quadratic Programming
RF	Radio Frequency
RFP	Reversed Field Pinch
RD	Ramp-Down
RU	Ramp-Up
RWM	Resistive Wall Modes
SC	Saddle Coils
SISO	Single-Input-Single-Output

TF	Toroidal Field
VDE	Vertical Displacement Event
VS	Vertical Stabilization
XSC	eXtreme Shape Controller

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List of Symbols

The following symbols are used within the thesis

β_p	Poloidal beta
μ_0	Vacuum magnetic permeability
e	Neper's number
l_i	Plasma internal inductance

Introduction

Over the last century, there has been a continuing increase in energy demand, calling for new solutions to the energy problem. At first, fossil-based plants were used to produce the electricity needed to satisfy the rising demand. At the beginning of years 2000s, the effects of the burning of fossil fuels on the climate and global environment began to be widely recognized. Since then, increasingly catastrophic climate-related events happened, such as the 2003 European heatwave, which caused tens of thousands of deaths. Since 2010, large investments have been made to significantly increase the energy produced by renewable sources. The costs of the technology needed for solar and wind farms had been reduced, pushing governments to encourage the use of renewable energies. In fact, in 2015, the Paris Agreement, signed by 196 countries, marked a historic turning point in global efforts to combat climate change, paving the way to zero net CO₂ emissions for many countries.

This goal can only be achieved through a portfolio of options, including further improvements in renewable energy, nuclear fission, and nuclear fusion. These are the only non-fossil-based solutions for clean energy production. Although renewable energy resources are inexhaustible and fundamental to reducing the CO₂ emissions, low energy densities and fluctuations in generation over time imply a need for storage. This reduces the efficiency of renewables, and could lead to extra costs. In the case of nuclear fission power plants, there is increasing concern regarding waste management and operational safety. However, their contribution to low-emission energy is invaluable at the moment, and a complete shutdown would not be wise without a proper substitution of the nuclear share.

Nuclear fusion is the least developed alternative; however, it offers valuable advantages that would ensure sustainability and safety. In fact, fuels are geographically widely available and virtually unlimited, providing energy independence while being inherently safe and without producing long-term radioactive waste. In detail:

1. **Clean and Sustainable:** Fusion produces no harmful greenhouse gas emissions and generates minimal nuclear waste compared to fission reactors. The fuel (like deuterium and tritium) is abundant and can be sourced from water and lithium, making fusion a sustainable energy source.
2. **High Efficiency:** Fusion reactions release vast amounts of energy from small amounts of fuel, making it far more efficient than fossil fuels or even current nuclear fission reactors. A gram of fusion fuel can produce as much energy as several tons of coal or oil.
3. **Safety:** Fusion reactions are inherently safer than fission. There's no risk of a meltdown (like in traditional nuclear reactors), and the reaction cannot sustain itself without constant input, so it's less prone to catastrophic failures. Moreover, fusion doesn't create long-lived, high-level radioactive waste.

However, researchers still have to prove that these advantages can actually be reached by a fusion power plant. A big effort is being made to do so and to prove that nuclear fusion is a feasible source of energy. This goal may eventually be achieved in the future, but it may still require several decades for nuclear fusion to have an impact globally. This is why investments for the improvement of solar and wind farms, as well as nuclear fission power plants, should not be stopped.

Chapter 1

Nuclear Fusion

1.1 Introduction to Nuclear Fusion

Nuclear fusion represents one of the most promising avenues for generating nearly limitless and clean energy on Earth.

The idea of nuclear fusion as a viable energy production method dates back to the year 1920. It was the English scientist Arthur Eddington who first put forward the hypothesis that the large amount of energy needed to light up a star could have its origin in fusion reactions of light elements. The key findings were the existence of isotopes and that the mass of these isotopes was not an exact multiple of that of the base form of the element. Regarding hydrogen, in the same way, it was found that a helium nucleus was lighter than the sum of four hydrogen nuclei.

The first laboratory nuclear fusion reaction was obtained in 1933 thanks to Mark Oliphant. He used an accelerator to fire deuterium nuclei against several targets consisting of metal foils containing deuterium, tritium, and other elements. In this way, not only he discovered that tritium is a product of the reaction between deuterium and lithium (tritium breeding is still a big challenge to be overcome for a future fusion power plant), but he also noticed that deuterium nuclei can be made to react between each other, producing particles that had more energy than the initial ones. Further studies related to the cross-section of fusion reactions revealed that the most favorable one is the deuterium-tritium one. The temperature needed to have the maximum probability of occurrence for this reaction is

100 keV (around 1 billion K). Anyway, Enrico Fermi calculated that the reactions could self-sustain at the much lower temperature 50 million K.

At such high temperatures, materials undergo a transition into a state of matter known as plasma. A plasma is a fully ionized gas, which is hence a very good conductor and is strongly affected by electric and magnetic fields. If enough kinetic energy is provided to the nuclei, they become able to overcome the repulsive Coulomb forces, and hence collide to give rise to fusion events. The main problem, thus, remained that of holding the plasma together (since it has a natural tendency to expand, like any other hot gas) without having it touch any physical wall (since any wall of any material would invariably melt at a hundred million degrees). The first breakthrough into the peaceful use of fusion as an energy source came from the former Soviet Union in 1958 when a magnetic confinement device was invented with an excellent capacity for containing high-temperature plasmas. The device is called a tokamak and has been the focus of mainstream fusion research ever since. The Russian word *tokamak* stands for “toroidal chamber with magnetic coils”. In fact, in a tokamak, strong magnetic fields are used to confine the plasma in a doughnut-shaped machine. At the same time, a plasma current is induced to form helical magnetic field lines that effectively confine the particles. The first tokamak was the Soviet Union’s T-3 tokamak (Figure 1.1) [114], which demonstrated that high-temperature plasma confinement was feasible, achieving temperatures and confinement times far exceeding previous attempts.

Since then, many experimental tokamak devices have been built. The most important ones being: the Joint European Torus (JET, [142, 106, 72]), Tokamak Fusion Test Reactor (TFTR [55]), Korea Superconducting Tokamak Advanced Research (KSTAR, [98, 75]), Experimental Advanced Superconducting Tokamak (EAST, [59, 52]), and Japan Torus-60 Super Advanced (JT-60SA, [37]). While the first two experimental devices are the only one that have implemented tritium campaigns, the latter is currently the world’s largest tokamak in operation.

In 2006, the construction of the ITER (The Way, [101, 22]) tokamak began, in Figure 1.2 its cross-section is shown. ITER is one of the most significant and ambitious projects in the field of nuclear fusion research. It is a large-scale scientific experiment aimed at demonstrating the feasibility of nuclear fusion as an energy source. ITER is being constructed

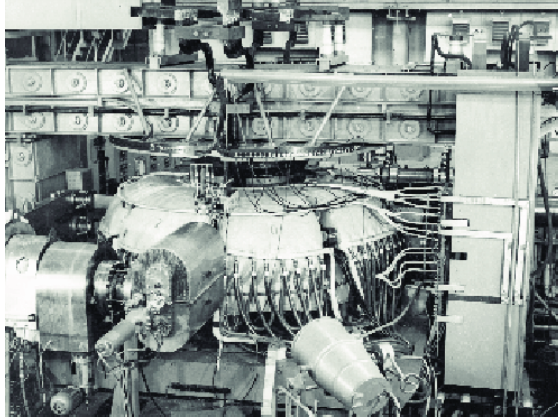


Figure 1.1: View of tokamak T-3 (Source [90]).

in Cadarache, France, and represents a collaboration between 35 countries [57]. ITER aims to produce 500 MW of fusion power, while consuming only 50 MW of input power. This is a tenfold return on energy, a major milestone in fusion research. Moreover, it will demonstrate the ability to keep plasma stable and hot enough for extended periods (up to 1,000 seconds), an essential step toward future commercial fusion reactors. The reactor will also test materials that can withstand the extreme conditions inside a fusion reactor, providing valuable data for the design of future fusion power plants. While ITER itself is not designed to be a power plant, the lessons learned from it will pave the way for future reactors, such as DEMO, which will aim to generate electricity for the grid using fusion.

In recent years, many promising results in view of a reactor have been achieved. In 2022, during its last D-T campaign, the JET facility set a new world energy record of 59 megajoules released in sustained and controlled fusion energy [83]. In 2024, a new plasma duration record of 6 minutes and 4 seconds was achieved in WEST tokamak, certified by Princeton Plasma Physics Laboratory, with a 50 million degrees plasma [96]. In KSTAR's most recent plasma operations, plasma at high confinement mode was held at 100 million degrees for 48 seconds [67]. In 2021 EAST tokamak reached the record temperature of 120 million degrees Celsius, and in recent progress maintained a high temperature and high-confinement plasma for

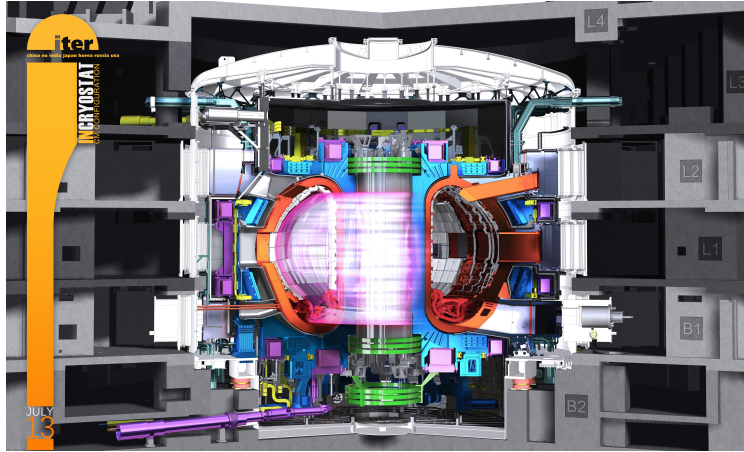


Figure 1.2: Cross-section of the ITER tokamak (retrieved from [101]).

403 seconds [52].

Regarding magnetically confined nuclear fusion, other device concepts arose during the years, more details will be given in the next sections. In fact, apart from the tokamak, two other promising designs are the Stellarator and Reversed Field Pinch (RFP). The advantage of the first one lies in improved plasma stability, thanks to the absence of current flowing inside the plasma. In fact, Wendstein-7X (W-7X) recently achieved a 17-minute plasma pulse, setting the record for the longest plasma pulse ever recorded [66]. However, since the machine itself is not axisymmetric, stellarators' construction and modelling are highly demanding and represent the biggest limit of this approach. On the other hand, the RFP design (which is the least developed among the three), relying on a particular configuration of the magnetic field, allows for a much simpler and compact design with respect to tokamaks and stellarators. The most important RFP machine is RFX-mod2 [120], located in Padova, which is the upgrade of RFX-mod. RFX-mod, although being able to operate also in tokamak configuration, achieved significant advances in plasma confinement within the RFP configuration [149], particularly by mitigating some of the instabilities typically associated with RFPs, such as magnetic islands and magnetohydrodynamic (MHD) turbulence.

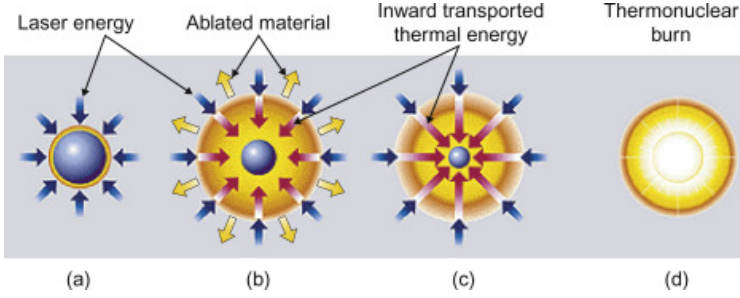


Figure 1.3: Inertial confinement fusion stages with (a) irradiation of the outer layer, (b) ablation of the outer material and implosion of the payload, (c) central ignition, and (d) burn and explosion (Source [89]).

As an alternative to magnetic confinement fusion, inertial confinement fusion is a method of achieving fusion by compressing a small pellet of fusion fuel (a mixture of deuterium and tritium) to extremely high pressures and temperatures (in Figure 1.3, a conceptual explanation of how this method works). This compression is achieved by using powerful lasers or other forms of energy to rapidly heat and compress the fuel. The key idea is to overcome the repulsive force between the positively charged nuclei and force them to fuse, releasing energy in the process. The most well-known facility is the National Ignition Facility at Lawrence Livermore National Laboratory (LLNL) in California. On December 5, 2022, LLNL made a groundbreaking achievement in inertial confinement fusion, setting a significant milestone in fusion research: they were able to generate, from a fusion reaction, more energy than the injected one for the first time. In this experiment, about 2.5 MJ of fusion energy was released, which is about 1.5 times the amount of energy delivered by the laser system to the pellet [92]. It is important to remark that the net energy production goal has not been achieved yet. In fact, the energy gained from the fusion reaction was almost 100 times lower than the one fed to the lasers.

Since this thesis is focused on magnetic control for fusion devices, the next sections introduce in more detail how the magnetic confinement devices work, with a particular attention to tokamaks.

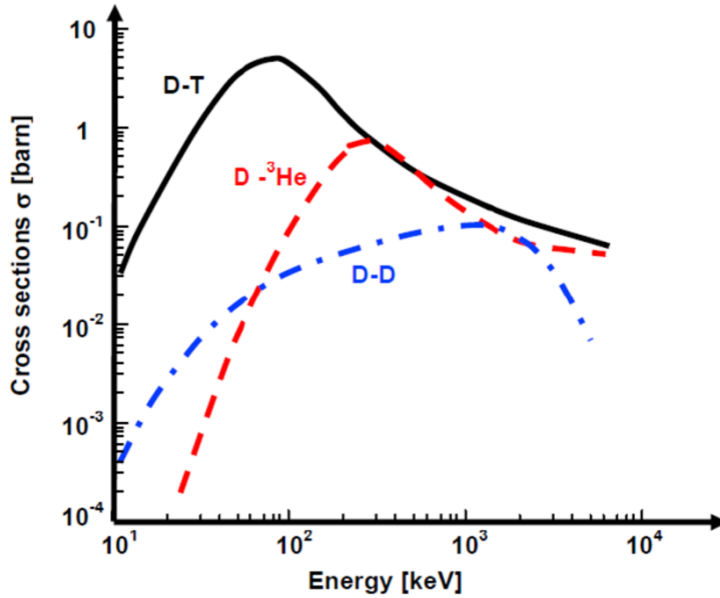


Figure 1.4: Cross section of different fusion reactions. In black, the D-T one. It has the highest cross-section among the three. The maximum occurs at the lowest temperature (Source [74]).

1.2 Magnetic confinement in fusion devices

As briefly anticipated, nuclear fusion occurs when nuclei of light elements overcome their mutual electrostatic repulsion, known as the Coulomb barrier, and collide with sufficient energy to combine into a heavier nucleus. This process releases a substantial amount of energy, primarily in the form of kinetic energy. The probability of the fusion reaction to occur under specific conditions is represented by the cross-section and, the behavior for typical reactions shown in Figure 1.4, justifies the wide preference of the two hydrogen isotopes, deuterium (D, H2) and tritium (T, H3), that have high cross-section at relatively low temperatures.

The fusion reaction between D and T is here reported:



It is now clear why fusion reactions are appealing to produce energy: the products of the reaction are different from the reagents; therefore, there is no risk of triggering a chain reaction that becomes difficult to stop. Moreover, there is no emission of greenhouse gases and few amount of fuel is needed to generate a large amount of energy. The problem of nuclear fusion is that the reaction D-T happens only at very extreme conditions, which are difficult to reach.

To achieve these conditions, the D-T gas mixture, with density n , needs to be heated to extremely high temperatures, $T \geq 10^8 C$, and maintained for a sufficient time, denoted as confinement time τ_E , to provide sufficient kinetic energy to the nuclei to overcome the Coulomb forces. According to Lawson criteria, the following condition on the triple product need to be met to reach the ignition condition:

$$nT\tau_E \geq 3 \times 10^{21} m^{-3} keVs.$$

As already anticipated, at these temperatures, the gas mixture becomes a plasma. In the plasma state, the electrons are separated from the nuclei; it is an ionized gas that reacts to magnetic fields, and it is a good conductor. Anyway, it suffers from energy losses that can be sustained either by external power sources or by the energy produced by the reaction itself. A measure of the reaction efficiency is the fusion gain:

$$Q = \frac{P_{Fus}}{P_{in}},$$

where P_{Fus} is the power gained from the fusion reaction and P_{in} the input power needed to start it. The ideal goal is to build a reactor capable of sustaining the reactions, reaching $Q \rightarrow \infty$. ITER is designed to reach $Q = 10$.

1.3 Tokamaks

As it was already mentioned in the previous section, the tokamak is a toroidal magnetic confinement system, invented by Russian scientists. In a tokamak, the confinement is mainly provided by means of a strong toroidal magnetic field. Bending the field lines into a torus has the main advantage

of avoiding particle end-losses. However, the plasma ring would tend to naturally expand, and a purely toroidal field is not enough to provide a balancing force. To achieve an equilibrium configuration, a poloidal field is needed in addition to the toroidal one. In a tokamak, this poloidal field is obtained by inducing a toroidal current into the plasma. Typically, in tokamaks, the toroidal magnetic field is greater than the poloidal field. The combination of the poloidal and toroidal magnetic fields generates helical magnetic field lines, which succeed in confining the plasma. In fact, the plasma particles, subject to the Lorentz Force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

where the force applied on the particle with charge q depends on its velocity $\mathbf{v}$, magnetic field $\mathbf{B}$ and electrical field $\mathbf{E}$, spin around the magnetic field lines. This is why in a tokamak, the stability and confinement are greatly improved with respect to θ -pinches or Z -pinches, where only one of the two types of magnetic field is applied. Experimental linear devices exist nowadays, but they are not used to study plasma physics and stability, but rather plasma-material interactions.

More details regarding plasma modelling in tokamaks will be given in the next chapter. Let us now introduce the structure of a tokamak.

1.3.1 Components of a tokamak

In a tokamak, the required strong toroidal magnetic field is generated through a set of coils known as Toroidal Field Coils (TFC). The maximum achievable magnetic field strength is constrained by two primary factors: the maximum forces the coils can withstand and the associated Joule losses. To reduce these losses, superconducting TFCs are commonly used in modern devices. The plasma current, which generates the poloidal magnetic field, is typically induced by a central primary winding, referred to as the Central Solenoid (CS). To maintain radial force balance, external circuits are employed to supply a vertical magnetic field. Additionally, adjusting the plasma shape is often necessary to enhance the machine's performance. Vertical field and shaping circuits, and sometimes even the CS itself, are collectively known as Poloidal Field (PF) Coils, which can be utilized to control the plasma's shape and position.

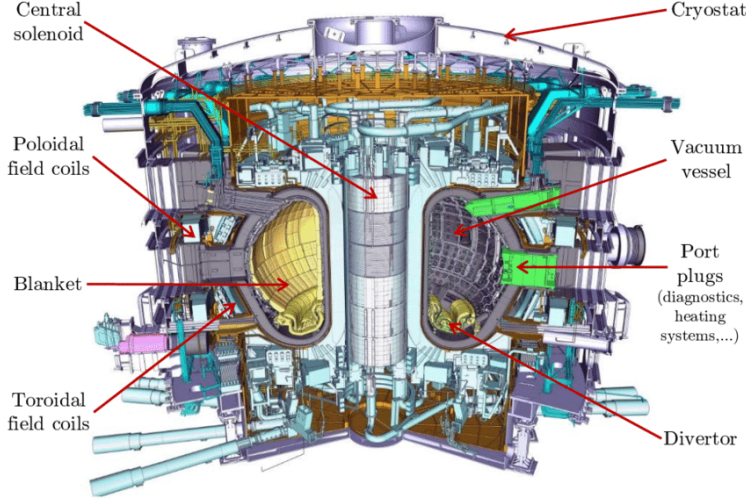


Figure 1.5: Components of a superconducting tokamak (Source [43]).

In some cases, additional coils are placed inside the vacuum chamber to provide more rapid control of the plasma, thanks to the shorter magnetic field penetration time. These coils are typically constructed from copper alloys, as superconductors cannot be efficiently cooled close to the hot plasma. These coils, referred to as In-Vessel coils, are commonly used for vertically stabilizing elongated plasmas.

While ohmic heating alone can achieve plasma temperatures in the range of a few keVs, Joule losses in the plasma decrease as the temperature increases. To reach temperatures of 10 keV and beyond, additional heating methods are required. Neutral Beam Injection (NBI) and Radio Frequency (RF) heating are the primary techniques employed for this purpose.

The plasma in a tokamak has a very low density, typically around 10^{20} particles per cubic meter. To maintain this low pressure, the plasma is confined within a vacuum vessel. Impurities in the plasma can lead to radiation losses and degrade confinement, making the prevention of plasma dilution crucial for tokamak operations. Therefore, the plasma must be kept separated from the vacuum chamber walls. Moreover, the high heat loads produced by a fusion plasma must be effectively managed. These

challenges are typically addressed in two main ways: first, by using appropriate materials for the plasma-facing components, especially the region where the plasma makes contact with the first wall, which is known as the limiter; second, by utilizing a magnetic configuration that keeps the main plasma away from the vessel walls. In this latter case, power exhaust is concentrated on a specific region of the chamber, called the divertor.

Finally, a fully functional reactor will require numerous additional components, such as diagnostic systems, vacuum pumps, and a cryostat for the superconducting coils. Among these, the blanket is essential for slowing down the neutrons produced during fusion and breeding tritium. Since not all neutrons react with the lithium in the blanket, a neutron multiplier, such as beryllium, is also necessary to optimize the fusion process. Figure 1.5 reports the main components of a superconducting tokamak.

1.3.2 Tokamak plasma pulse

A tokamak is a pulsed device, this is because only limited currents can flow in the active coils. In fact, because of the resistivity losses of the plasma, currents in the external coils need to ramp up throughout the pulse to keep the plasma current constant, with the central solenoid being the most effective one. Figure 1.6 shows the several phases of a tokamak plasma pulse. Throughout the discharge the PF coils currents need to be properly designed to study the plasma physics of interest [32].

Breakdown

Before the breakdown time, the tokamak vacuum vessel is prefilled with neutral particles up to a certain pressure. A seed of free electrons within the neutral prefill gas can be accelerated by means of an electric field, to provide them with sufficient energy to further ionize neutrals by collisions, generating more electrons and ions in a process that is similar to a Townsend avalanche discharge. In ohmic breakdowns, a toroidal electric field is induced by transformer action, using the CS coils. The loop voltage, V_{loop} , around the torus is proportional to the rate of change of the magnetic flux in the CS coils (by changing the coil current), but it is also affected by currents in other PF coils and currents flowing in passive conducting structures (e.g. vacuum vessel). This is why the currents in

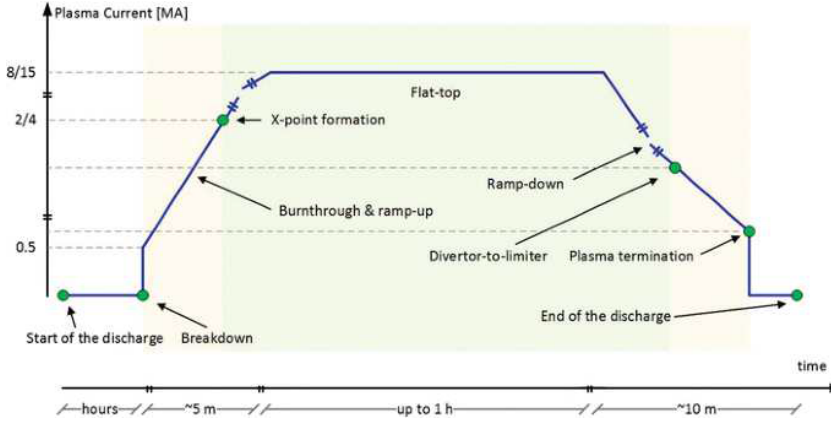


Figure 1.6: Representation of the different phases of a tokamak pulse (Source [121]).

the PF coils need to be optimized such that the poloidal magnetic field is null in a wide region inside the chamber, so that the accelerated charged particles can flow in the toroidal direction only. Figure 1.7 shows the relation between the CS and V_{loop} . Continuous heating, whether ohmic or auxiliary, sustains ionization, enabling further temperature increases and plasma current ramp-up.

Recently, other breakdown techniques have proven their effectiveness by exploiting the bottleneck effect of magnetic fields. This technique, called Trapped Particle Configuration, makes extensive use of RF heating, in particular Electron Cyclotron Resonance Heating. The key idea is to heat the prefill gas in a precise region of the chamber while at the same time optimizing the PF current such that a magnetic bottleneck effect is created for the ionized particles. More details can be found in [15].

Ramp-Up

The plasma Ramp-Up (RU) phase, with the plasma current and the kinetic profiles evolving dynamically, is important for tokamak operation due to its effect on the available magnetic flux for driving inductive current during the flat-top, the performance of the flat-top plasma, and the plasma controllability with the given hardware constraints. In fact, once the coil

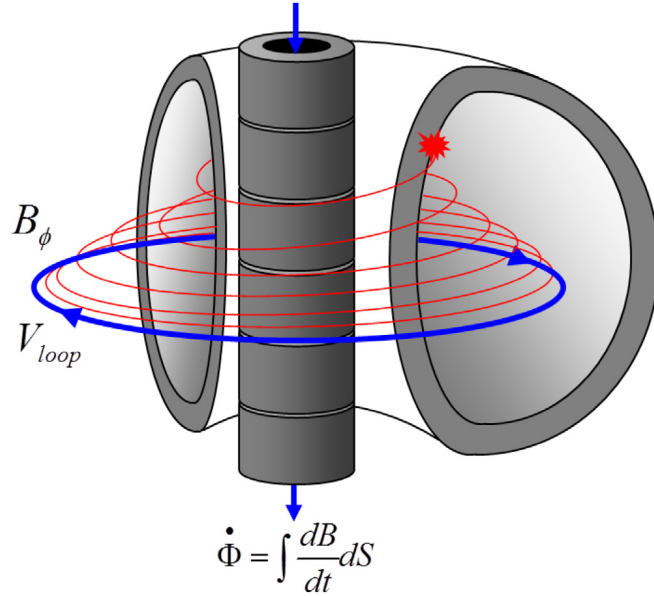


Figure 1.7: The change in flux or magnetic field through the central solenoid (CS) coils, initiated by a discharge of the CS coils, provides a loop voltage around the torus. Electrons, created by the ionization of the prefll gas, will be toroidally accelerated by the toroidal electric field (loop voltage) and follow the magnetic field lines. Because of poloidal field errors (stray field), these field lines are not purely toroidal and eventually reach the wall (Source [137]).

current saturation is reached, the PF coils cannot generate flux anymore. Therefore, it is important to optimize the current ramps such that the desired plasma current and plasma equilibrium is reached at the end of this phase, while minimizing the flux consumption.

Depending on the desired plasma equilibrium to be reached, during this phase several plasma configuration are obtained. Usually, juts after the breakdown the plasma column is in a configuration called *limiter*. It touches the first wall in a point and it is usally a circular and stable plasma. Then, during the ramp-up, the PF coils keep driving currents, computed by the magnetic control system, such that the plasma shape and equilibrium change in a diverted configuration. Here, the plasma cannot touch any part of the first wall due to its temperature; the energy produced, due to the flux

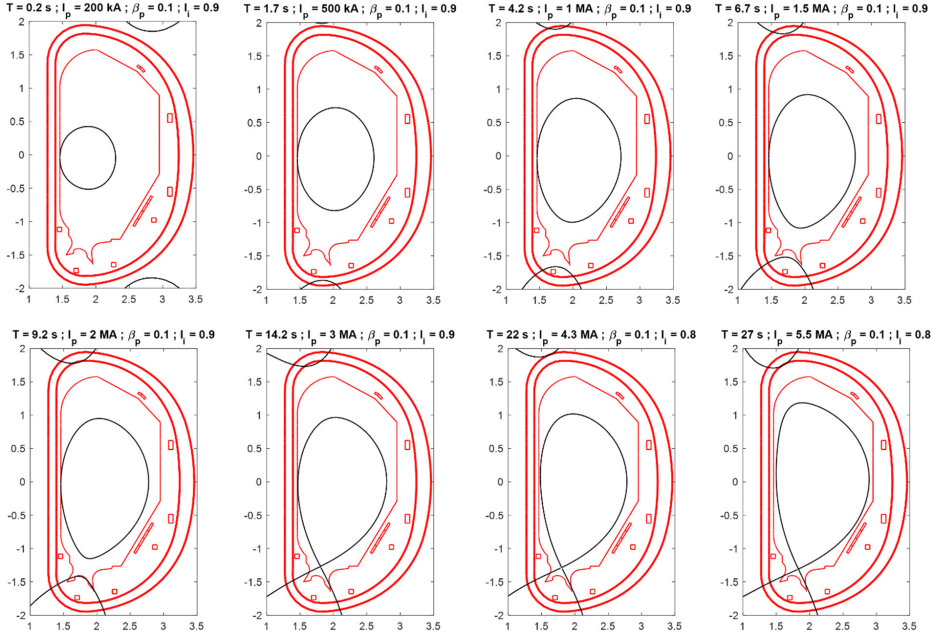


Figure 1.8: Time evolution of the plasma shape for a typical pulse of the DTT tokamak (Source [32]).

surfaces, is concentrated in the divertor, which is specifically designed to handle a high amount of released power. Usually the diverted configuration is preferred since has been observed an increased plasma performance. In fact, diverted plasmas can enter the H-mode that is achieved during the ramp-up. H-mode plasmas see an increase in the confinement time due to peaked pressure and temperature profiles at the boundary.

Finally, the magnetic control system not only has to ensure the plasma current ramp but also handle the transition between limiter and diverted plasma and between L-mode and H-mode. In Figure 1.8, typical ramp-up plasma shapes and the transition between limiter and diverted plasma are shown.

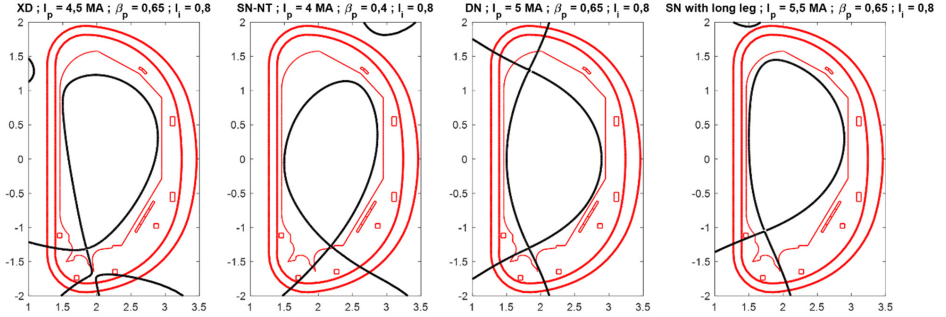


Figure 1.9: Plasma shapes at the flat-top for different plasma configurations for the DTT device (Source [32]).

Flat-top

During the Flat-Top (FT) phase, the plasma current and the main plasma parameters, such as temperature, density, and pressure, are kept constant. The diagnostic systems monitor the plasma quantities of interest, and the relative data is acquired to carry on the physics studies relative to the desired plasma configuration.

Plasma stability and performance in terms of power, temperature, density, and pressure can be studied during this phase, according to the achieved equilibrium. In Figure 1.9, several possible plasma equilibria to be studied with the DTT tokamak [107] are shown. The plasma is subject to many instabilities, mainly due to the plasma-wall interactions, the so-called Resistive Wall Modes (RWM) or MHD turbulence like the Edge-Localized-Modes happening in the H-mode configuration. It is important that the plasma magnetic control system rejects these disturbances in order to avoid sudden plasma disruptions. The disruption phenomena happen on a very short time scale; therefore, the fast loss of a very high flat-top plasma current could lead to huge electro-mechanical and thermal stress on the coil system, passive structures, and first wall, possibly damaging the machine [7].

Ramp-Down

During the Ramp-Down (RD) phase, the plasma current needs to be taken to a low value such that a plasma disruption is not harmful to the machine. Usually, together with plasma current also the shape is also affected and the magnetic controllers need to give the proper current references [125]. Sometimes, it may be convenient to end the plasma pulse before than designed due to sudden MHD unstable modes. Therefore, if an unstable mode is predicted based on the observation of the plasma parameters, the ramp-down phase can be triggered beforehand [64].

1.3.3 JT-60SA

Part of this thesis will be focused on the magnetic control system of the JT-60SA tokamak [104] and the recent results achieved during the Integrated Commissioning (IC) and First Operation (OP1) campaigns [112]. JT-60SA is currently the largest superconducting tokamak in the world, its main parameters are reported in Table 1.1 and compared with other tokamaks in Figure 1.10. The scientific objectives are to explore fundamental fusion technology that will support the ITER reactor, such as the achievement of the self-ignition condition, the demonstration of long pulse (thousands of seconds) burning plasma, disruption avoidance, and mitigation, Edge Localized Modes control, heat load mitigation by a radiative divertor, and controllability of the plasma via superconducting coils.

In the successful IC & OP1 campaigns, the JT-60SA plasma reached a 1 MA plasma current for several seconds, which is a relevant result considering that the CS coils were not operating to their maximum saturation level. It was also proven that the TPC breakdown is feasible also in large superconductive tokamaks [140], which is an important result in view of the first ITER operations.

After the IC & OP1 campaigns, JT-60SA is currently being upgraded. In the next operations, not only will the CS limit be its nominal one, but further diagnostics and heating systems will be installed to reach the foreseen scientific objectives.

1.3.4 Divertor Tokamak Test

The 2012 European Roadmap for Fusion Energy identified eight missions to address key challenges in developing the DEMO fusion power plant. Mission #2 targets the development of a robust heat and power exhaust system capable of managing the intense loads in the divertor region of a fusion reactor.

Although work is underway to refine a conventional detached divertor for ITER, the roadmap also proposed the creation of a dedicated Divertor Tokamak Test (DTT) facility. This facility would serve to investigate alternative solutions for DEMO, including novel magnetic configurations and liquid metal divertors.

In 2015, an international team presented the DTT concept, demonstrating its potential as a flexible and effective platform to close the gap between current fusion experiments, ITER, and DEMO. The importance of DTT was further recognized in the 2018 update of the EUROfusion Roadmap, which endorsed it as a key collaborative initiative to tackle the critical issue of power exhaust in future fusion reactors.

DTT has been conceived as a flexible test bed capable of tackling plasma exhaust issues in a fairly integrated fashion, according to the EU Fusion Roadmap Mission. In Table 1.2, the main parameters of DTT are reported.

Table 1.1: Value of the main parameters of JT-60SA

Parameter	Value
Plasma current	5.5MA
Toroidal field	2.25T
Major radius	2.96m
Minor radius	1.18m
Elongation	1.95
Triangularity	0.53
Aspect ratio	2.5

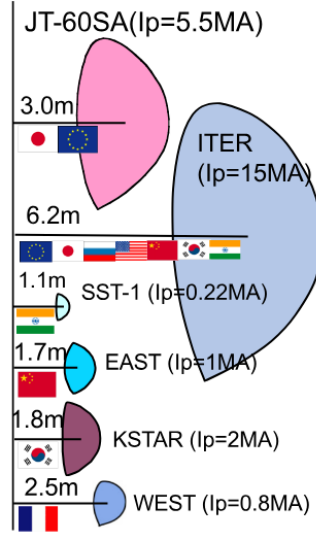


Figure 1.10: JT-60SA major radius and plasma current compared to other tokamaks. Until ITER completion, JT-60SA will be the largest tokamak in the world (Source [70]).

1.4 Reversed Field Pinch

The main difference between RFPs and tokamaks lies in the magnetic field configuration. This is highlighted in Figure 1.11. Contrary to tokamaks, in RFPs the toroidal magnetic field is also mainly self-generated by the plasma itself and has an amplitude similar to the poloidal one. Actually, with respect to the radial profile, the toroidal field even changes sign. This feature makes the plasma ring prone to MHD instabilities, which break the toroidal symmetry of the magnetic field. These instabilities are the drive, through a solenoidal effect, of the reversed toroidal field regeneration mechanism [45].

Another effect of these so-called *kink modes* is the helical plasma state (shown in Figure 1.12) that allows the plasma current itself to generate the toroidal field. In general, the RFP configuration allows for a much larger plasma current with respect to tokamaks, possibly removing the need for additional external heating, resulting in a device much easier and cheaper to

construct.

The main RFP advantage lies in its greatly reduced costs. However, the loss of the toroidal symmetry is its biggest disadvantage. In fact, 3D modelling of the plasma is a very challenging task, moreover it triggers these 3D kink instabilities which are very hard to monitor and control.

1.4.1 RFX-mod2

As anticipated, the world's largest RFP device is RFX-mod which, although being fairly small with respect to modern tokamaks, during its scientific exploitation phase, reached 2 MA plasma current. The main scientific contribution of the RFX-mod machine regards control of 3D modes [119, 105].

RFX-mod is currently being refurbished, and its enhanced version will be the RFX-mod2 RFP. Since part of this thesis regards the RFX-mod2 magnetic control system, the details of the upgrades will be discussed in a following chapter. Mainly, all the changes to the old machine result in improved plasma stability, allowing for better investigation of the RFP configuration properties. Finally, RFX-mod2, as its predecessor, will be an extremely flexible device, allowing it to operate as a tokamak device. In the following, a possible magnetic control system architecture for RFX-mod2 tokamak pulses is proposed. The typical parameters of an RFX-mod2 tokamak pulse are reported in Table 1.3.

Table 1.2: Value of the main parameters of DTT

Parameter	Value
Plasma current	5.5MA
Toroidal field	6T
Major radius	2.19m
Minor radius	0.7m
Aspect ratio	3.1

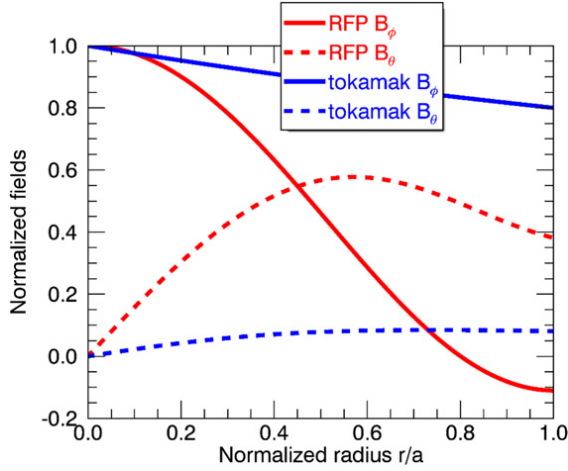


Figure 1.11: Relation between toroidal and poloidal magnetic field in RFPs and in tokamaks (Source [86]).

1.5 Stellarator

For the sake of completeness, in this section, the stellarators' characteristics are briefly summarized. Magnetic confinement of plasma in toroidal devices requires magnetic field lines that twist both toroidally and poloidally to prevent particle losses and ensure effective confinement. Tokamaks and reversed-field pinches accomplish this by inducing a toroidal plasma current. Stellarators, in contrast, generate the necessary helical magnetic field structure through carefully shaped external magnets (see Figure 1.13).

Table 1.3: Value of the main parameters of RFX-mod2

Parameter	Value
Toroidal field	0.55T
Plasma current	50kA
Major radius	2m
Minor radius	0.46m
Elongation	1.26

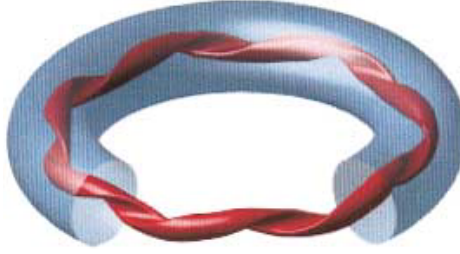


Figure 1.12: Schematic view of an helical structure inside the RFX vessel (Source [45]).

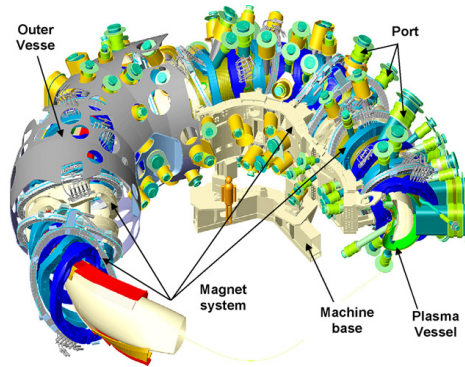


Figure 1.13: CAD view of the main components of W7-X, with the twisted magnets shown in blue and light blue (Source [28]).

Stellarators, a general term for all helical devices, offer several advantages over tokamaks, making them a promising alternative for future fusion power plants. The most significant benefit is their minimal plasma current, which leads to inherently better stability and supports steady-state operation. Because stellarators rely entirely on externally generated magnetic fields for plasma initiation and control, they are far less prone to major disruptions. Additionally, stellarators have demonstrated the ability to operate at higher plasma densities, as no clear density limit has been observed so far. Higher densities help suppress fast ion-driven instabilities and make divertor operation more manageable.

However, these advantages come with trade-offs. The three-dimensional magnetic coil system is extremely complex, making it challenging to de-

sign, build, and align accurately. Since the plasma shape is tightly linked to the magnetic coil geometry, stellarator configurations lack flexibility once constructed. Furthermore, the absence of plasma current makes it difficult for stellarators to access high-performance regimes like the H-mode, which significantly limits their overall efficiency in the context of energy-producing reactors.

Despite these challenges, ongoing research is addressing these limitations with the aim of demonstrating that stellarators can be a viable and competitive alternative to tokamaks for future fusion energy production ([26, 143]).

1.6 Contribution

As briefly mentioned in the previous sections, the plasma magnetic control system is fundamental for tokamaks' operations. The poloidal and toroidal field coils provide the required magnetic field for plasma equilibrium. Diverted elongated plasmas, which are usually preferred for their particle confinement characteristics, are not only subject to MHD turbulence but also vertically unstable. The magnetic control system needs to apply corrections to the equilibrium magnetic field to cope with these instabilities. Moreover, the controlled quantities (e.g., plasma shape, plasma current, centroid position) usually are not directly measured. Therefore, efficient algorithms, on the basis of the magnetic field and flux sensor readings, provide the values of such quantities.

In the initial operational phases of tokamaks, not all components and diagnostics are typically available. For example, during the JT-60SA IC & OP1, the in-vessel coils designed for plasma vertical stability were not yet installed. Additionally, the complete set of magnetic diagnostics will only become operational in later phases. A similar situation is expected for ITER during its first plasma and IC campaigns [29]. Despite these limitations, there is often a desire to generate elongated plasmas during these phases to explore the full capabilities and flexibility of the machine, as was demonstrated during the JT-60SA IC campaign [145]. The absence of complete diagnostic data limits the effectiveness of reconstruction algorithms, while the lack of active stabilization systems complicates the control of elongated plasmas.

On the other hand, the upgraded RFX-mod2 device introduces further challenges compared to its predecessor, RFX-mod, including the need for updated machine modeling and the management of tight constraints imposed by its power supply system, which tracks only positive voltage references. Since the power supply system remains unchanged from RFX-mod, these constraints—previously not explicitly accounted for—must now be addressed. The RFX-mod2 upgrade thus provides a key opportunity to apply advanced control strategies, such as the design of robust control systems operating under strict saturation limits or reduced actuator conditions.

While the primary motivation for addressing these issues stems from the initial start-up phases and the associated limited sensor and actuator availability, the scope of this thesis extends significantly beyond these early stages. The methodologies and algorithms presented herein, including a new fast plasma boundary reconstruction algorithm based on Neural Networks, are highly relevant over the entire lifetime of fusion devices. Indeed, they provide a necessary framework to manage contingency situations, such as actuator and sensor failures, which can occur at any stage of operation.

The development of such robust and efficient algorithms is not only a technical necessity but also a critical factor for the long-term economic viability of next-generation tokamaks, as minimizing downtimes caused by hardware failures is essential to reduce serious economic losses. In this context, and in view of ITER operations, the specific contributions of this thesis are the following:

- The successful JT-60SA IC gives the possibility to do a preliminary validation of ITER real-time reconstruction algorithms for the plasma current, centroid position, boundary, and for the poloidal beta (a parameter representing magnetic pressure). The accuracy of the reconstruction, which is crucial for the proper functioning of the entire magnetic control system, is evaluated by comparing it to plasma equilibria computed with the CREATE-NL nonlinear magnetic equilibrium code [8], a tool capable of reconstructing all the plasma parameters of interest with high accuracy. Moreover, the algorithms' reconstruction capabilities with a reduced set of magnetic sensors can be evaluated.

- An alternative fast boundary reconstruction algorithm based on Extreme Learning Machines (ELMs), a class of neural networks with reduced training time, is developed and presented. This approach offers a promising alternative to more computationally intensive methods, enabling real-time application of neural networks in future tokamak operations. The performances of this algorithm are compared to the XLOC algorithm (which has been used at JET for several years), considering as case study the DTT tokamak.
- In the absence of dedicated actuators, the magnetic control system shall perform the essential fast control actions by using the out-vessel superconductive coils, if needed. These are typically less efficient in reacting to fast transients, due to the shielding effect of the vessel, and imply a coupling with other control tasks relying on the same actuators, such as plasma current, position, and shape control. Hence, effective actuator-sharing strategies must be put in place. This thesis presents an architecture and a possible control strategy that is able to cope with vertically unstable elongated plasmas subject to fast varying disturbances, in the absence of dedicated in-vessel coils. The architecture exploits a model-based actuator-sharing approach to effectively accomplish the main magnetic control objectives while minimizing the cross-couplings among the various tasks. The effectiveness of the approach is demonstrated by means of nonlinear simulations of realistic JT-60SA scenarios [133].
- The RFX-mod2 main refurbishments concern the removal of the Inconel vacuum vessel, allowing the plasma to be closer to the highly conducting copper shell, improving the plasma stability, and the introduction of a cage composed of toroidal and poloidal passive conducting elements, which allow to keep the graphite first-wall equipotential. At least at the beginning of the operations, the power supply system will not be upgraded. Therefore, the old one-quadrant a.c.-d.c. converters will be used for early operations. This translates in non-symmetric saturation limits on the possible voltages fed to the system. The upgrade of the machine's hardware components is an opportunity to also update the plasma magnetic control system. Hence, following a model-based design approach, the first step

is to develop a new electromagnetic model of the machine, which is used to develop the PF coil currents controller. Finally, a whole new magnetic control architecture, based on the Model Predictive Control (MPC) capable of taking into account the constraints related to the machine design is proposed.

This thesis is structured as follows:

Chapter 2 presents the electromagnetic modeling of tokamak plasmas. It introduces the fundamental equations governing plasma electromagnetic behavior and outlines the most relevant plasma parameters. The chapter concludes with the derivation of a linearized plasma model around a generic equilibrium point, which will be extensively used in the design of model-based controllers and in simulations.

Chapter 3 addresses the problem of reconstructing controlled plasma parameters from magnetic measurements. In particular, ITER's reconstruction algorithms are described and tested under a reduced-sensor scenario using data from the JT-60SA IC campaign. Three boundary reconstruction algorithms are then presented: a simple filament-based method, an adaptation of the XLOC code for DTT, and a fast neural network-based approach. The latter two are compared in terms of performance, using the DTT tokamak as a case study.

Chapter 4 introduces a model-based actuator sharing technique aimed at stabilizing elongated plasmas in the absence of in-vessel coils. The proposed method is evaluated on the JT-60SA setup during the IC phase. The effectiveness of the approach is demonstrated through extensive simulations, including both linear and nonlinear models.

Chapter 5 presents proposals for the magnetic control system of the new RFX-mod2 machine. First, a new electromagnetic model is derived, incorporating the structural changes introduced in comparison to its predecessor. Then, a Model Predictive Control based scheme for active current control is proposed, which is particularly useful for operation during plasmaless pulses. Finally, possible architectures for an equilibrium control system are discussed. All proposed control strategies take into account the stringent non-symmetrical saturation limits of the power supplies.

Chapter 6 summarizes the main findings of the thesis across the various topics addressed and outlines possible future work.

Chapter 2

Tokamak plasmas modelling

This chapter introduces the electromagnetic modeling of tokamak plasmas. The Grad-Shafranov equation is briefly explained, along with the assumptions that lead to it. Moreover, the physical quantities that are usually considered for the magnetic control are presented. Then, the elongated plasmas vertical instability is described, and it is shown how to derive a control-oriented linear model of the plasma behaviour around an equilibrium. This is useful for the next chapters, which will present the main contribution of the thesis. In fact, Chapter 3 uses the relation between currents and flux explained in this chapter, while Chapters 4 and 5 extensively use linearized models for design and controller validation purposes.

2.1 Ideal MHD

Ideal-MHD [51] describes the basic behaviour of a magnetically confined plasma. In this theory, the plasma is considered as a single fluid, that is, no distinction is made among the various particles constituting the plasma. The key equation that describes the plasma behaviour is the Grad-Shafranov equation.

The dynamics of the quantities of interest for magnetic control (such

as position and shape) in a tokamak are governed by Maxwell's equations

$$\begin{cases} \nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} \\ \nabla \times \mathbf{H} = \mathbf{J} \\ \nabla \cdot \mathbf{B} = 0 \end{cases} \quad (2.1)$$

Equation (2.1) represent the Faraday's law for the electrical field $\mathbf{E}$, the Ampere's law, representing the relation between the current density $\mathbf{J}$ and magnetic field intensity $\mathbf{H}$, and finally the Gauss law for the induction magnetic field $\mathbf{B}$. The constitutive relations,

$$\begin{cases} \mathbf{B} = \mu_0 \mathbf{H} \\ \eta \mathbf{J} = \mathbf{E} + \mathbf{v} \times \mathbf{B} \end{cases}$$

complete the set of equations relative to the magnetic and electrical field, where η is the plasma resistivity and $\mathbf{v}$ the plasma particle velocity. In this theory, the plasma is modeled as a single fluid, without accounting for the differences in the particles that compose the plasma.

The electromagnetic equations need to be coupled with the ones that describe the behaviour of the fluid, subject to external forces and pressure. The fundamental laws that link velocity, plasma density (ρ), and pressure (p) are the following:

$$\begin{cases} \frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \\ \frac{d}{dt} (p \rho^{-\gamma}) = 0 \\ \rho \frac{d}{dt} \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla p \end{cases} \quad (2.2)$$

In Equation (2.2) the first equation is the local expression of the mass conservation law, the second is the adiabatic state equation, and the last one is Newton's law.

In ideal MHD, the plasma is assumed to be a massless superconductor, i.e., $\eta = 0$ and $\rho = 0$. Hence, it is subject to two interacting forces: the magnetic force $\mathbf{J} \times \mathbf{B}$, and the pressure gradient ∇p .

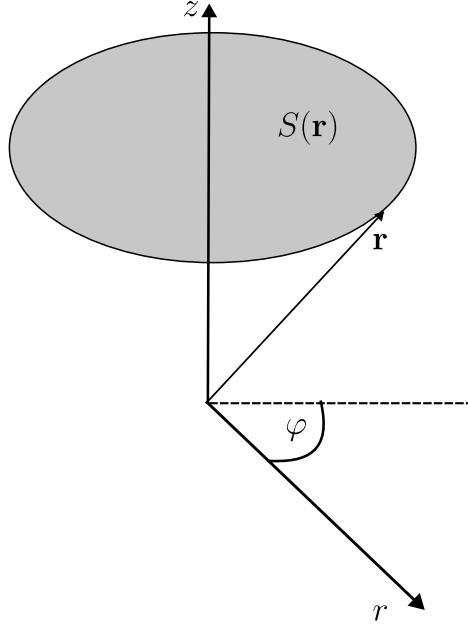


Figure 2.1: The cylindrical coordinate system and the surface $S(\mathbf{r})$ obtained by rotating $\mathbf{r}$ around the $r = 0$ axis.

2.2 The Grad-Shafranov equation

Let (r, φ, z) be a three-dimensional cylindrical coordinate system. Since, a tokamak is a axisymmetric toroidal machine, it is assumed for the generic vector $\mathcal{A}$

$$\frac{\partial}{\partial \varphi} \mathcal{A} = 0, \quad (2.3)$$

reducing the plasma equilibrium to a two-dimensional problem.

Let us now introduce the poloidal flux function. It is defined as

$$\psi(\mathbf{r}) = \frac{1}{2\pi} \int_{S(\mathbf{r})} \mathbf{B} \cdot d\mathbf{S} \quad (2.4)$$

where $S(\mathbf{r})$ is a surface such that its contour ∂S is a circumference obtained by rotating the point $\mathbf{r}$ around the $r = 0$ axis (see Figure 2.1). Hence, it is $\psi(\mathbf{r}) = \psi(r, z)$. By choosing $S(\mathbf{r})$ perpendicular to the $r = 0$

axis, it is possible to write the surface integral in Equation (2.4) as

$$\psi(\mathbf{r}) = \frac{1}{2\pi} \int_0^r \int_0^{2\pi} B_z(\rho, z) \rho d\rho dz = \int_0^r \rho B_z(\rho, z) d\rho, \quad (2.5)$$

where $\mathbf{B} = B_r \hat{\mathbf{r}} + B_\varphi \hat{\boldsymbol{\varphi}} + B_z \hat{\mathbf{z}} = \mathbf{B}_p + B_\varphi \hat{\boldsymbol{\varphi}}$, where $\mathbf{B}_p$ is the poloidal magnetic field.

When 2.3 holds, then Gauss's Law for the magnetic field in cylindrical coordinates becomes

$$\frac{1}{r} \frac{\partial}{\partial r} r B_r + \frac{\partial}{\partial z} B_z = 0. \quad (2.6)$$

Differentiating Equation (2.5) with respect to r and z , and taking into account Equation (2.6), the following relations between the poloidal flux function and the magnetic field hold:

$$\begin{cases} \frac{\partial}{\partial r} \psi = r B_z \\ \frac{\partial}{\partial z} \psi = -r B_r \end{cases}. \quad (2.7)$$

With the massless assumption, the Newton's law is simplified as follows:

$$\nabla p = \mathbf{J} \times \mathbf{B},$$

from which

$$\begin{cases} \mathbf{B} \cdot \nabla p = 0 \\ \mathbf{J} \cdot \nabla p = 0 \end{cases}. \quad (2.8)$$

Equation (2.8) shows that the magnetic field lines and the current density lie on surfaces of constant pressure (isobaric surfaces). In most plasma equilibria, the pressure reaches its maximum near the center of the plasma's poloidal cross-section, with the isobaric surfaces forming nested toroidal shapes. As a consequence of the fact that the magnetic field lines lie on the isobaric surfaces, these surfaces are also called magnetic or flux surfaces. The limiting magnetic surface, which approaches a single magnetic line where the pressure and flux are maximum (assuming a positive plasma current), is called the magnetic axis. An example of nested magnetic surfaces is shown in Figure 2.2.

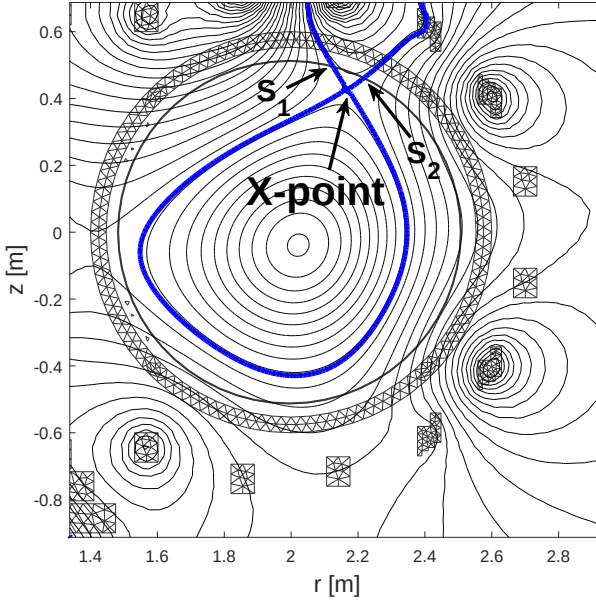


Figure 2.2: Magnetic surfaces for a plasma equilibrium in the RFX-mod2 device. The last closed magnetic surface, which determines the plasma boundary, is highlighted in blue.

Moreover, from Equation (2.7) it also follows that

$$\mathbf{B} \cdot \nabla \psi = 0.$$

Hence, the magnetic surfaces also coincide with constant poloidal flux surfaces. As a consequence, on the poloidal plane, the magnetic induction, current density and pressure are constant on each line level of the poloidal flux function, and the plasma equilibrium is characterized completely by the ψ function ($p = p(\psi)$ and $\mathbf{J} = \mathbf{J}(\psi)$).

Let us define the Δ^* operator as:

$$\Delta^* f = r \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} f \right) + \frac{\partial^2}{\partial z^2} f, \quad (2.9)$$

using Equation (2.7) in Ampere's Law,

$$\Delta^* \psi = -\mu_0 r J_\varphi, \quad (2.10)$$

where J_φ is the toroidal current density whose expression is different depending on the region of the poloidal plane of interest. Clearly, in the vacuum the poloidal flux function satisfies the homogeneous Grad-Shafranov (GS) equation

$$\Delta^* \psi = 0. \quad (2.11)$$

In the active coils region, J_φ is determined by the toroidal current flowing through them. Finally, in the plasma region, the toroidal current depends on the pressure profile and the toroidal component of the magnetic induction field.

The Grad-Shafranov equation can be expressed as follows:

$$\begin{cases} \Delta^* \psi = 0 & \text{in the vacuum region} \\ \Delta^* \psi = -\mu_0 r J_{ext} & \text{in the active coils} \\ \Delta^* \psi = -\mu_0 r^2 \frac{d}{d\psi} p - F \frac{d}{d\psi} F & \text{in the plasma region} \\ \psi(0, z) = 0 \\ \lim_{r \rightarrow \infty} \psi(r, z) = 0 \end{cases} \quad (2.12)$$

where $F = F(r, z)$ is the toroidal component of the magnetic induction field, and J_{ext} is the toroidal current density in the active coils. Usually, numerical codes can solve these non-linear partial differential equations where the boundary of the plasma region is part of the unknown to be determined. They take as inputs the functions J_{ext} , $J_{p_\phi} = \mu_0 r^2 \frac{d}{d\psi} p - F \frac{d}{d\psi} F$, where J_{p_ϕ} is commonly determined based on experimental observations. In [3] the following parametrization is assumed

$$J_{p_\phi} = \iota (\beta_0 r / R_0 + (1 - \beta_0) R_0 / r) (1 - \bar{\psi}^{\alpha_m})^{\alpha_n}, \quad (2.13)$$

with $\bar{\psi} = \frac{\psi - \psi_a}{\psi_b - \psi_a}$, (α_m, α_n) are related to the current density profile, ι is mainly related to the total plasma current, and β_0 to the plasma pressure. R_0 is a geometrical plasma parameter that will be introduced later. Finally, it is important to remark that Equation (2.12) is satisfied at each time instant, meaning that the plasma equilibrium is evolutionary and changes

throughout a plasma discharge.

A more detailed explanation of the derivation of the Grad-Shafranov equation is presented in [51] and [21].

2.2.1 Green's functions for the homogeneous GS equation

The use of Green's functions offers an analytical method for solving the GS equation, particularly in the context of magnetic reconstruction and plasma boundary identification. Let us consider the following equation:

$$\Delta^* G(\mathbf{r}, \tilde{\mathbf{r}}) = -\mu_0 \delta(\mathbf{r} - \tilde{\mathbf{r}}),$$

where $G(\mathbf{r}, \tilde{\mathbf{r}})$, and δ is the Dirac function. Given appropriate boundary conditions, the solution of the GS equation can be written as:

$$\psi(\mathbf{r}) = \int_D G(\mathbf{r}, \tilde{\mathbf{r}}) J_\varphi(\tilde{\mathbf{r}}) d\tilde{S} - \oint_{\partial D} \frac{1}{\mu_0 \tilde{\mathbf{r}}} \left(\psi(\tilde{\mathbf{r}}) \frac{\partial}{\partial n} G - G \frac{\partial}{\partial n} \psi \right) d\tilde{l},$$

This formulation separates the contribution to the flux from the internal current density (within domain D) and the external sources or boundary effects. By considering the full poloidal plane as the domain and applying appropriate boundary conditions that vanish at infinity, a free space Green's function $G_0(\mathbf{r}, \tilde{\mathbf{r}})$ is obtained. The solution simplifies to

$$\psi(\mathbf{r}) = \int_{\mathbb{R}^2} G_0(\mathbf{r}, \tilde{\mathbf{r}}) J_\phi(\tilde{r}) d\tilde{S}.$$

Therefore, $G_0(\mathbf{r}, \tilde{\mathbf{r}})$ represents the flux produced by an infinitesimal toroidal filament carrying a unitary current. The analytical expression of G_0 is

$$G_0(\mathbf{r}, \tilde{\mathbf{r}}) = \mu_0 \frac{\sqrt{r\tilde{r}}}{\pi k} \left[\left(1 - \frac{k^2}{2} \right) K(k^2) - E(k^2) \right],$$

where

$$k^2 = \frac{4r\tilde{r}}{(r - \tilde{r})^2 + (z - \tilde{z})^2}.$$

2.2.2 The plasma boundary

The identification of the plasma boundary is essential for real-time control. In fact, plasma shape descriptors are needed to let the control system achieve the desired plasma configuration. The plasma boundary is identified with one of the constant level lines of the poloidal flux function (i.e., the edge of magnetic surfaces), specifically with the last closed flux surface (LCFS) entirely contained within the vacuum vessel.

The plasma can be either *limiter* or *diverted*. If the plasma boundary and the tokamak first wall (also called limiter) have a point in common, the plasma is called a limiter plasma. On the other hand, if the plasma boundary and the first wall do not share any point in common, the plasma is called a diverted plasma. In Figure 2.2, the thick blue line shows the boundary of a diverted plasma. In this case, the boundary separates the closed and open flux surfaces. Moreover, the LCFS is characterised by a saddle point (also called X-point or null-point) for the poloidal flux function, therefore, in this point, the poloidal magnetic field is zero. For diverted plasmas, the plasma boundary continues after the X-point with two baffles. The intersections between the baffles and the first wall are called strike points. Most of the power produced by the plasma is concentrated here; therefore, precise estimation and control of the strike points is fundamental.

To identify the plasma boundary, it is sufficient to solve the Grad-Shafranov equation in the vacuum on a grid of points, knowing the flux and the magnetic field at the measurement points $(\mathbf{r}_\psi, \mathbf{z}_\psi)$ and $(\mathbf{r}_b, \mathbf{z}_b)$:

$$\begin{cases} \Delta^* \psi = 0 \\ \psi(\mathbf{r}_\psi, \mathbf{z}_\psi) = \psi^*(\mathbf{r}_\psi, \mathbf{z}_\psi) \\ \mathbf{B}_p(\mathbf{r}_b, \mathbf{z}_b) = \mathbf{B}_p^*(\mathbf{r}_b, \mathbf{z}_b) \end{cases}, \quad (2.14)$$

where $\psi^*(\mathbf{r}_\psi, \mathbf{z}_\psi)$ and $\mathbf{B}_p^*(\mathbf{r}_b, \mathbf{z}_b)$ are the flux and magnetic field sensors readings.

Since the poloidal flux monotonically increases from the edge towards the center of the plasma, the value of the flux at the boundary ψ^B can be easily obtained by taking the maximum between the flux at the first wall and the flux at the X-points.

Once the plasma edge is determined, some shape descriptors need to

be computed. Namely, the so-called *gaps* are the distances between the plasma boundary and the first wall, along pre-defined segments. The gaps can be controlled to the desired values to control the plasma shape. Alternatively, to impose a particular shape, the flux at some control points can be imposed to be equal to ψ^B . In fact, by controlling to zero the difference $\psi(\mathbf{r}^*, \mathbf{z}^*) - \psi^B$ the plasma boundary will pass through the points $(\mathbf{r}^*, \mathbf{z}^*)$.

To this aim, the real-time reconstruction of the poloidal flux function in the vacuum is crucial. Boundary reconstruction codes only solve Equation (2.14), which yields an accurate reconstruction of the flux map in the vacuum, and are hence simpler and faster, making them suitable for real-time control purposes only. On the other hand, full-equilibrium codes solve Equation (2.12) and are generally more accurate and reliable. Nevertheless, being a free-boundary problem, these are iterative codes and consequently slower; therefore, they need to be optimized for real-time deployment, and enough computational resources need to be available.

2.3 Plasma quantities relevant for magnetic control

This section presents the plasma quantities of interest for magnetic control. Since these quantities are usually not directly measured but are instead provided to the controller through post-processing of the direct measurements, such reconstruction must be as accurate as possible. The next chapter will go into more detail on the problem of reconstructing these quantities, showing an application with data collected during the JT-60SA IC & OP1 campaigns.

Plasma current

The induced plasma current is one of the main parameters that defines the whole plasma equilibrium. It is the main source of the poloidal magnetic field, and its value is crucial for particle confinement. The total plasma current is the current flowing through the poloidal plane in the plasma region, and it is defined as follows:

$$I_p = \int_{\Omega_p} J_\varphi dS,$$

where Ω_p is the plasma region (referring to Figure 2.2, the plasma region is the one contained inside the LCFS). The plasma current can be either reconstructed via the magnetic field measurements or directly measured.

An algorithm to estimate the plasma current via magnetic measurements will be presented in the next chapter.

Plasma current centroid

The plasma current centroid is the centre of the plasma current distribution. In particular,

$$z_c = \frac{1}{I_p} \int_{\Omega_p} z J_\varphi dS$$

is used to characterize the plasma vertical position, while

$$r_c = \sqrt{\frac{1}{I_p} \int_{\Omega_p} r^2 J_\varphi dS}$$

characterizes the plasma radial position.

Estimating and controlling the plasma current centroid position is critical for the proper operation of a tokamak. In fact, this quantity plays a key role in controlling the plasma's position within the chamber. It's also important to note that, during the initial phase of a tokamak pulse, the plasma current centroid position is often controlled independently of shape control. Furthermore, for vertically unstable plasmas, controlling the centroid position z_c is crucial for maintaining stability.

Shape descriptors

As briefly anticipated, the gaps are the distances between selected points of the first wall and the plasma boundary, measured along given directions. Referring to Figure 2.3, the segment is given by points $\mathbf{p}_1$ and $\mathbf{p}_2$, while the gap g is the distance along that segment between the plasma boundary and the first wall. They can be used for an accurate shape control of the plasma column.

Other shape-descriptors relevant for control purposes of diverted plasmas are the X-point position (r_x, z_x) and the position of the two strike

points. These geometrical descriptors can be evaluated once the poloidal flux is assigned.

Another key descriptor of the plasma shape is the elongation κ . It is defined as

$$\kappa = \frac{Z_{max} - Z_{min}}{R_{max} - R_{min}},$$

where:

- Z_{max} is the maximum value of z along the LCFS;
- Z_{min} is the minimum value of z along the LCFS;
- R_{max} is the maximum value of r along the LCFS;
- R_{min} is the minimum value of r along the LCFS

as depicted in Figure 2.3. Finally, the minor and major radius are defined as

$$a = \frac{R_{max} - R_{min}}{2}$$

and

$$R_0 = \frac{R_{max} + R_{min}}{2}$$

respectively.

Poloidal beta and internal inductance

The parameter poloidal beta is a measure of the efficiency of the plasma confinement, indeed it is the ration between the kinetic pressure and the magnetic energy inside the plasma. In literature, several definitions of β_p exist. Let us consider the following β_p definition

$$\beta_p = \frac{\iiint_{V_p} p dV}{\iiint_{V_p} dV \frac{B_{pa}^2}{2\mu_0}}, \quad (2.15)$$

where

$$B_{pa} = \frac{\oint_{\Gamma_p} \mathbf{B}_p \cdot d\mathbf{l}}{\oint_{\Gamma_p} d\mathbf{l}},$$

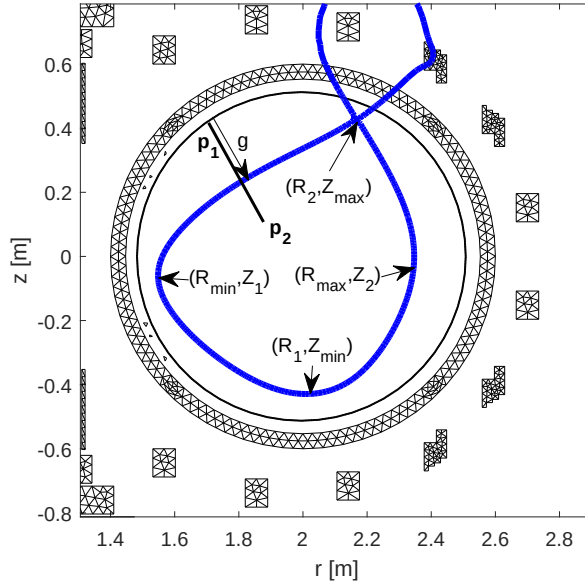


Figure 2.3: Geometrical descriptors of the plasma shape. The upper, the inner, the lower, the outer point, and a sample gap are shown.

V_p denotes the plasma volume, while Γ_p the plasma boundary. The internal inductance is dimensionless and it is linked to the magnetic energy inside the plasma region

$$l_i = \frac{4}{\mu_0 r_c I_p^2} \iiint_{V_p} \frac{\|\mathbf{B}_p\|^2}{2\mu_0} dV.$$

These two parameters, as well as the plasma current, are used to characterize the toroidal current density. Therefore, a variation of β_p and l_i means that the plasma equilibrium changes as well. At steady state, a predefined equilibrium is chosen and needs to be kept throughout the flat-top phase. This is why a change of these two quantities is seen as a disturbance as far as the magnetic control system is concerned.

2.4 Plasma vertical instability

Vertically elongated plasmas are commonly used in fusion devices to enhance performance, particularly in terms of MHD stability. However, as previously noted, such plasmas are inherently unstable in the vertical direction, which imposes a limit on the maximum achievable elongation. This instability arises because the elongated shape results from vertical forces applied by currents in surrounding conductors, pulling the plasma upward and downward. When the plasma experiences a small vertical displacement, these forces generate a net vertical force in the same direction as the displacement, thus reinforcing it and leading to instability. Figure 2.4 shows a simple model of the phenomenon.

In theory, surrounding the plasma with a perfectly conducting wall would suppress this instability. In practice, however, real walls are resistive and can only slow down the growth of the instability, also referred to as RWM. Hence, it turns out that the Vertical Stabilization system is essential, since, in the case of elongated plasmas, without this system, the tokamak discharge cannot be run. It should also be remarked that, without this passive stabilizing action, it would not be possible to actively stabilize the plasma.

The simplest model for control design of vertical instability is the rigid displacement model [81, 68], which idealizes the plasma as a rigid, filamentary current-carrying wire. However, as shown in [2], this model provides inaccurate estimates of the instability growth rate and does not correctly represent the magnetic signals detected by diagnostics used for feedback control.

A more accurate representation of the vertically unstable plasma can be achieved through a perturbed equilibrium approach, which accounts for the plasma's response to vertical displacement more realistically.

2.5 Control-oriented linearized plasma model

As discussed earlier in this chapter, the GS equation enables the determination of two-dimensional magnetic equilibria under the assumption of axisymmetry. This partial differential equation is solved using numerical codes [4, 56], which compute the poloidal flux ψ , particularly its value at

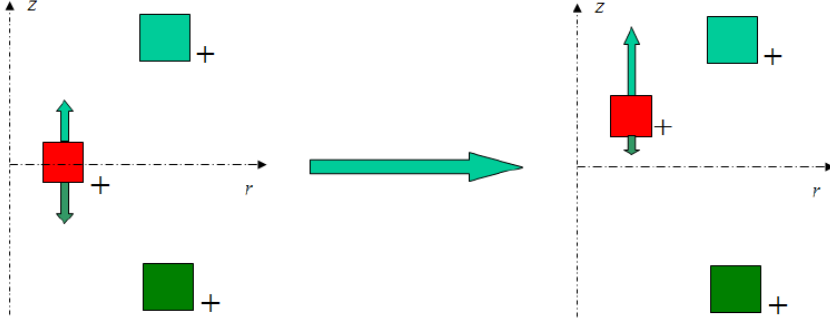


Figure 2.4: Simplified representation of the plasma vertical instability. The plasma is modeled as a rigid conductor in red. The active coils are represented in green. It is important to note that the current in the two active coils of this simplified model represents differential control currents. These additional currents act on top of the equilibrium ones that counteract plasma radial expansion (source [123]).

the conductive structures surrounding the plasma, including both passive and active components (e.g., PF coils). In fact, the solution domain encompasses several regions: (a) plasma; (b) vacuum; (c) active coils; and (d) passive coils and other conducting structures. From an electromagnetic standpoint, the primary distinction between the two types of conductors lies in their power supply: passive coils have zero applied voltage.

First, let us consider the plasmaless case. In this scenario, passive structures are typically discretized, with the induced currents modeled as flowing through a series of conductors that make up the entire structure. When active coils are included, the time evolution of the toroidal currents in all conductors can be described using a straightforward circuit equation

$$\mathbf{L}_c \dot{\mathbf{I}}(t) + \mathbf{R}_c \mathbf{I}(t) = \mathbf{S}_c \mathbf{V}, \quad (2.16)$$

where $\mathbf{I}(t)$ is the vector of active ($\mathbf{I}_a(t)$) and passive currents ($\mathbf{I}_e(t)$), $\mathbf{L}_c$ is the inductance matrix whose off-diagonal terms describe the mutual inductance between the circuits, $\mathbf{R}_c$ is the resistance matrix, $\mathbf{V}$ is the vector of the applied voltages (the entries relative to the passive conductors currents are equal to zero), and $\mathbf{S}_c$ is a matrix that takes into account the sign

between the voltage and the currents for each circuit. Usually, $\mathbf{R}_c$ is known while $\mathbf{L}_c$ can be computed taking into account the particular machine geometry. Generally speaking, each time sample of the vector $\mathbf{I}(\cdot)$ contains between 100-200 elements, depending on how the passive structures have been discretized and on the size of the machine.

When the effect of the plasma is taken into account, an additional term appears in Equation (2.16)

$$\mathbf{L}_c \dot{\mathbf{I}}(t) + \mathbf{R}_c \mathbf{I}(t) + \dot{\boldsymbol{\Psi}}_p(t) = \mathbf{S}_c \mathbf{V},$$

where $\dot{\boldsymbol{\Psi}}_p$ is a vector that depends only on the flux produced by the plasma on these structures. Its values can be computed by solving the non-linear equilibrium problem described by Equation (2.12), and is a function of the external active currents, the total plasma current, and the vector $\mathbf{w} = (\beta_p \ l_i)^T$ which parameterize the plasma current distribution, therefore,

$$\dot{\boldsymbol{\Psi}}_p(t) = \lambda(\mathbf{I}_{\mathbf{PF}}(t), \mathbf{I}_p(t), \mathbf{w}(t)).$$

Once $\lambda(\cdot)$ is obtained numerically, with a straightforward linearization procedure a linearized plasma response model can be computed around the equilibrium point $(\mathbf{I}_0, \mathbf{w}_0, I_{p0})$. Let

$$\begin{aligned} \mathbf{I} &= \mathbf{I}_0 + \delta \mathbf{I} \\ \mathbf{w} &= \mathbf{w}_0 + \delta \mathbf{w} \\ I_p &= I_{p0} + \delta I_p \\ \mathbf{V} &= \mathbf{V}_0 + \delta \mathbf{V} \end{aligned}$$

The linearized model can be written as

$$\left(\mathbf{L}_c + \frac{\partial}{\partial \mathbf{I}} \lambda \Big|_0 \right) \delta \dot{\mathbf{I}} + \mathbf{R}_c \delta \mathbf{I} + \frac{\partial}{\partial \mathbf{w}} \lambda \Big|_0 \delta \dot{\mathbf{w}} + \frac{\partial}{\partial I_p} \lambda \Big|_0 \delta \dot{I}_p = \mathbf{S}_c \delta \mathbf{V}. \quad (2.17)$$

In Equation (2.17), the plasma current appears as an input, while being a variable to be controlled, it would be convenient to express it as a state variable. This can be done by coupling Equation (2.17) with

$$\mathbf{L}_p \delta \dot{I}_p + \mathbf{M}_{pi} \delta \dot{\mathbf{I}} + \mathbf{M}_{pw} \delta \dot{\mathbf{w}} = 0, \quad (2.18)$$

which shows that, from the total plasma current point of view, the plasma can be seen just as another circuit added to the ones used to model the conducting structures of the machine. More details on the derivation of Equation (2.18) and of the linear model can be found in [3].

Defining the state variable vector as $\mathbf{x} = (\mathbf{I} \ I_p)^T$ and $\mathbf{u} = \mathbf{V}$, Equations 2.17-2.18 can be coupled, yielding

$$\mathbf{L}^* \delta \dot{\mathbf{x}} + \mathbf{R}^* \delta \mathbf{x} + \mathbf{E}^* \delta \dot{\mathbf{w}} = \mathbf{S}^* \delta \mathbf{u}, \quad (2.19)$$

where,

$$\begin{aligned} \mathbf{L}^* &= \begin{pmatrix} \mathbf{L}_c + \frac{\partial}{\partial \mathbf{I}} \lambda \Big|_0 & \frac{\partial}{\partial I_p} \lambda \Big|_0 \\ \mathbf{M}_{pi} & \mathbf{L}_p \end{pmatrix} \\ \mathbf{R}^* &= \begin{pmatrix} \mathbf{R}_c & 0 \\ 0 & 0 \end{pmatrix} \\ \mathbf{E}^* &= \begin{pmatrix} \frac{\partial}{\partial \mathbf{w}} \lambda \Big|_0 \\ \mathbf{M}_{pw} \end{pmatrix} \\ \mathbf{S}^* &= \begin{pmatrix} \mathbf{S}_c \\ 0 \end{pmatrix}. \end{aligned}$$

By multiplying each term of Equation (2.19) for L^{*-1} the state space form can be derived

$$\begin{cases} \delta \dot{\mathbf{x}} = \mathbf{A} \delta \mathbf{x} + \mathbf{B} \delta \mathbf{u} + \mathbf{E} \delta \dot{\mathbf{w}} \\ \delta \mathbf{y} = \mathbf{C} \delta \mathbf{x} + \mathbf{F} \delta \dot{\mathbf{w}} \end{cases}, \quad (2.20)$$

where $\mathbf{y}$ is the output of the linear system. Usually, its entries are the variable that need to be controlled, such as PF currents, plasma current, gaps, X-point position, *etc.* Furthermore, this linear model can effectively characterize the vertical instability of the plasma. Indeed, for unstable plasmas, the plasma growth rate ($\gamma [s^{-1}]$) is the unstable eigenvalue of matrix $\mathbf{A}$. Note that in (2.20), $\mathbf{B}$ denotes the input matrix describing the relationship between the control input and the state vector, and does not represent the magnetic field. The intended meaning is clear from the context.

The matrices in Equation (2.19) are calculated by using numerical

codes, for instance CREATE-L [3] and CREATE-NL+ [8], which have already been tested on several machines [4, 136, 9, 127].

Though simpler than physics-oriented simulation codes like ASTRA [103], MECS [91] and JINTRAC [108], the engineering-oriented linear model (2.20) is essential not only for model-based control system design, but also for automating both the validation and deployment of axisymmetric magnetic control of the plasma.

Chapter 3

Tokamak magnetic diagnostics and algorithms for real-time control

In the previous chapter, we introduced the fundamental aspects of tokamak plasma modeling, focusing on the physical principles that govern plasma behavior and the key parameters essential for magnetic control. Effective magnetic control relies heavily on the ability to accurately measure and reconstruct the controlled plasma quantities such as current density, current centroid position, and the plasma boundary. These reconstructions are crucial for ensuring the stability and performance of the plasma.

In this chapter, we investigate the methodologies used to achieve precise magnetic reconstruction. We begin by discussing the general principles of magnetic reconstruction and the types of diagnostic inputs required, such as magnetic probes, flux loops, and poloidal field coil currents.

Particular attention is given to the magnetic reconstruction algorithms planned for use in ITER. These algorithms are designed to operate in real-time, coping with the demanding performance and reliability requirements of ITER's control system. A preliminary performance evaluation of these algorithms is done exploiting the magnetic sensors data from JT-60SA IC & OP1 pulses. In addition to these, we introduce three fast boundary reconstruction algorithms that offer simplified yet efficient approaches to estimating the plasma boundary with low computational

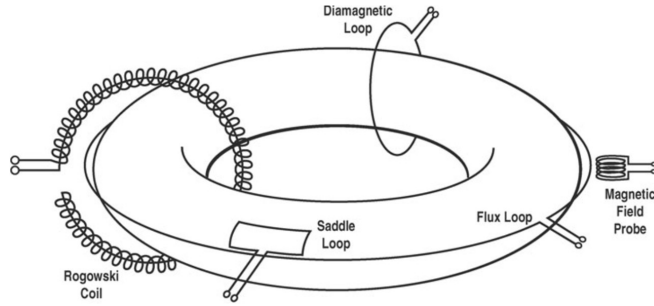


Figure 3.1: Magnetic sensors used for estimation of the controlled plasma parameters.

cost, making them suitable for real-time control applications or initial guesses in more detailed reconstruction routines.

The analysis relative to the performance assessment of the magnetic reconstruction ITER algorithms are also found in [50].

3.1 Magnetic diagnostics relevant for magnetic control

Many controlled variables, as the shape descriptors and the plasma centroid position, cannot be directly measured; they need to be estimated starting from the available measurements. This section briefly summarizes the relevant magnetic sensors for plasma control. Figure 3.1 shows the main magnetic sensors used by magnetic reconstruction algorithms.

Flux loop

Flux loops, as well as all the magnetic sensors that will be described, consist of an open coil whose voltage across the two terminals is measured by means of a differential amplifier. It is a single open loop of wire, making a complete toroidal turn, and crossing the poloidal plane at a point $\mathbf{r}$. According to Faraday's law, this voltage is proportional to the time derivative of the total magnetic flux linked with the coil.

$$V_\psi(t) = -2\pi \frac{\partial}{\partial t} \psi(\mathbf{r}, t). \quad (3.1)$$

By integrating the measured voltage V_ψ , from Equation (3.1)

$$\psi(\mathbf{r}, t) = -\frac{1}{2\pi} \int_{t_0}^t V_\psi(\tau) d\tau + \psi(\mathbf{r}, t_0).$$

Therefore, flux loops, together with an integrator circuit, give the time evolution of the poloidal flux function at point $\mathbf{r}$.

Diamagnetic loop

The diamagnetic loop works in the same way as a flux loop, but it measures the toroidal magnetic flux. In fact, it is installed on a poloidal cross-section to minimize the poloidal field coupling. As it will be shown later in this section, it is essential to estimate the thermal energy of the plasma.

Pick-up coil

A pick-up coil consists of multiple windings of wire of small radius located at a point $\mathbf{r}$. Since the section A is very small, the induced voltage is proportional to the time derivative of the magnetic field passing through the solenoid:

$$V_b = -NA \frac{\partial}{\partial t} \mathbf{B}_\theta(\mathbf{r}, t) \quad (3.2)$$

where N is the number of windings, θ is the angle with respect to the radial axis at which the pick-up coil is mounted. Again, by integrating Equation (3.2), the magnetic field along the direction given by θ is estimated.

Rogowski coil

The direct measurement of the plasma current can be done via the Rogowski coil. The coil is placed around the plasma or the conductor carrying the current. It works by sensing the magnetic field created by the current flowing through the plasma. As the plasma current changes, it generates a corresponding magnetic field, and since the coil is positioned around it, the changing magnetic field induces a voltage in the coil. This induced voltage is directly proportional to the rate at which the current is

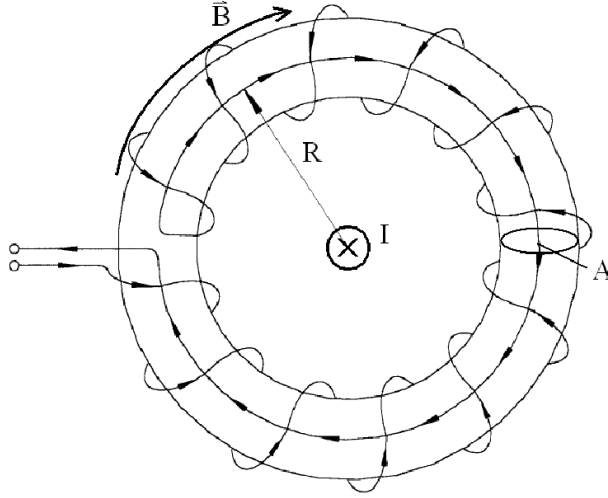


Figure 3.2: Diagram of a back-wound Rogowski coil.

changing over time,

$$V_I = -M \frac{d}{dt} I_p(t),$$

where M is the mutual inductance between the plasma current and the coil. To get the actual current, the signal from the coil needs to be processed by an integrator. In a tokamak, the environment is also filled with a toroidal magnetic field, which can interfere with the measurements, this is why many Rogowski coils used in tokamaks are designed with a back-wound configuration (see Figure 3.2). This means that one part of the coil is folded back, so it essentially cancels out the effects of the toroidal field.

3.2 Overview of ITER Reconstruction algorithms

In this section, we will describe the reconstruction algorithms and codes envisaged for ITER operations. As far as the reconstruction of plasma current and centroid position is concerned, they exploit a Quadratic Programming (QP) optimization problem to compute offline a set of weights that,

multiplied by the vector of magnetic measurements, best predict the quantity of interest. Concerning the plasma boundary reconstruction, we consider the LIUQE reconstruction code [93] that, together with PEFIT [147], is one of the two options currently considered by ITER. Finally, it is analyzed how these algorithms perform using data from the JT-60SA IC & OP1 campaign.

3.2.1 Plasma current reconstruction

Before introducing the proposed optimization-based approach, we recall that a set of weights to compute the plasma current as linear combination of measurements can be easily determined starting from a linear approximation of the poloidal magnetic field line integral along the integration path; these weights depend only on the geometry of the sensors. In fact,

$$\mu_0 I_p = \oint_{\delta S} \mathbf{B}_p \cdot d\mathbf{r} = \tilde{\mathbf{w}}_{I_{p0}}^T \mathbf{m}_{B_t}, \quad (3.3)$$

where $\mathbf{m}_{B_t}$ is the vector of the tangential magnetic field measurements (blue arrows in Figure 3.3), and $\tilde{\mathbf{w}}_{I_{p0}}$ the unknown weights. One can approximate the curve δS with segments connecting the tangential magnet field sensors. Therefore, the line integral in (3.3) can be replaced by its approximation, yielding

$$\begin{aligned} I_p &= \frac{1}{\mu_0} \sum_{i=1}^{n-1} \frac{d_{i,i+1}}{2} (m_{B_{i+1}} - m_{B_i}) \\ &+ \frac{d_{n,1}}{2} (m_{B_n} - m_{B_1}) = \mathbf{w}_{I_{p0}}^T \mathbf{m}_{B_t}, \end{aligned}$$

where n is the number of sensors, $d_{i,i+1}$ is the distance between two consecutive sensors, and $\mathbf{w}_{I_{p0}}$ is a vector of weights.

This method does not take into account that the magnetic measurements also include the contribution of the active and passive currents. To let the estimation of the plasma current be insensitive to active currents, it is possible to set up a QP optimization problem, where the weights $\mathbf{w}_{I_p}$ are the unknowns, whose initial guess is taken equal to $\mathbf{w}_{I_{p0}}$. Neglecting

the effect of the passive currents, it is

$$\mathbf{m}_{B_t} = \mathbf{m}_{B_t}^{I_p} + \mathbf{m}_{B_t}^{I_a} = \mathbf{m}_{B_t}^{I_p} + \mathbf{C}_{I_a} \mathbf{I}_a,$$

where matrix $\mathbf{C}_{I_a} \in \mathbb{R}^{n \times n_a}$ is derived from the Green functions and links the active currents to their contribution in terms of the magnetic field at the measurement points.

Magnetic diagnostics set-up and filaments distribution

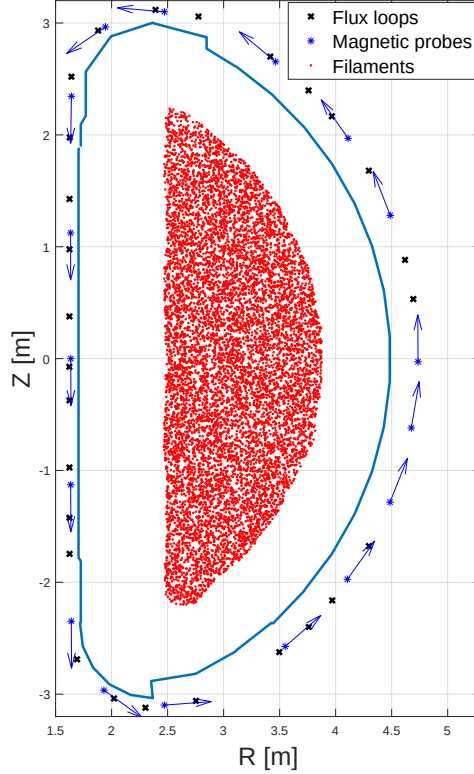


Figure 3.3: Filaments distribution considered in the optimization problems used to compute the weights for the plasma current and centroid position reconstruction.

Let us denote matrix $\mathbf{E} \in \mathbb{R}^{n \times (n-n_a)}$ as $\mathbf{E} = \mathcal{N}(\mathbf{C}_{I_a}^T)$, i.e. the columns of $\mathbf{E}$ are a basis in the null-space of $\mathbf{C}_{I_a}^T$ and consequently $\mathbf{C}_{I_a}^T \mathbf{E} = \mathbf{0}$.

Moreover, let us fill the vacuum chamber with $N \gg n$ fictitious filaments, and solve a QP problem that minimizes the following objective function

$$f_{I_p} = \left\| \mathbf{C}_{I_f}^T \mathbf{E} \mathbf{x}_{I_p} - \mathbf{1} \right\|^2, \quad (3.4)$$

where $\mathbf{C}_{I_f} \in \mathbb{R}^{n \times N}$ has the equivalent meaning of $\mathbf{C}_{I_a}$, $\mathbf{1} \in \mathbb{R}^n$ is the unitary vector, and $\mathbf{x}_{I_p} \in \mathbb{R}^{n-n_a}$ is the actual unknown, then $\mathbf{w}_{I_p} = \mathbf{E} \cdot \mathbf{x}_{I_p}$. The minimization of the objective function (3.4) aims to find a set of weights that best predict the current flowing in each of the fictitious filaments in the chamber and at the same time this estimation is insensitive of the contribution of the active currents $\mathbf{I}_a$. Finally, $\mathbf{w}_{I_{p0}}$ can be used to fix the sign of the weights $\mathbf{w}_{I_p}$ and a regularization term in (3.4) can be added, leading to the following optimization problem

$$\min_x f_{I_p} + \epsilon \left\| \mathbf{E} \mathbf{x}_{I_p} \right\|^2 \quad (3.5)$$

$$\text{s.t. } \text{sgn}(\mathbf{w}_{I_{p0}}) \odot (\mathbf{E} \mathbf{x}_{I_p}) \geq 0, \quad (3.6)$$

where $\odot$ denotes the Hadamard product between vectors. Once this optimization problem has been solved offline, the real-time estimation of the plasma current is given by

$$I_p = \mathbf{w}_{I_p}^T \cdot \mathbf{m}_{B_t}.$$

3.2.2 Current centroid position reconstruction

For the plasma current centroid position, a similar approach can be followed. In fact, the following equations hold¹

$$\begin{aligned} z_p I_p &= \frac{1}{\mu_0} \oint_{\delta S} (f \mathbf{B}_t - g \mathbf{B}_n) dl, \\ (r_p^2 - r_0^2) I_p &= \frac{1}{\mu_0} \oint_{\delta S} (g \mathbf{B}_t - f \mathbf{B}_n + 0.5 g^2 \mathbf{B}_t + f g \mathbf{B}_n) dl, \end{aligned}$$

¹For further details, the interested reader can refer to [80].

where r_0 is a reference radial coordinate, typically taken in the center of the chamber, r_p and z_p are the plasma current centroid radial and vertical position respectively and finally f and g are the coordinates with respect to a reference system centered in r_0 . It is worth noticing that for these estimations normal magnetic field ($\mathbf{B}_n$) sensors are needed. If not available, as in the JT-60SA IC campaign, it is possible to use virtual normal magnetic sensors by combining the measures of the flux loops, $\mathbf{m}_{B_n} = \mathbf{M} \mathbf{m}_\psi$. Let us now define

$$\mathbf{m}_B = \begin{pmatrix} \mathbf{m}_{B_t} \\ \mathbf{m}_{B_n} \end{pmatrix},$$

with similar arguments as before, we can estimate the plasma position as follows

$$z_p = \frac{\mathbf{w}_{z_p}^T \mathbf{m}_B}{\mathbf{w}_{I_p}^T \mathbf{m}_{B_t}},$$

$$r_p = \sqrt{\frac{\mathbf{w}_{r_p}^T \mathbf{m}_B + r_0^2 \mathbf{w}_{I_p}^T \mathbf{m}_{B_t}}{\mathbf{w}_{I_p}^T \mathbf{m}_{B_t}}}.$$

The weights $\mathbf{w}_{z_p}$ and $\mathbf{w}_{r_p}$ can be computed solving a QP optimization problem with a similar function as in Equation (3.4). In particular

$$f_{z_p} = \left\| \bar{\mathbf{C}}_{I_f}^T \mathbf{E} \mathbf{x}_{z_p} - \mathbf{z}_f \right\|^2,$$

$$f_{r_p} = \left\| \bar{\mathbf{C}}_{I_f}^T \mathbf{E} \mathbf{x}_{r_p} - (\mathbf{r}_f^2 - r_0^2 \mathbf{1}) \right\|^2,$$

where $\bar{\mathbf{C}}_{I_f}$ links the currents in the N filaments to the magnetic field at the measurement points. Consequently, $\mathbf{w}_{z_p} = \mathbf{E} \mathbf{x}_{z_p}$ and $\mathbf{w}_{r_p} = \mathbf{E} \mathbf{x}_{r_p}$.

3.2.3 Shape reconstruction

One of the real-time plasma shape reconstruction code envisaged for ITER is LIUQE [93], which was first developed for TCV but can be easily adapted and used for studies on other tokamaks.

LIUQE solves the full Grad-Shafranov equation (2.12) iteratively starting from an initial guess of the plasma current density distribution, which

is expressed as a 1D function depending only on the poloidal flux. The reconstruction procedure aims at finding the coefficients of a polynomial expansion of such profile functions that best fit the available measurements. This computation is based on an inversion of a matrix whose dimensions depend on the number of active currents, free parameters, and the discretization of the vessel. This task is computationally heavy since at every iteration the matrix changes and needs to be recomputed. Therefore, for the real-time implementation, LIUQE makes only one step of the algorithm, achieving a cycle time of less than 200 μ s. This approach works if the sampling frequency is high enough, so that at each step it starts from a similar plasma equilibrium.

3.2.4 Poloidal beta estimation

In literature, several definitions of β_p exist. Let us recall Equation (2.15) which is the β_p definition used by the EFIT code:

$$\beta_p = \frac{\iiint_{V_p} p dV}{\iiint_{V_p} dV \frac{B_{pa}^2}{2\mu_0}}. \quad (3.7)$$

The following simplified formula for β_p estimation can be derived [79]:

$$\beta_p = 1 + \frac{(\kappa^2 + 1)B_{t_0}\Delta\Phi}{2\kappa 20\pi I_p^2}, \quad (3.8)$$

where κ is the plasma elongation, B_{t_0} the vacuum toroidal field at the machine axis, and $\Delta\Phi$ the diamagnetic flux (measured with the diamagnetic loop). Note that this simplified formula does not require the computation of Shafranov's integrals². Moreover, the estimation given by (3.8) assumes elliptic geometry of the magnetic field lines, and it is valid only for sufficiently elongated plasmas.

²More details are reported in [65].

3.3 Validation of ITER reconstruction algorithms using JT-60SA IC & OP1 data

The IC & OP1 of the JT-60SA tokamak have been successfully completed in 2023 by the joint EU-Japan team [113, 99, 117]. JT-60SA is the world's largest operating tokamak and, being fully superconductive, represents the ideal testbed for ITER-relevant tasks, such as the validation of real-time plasma magnetic reconstruction algorithms.

Magnetic diagnostics, together with the related control system, are fundamental components needed since first tokamak operation. Indeed, Mirnov coils, Rogowski coils and flux loops enable the estimation of the magnetic flux and field inside the vacuum vessel, as well as the estimation of many relevant plasma parameters to be controlled in real-time, such as the plasma current, the centroid position and the plasma boundary [10, 48].

However, any estimated plasma parameter is characterized by an accuracy that depends both on the quality of the raw measurements, as well as on the adopted estimation algorithms [33]. Therefore, real-time estimation relies on diagnostic algorithms that should meet the requirements in terms of latency needed by the control system, which typically deals with vertically elongated and therefore unstable plasmas [133]. Consequently, the requirements related to real-time may imply developing computationally efficient algorithms, for which non-iterative solutions are usually preferred, and/or deploying them on high-performance hardware architectures, such as real-time nodes equipped with GPUs [60].

Along these lines, this Chapter presents a preliminary validation of the magnetic diagnostic algorithms that have been designed for ITER [95], by making use of JT-60SA data, also presented in [50]. To this aim, the reconstruction accuracy of such algorithms is evaluated against plasma equilibria obtained with the CREATE-NL nonlinear magnetic equilibrium code [8]. In what follows, the CCS algorithm will be used as a benchmark. It solves the GS equation in the vacuum by assigning the Cauchy conditions on a pre-determined surface, whose boundary is usually an ellipse. The center of this ellipse gives the plasma current centroid position, and it is chosen to maximize the correlation coefficient with respect to the magnetic measurement. For more details about the CCS method, see [78, 73].

3.3.1 Requirements for plasma reconstruction

This section summarizes the requirements for the ITER plasma magnetic diagnostic algorithms that are considered in Section 3.3.3; such requirements have been originally specified in [138]. In particular:

- i) Regarding the plasma shape, the reconstruction error along the gaps should be less than 1 cm. This can be translated to an equivalent overall requirement on the RMSE along the plasma boundary.
- ii) Following the previous point, it can be derived that the reconstruction error on the plasma current centroid vertical and radial position should be less than 1 cm.
- iii) The reconstruction error on the plasma current should be less than 1% if $I_p > 1$ MA, or less than 10 kA otherwise.
- iv) The error on poloidal beta reconstruction should be less than $0.05 + 0.05\%$.
- v) Time resolution poloidal beta estimation during I_p quenches shall be 0.1 ms.

It is worth noticing that the choice of the poloidal beta estimation given by (3.8), although less accurate if compared to the ones that make use of Shafranov's integrals, permits to guarantee the time resolution required during I_p quenches, as specified by requirement v). Indeed, equilibrium codes struggle to follow plasma evolution during these phenomena. It is worth to remark that the time resolution needed for real-time control of β_p during the discharge is usually less demanding (see also [54]); however, in this section we focus on the ITER requirement for β_p reconstruction during I_p quenches.

3.3.2 Validation setup

As anticipated before, the considered reconstruction algorithms are tested against the data given by CREATE-NL, which is a free-boundary magnetic equilibrium code that iteratively solves the GS equation. It allows to compute an *ideal* value for all the plasma parameters of interest [8], as well as a complete magnetic diagnostic data set [60, 61]. A

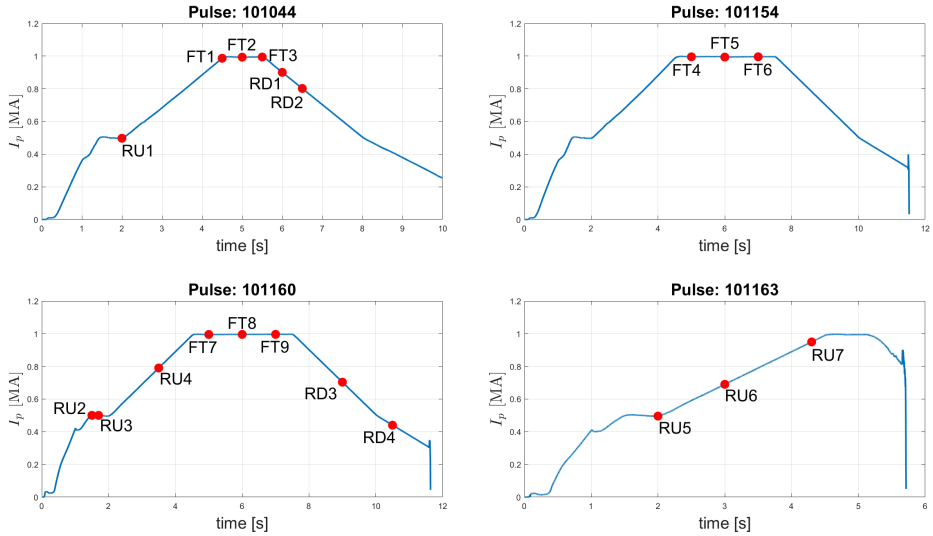


Figure 3.4: Plasma equilibria included in the considered analysis. For each pulse, the red dots represent the time instants for which a CREATE-NL equilibrium has been computed.

set of plasma equilibria have been generated with CREATE-NL, starting from the JT-60SA magnetic measurements at given time instants, and for selected pulses. The corresponding plasma centroid positions, currents, and shapes have been taken as ground truth for performance evaluation. However, since CREATE-NL slightly varies the magnetic measurements to converge, these slightly varied measurements have been used as input to the considered reconstruction codes in order to perform the correct comparison with ideal values.

The considered validation has been performed by considering a set of 20 plasma equilibria has been generated starting from the experimental data of four different JT-60SA pulses. The time instants at which the plasma equilibria are computed have been chosen during the various phases of the discharges, and are reported as red dots in Figure 3.4. In particular: 9 equilibria are computed during the FT phase, 7 during the RU, while 4 during the RD.

For the proposed poloidal beta estimation algorithm, a comparison with the estimation computed by the CCS is presented in Section 3.3.3. It

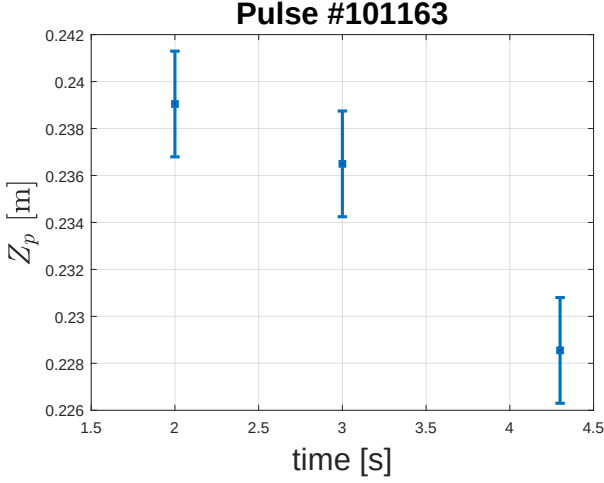


Figure 3.5: Given the three equilibria taken from pulse #101163, the 2σ error bands on the plasma vertical position are reported. It can be observed that the precision of the algorithms even in the presence of Gaussian noise is very high.

is important to note that the reconstruction of β_p made by CCS is obtained with the computation of the Shafranov's integrals.

With respect to the magnetic diagnostics, the effect of measurement noise has been neglected after an accurate analysis. Firstly, the characterization of the noise has been considered as in [63], therefore as a Gaussian noise with zero mean and $\sigma_B = 0.033$ mT for the magnetic field sensors and $\sigma_\psi = 0.17$ mWb for the flux loops. Considering a thousand realizations of the noise for each equilibrium, the resulting standard deviation in the output of the algorithms is very low. For example, as reported in Figure 3.5, the resulting standard deviation on the reconstructed plasma current centroid vertical position Z_p computed with the experimental data from pulse #101163 is about 1 mm and would not affect the outcome of the analysis if considered.

In conclusion, it is worth noting that during the JT-60SA IC, only a limited number of sensors were available: 17 tangential magnetic field sensors and 27 flux loops, as shown in Figure 3.3.

3.3.3 Reconstruction results

The results obtained with the ITER reconstruction algorithms given the data set introduced in Section 3.3.2 are discussed in what follows. For each quantity, the error with respect to the equilibrium and the corresponding ITER requirement are highlighted.

In Figure 3.6 the errors of the plasma current reconstruction are reported. The accuracy of the proposed ITER magnetic diagnostic algorithm is very high. It always fits within the requirements in the considered cases. As a benchmark, a very good accuracy in reconstructing the plasma current, shape and current centroid vertical position is obtained also with the CCS code, which is used at JT-60SA to estimate in real-time the plasma quantities, and that includes a model for the eddy current evolution. Figure 3.7 reports the obtained accuracy when plasma current is reconstructed with CCS for some of the considered equilibria.

It is worth noticing that normal field sensors are not needed to compute the estimate of the plasma current. On the other hand, for the plasma position and shape descriptors, the availability of such additional measurements is essential. Indeed, except for the vertical plasma position (which does not reach the desired accuracy only for equilibrium RD4), the proposed algorithms fail to meet ITER requirements (see Figure 3.8).

Note that, as anticipated in Section 3.2.3, the iterative version of LIUQE has been used to obtain these reconstructions; in particular, the code converged in about 15 iterations. Hence, the obtained performance represents a best case with respect to the one expected in real-time. Figure 3.9 shows the worst case for boundary reconstruction, among the considered equilibria, which corresponds to RU2. In this case LIUQE reconstructs a plasma boundary that mainly differs from the equilibrium in the lower part of the vacuum chamber.

Also for the plasma position and shape descriptors, the CCS benchmark reported in Figure 3.10 confirms the results obtained with the ITER reconstruction algorithms. In particular, for the considered flat-top equilibria, the vertical position is estimated with very good accuracy, as well as the overall plasma shape. On the other hand, the CCS accuracy in estimating the centroid radial position decreases as soon as the plasma internal inductance l_i decreases. This is expected, since CCS computes the plasma centroid position assuming a single filament current source (see [73, Sec. 4]

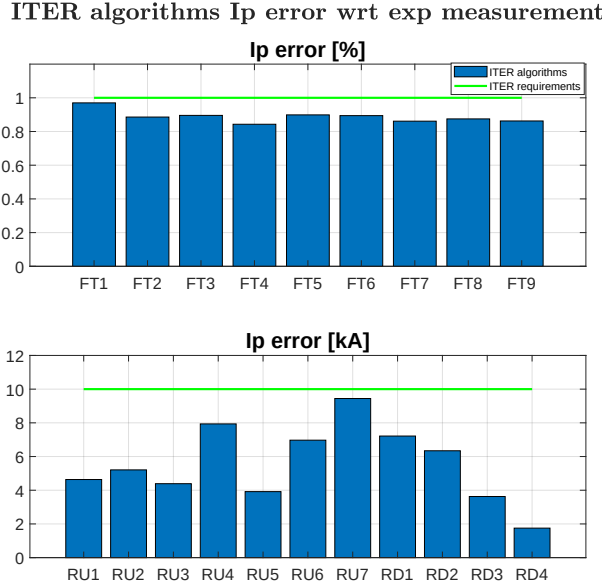


Figure 3.6: Plasma current reconstruction error with respect to the corresponding criteria.

for more details). Therefore, as soon as the current distribution broadens, the CCS accuracy naturally worsens. On the other hand, the ITER reconstruction algorithm solves an optimization problem that chooses the weights that minimize the position estimation error for all the filaments placed in the area covered by the distribution shown in Figure 3.3. Consequently, it turns out to be less sensitive to the shape of the current profile, given that the actual plasma current centroid position stays in the same area as the fictitious filaments. Hence, although it does not meet the requirements, it shows a better performance.

Concerning the reconstruction of β_p , Figure 3.11 shows the estimate obtained by the proposed approach and the one given by the CCS code. The estimated quantities are not very different in absolute value and have similar behaviors, even though Equation 3.8 is a simplified formula. Finally, in Figure 3.12 and Figure 3.13, the estimation errors are reported. As expected, the estimation errors of the β_p made by Equation 3.8 are higher compared to those made by CCS. In fact, CCS derives β_p by ex-



Figure 3.7: CCS plasma current reconstruction error with respect to the corresponding criteria.

actually computing Shafranov’s integrals. However, although the errors are significant, the benefits of applying Equation 3.8 lie in its simplicity and fast execution time, which allows the strict temporal requirements during plasma current quenches to be met.

3.4 Conventional approaches for plasma shape identification with magnetic measurements

As mentioned in the previous chapter, the real-time full-equilibrium codes are computationally demanding and require resources that may not be available. This is why, for shape control purposes, non-iterative boundary reconstruction codes are still relevant for tokamak operations. In this section, two conventional algorithms for shape identification are described:

- A filamentary reconstruction code that approximates the plasma column as a set of filaments carrying currents.

ITER algorithms shape and position error wrt CREATE-NL equilibria

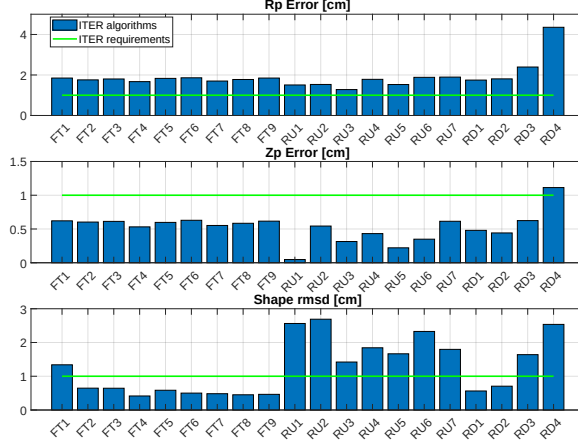


Figure 3.8: ITER algorithms plasma shape and position reconstruction error with respect to the corresponding criteria.

- XLOC_DTT, a modified version of the XLOC code which has been used at JET for several years [109].

3.4.1 Filamentary reconstruction

In this section, we will briefly describe a simplified filamentary reconstruction code. The starting point of the algorithm is to place a number of filaments $n_f \ll n$ inside the chamber on an ellipse around the center of the chamber (as in Figure 3.14). Then, neglecting again the passive currents and making use again of the Green's functions,

$$\mathbf{m} = \begin{pmatrix} \mathbf{m}_B \\ \mathbf{m}_\psi \end{pmatrix} = \tilde{\mathbf{C}}_{\mathbf{I}_f} \cdot \mathbf{I}_f + \tilde{\mathbf{C}}_{\mathbf{I}_a} \cdot \mathbf{I}_a, \quad (3.9)$$

where matrix $\tilde{\mathbf{C}}_{\mathbf{I}_f}$ ($\tilde{\mathbf{C}}_{\mathbf{I}_a}$) links the currents in the n_f (n_a) filaments to vector $\mathbf{m}$. By slightly modifying Equation (3.9) in

$$\begin{pmatrix} \mathbf{m} \\ \bar{\mathbf{I}}_a \\ \mathbf{0} \end{pmatrix} = \Theta^{-1} \begin{pmatrix} \tilde{\mathbf{C}}_{\mathbf{I}_a} & \tilde{\mathbf{C}}_{\mathbf{I}_f} \\ \mathbf{I}_{n_a} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{n_a} \end{pmatrix} \begin{pmatrix} \mathbf{I}_a \\ \mathbf{I}_f \end{pmatrix}, \quad (3.10)$$

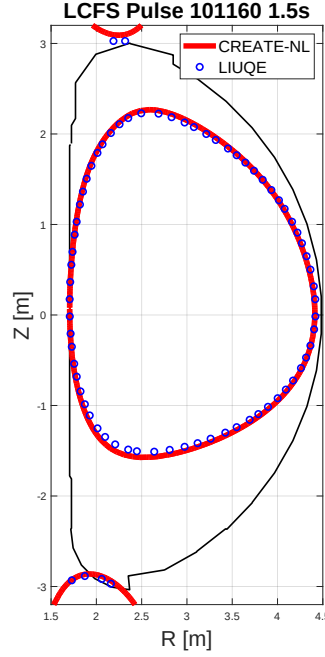


Figure 3.9: LIUQE plasma boundary reconstruction compared to CREATE-NL.

where Θ is a weighting matrix based on the measurement error standard deviations, $\bar{\mathbf{I}}_a$ are the measured currents, and $\mathbf{0}$ zero matrix of appropriate dimensions. From Equation (3.10) $\mathbf{I}_f$ and $\mathbf{I}_a$ can be easily derived. Then, the flux map can be reconstructed by summing at all points of a predefined grid, all the sources' (represented by $\mathbf{I}_f$ and $\mathbf{I}_a$) contributions to the poloidal flux. It is worth noticing that the position of the filaments can be changed during the execution of the algorithm to cope with the changes in the plasma position. Moreover, from this method, one can compute the

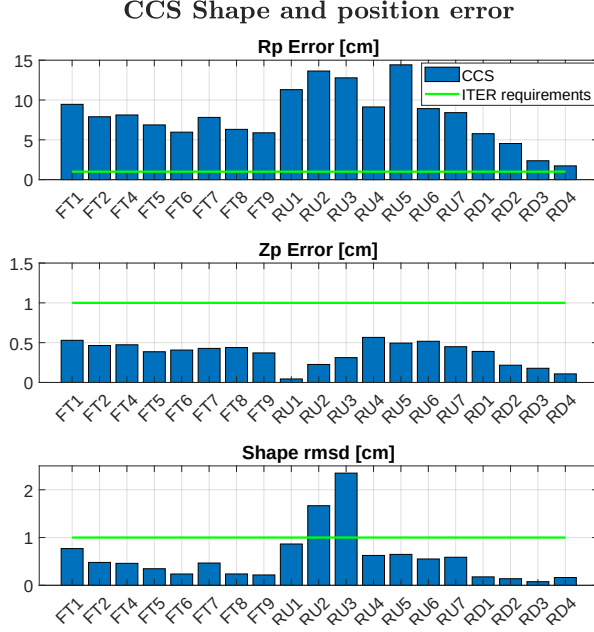


Figure 3.10: CCS plasma shape and position reconstruction error with respect to the corresponding criteria.

plasma current as $I_p = \sum_{i=1}^{n_f} I_{f_i}$ and the current centroid position as

$$z_p = \frac{\sum_{i=1}^{n_f} z_{f_i} I_{f_i}}{\sum_{i=1}^{n_f} I_{f_i}}, \quad (3.11)$$

$$r_p = \sqrt{\frac{\sum_{i=1}^{n_f} r_{f_i}^2 I_{f_i}}{\sum_{i=1}^{n_f} I_{f_i}}}. \quad (3.12)$$

3.4.2 XLOC_DTT

The XLOC_DTT code computes the poloidal flux function from the magnetic measurement by satisfying the GS equation in the vacuum region. Similarly to XLOC, the starting point is to choose a basis of functions that solve (2.11). Polynomial solutions to the homogeneous GS equation [27]

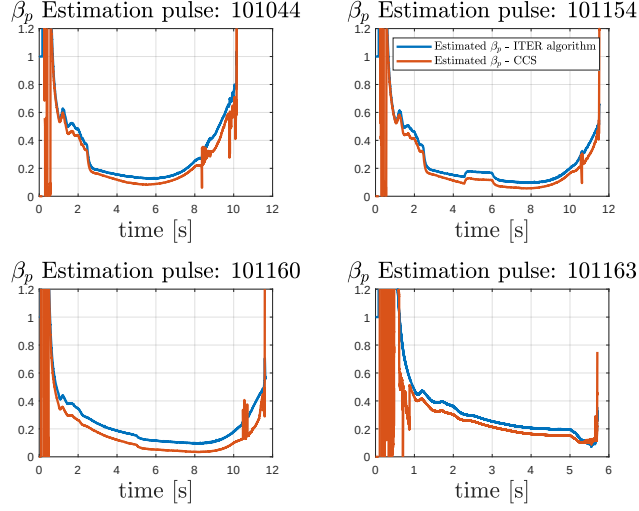


Figure 3.11: Estimation of β_p by the ITER algorithm and by CCS.

are:

$$\chi_n(r, z) = \begin{cases} 1 & n = 1 \\ \sum_{k=0}^{\lfloor n/2 \rfloor - 1} (-4)^{-k} \frac{(n-1)!}{2k!(k+1)!(n-2k-2)!} r^{2k+2} z^{n-2k-2} & n \geq 2 \end{cases}, \quad (3.13)$$

where r and z are the coordinates in the poloidal plane. Let

$$\boldsymbol{\chi} = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \vdots \\ \chi_{\bar{N}} \end{pmatrix}$$

be the chosen vector of basis functions, then the poloidal flux function can be approximated by using a linear combination of the elements of $\boldsymbol{\chi}$:

$$\psi(r, z) = \boldsymbol{\chi}^T(r, z) \cdot \bar{\mathbf{a}}, \quad (3.14)$$

where $\bar{\mathbf{a}} \in \mathbb{R}^{\bar{N}}$ is the vector of coefficients to be determined. Since one polynomial does not fit the whole flux distribution outside the plasma well enough, more polynomials are used, each one valid in a different region of

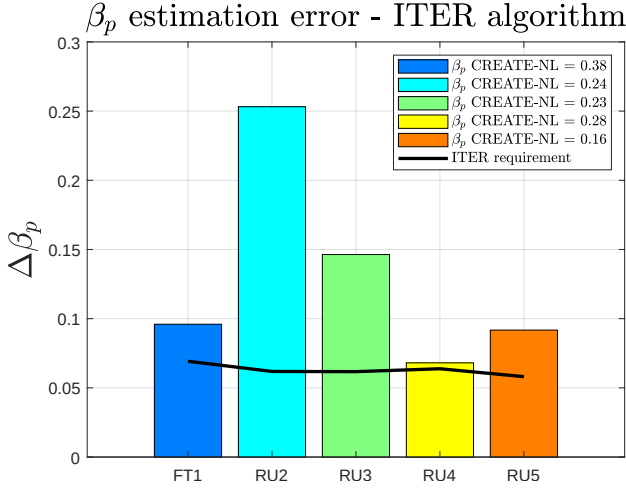


Figure 3.12: ITER algorithm β_p estimation error with respect to equilibrium value computed by CREATE-NL. The β_p equilibrium values are reported in the legend.

the chamber. Note that the number of basis functions to be used may vary by region. For instance, if the chamber is divided into K regions, the total number of coefficients to be computed is $N = \sum_{i=1}^K \bar{N}_i$. Then, exploiting Equations (2.7), also the magnetic sensors can also be used to determine the coefficients vector $\mathbf{a}$. Thus, Equation (3.14) can be reworked as

$$\mathbf{M} \cdot \mathbf{a} = \mathbf{b}, \quad (3.15)$$

where $\mathbf{a} \in \mathbb{R}^N$, $\mathbf{b} \in \mathbb{R}^n$ is the vector of the measurements of the poloidal flux and the magnetic field, and $\mathbf{M} \in \mathbb{R}^{n \times N}$ is the matrix of the basis functions and their derivatives computed offline in the points where the sensors are located.

Equation (3.15) alone does not guarantee the continuity of the poloidal flux function at the boundary of the regions. This is why a point on each segment dividing the regions is selected. In this point, the constraint is added that the flux computed with two polynomials of adjacent regions is equal:

$$\chi_i^T(r_p, z_p) \cdot \bar{\mathbf{a}}_i = \chi_{i+1}^T(r_p, z_p) \cdot \bar{\mathbf{a}}_{i+1}. \quad (3.16)$$

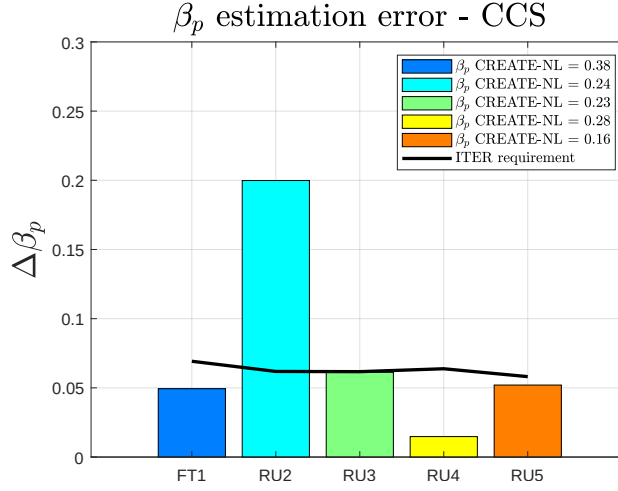


Figure 3.13: CCS β_p estimation error with respect to equilibrium value computed by CREATE-NL. The β_p equilibrium values are reported in the legend.

While Equation (3.16) is used to provide hard constraints, some additional points on each segment can be chosen, and continuity of the flux function in these points can be used as soft constraints. These constraints can be described as a set of linear equations as well, therefore, the unknown coefficients can be evaluated by standard least square methods.

3.5 CARONTE: fast Neural Network-based plasma boundary reconstruction

In this section, CARONTE (Create Ai RecONsTructor of plasma surfacE), a boundary reconstruction algorithm based on a particular structure of Neural Networks (NN), namely the Extreme Learning Machines (ELMs), is proposed. Neural networks are non-linear approximators and have great generalization capabilities, and are widely used in literature for transport simulation or full-equilibrium solvers [34, 141, 135, 69]. Most of these approaches are based on a multi-layer approach that requires long training times. This section presents CARONTE, and it is also one of the main

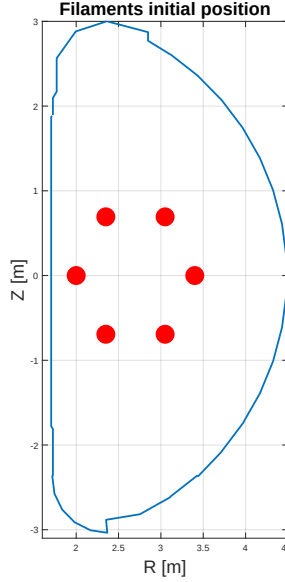


Figure 3.14: Filaments position for flux map reconstruction.

contributions of this thesis.

3.5.1 Extreme Learning Machines

The basic architecture of an ELM network is reported in Fig. 3.15.

ELM training differs from traditional gradient-based learning methods because of the random initialization of the weights between the input and the hidden layer, as well as the biases of the hidden neurons. These parameters remain unchanged during training. The use of nonlinear activation functions in the hidden layer introduces the necessary nonlinearity into the network. With these fixed random values, the learning process reduces to solving a linear system. In fact, only the output weights (between the hidden and output layers) are learned. Hence, in case of a single output network, as the one proposed in this paper to reconstruct the poloidal flux $\psi(r, z)$ (see Section 3.5.2), the ELM architecture can be expressed by:

$$\mathcal{F}(\mathbf{x}, \boldsymbol{\alpha}) = \boldsymbol{\alpha}^T \cdot \Lambda(\mathbf{W} \cdot \mathbf{x} + \mathbf{b}),$$

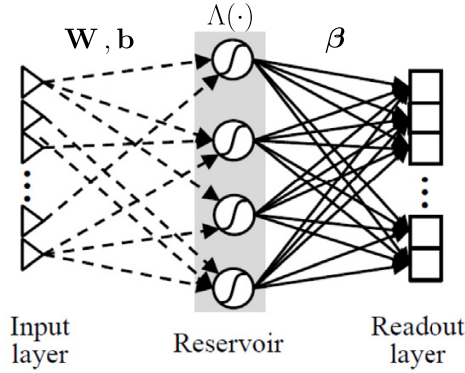


Figure 3.15: The basic ELM is made up of the reservoir and the output layer. The reservoir consists of nonlinear neurons randomly connected to the network input, while the output layer is connected to the nonlinear neurons through the weights to be optimized during the network’s training.

where $\mathbf{W} \in \mathbb{R}^{\omega \times h}$ and $\mathbf{b} \in \mathbb{R}^{\omega}$ are randomly initialized, being h and ω the number of network’s inputs and neurons, respectively. The vector $\mathbf{x} \in \mathbb{R}^h$ is the input vector, while $\boldsymbol{\alpha} \in \mathbb{R}^{\omega}$ is the vector of unknown output weights. Note that in general, depending on the network output, $\boldsymbol{\alpha}$ could be a two-dimensional matrix. The function $\Lambda(\cdot)$ is the activation function, and can be chosen as any of the commonly used activation functions (**tanh**, **sigmoid**, **logsig**, etc.), as long as they are twice differentiable in order to set the physics-informed constraints corresponding to (2.7) and (2.11).

As preliminary discussed before, the ELM network trains much faster than conventional NNs, since it avoids iterative optimization. Despite the randomness, it retains the universal approximation capability, and theoretical analyses suggest that it has a higher potential to reach a globally optimal solution, if compared to traditional methods that train all parameters. These networks have been widely used for clustering, object recognition, and feature learning. Recently, some studies also showed how to use ELMs to solve Partial Differential Equations with applications to Lyapunov’s stability [148]. Therefore, here it is proposed to use an ELM to solve the homogeneous GS equation, which gives the poloidal flux function in the vacuum.

3.5.2 Plasma boundary reconstruction via Physics-Informed ELMs

Let $\psi_{ELM}(\mathbf{x}, \boldsymbol{\alpha})$ be the physics-informed ELM network used for the boundary reconstruction. For this application, the input vector consists of the coordinates of a generic point in the poloidal plane, i.e., $\mathbf{x} = (r \ z)^T$, while the output is a scalar value corresponding to the poloidal flux at $\mathbf{x}$. Given the definition of the Δ^* operator reported in Eq. (2.9), by applying it to $\psi_{ELM}(\mathbf{x}, \boldsymbol{\alpha})$, the homogeneous GS equation is linear with respect to the parameters vector $\boldsymbol{\alpha}$, yielding

$$\boldsymbol{\alpha}^T \cdot D_{GS}(\mathbf{x}) = 0, \quad (3.17)$$

where $D_{GS}(\mathbf{x}) \in \mathbb{R}^m$ represents $\Delta^* \Lambda(W\mathbf{x} + \mathbf{b})$. Therefore, once the required derivatives have been computed and enough points in the vacuum chamber are chosen, exploiting Eq. (3.17), a homogeneous linear system can be put in place to satisfy the homogeneous GS equation. Considering also the sensor readings, the physics-informed ELM can be trained to reconstruct the poloidal flux function by minimizing the following loss function:

$$\begin{aligned} \text{Loss}(\boldsymbol{\alpha}) = & \frac{1}{N} \sum_{i=1}^N \lambda_0 \cdot \|\boldsymbol{\alpha}^T \cdot D_{GS}(\mathbf{x}_i)\|^2 + \\ & \lambda_1 \cdot \|\boldsymbol{\alpha}^T \cdot \Lambda(\mathbf{W} \cdot \mathbf{x}_\psi + \mathbf{b}) - \mathbf{m}_\psi\|^2 + \lambda_2 \cdot \|\boldsymbol{\alpha}^T \cdot D_{\mathbf{B}}(\mathbf{x}_{\mathbf{B}}) - \mathbf{m}_{\mathbf{B}}\|^2, \end{aligned} \quad (3.18)$$

where:

- $D_{\mathbf{B}}(\mathbf{x}_{\mathbf{B}}) \in \mathbb{R}^{\omega \times n_{\mathbf{B}}}$ takes into account the derivatives (2.7) evaluated at magnetic field sensors points $\mathbf{x}_{\mathbf{B}}$;
 - $\mathbf{m}_\psi$ is the vector of poloidal flux measures at points $\mathbf{x}_\psi$;
 - $\mathbf{m}_{\mathbf{B}}$ is the vector of poloidal magnetic field measures at points $\mathbf{x}_{\mathbf{B}}$;
 - $\mathbf{x}_i \in \mathbb{R}^g$ are the collocation points placed in the poloidal plane to satisfy Eq. (3.17);
 - λ_i , with $i = 0, 1, 2$, are weights for the different components of the loss function (3.18).
-

Since (3.18) is a quadratic function of α , the unknown set of weights can be determined by standard least squares methods. Hence, once the optimal vector α^* is computed, $\psi_{ELM}(\mathbf{x}, \alpha^*)$ represents an approximation of the poloidal flux function and can be used to identify the plasma boundary. Due to its simplicity, the neural network can be trained every time new sensor data is available, making it suitable for real-time implementation. It follows that, given the loss function (3.18), the proposed physics-informed ELM network approximates the solution of the GS PDE.

The solution of the homogeneous GS equation can also be imposed as a hard constraint by choosing α in the null-space of $D_{GS}(\mathbf{X}_c)^T$, being $\mathbf{X}_c \in \mathbb{R}^{N \times 2}$ the matrix formed by the N collocation points $\mathbf{x}_i$. Let $E \in \mathbb{R}^{\omega \times (\omega - \text{rank}(D_{GS}(\mathbf{X}_c)))}$ be a matrix such that

$$\begin{cases} D_{GS}(\mathbf{X}_c)^T E = 0 \\ E^T E = I \end{cases},$$

therefore, it represents an orthonormal basis in the null-space of $D_{GS}(\mathbf{X}_c)^T$. By choosing $\alpha = E\tilde{\alpha}$, then Eq. (3.17) is automatically satisfied at points $\mathbf{x}_i$. As a consequence, the choice of the collocation points becomes crucial. In fact, to have a good reconstruction of the plasma shape, they need to be placed in the vacuum vessel where Eq. (2.11) is always satisfied. One possibility is to place the points $\mathbf{x}_i$ along the first wall (or in its near proximity).

Note that $\tilde{\alpha} \in \mathbb{R}^{(\omega - \text{rank}(D_{GS}(\mathbf{x}_c)))}$, reducing the number of free parameters to be optimized. On the other hand, the loss function to be minimized with the training needs now to take into account only the flux and magnetic field measurements:

$$\begin{aligned} \text{Loss}(\alpha) = \frac{1}{N} \sum_{i=1}^N \lambda_1 \cdot |\tilde{\alpha}^T \cdot E^T \cdot \Lambda(\mathbf{W} \cdot \mathbf{x}_\psi + \mathbf{b}) - \mathbf{m}_\psi|^2 + \\ + \lambda_2 \cdot \|\tilde{\alpha}^T \cdot E^T \cdot D_B(\mathbf{x}_B) - \mathbf{m}_B\|^2. \end{aligned} \quad (3.19)$$

Finally, in order to properly identify the neural network weights, the number of degrees of freedom $\omega - \text{rank}(D_{GS}(\mathbf{x}_c))$ needs to be less than or equal to the number of available measurements n_m .

As anticipated in the Introduction, the main advantage of this method

over the others presented in literature lies in its simplicity. The nonlinearities introduced by the NN activation functions allows to estimate accurately the whole poloidal flux function, without using local approximations, as the XLOC algorithm. Specifically, the minimization of the loss function (3.19) requires only single matrix pseudo-inversion that is performed offline, while in real-time a single matrix multiplication is sufficient at each sampling period to reconstruct the flux map in the vacuum region. An estimation of the poloidal magnetic field in the vacuum can be given by exploiting Eq. (2.7).

In contrast, other approaches typically involve either locally approximating the poloidal flux function and then putting together the results, or identifying current sources and summing their contributions. With the latter methods, accounting for the eddy currents is a challenging operation, indeed, current sources need to be placed inside the whole vessel. On the other hand, the proposed PINN-based method, similarly to the XLOC_DTT algorithm, directly approximates the poloidal flux function using a set of basis functions. Therefore, the passive currents' contribution is directly taken into account.

Moreover, many of these methods depend on a fixed grid of points. Instead, the proposed approach allows for the evaluation of the poloidal flux function at arbitrary points once the parameters have been identified, giving a continuous approximation of the poloidal flux function. Finally, Fig. 3.16 shows the workflow of CARONTE. Once the hyper-parameters of the physics-informed ELM network are set, i.e. the number of neurons and the corresponding activation functions, as well as the randomly chosen input weights and biases, the CARONTE algorithm consists into the following steps:

- i) offline computation of the matrices that allows to compute the optimal output weights given the actual value of the magnetic measurements;
 - ii) the online training of the network output weights at each sampling time;
 - iii) the computation of the flux at the plasma boundary;
 - iv) the computation of the flux at the desired control points, by evalu-
-

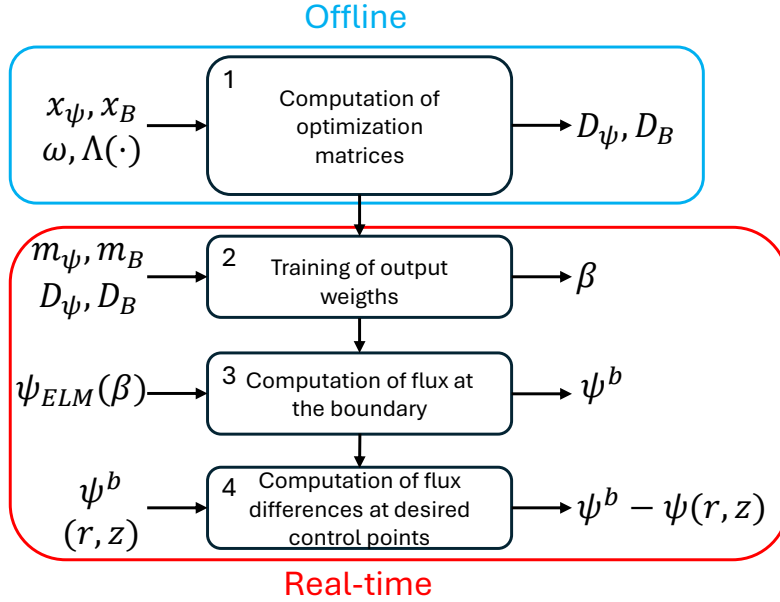


Figure 3.16: Simplified scheme of the CARONTE algorithm workflow.

ating the trained ELM NN at the corresponding coordinates.

Note that steps ii) and iv) are simple matrix multiplications, and the evaluation of the estimated flux at a given point can be done straightforwardly in one single operation. On the other hand, Step iii) is required by any reconstruction algorithm used for control purposes.

3.5.3 The JT-60SA case study: reduced sensors scenario

As previously anticipated and shown in Section 3.3.3, the JT-60SA IC & OP1 campaign, due to the few magnetic sensors available, represents a challenging case from the magnetic reconstruction point of view. The scope of this section is again to test the performance of a magnetic reconstruction algorithm in this challenging environment. In particular, making use of the plasma equilibria shown in Figure 3.4, the boundary reconstruction algorithm presented in Section 3.5.2 is tested using the data from the JT-60SA IC & OP1 campaign. The magnetic setup is again the

Physics-Informed ELMs boundary reconstruction

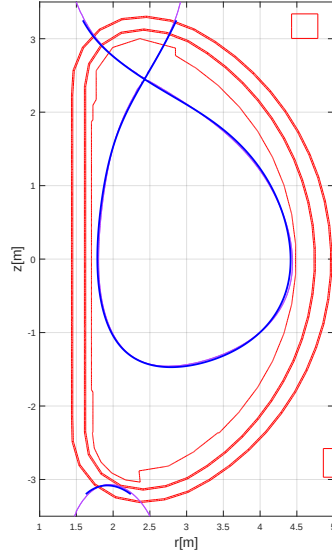


Figure 3.17: Plasma boundary reconstruction of equilibrium RU1 using the Physics-Informed ELM.

one shown in Figure 3.3, while regarding the ELM, the activation function is the logistic sigmoid

$$\Lambda(x) = \frac{1}{1 + e^{-x}},$$

and the total number of neurons is 80. However, the homogeneous GS equation is imposed on 39 points along the first wall; hence, the actual number of weights to be determined is 41. An example of reconstruction using this method is shown in Figure 3.17. The results of the reconstruction of all the equilibria are shown in Figure 3.18. The overall RMSD remains mostly under the 1.5 cm range, which is less performant with respect the other reconstruction methods analyzed in Section 3.3.3. However, considering that:

- the analysis performed on the overall plasma shape represents a worst-case scenario, as it assumes a high number of plasma-wall gaps, not all of which are perpendicular to the plasma boundary;

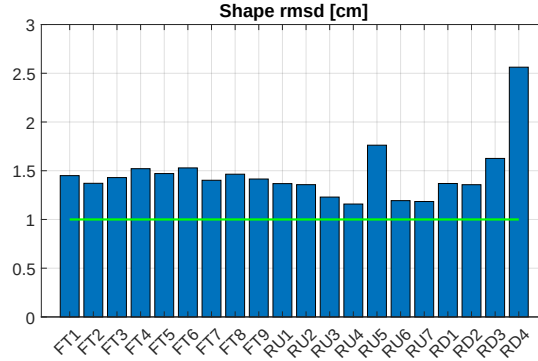


Figure 3.18: Plasma boundary reconstruction of equilibrium RU1 using the Physics-Informed ELM.

- unlike LIUQE, which is an equilibrium reconstruction code, the proposed algorithm focuses solely on boundary reconstruction and is therefore less accurate;
- the limited number of sensors constrains the size of the network, leaving it with relatively few parameters to identify the entire boundary with a single function,

and taking into account the advantages discussed at the end of the previous section, the results obtained so far are promising.

The following section provides a more detailed analysis based on the DTT tokamak, where the number of available measures is significantly increased. In this scenario, not only is the RMSD evaluated over the entire plasma shape, but the effectiveness of the algorithm is also demonstrated using gap configurations suitable for magnetic control. Additionally, the algorithm is tested under non-negligible sensor noise conditions. Finally, in Chapter 5, it will also be used on RFX-mod2 and integrated within a control loop.

3.5.4 The DTT tokamak case study

This test case deals with the advanced tokamak scenario of the DTT device, in which the full set of magnetic diagnostics is available. Similarly to RFX-mod2, DTT is highly flexible and capable of sustaining single null

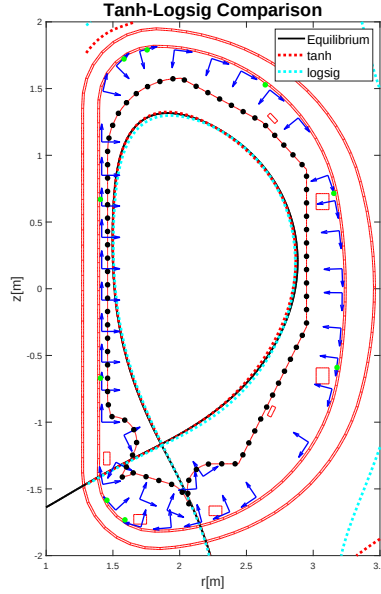


Figure 3.19: Comparison between reconstruction using `tanh` (red) or `logsig` (cyan) functions. In blue the magnetic field sensors and the green dots the flux loops. The black dots represent the points at which the homogeneous GS condition is imposed.

plasma equilibria with both positive (SN) and negative triangularity (SN-NT), as well as X-Divertor (XD) configurations featuring two X-points. This flexibility motivates the need for a fast and robust algorithm capable of accurately identifying the LCFS. Moreover, given the high energies involved, precise reconstruction of the plasma boundary is even more critical. Indeed, as reported in [50], the accepted accuracy for these large-scale machines is 1 cm, which represents a strict requirement. At the end of this section, we will demonstrate that our method satisfies this criterion.

First, using a SN equilibrium, we show that the `logsig` activation function, combined with the a random distribution for the input weights and biases, yields an acceptable reconstruction of the flux map. The number of

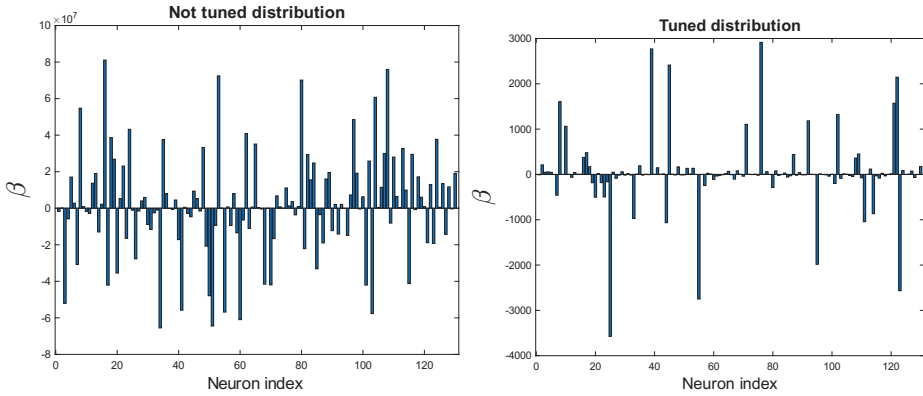


Figure 3.20: On the left the resulting output weights α of the physics-informed ELM network, when the input weights and biases are randomly generated gaussian distributions centered in the origin and with amplitude equal to 1. In this case, the order of magnitude for the elements of β is 10^7 , which is due to matrices that are ill-conditioned. By choosing the random distribution for the input weights and biases as described in Section 3.5.4, the maximum value for the β components is in order of 10^3 , due to a better conditioning of the optimization matrices.

Table 3.1: Results of plasma boundary reconstruction of DTT plasmas carried out with the XLOC_DTT algorithm. This table reports the error on the gap values with respect to the corresponding equilibrium computed with the CREATE codes. The gaps used for this evaluation are shown in green in Figure 3.21. All the values reported here are expressed in cm.

	XLOC_DTT					
	SN		SN-NT		XD	
Gaps	μ_e	σ_e	μ_e	σ_e	μ_e	σ_e
1	0.08	0.10	-0.37	0.20	0.96	0.30
2	-0.01	0.09	-0.03	0.40	-0.70	0.83
3	-0.37	0.63	0.13	0.50	-0.20	0.73
4	-0.50	0.74	0.62	1.27	0.17	1.07
5	-0.02	1.33	0.72	1.03	1.29	1.03
6	0.75	0.56	0.75	0.81	0.14	0.48
RMSD	0.96	0.28	1.42	0.52	1.63	0.44

Table 3.2: Results of plasma boundary reconstruction of DTT plasmas carried out with CARONTE code. This table reports the error on the gap values with respect to the corresponding equilibrium computed with the CREATE codes. The gaps used for this evaluation are shown in green in Figure 3.21. All the values reported here are expressed in cm.

	CARONTE					
	SN		SN-NT		XD	
Gaps	μ_e	σ_e	μ_e	σ_e	μ_e	σ_e
1	0.05	0.12	-0.15	0.19	-0.47	0.33
2	-0.46	0.11	-0.53	0.18	-0.12	1.15
3	-0.18	0.24	0.59	0.17	0.15	0.27
4	-0.18	0.15	-0.93	0.28	-0.25	0.12
5	-0.08	0.28	0.07	0.24	0.14	0.18
6	-0.05	0.08	0.02	0.13	0.02	0.07
RMSD	0.71	0.15	0.55	0.09	0.82	0.19

neurons used in this case is set equal to $\omega = 130$. Indeed, referring to the magnetic sensor layout of DTT reported in Fig. 3.19, 40 biaxial magnetic field sensors returning 80 measurements of both the tangential and normal

field components, and 9 flux loops are available. Along the first wall, 59 points are selected to impose the GS equation in the vacuum region, resulting in a total of 71 degrees of freedom for the physics informed ELM network.

With this setup, the network manages to reconstruct the LCFS. However, the resulting RMSD, computed as described in the previous section, is 1.6 cm, which is close to the 1 cm requirement. Therefore, a further improvement is needed, and can be obtained by simply changing the activation function, while keeping all other network's hyper-parameters unchanged, even the random ones. Hence by choosing

$$\Lambda(x) = \tanh(x),$$

the RMSD decreases to 0.6 cm, fully satisfying the accuracy requirement. The two reconstructed shapes are shown in Fig. 3.19. Furthermore, since $\Lambda(x) \in (-1, 1)$, each neuron carries higher information content, allowing for a reduction in the number of degrees of freedom required by the ELM. Indeed, comparable performance is also achieved with only 48 degrees of freedom, hence increasing the points for the homogeneous GS hard-constraints up to 82. The location of these 82 points is reported in Fig. 3.19.

The conditioning of the $D_B(\mathbf{x}_B)$ and $\Lambda(\mathbf{W}\mathbf{x}_\psi + \mathbf{b})$ matrices strongly depend on the parameters $\mathbf{W}$ and $\mathbf{b}$. If these two matrices are not well-conditioned, the resulting α weights can assume values in a very large range, which may also affect the robustness against noise. If the centers of the activation functions are better scattered on the poloidal plane, the entries of these matrices will result in more heterogeneous values and hence are better conditioned. The weights and biases generated by such tuned random distributions permit to obtain much better conditioned matrices, therefore smaller output weights α . Note that this tuning process depends on the position of the sensors, therefore, the optimization of the random weights can be done based on the machine geometry (and on the size of the network), and it is independent of the particular plasma configuration to be achieved. In Fig. 3.20 it is shown how the choice of $\mathbf{W}$ and $\mathbf{b}$ can affect α . In particular, in the tuned case, the activation functions have been translated in a point of the poloidal plane $\mathbf{x}_0 = [2.1 \ 0]^T$, while $\mathbf{W}$ and α are chosen with a gaussian distribution contained in the interval

$[-1.5 \ 1.5]$.

We now compare the performance of the proposed method with that of XLOC_DTT, using three entirely different plasma configurations. In addition to evaluating the RMSD with respect to the equilibria computed with the CREATE codes, we assess the quality of reconstruction along the critical gaps highlighted in green in Fig. 3.21, which were the same as those chosen in [139], where the performance of the XLOC_DTT algorithm on DTT has been evaluated.

With 48 degrees of freedom and tuned distribution for the random weights, the proposed approach can accurately estimate the three different plasma boundaries without modifying network hyper-parameter. In Fig. 3.21 the shapes reconstructed by the two algorithms are shown. Although the performance of the XLOC_DTT algorithm is also somewhat satisfactory, it is challenging to reconstruct the plasma shape of these three equilibria using the same algorithm setup. This implies that for different plasma configurations, retuning of the algorithm may be necessary, including adjustments to the number of regions, their positions, and the number of sensors used per region. Indeed, in Fig. 3.21, the blue lines represent the segments defining the different regions considered by the XLOC_DTT algorithm, while the red dots indicate the points where continuity of the different polynomials is imposed, according to Eq. 3.16. When switching from the two SN equilibria to the XD configuration, both the regions and sensor weights require modification, whereas such changes are unnecessary for CARONTE reconstruction. To systematically assess the performance of these two algorithms, noise is added to the magnetic field measurements. The total measurement error consists of both a relative and an absolute term. Specifically, it is assumed that

$$m = m_0(1 + n_r) + n_a ,$$

where

- m_0 is the ideal (noise-free) value measured by a sensor;
- m is the actual measured value;
- n_r is the relative error, modeled as a zero-mean Gaussian random variable with standard deviation σ_r ;

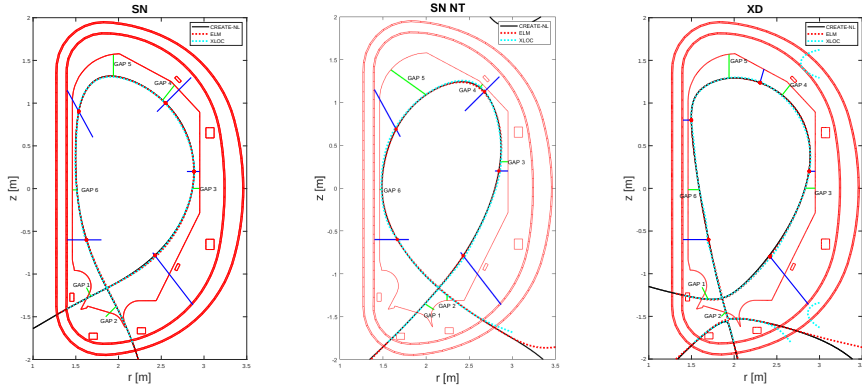


Figure 3.21: These figures show the plasma equilibrium computed by the CREATE equilibrium code, the reconstruction with the XLOC_DTT approach (cyan) and with the physics-informed ELM network (red). On the left the SN configuration, in the center the SN-NT configuration, and on the right the XD configuration. The agreement with the equilibrium is good with both reconstruction methods, but the ELM show superior performances. The green dots represent the flux loops, the arrows represent the magnetic probes. The points at which the homogeneous GS is imposed are shown in Fig. 3.19.

- n_a is the absolute error, modeled as a zero-mean Gaussian random variable with standard deviation σ_a .

The parameters σ_a and σ_r are selected to realistically model sensor positioning errors and misalignments. For DTT, these are estimated as $\sigma_r = 0.00079$

and $\sigma_a = 0.64 \text{ mT}$. In the analyses presented here, more conservative values of $\sigma_r = 0.003$ and $\sigma_a = 1.5 \text{ mT}$ were used.

After 500 noise realizations, the reconstruction errors of the chosen gaps are summarized in Tab. 3.1 and Tab. 3.2, reporting the mean (μ_e) and standard deviation (σ_e) in centimeters. These results clearly demonstrate the superior performance of the ELM reconstruction. Along the defined gaps, although the requirement is met on the mean, considering the resulting standard deviation, the XLOC_DTT algorithm frequently fails to meet the 1 cm accuracy requirement. On the other hand, the CARONTE algorithm satisfies the accuracy criterion at a 95% confidence level in all but very few cases. Nevertheless, in these exceptional cases, the mean error remains within the acceptable limit anyway.

Actuator sharing techniques for early-stage operations in superconductive tokamaks

4.1 Introduction

Broadly speaking, the control problem in a tokamak reactor can be divided into two main areas: electromagnetic control and kinetic control. Kinetic control primarily involves manipulating particle fueling rates and/or auxiliary heating and current drive systems to adjust the plasma's internal profiles, such as density, temperature, pressure, and current density. While some foundational aspects of kinetic control have been addressed (e.g., plasma density control, or achieving H-mode via additional heating systems), many challenges remain open research topics. These become especially relevant in the context of advanced tokamak scenarios. A comprehensive overview can be found in [46].

Electromagnetic control focuses on the use of the PF coil system to maintain or modify the plasma current, position, and shape. In addition to these primary objectives, several secondary but critical requirements must be met. These include vertical stabilization for elongated plasmas and effective regulation of the currents flowing in the PF coils. In particular, the magnetic control system is often divided in the following subsystems:

- **The Vertical Stabilization (VS) controller:** as shown in Sec-

tion 2.4, elongated plasmas are vertically unstable. In modern tokamaks, the vertical stabilization system typically relies on a set of dedicated actuator circuits, which often include at least one pair of in-vessel coils. To decouple vertical stabilization from position and shape control, only the vertical velocity of the plasma is fed back to this system. In other words, the VS system is responsible solely for stopping vertical motion, while the task of bringing the plasma to its desired vertical position is handled by the position and shape control system;

- **PF Current (PFC) Decoupling Controller:** this controller forms the inner control loop of a nested architecture, in which the plasma current and position/shape controllers operate as outer loops. Its role is to track the PF coils' current references, which are computed as the sum of the preprogrammed scenario currents and the corrections provided by the outer controllers. The controller is designed using a Multi-Input Multi-Output (MIMO) approach, with the objective of decoupling the dynamic responses of the various PF coil circuits;
- **Plasma Current Controller:** it tracks the plasma current reference by generating additional requests for the PF Current Decoupling Controller;
- **Plasma Shape Controller:** which controls the shape of the LCFS by generating additional requests for the PFC Decoupling Controller. This block could be replaced by a simpler Position Controller, which acts only on z_c and r_c , especially in the early phase of the pulse.

Figure 4.1 shows the block scheme of the structure of a magnetic control system, similar structures have been extensively used and proposed [36, 9, 13, 47, 5]. On the plant side, other than the tokamak itself, the PF coil and VS controllers feed voltages requests to the Power Supply system, which, for model-based controller design purposes, can be integrated in the linear model (2.20). Finally, from the magnetic diagnostics, as discussed in the previous chapter, fast reconstruction algorithms need to provide the values of the controlled plasma quantities, i.e., the shape descriptors, the plasma current, plasma position, and current centroid vertical speed.

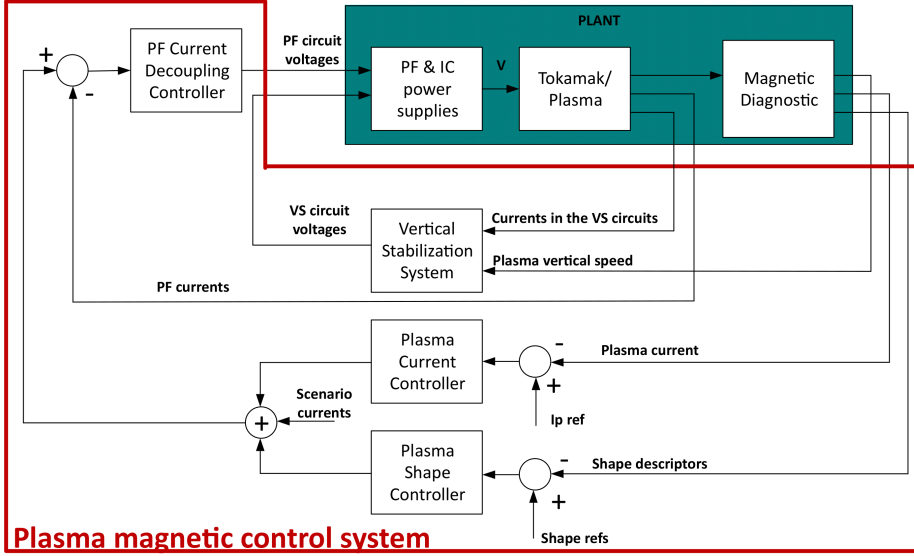


Figure 4.1: A possible architecture of a plasma magnetic control system. Here the main controllers are shown, and the scenario currents represent the nominal PF currents. The PF Current Controller is responsible for the tracking of these currents, along with the corrections given by the Shape and Plasma current controllers (source [121]).

It is worth noting that recently, even though model-based controllers have proven their effectiveness, many efforts have been made to let the magnetic controllers be independent of the particular model of the plasma. For an extensive overview of model-based magnetic control and modelling for tokamak devices, the interested reader is also referred to [21, 121]. The downside of the model-based approach is that the plasma equilibrium is evolutionary; therefore, it changes throughout the pulse, and the control system needs to be extensively tested with different models of the plasma. In fact, control performances may be worsened by changing from one plasma equilibrium to another. Model-free approaches to the VS problem include the usage of the Extremum Seeking optimization algorithm [39, 129, 41, 42, 131], and would not suffer from such disadvantages, being independent of a particular plasma model. Finally also data-driven approaches using Reinforcement Learning algorithms are currently under

study, more details can be found in [40, 130, 38].

This part of the Thesis is focused on model-based design approaches for magnetic control. In particular, the magnetic control system structure shown in Figure 4.1 is adapted for early-stage operations of superconductive machines such as JT-60SA, in which the absence of in-vessel components do not allow the application of the classic magnetic control architecture.

In fact, among the various challenges to be faced, the magnetic control one plays an essential role since the very first operations of every magnetically confined fusion device [115, 62, 16, 87, 17]. Indeed, some basic components of the magnetic control systems, such as the coils currents controller, are needed even before plasma operation, during the commissioning phase, to put the coils in safe operation [115, 132].

The main magnetic control tasks during plasma operations include tracking of the plasma current, position, and shape. Furthermore, since the shapes typically pursued are elongated, and therefore vertically unstable [100], an active VS control system is needed to stabilize the plasma column [20, Ch. 7]. VS systems typically rely on dedicated actuators, which are toroidal coils either placed inside the vessel (in-vessel coils [94, 91, 9]) and/or outside (out-vessel coils [6]), and that are fed with dedicated power supplies. When dedicated actuators are available, different strategies are possible and have proved to be effective in practice (among the various, see [81, 110, 111, 14, 102, 9, 76]).

Nevertheless, the roadmap of superconductive tokamaks, such as ITER [25] envisages the possibility of running discharges with elongated plasmas before the complete installation of the in-vessel components, and hence without dedicated actuators for the VS [144, 62]. However, when a *current-driven* control scheme is adopted (see [110, 146, 62, 36, 87]), the presence of the PF current controller itself is equivalent to an increase of the coil resistances, which in turn implies a reduction of their passive stabilization effect, and therefore to an increase of the growth rate. This worsening of the vertical instability makes the VS task even more challenging in the absence of a dedicated fast actuator, calling for an effective control system that relies only on the superconductive coils.

As for the ITER case, the first operations of JT-60SA do not envisage the presence of dedicated in-vessel coils for the VS. Specifically, JT-60SA

is a superconductive tokamak constructed within the Broader Approach agreement [134] that will operate as a ITER “satellite” facility. Indeed, it will develop ITER relevant operational scenarios and address the corresponding physics issues. Furthermore, it is expected that the operation of JT-60SA will complement the ones of ITER in R&D areas relevant for the design and realization of DEMO [144].

This Chapter is motivated by the need for an architecture able to cope with all magnetic control tasks in the absence of in-vessel coils, as arisen in [62], where realistic scenarios relevant for the first operation of JT-60SA have been considered. In these scenarios, there is the need not only to accomplish the usual magnetic control tasks, but also to ensure vertical stability and counteract fast plasma deformations due to poloidal beta variations, while avoiding voltage saturation of the power supplies that feed the coils. In [62], the authors propose an Adaptive Voltage Allocation (AVA) scheme that *relaxes* the plasma current control to avoid such saturation and to guarantee closed-loop vertical stability. The architecture proposed in this Chapter represents a possible alternative to such AVA scheme, which is based on the offline identification of practically decoupled control directions to be assigned to each control task, i.e. plasma current, shape and VS control. Although saturation management is out of the scope of this Chapter, it is worth to notice that the proposed approach has similarities with real-time allocation techniques such as those proposed in [124, 53], hence they can be integrated to increase the robustness against actuator saturation. Next Chapter presents a possible control architecture to cope with strict saturation limits.

Furthermore, the proposed architecture is anyhow machine-agnostic and extends the one currently proposed for the ITER tokamak [87], by explicitly taking into account a VS system that shares the same superconductive circuits used as actuators for the other magnetic control tasks, i.e. shape/position and plasma current control. Actuator sharing among the various control tasks is achieved by means of a model-based approach that relies on a linear approximation of the plasma response. Indeed, as has been proved in several tokamaks, the use of linearized models of the plasma/circuit dynamics is an effective means for model-based design of magnetic control actions; the interested reader can refer to [18] for an application of $\mathcal{H}_\infty$ plasma magnetic control at TCV, to [12] for the plasma

shape control at JET, and [9] for the VS at EAST. As usual, when relying on a model-based design approach, the robustness of the control architecture is first assessed via linear simulations, by using a collection of models that are different from the one used for the design. After such a preliminary assessment, the proposed control algorithms are then validated via nonlinear simulations based on a free boundary GS equations solver. Note that analyses presented in this Chapter are also found in [133].

4.2 Decoupled magnetic control architecture

The proposed magnetic control architecture is presented in this section, together with the control algorithms implemented by the various components. The proposed setup allows an effective sharing of the PF coil actuators among the main plasma magnetic control objectives, which are plasma current and boundary control, as well as the counteraction of fast plasma movements and the vertical stabilization. More in detail, the proposed actuator sharing policy is based on a geometric approach that relies on a linear description of the plasma behavior, which in turn allows to define a set of virtual circuits to be used by each magnetic control task.

A simplified block diagram that reports the main functional components of the proposed architecture is reported in Figure 4.2. Note that with respect to Figure 4.1, there VS controller does not act on a dedicated circuit and the Plasma Shape Controller is divided into two components. The architecture relies on the so-called *current-driven* approach [87], which consists of a nested control system, whose inner loop is the PFC Decoupling Controller. This block ensures the tracking of the required currents in the superconductive PF circuits whose references are obtained as the sum of the *scenario* (i.e., the nominal) currents and the corrections requested by the outer control loops to track the desired plasma shape and current. Indeed, the proposed architecture includes the following additional components:

- the **Plasma Current Controller**, which tracks the plasma current reference by sending the corresponding requests to the PFC Decoupling Controller;

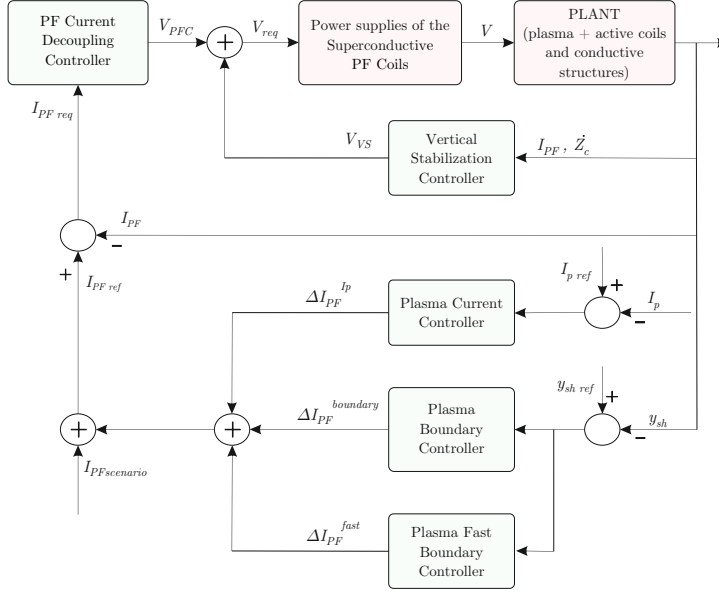


Figure 4.2: Block diagram of the proposed plasma magnetic control architecture that shares the PF superconductive coils system to perform all the magnetic control tasks.

- the **Plasma Boundary Controller**, which controls the shape of the LCFS within the vacuum chamber by minimizing the differences of the flux at a set of control points chosen along the desired plasma boundary and the flux at the X-point. This block also computes the PF current corrections to be tracked by the PFC Decoupling Controller;
- The **Plasma Fast Boundary Controller**, which assures a *faster* response against the plasma deformations that are typically induced by the poloidal beta variations due to the switch on and off of the additional heating, such as the Neutral Beam Injectors or the Electron Cyclotron.
- the **VS Controller**, which is needed to vertically stabilize the plasma column and counteract relevant disturbances. Given the characteristic time of the vertical instability, this controller must act on a time

scale that is faster than the dynamics imposed by the PFC. Therefore, the VS is designed to directly compute the additional voltage request for the PF coils.

As anticipated in Section 4.1, the peculiarity of the proposed architecture is the sharing of the superconductive PF circuits as actuators for both the VS and the other magnetic control tasks. Indeed, in Figure 4.2 the VS, although with voltage requests, acts on the same actuators as the PFC Decoupling Controller, therefore there is a need to adopt a model-based design approach that ensures effective decoupling between the various tasks, to avoid interactions that may lead to poor closed-loop performance and, in the worst case, instability.

In the following sections, a detailed description of the control algorithms implemented by each of the blocks shown in Figure 4.2 is given. Since a model-based approach to design the control algorithms is proposed, this section concludes by briefly recalling the linear model of the plasma response, which will be referred to in what follows. Indeed, around a given plasma equilibrium, the linear model is specified by means of

- nominal plasma current $I_{peq} \in \mathbb{R}$;
- nominal currents in the n_{PF} PF circuits¹ $I_{PF_{eq}} \in \mathbb{R}^{n_{PF}}$;
- expected values of poloidal beta β_{peq} and plasma internal inductance l_{ieq} , whose variation can be regarded as disturbances, i.e. exogenous inputs, as far as the plasma magnetic control is concerned;

the behavior of the plasma and of the currents flowing in the surrounding conductive structures can be described (with a slight abuse of notation wrt (2.20), i.e. δ is omitted) by the state space model [11, 20]

$$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x}(t) + \mathbf{B} \cdot \mathbf{u}(t) + \mathbf{E} \cdot \dot{\mathbf{w}}(t) \quad (4.1a)$$

$$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x}(t) + \mathbf{F} \cdot \mathbf{w}(t), \quad (4.1b)$$

where:

- $\mathbf{x} = (\delta \mathbf{I}_{PF}^T \quad \delta \mathbf{I}_e^T \quad \delta I_p)^T$ is the vector that holds the variations of the currents in the PF circuits $\delta \mathbf{I}_{PF}$, in the passive structures $\delta \mathbf{I}_e$, and of the plasma current δI_p ;

¹In this chapter all the active circuits are assumed to be superconductive.

- $\mathbf{u}$ is the vector of the voltages applied to the superconductive PF circuits;
- $\mathbf{w} = (\delta\beta_p \ \delta l_i)^T$ is the disturbances vector, i.e. the variations of β_p and l_i ;
- $\mathbf{y}$ is the output vector that contains the variations with respect to the equilibrium values of all the quantities of interest; e.g. the plasma shape descriptors like flux differences or plasma-to-wall distances (*gaps*), the magnetic field at the X-point, radial and vertical position of the X-point, PF circuit currents, plasma current, plasma centroid vertical position².

4.2.1 PFC Decoupling Controller

The proposed architecture relies on a *current-driven* control scheme, therefore a controller for the currents flowing in the PF circuits is needed. The design of the PFC Decoupling Controller exploits a well-assessed approach for the design of a Multi-Input-Multi-Output (MIMO) controller based on the plasmaless model that describes the behavior of the mutually coupled PF superconductive circuits [121]. Let us now consider

$$\mathbf{V}_{PFC} = \mathbf{M} \cdot \dot{\mathbf{I}}_{PF},$$

where $\mathbf{V}_{PFC}, \mathbf{I}_{PF} \in \mathbb{R}^{n_{PF}}$ are the voltages applied by the PFC Decoupling controller and the currents flowing in PF superconductive circuits, respectively, while $\mathbf{M} \in \mathbb{R}^{n_{PF} \times n_{PF}}$ is the inductance matrix computed in absence of plasma, i.e. the so-called *plasmaless* mutual inductance matrix. Let $\mathbf{I}_{PF_{ref}}$ be the reference waveforms to be tracked by the PFC Decou-

²It is worth to notice that the output of model (4.1) are *ideal*, i.e. measurement noise is not taken into account.

pling Controller, and let us consider the following diagonal matrix

$$\mathbf{\Lambda} = \begin{pmatrix} \frac{1}{\tau_1} & \cdots & \cdots & 0 \\ 0 & \frac{1}{\tau_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{1}{\tau_{n_{PF}}} \end{pmatrix}.$$

If the PFC Decoupling controller output is computed as follows

$$\mathbf{V}_{PFC} = \mathbf{K}_{PF} \cdot (\mathbf{I}_{PF_{ref}} - \mathbf{I}_{PF}),$$

where the matrix gain $\mathbf{K}_{PF}$ is set equal to

$$\mathbf{K}_{PF} = \mathbf{M} \cdot \mathbf{\Lambda},$$

then the closed-loop behaviour of the n_{PF} PF currents becomes equal to

$$\dot{\mathbf{I}}_{PF} + \mathbf{\Lambda} \cdot \mathbf{I}_{PF} = \mathbf{\Lambda} \cdot \mathbf{I}_{PF_{ref}}, \quad (4.2)$$

i.e. decoupled control on each PF circuit is achieved, and the closed-loop first order time constant can be set by means of the design parameter $\mathbf{\Lambda}$. It is worth noticing that, although the design of the control matrix $\mathbf{K}_{PF}$ is based on plasmaless mutual inductance when mutual coupling with passive toroidally continuous conducting structures is neglected, the robustness of such an approach has also been assessed in the presence of various plasma operating tokamaks, such as JET and EAST [88, 128].

However, the adoption of a *current-driven* approach constraints the currents flowing in the active coils, whose contribution to the passive stabilization of elongated plasmas is not negligible in the absence of in-vessel coils and or passive stabilizers. As will be discussed in more detail in Section 4.3, the presence of a PFC Decoupling Controller significantly contributes to the increase of the actual plasma growth rate.

4.2.2 VS Controller

Similarly to what is currently implemented at the JET tokamak [94], and to what is envisaged for ITER once the in-vessel coils will be installed [9, 87], the proposed VS controller aims at controlling the plasma vertical velocity, by letting the plasma boundary control to keep the desired shape and position. However, since the control architecture shown in Figure 4.2 is meant to operate without in-vessel coils, the proposed VS controller acts on the plasma column by directly computing additional voltages $\mathbf{V}_{VS} \in \mathbb{R}^{n_{PF}}$ to be summed to the output voltages $\mathbf{V}_{PFC}$ computed by the PFC Decoupling Controller, and then applied to the superconductive circuits by the power supplies.

Such voltages $\mathbf{V}_{VS}$ are chosen to ideally drive the PF currents along a specific *direction* given by the linear combination $\mathcal{J}_z \in \mathbb{R}^{n_{PF}}$. The *direction* $\mathcal{J}_z$ is chosen to somehow maximize the radial field able to exert a vertical force on the plasma column, and can be regarded as a virtual circuit to be used by the VS system. Given the typical number of degrees of freedom available on existing tokamaks, which are equal to the number of independent PF circuits, there are different possible choices for the vector $\mathcal{J}_z$.

A possible approach to compute $\mathcal{J}_z$ consists in choosing a point P in the center of the vacuum chamber, a pair of up-down symmetric coils and the corresponding currents so that the vertical field in P be equal to 0. However, in this paper, we briefly describe an alternative approach based on the solution of a Quadratic Programming (QP) optimization problem, which gives more flexibility on the choice of the control coils, and allows to set constraints both on the PF currents and on the field in more than one point inside the vacuum chamber. To this aim, let

$$\begin{pmatrix} \delta \tilde{\mathbf{B}}_z \\ \delta \tilde{\mathbf{B}}_r \end{pmatrix} = \begin{pmatrix} \mathbf{C}_{\tilde{\mathbf{B}}_z} \\ \mathbf{C}_{\tilde{\mathbf{B}}_r} \end{pmatrix} \cdot \delta \tilde{\mathbf{I}}_{PF}, \quad (4.3)$$

where $\mathbf{C}_{\tilde{\mathbf{B}}_z}, \mathbf{C}_{\tilde{\mathbf{B}}_r} \in \mathbb{R}^{m \times n}$ are the linear relationship between the $n \leq n_{PF}$ currents in the selected PF circuits $\tilde{\mathbf{I}}_{PF}$ and both the vertical $\tilde{\mathbf{B}}_z$ and radial components $\tilde{\mathbf{B}}_r$ of the poloidal magnetic field on a grid of $m > n$ points, typically chosen in the plasma centroid region. The two matrices $\mathbf{C}_{\tilde{\mathbf{B}}_z}, \mathbf{C}_{\tilde{\mathbf{B}}_r}$ can be derived from the linear model (4.1) of the reference equilibrium,

which includes the presence of the passive structures. Given the linear relationship (4.3), the vector $\mathcal{J}_z$ can be obtained as the solution of the following constrained QP problem

$$\min \frac{1}{2} \cdot \delta \tilde{\mathbf{I}}_{PF}^T \cdot \mathbf{C}_{\tilde{B}_z}^T \cdot \mathbf{C}_{\tilde{B}_z} \cdot \delta \tilde{\mathbf{I}}_{PF}, \quad (4.4)$$

subject to

$$\mathbf{C}_{\tilde{B}_r} \cdot \delta \tilde{\mathbf{I}}_{PF} \geq B_{\min} \cdot \mathbf{1}_m, \quad (4.5a)$$

$$\overline{\delta \mathbf{I}}_{PF} \geq \delta \tilde{\mathbf{I}}_{PF} \geq \underline{\delta \mathbf{I}}_{PF}, \quad (4.5b)$$

where $\overline{\delta \mathbf{I}}_{PF}$ and $\underline{\delta \mathbf{I}}_{PF}$ can be used to weight the control effort on some circuits and/or to force some of the PF coils currents to be positive or negative (e.g. outer currents above/below the poloidal mid-plane forced to be positive/negative, respectively).

Thanks to the objective function (4.4), the solution of (4.4)–(4.5) minimizes the vertical field on the grid while trying to obtain a corresponding radial field which is greater than the chosen threshold $B_{\min}$, as required by constraint (4.5a). It is worth to remark that the threshold $B_{\min}$ does not limit the magnetic field that can be obtained with the control, since the current in the obtained virtual circuit is scaled by the controller as required to achieve the objective.

As an example, we apply the proposed approach to the JT-60SA 1 MA upper single-null equilibrium described in [144, 71] and considered for first plasma operation, whose poloidal cross-section is shown in Figure 4.3a.

As far as the PF coils system is concerned, 10 superconductive coils are installed, 4 in the CS, and 6 Equilibrium Field (EF) coils. Each coil is independently fed by a dedicated power supply, and therefore 10 PF circuits are available for the magnetic control system. In particular, only the $n = 6$ EF coils are considered in the static map (4.3), while the 8×9 grid shown in Figure 4.3b is used to constrain the magnetic field components, hence $m = 72$. Moreover, $B_{\min}$ in (4.5a) is set equal to $0.1 T$, while up-down symmetry is enforced across the mid-plane, letting EF1-EF3 positive and EF4-EF6 negative by means of constraint (4.5b). By solving the QP problem (4.4)–(4.5) with the Matlab 2022b `quadprog` solver on a MacBookPro, the resulting normalized vector $\delta \tilde{\mathbf{I}}_{PF}$, i.e. with

unitary norm, turns out to be equal to

$$\delta \tilde{\mathbf{I}}_{PF} = (0.0069 \quad 0 \quad 0.76 \quad -0.65 \quad 0 \quad 0)^T.$$

Therefore, once it is recognized that the contribution given by EF1 in the computed solution is negligible, the corresponding virtual circuit for VS control can be set equal to³

$$\mathcal{J}_z = (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.76 \quad -0.65 \quad 0 \quad 0)^T \quad A, \quad (4.6)$$

which correspond to:

$$\tilde{\mathcal{J}}_z \cong (0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 188 \quad -229 \quad 0 \quad 0)^T \quad A \cdot t,$$

if we consider the turns of each JT-60SA coil.

Another possible choice for the VS virtual circuit is obtained by solving the QP on the same optimization grid chosen for $\mathcal{J}_z$, but removing the constraints that enforce up-down symmetry. In this case the solution

$$\mathcal{J}'_z = (0.04 \quad -0.07 \quad 0.24 \quad -0.63 \quad 0.71 \quad -0.21)^T \quad A,$$

is obtained, which allow to further minimize the vertical components of the magnetic field on the optimization grid with respect to $\mathcal{J}_z$, as summarized in Table 4.1, while obtaining practically the same vertical displacement, as shown in Figure 4.4.

³The following order is considered for the JT-60SA PF circuits when using vectors: CS1, CS2, CS3, CS4, EF1, EF2, EF3, EF4, EF5, EF6.

Table 4.1: Minimum and maximum magnetic field components on the optimization grid used to solve the QP problem (4.4)-(4.5). The current in each virtual actuator has been set equal to the value that correspond to the plasma vertical movement shown in Figures 4.3b and 4.4, respectively.

VS virtual circuit	$\tilde{B}_{r_{\min}}$ [mT]	$\tilde{B}_{r_{\max}}$ [mT]	$\tilde{B}_{z_{\min}}$ [mT]	$\tilde{B}_{z_{\max}}$ [mT]
$\mathcal{J}_z$	4.1	4.7	-0.6	0.4
$\mathcal{J}'_z$	4.1	5.6	-0.1	0.1

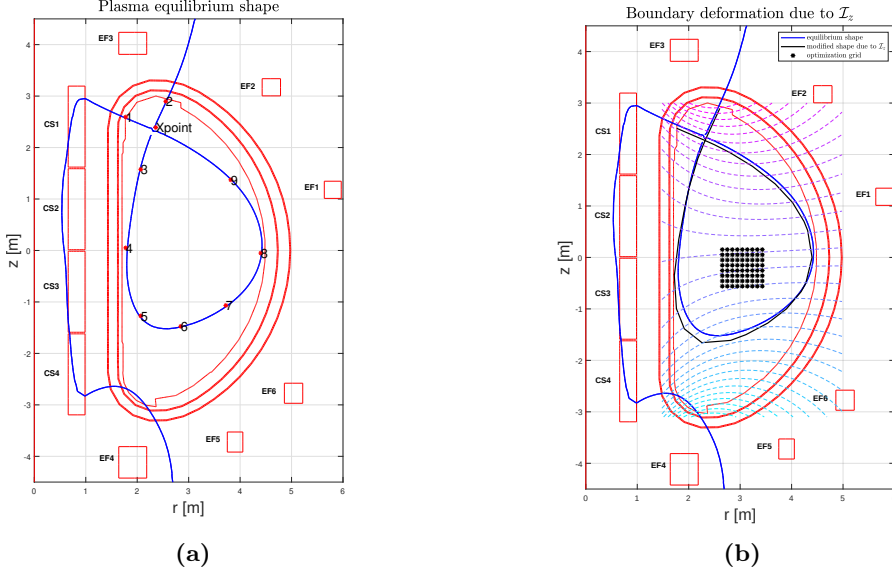


Figure 4.3: (a) Plasma equilibrium shape for the linearized model referred to as *Eq#2* in Table 4.2 and quantities controlled by a possible implementation of the proposed Plasma Boundary Controller for JT-60SA. The chosen controlled variables are 9 flux differences plus the radial and vertical components of the magnetic field at the X-point. In the proposed architecture shown in Figure 4.2, these control quantities are sent also to the Plasma Fast Boundary Controller. (b) Plasma displacement and the magnetic field generated by the VS virtual circuit when set equal to $\mathcal{I}_z$. The black dots show the optimization grid used to solve problem (4.4)-(4.5).

It is worth to remark that the proposed approach to compute VS virtual circuits is just one of the possible options. Indeed, also the complementary alternative that maximizes the radial field on the grid while constraining the vertical one to be below a given threshold is viable.

For a given choice of the vector $\mathcal{I}_z$, the simplified scheme for the VS controller is shown in Figure 4.5, and the corresponding control action is given by

$$\mathbf{V}_{VS} = \mathcal{V}_z \cdot \left[I_{pref} \cdot K_v \cdot \dot{Z}_c + K_I \cdot \mathcal{I}_z^T \cdot (\mathbf{I}_{PF} - \mathbf{I}_{scenario}) \right], \quad (4.7)$$

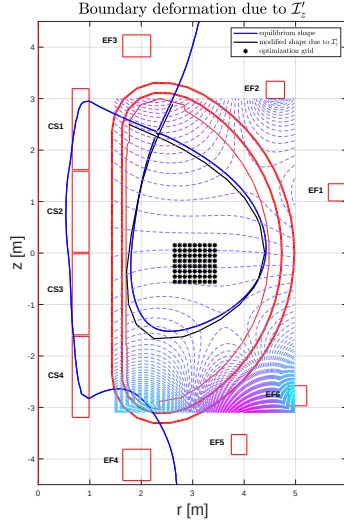


Figure 4.4: Plasma displacement and the magnetic field generated by the VS virtual circuit when set equal to $\mathcal{I}'_z$.

where

- $\mathbf{I}_{PF}$ are the currents flowing in the superconductive circuits;
- $\mathbf{I}_{scenario} \in \mathbb{R}^{n_{PF}}$ are the reference scenario currents;
- $\dot{Z}_c \in \mathbb{R}$ is the vertical velocity of the plasma centroid;
- $K_v, K_I \in \mathbb{R}$ are the velocity and current gains, respectively;
- $I_{pref} \in \mathbb{R}$ is the reference plasma current which is used to scale the velocity gain K_v ;
- the vector $\mathcal{V}_z \in \mathbb{R}^{n_{PF}}$ is computed exploiting the plasmaless mutual inductance matrix $\mathbf{M}$ as follows

$$\mathcal{V}_z = -\mathbf{M} \cdot \mathcal{I}_z.$$

Indeed, since the VS controller acts directly on the PF voltages, the $\mathcal{V}_z$ vector allows to generate voltages that produce currents in the directions specified by the vector $\mathcal{I}_z$.

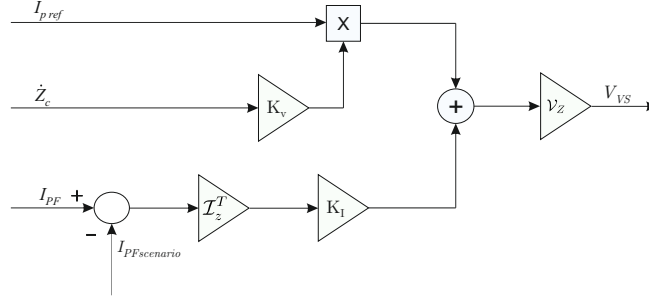


Figure 4.5: Simplified scheme of the proposed VS controller.

Since in the proposed architecture vertical stabilization is achieved by using slow circuits/power supply as actuators, which are typically designed to perform other tasks, such as I_p and equilibrium control, neglecting the passive structures when deriving $\mathcal{V}_z$ has no major impacts, as it is shown in the case study presented in Section 4.4. Furthermore, it is also worth noticing that other to bring the plasma velocity to zero, the control law (4.7) aims also at regulating to zero the projection along the direction specified by $\mathcal{J}_z$ of the difference between the actual currents $\mathbf{I}_{PF}$ and the scenario ones $\mathbf{I}_{scenario}$, being $\mathcal{J}_z^T \cdot (\mathbf{I}_{PF} - \mathbf{I}_{scenario})$ the projection of $\mathbf{I}_{PF} - \mathbf{I}_{scenario}$ along the $\mathcal{J}_z$ direction. In other words, the VS controller aims at controlling to zero the control effort while stabilizing the plasma. This objective is achieved thanks to the presence of the Plasma Boundary Controller described in Section 4.2.4, which brings the plasma to an equilibrium position on a slower timescale compared to the one of the VS controller. Moreover, it is also important to remark that, if there is a need to improve the stability margins, the control law (4.7) can be augmented with a Single-Input-Single-Output (SISO) dynamic compensator, typically a lead compensator; more details can be found in [9, 126].

4.2.3 Plasma Fast Boundary Controller

The Plasma Fast Boundary Controller is included in the proposed architecture to react to fast plasma shape variations due to external disturbances. This dedicated control loop is designed to induce a specific deformation to balance the one caused by the foreseen disturbance. Such

a deformation depends on both the disturbance to be rejected and the reference plasma equilibrium. In our architecture, we exploit this control loop to promptly counteract the Horizontal Displacement Events (HDE) induced by the switching of the additional heating systems, which in turn correspond to a poloidal beta variation.

The proposed control algorithm for the Plasma Fast Boundary Controller receives as input the same quantities controlled by the Plasma Boundary Controller. As it will be discussed in Section 4.2.4, when controlling single-null divertor plasmas, such control input vector, referred to as $\mathbf{y}_{sh} \in \mathbb{R}^{n_{sh}}$, holds

- both B_{r_X} and B_{z_X} that are the variations of the radial and vertical components of the poloidal magnetic field at the X-point, respectively;
- the flux differences $\Delta\psi_i, i = 1, \dots, p$ between the poloidal flux at the p control points, that are chosen along the desired plasma boundary, and the poloidal flux at the X-point.

As an example, Figure 4.3a shows a possible choice of the components of $\mathbf{y}_{sh}$ for the upper single-null equilibrium considered.

It is worth noticing that, given the above choice for $\mathbf{y}_{sh}$, the reference values for both the Plasma Fast Boundary and the Plasma Boundary controllers should be set equal to the null vector, since both the poloidal magnetic field at the X-point and the flux differences should be kept as small as possible to track the desired plasma shape and position.

For the considered JT-60SA equilibrium, the plasma model is exploited to compute the linear combination of PF currents $\mathcal{J}_r \in \mathbb{R}^{n_{PF}}$ corresponding to a virtual circuit that induces a plasma deformation capable to counteract the one caused by the β_p variation that corresponds to the switching of the additional heating systems.

Moreover, if $\mathbf{C}_{sh} \in \mathbb{R}^{n_{sh} \times n_{PF}}$ is the linear relationship between the currents in the PF circuits and the controlled plasma shape descriptors $\mathbf{y}_{sh}$, then $\mathcal{Y}_r = \mathbf{C}_{sh} \cdot \mathcal{J}_r$ is an estimation of the plasma deformation due to $\mathcal{J}_r$ in the n_{sh} -dimensional space defined by the controlled variables $\mathbf{y}_{sh}$. In other words, $\mathcal{Y}_r$ represents the *direction* in the space of the chosen plasma shape descriptors that is controlled by the linear combination $\mathcal{J}_r$, and is chosen

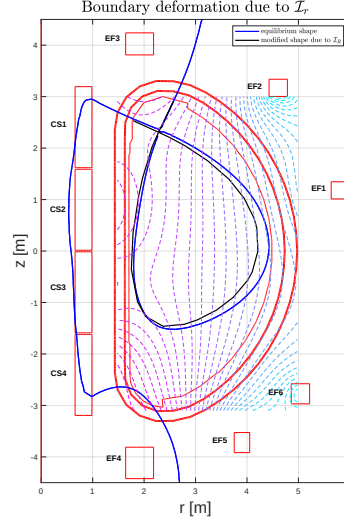


Figure 4.6: Plasma deformation corresponding to the current vector $\mathcal{I}_r$ given by (4.8). This is mainly a plasma outboard deformation, which is the one needed to counteract HDEs.

to be as equal as possible to the mainly outboard deformation induced by the β_p variation.

The following choice for $\mathcal{I}_r$ has been computed for the 1 MA upper single-null equilibrium of JT-60SA shown in Figure 4.3a

$$\mathcal{I}_r = (-0.06 \quad 0.38 \quad -0.18 \quad 0.58 \quad -0.1 \quad -0.24 \quad 0.42 \quad 0.28 \quad -0.38 \quad -0.08)^T A, \quad (4.8)$$

which correspond to:

$$\tilde{\mathcal{I}}_r \cong (-34 \quad 208 \quad -99 \quad 320 \quad -18 \quad -37 \quad 103 \quad 100 \quad -58 \quad -15)^T A \cdot t,$$

when we consider the ampere-turn. Moreover, the corresponding plasma outboard deformation is reported in Figure 4.6.

The control direction defined by the couple $(\mathcal{I}_r, \mathcal{Y}_r)$ allows to attain the control along the desired direction in the space of the controlled plasma boundary descriptors by means of a SISO controller. Indeed, by properly projecting the control error along the direction defined by $\mathcal{Y}_r$, and by properly distributing the control effort along the direction defined by $\mathcal{I}_r$,

the Plasma Fast Boundary Control is given by the following relationship in the Laplace domain, that relies on the SISO dynamic controller $K_{fast}(s)$

$$\Delta \mathbf{I}_{PF}^{fast}(s) = -\mathcal{J}_r \cdot K_{fast}(s) \cdot \mathcal{Y}_r^T \cdot \mathbf{Y}_{sh}(s), \quad (4.9)$$

where $\mathbf{Y}_{sh}(s) \in \mathbb{R}^{n_{sh}}$ is the Laplace transform of the plasma shape descriptors vector $\mathbf{y}_{sh}$, and $\Delta \mathbf{I}_{PF}^{fast}(s)$ is the Laplace transform of the corrections to be added to the scenario PF currents, i.e. is the Laplace transform of the Plasma Fast Boundary Controller output.

Although different solutions can be adopted once the fast boundary control problem is reduced to a SISO one, a possible choice for the controller $K_{fast}(s)$ is a Proportional Integral (PI) one, i.e.

$$K_{fast}(s) = K_P + \frac{K_I}{s},$$

which turned to be satisfactory for the case study considered in Section 4.4.

4.2.4 Plasma Boundary Controller

Different strategies are possible to control the shape of the plasma LCFS, the most common being the isoflux control [146, 91, 62] and the plasma-to-wall gap control [110, 35]. Although the proposed strategy relies on the so-called eXtreme Shape Controller (XSC) algorithm [19] applied to the case of isoflux control, it can be easily extended to the case of gap control, or the control of any other plasma shape descriptor, as long as a reliable linear model linking the currents in the PF circuits to the controlled variables is available.

Motivated by the considered JT-60SA case study, in this section, we focus on the case of isoflux control for a single-null plasma configuration. Therefore, as anticipated in Section 4.2.3, the proposed approach aims to control at zero the poloidal magnetic field at the X-point, as well as the flux differences between the flux at the control points and the flux at the X-point.

Since it relies on the XSC approach (more details can be found in [19]), the design of the proposed plasma boundary control algorithm is based on the following output equation of the linear model (4.1), that links the variations of the plasma shape descriptors $\mathbf{y}_{sh}$ to the variations of the

currents in the PF circuits:

$$\mathbf{y}_{sh}(t) = \mathbf{C}_{sh} \cdot \delta \mathbf{I}_{PF}(t).$$

The main advantage of resorting to the XSC approach is that we can set $n_{sh} > n_{PF}$, i.e. we can control a number of plasma shape descriptors that is greater than the number of independent PF control circuits. However, this implies that, at steady state, the error on the controlled variables $\mathbf{y}_{sh}$ is minimized in the least-mean-square sense. Moreover, such a minimization is performed on a weighted version of the shape descriptors by considering the pseudo-inverse of the following matrix

$$\mathbf{C}_{XSC} = \mathbf{Q} \cdot \mathbf{C}_{sh} \cdot \mathbf{N},$$

where $\mathbf{Q} \in \mathbb{R}^{n_{sh} \times n_{sh}}$ and $\mathbf{N} \in \mathbb{R}^{n_{PF} \times n_{PF}}$ are two diagonal matrices that are used to differently penalize the various shape descriptors ($\mathbf{Q}$) and to distribute the control effort among the various PF circuits ($\mathbf{N}$). It turns out that the following steady-state objective function is minimized

$$J = \lim_{t \rightarrow \infty} \|\mathbf{C}_{XSC} \cdot \delta \mathbf{I}_{PF}(t)\|_2^2. \quad (4.10)$$

The pseudo-inverse $\mathbf{C}_{XSC}^\dagger$ is obtained via the singular value decomposition $\mathbf{C}_{XSC} = \mathbf{U} \cdot \mathbf{S} \cdot \mathbf{V}^T$, where the matrix $\mathbf{S} \in \mathbb{R}^{n_{PF} \times n_{PF}}$ is a square diagonal matrix that holds the singular values in decreasing order and $\mathbf{U} \in \mathbb{R}^{n_{sh} \times n_{PF}}$ and $\mathbf{V} \in \mathbb{R}^{n_{PF} \times n_{PF}}$ are unitary matrices. As a matter of fact, minimizing the performance (4.10) index by retaining all singular values leads to a high control effort at steady state in terms of currents in the PF coil, which can lead to actuator saturations. For this reason, a trade-off condition is achieved, by minimizing the cost function while penalizing both the error on the controlled shape descriptors and the control effort. This is achieved by controlling to zero the error only for the $\bar{n} < n_{PF}$ linear combination related to the largest singular values [19].

However, to achieve effective actuator sharing between the plasma boundary and the other magnetic control tasks, the standard XSC approach needs to be extended by further exploiting the linear model. Indeed, the control directions set for both the Fast Boundary Control and the VS must be taken into account. In particular, to achieve decoupling with re-

spect to the Plasma Fast Boundary Controller, the control error $-\mathbf{y}_{sh}$ is projected on the subspace orthogonal to $\mathcal{Y}_r$ (see Section 4.2.3). Furthermore, to decouple the behavior of the Plasma Boundary Controller also from the VS, the pseudo-inverse matrix $\mathbf{C}_{XSC}^\dagger$ is projected in the orthogonal complement of $\text{span}(\mathcal{J}_z)$.

As a result, the Plasma Boundary Controller is a MIMO controller that computes the corrections to be applied to the scenario currents as follows:

$$\Delta \mathbf{I}_{PF}^{boundary}(s) = -K_{boundary}(s) \cdot \mathcal{J}_z^\perp \cdot \mathbf{V}_M \cdot \mathbf{S}_M^{-1} \cdot \mathbf{U}_M^T \cdot \mathcal{Y}_r^\perp \cdot \mathbf{Y}_{sh}(s)$$

where $\mathbf{V}_M \in \mathbb{R}^{n_{PF} \times \bar{n}}$, $\mathbf{S}_M \in \mathbb{R}^{\bar{n} \times \bar{n}}$, $\mathbf{U}_M \in \mathbb{R}^{n_{sh} \times \bar{n}}$, while $\mathcal{Y}_r^\perp$ is a matrix whose columns form an orthonormal basis of $\ker(\mathcal{Y}_r)$ and $\mathcal{J}_z^\perp$ is a matrix whose columns form an orthonormal basis of $\ker(\mathcal{J}_z)$. The diagonal transfer matrix $K_{boundary}(s)$ contains a set of n_{PF} regulators needed to improve the dynamic response of the controller, specified in the Laplace domain. As for the Plasma Fast Boundary Controller, a set of PI is typically sufficient to achieve the desired performance.

4.2.5 Plasma Current Controller

The Plasma Current Controller achieves robust control of the plasma current against uncertainties and disturbances by computing an additional correction to the scenario currents, which is then tracked by the PFC Decoupling Controller, together with the other requests from the outer loops. Similarly to the VS and the Plasma Fast Boundary Controller, the output of the Plasma Current Controller *moves* along a particular linear combination of currents in the PF circuits, namely $\mathcal{J}_{I_p}$, that provides a *transformer field*, which is a field that maximizes the effect on the plasma current by minimizing the effect on the plasma shape.

Similarly to what has been done for Plasma Boundary Control (see Section 4.2.4), to achieve effective actuator sharing, the output of the Plasma Current Controller is projected in the orthogonal subspace of $\text{span}[(\mathcal{J}_r, \mathcal{J}_z)]$, in order to decouple it from both the VS and the Plasma Fast Boundary Controller. Let $\mathcal{J}^\perp$ denote a matrix whose columns form an orthonormal basis of $\ker[(\mathcal{J}_z \ \mathcal{J}_r)]$, matrix $T = [\mathcal{J}_z \ \mathcal{J}_r \ \mathcal{J}^\perp]$ is an orthonormal basis in the space $\mathbb{R}^{n_{PF}}$. It is possible to force to zero the components of

$\mathcal{I}_{I_p}$ along $(\mathcal{I}_z \ \mathcal{I}_r)$, keeping only the ones along $\mathcal{I}^\perp$. Let

$$\mathcal{I}'_{I_p} = T^{-1} \begin{bmatrix} 0 & 0 & \mathcal{I}^\perp \cdot \mathcal{I}_{I_p} \end{bmatrix},$$

it turns out that the output of the Plasma Current Controller is given by

$$\Delta \mathbf{I}_{PF}^{I_p}(s) = \mathcal{I}'_{I_p} \cdot K_{I_p}(s) \cdot (I_{pref}(s) - I_p(s)),$$

where, similarly to all the other outer loops, $K_{I_p}(s)$ is a SISO dynamic regulator which is typically set equal to a PI controller.

4.3 Assessment of the effect of a current-driven control architecture on plasma vertical instability

As anticipated in Section 4.2.1, the adoption of a current-driven control scheme contributes to a worsening of the plasma vertical instability, which makes even more challenging the VS task in the absence of a dedicated and relatively fast actuator. As a case study, in this section we discuss the vertical stability of the considered JT-60SA equilibrium.

The analysis is carried out with reference to the 1 MA upper single-null and elongated equilibrium used during the first operation of JT-60SA, in the absence of in-vessel coils [144, 62], shown in Figure 4.3a. The following are examples of vessel time constants for the considered model: 0.0297 s, 0.0392 s, 0.0551 s, 0.0995 s. A set of different linear models (4.1) corresponding to small variations of plasma parameters around such an equilibrium have been considered. The main plasma parameters of the considered equilibria are reported in Table 4.2. Although, the shapes corresponding to the equilibria considered are similar, as shown in Figure 4.7, the resulting growth rates vary and cover a wide range.

It can be noticed that, under the assumption used to derive linear models with CREATE-NL [8], in some cases, the effect of the superconductive coil is sufficient to passively stabilize the equilibrium (as an example, see *Eq#5* in Table 4.2). However, such a stabilizing effect practically vanishes as a consequence of the parasitic circuit resistances.

Moreover, as anticipated in Section 4.1, the presence of the PFC De-

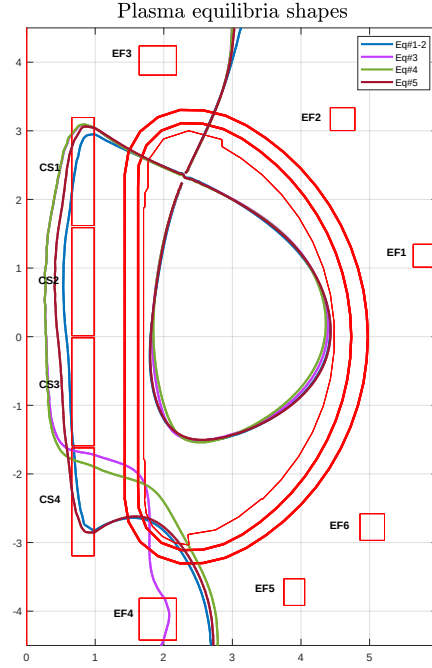


Figure 4.7: Equilibrium plasma shapes corresponding to the linear models listed in Table 4.2.

coupling Controller makes the VS task more challenging, by significantly reducing the passive stabilizing effect of the active coils. Indeed, by driving the desired currents in the coils, the PFC Decoupling Controller counteracts the currents induced within the coils themselves by plasma movements. Therefore, in practice, this effect causes an increase in plasma growth rate, calling for the deployment of an effective VS system.

To estimate such an effect on the considered JT-60SA equilibria, two different analyses have been carried out by exploiting the linear model (4.1). First, the superconductive coils have been *removed*, by removing the corresponding rows and the columns both from the mutual inductance matrix $\mathbf{\widetilde{M}}$ and from the resistance matrix $\mathbf{R}$ that appears in the following circuit

version of the of the linear model (4.1)

$$\widetilde{\mathbf{M}} \cdot \dot{\mathbf{x}}(t) + \mathbf{R} \cdot \mathbf{x}(t) = \mathbf{u},$$

in which the disturbance vector has been neglected, for the sake of simplicity. It is worth to remark that $\widetilde{\mathbf{M}}$ accounts not only for the active coils, but also the presence of plasma and eddy currents. Once the effect of the superconductive coils has been removed, obtaining the two matrices $\widetilde{\mathbf{M}}_{\text{mod}}$ and $\mathbf{R}_{\text{mod}}$, the dynamic matrix in (4.1), given by $\mathbf{A} = -\widetilde{\mathbf{M}}_{\text{mod}}^{-1} \cdot \mathbf{R}_{\text{mod}}$, is computed. A similar result would be obtained by considering a fictitious resistance in the superconducting coils. This additional resistance on the PF coils is an alternative way to emulate the effect of the PFC Decoupling Controller, since, as described in Section 4.2.1, it is designed to impose a given time constant on the various PF circuits – see also (4.2). The results of both analyses are reported in Table 4.3.

From the results shown in Table 4.3, we can conclude that the presence of the PFC Decoupling Controller can cause an increase of γ up to about one order of magnitude, calling for the presence of an effective VS system, even when this should share relatively slow actuators with the other magnetic tasks.

Table 4.2: Plasma parameters for the linear models corresponding to the considered 1 MA upper single-null equilibrium of JT-60SA. Poloidal beta β_p , internal inductance l_i , elongation κ , aspect ratio A , and an estimation of the growth rate γ , are reported.

JT-60SA equilibria					
	l_i	β_p	κ	A	γ [1/s]
<i>Eq#1</i>	0.78	0.14	1.46	2.42	0.36
<i>Eq#2</i>	0.78	0.13	1.47	2.43	1.13
<i>Eq#3</i>	1.16	0.15	1.50	2.52	4.02
<i>Eq#4</i>	1.16	0.15	1.54	2.55	7.64
<i>Eq#5</i>	0.85	0.83	1.45	2.55	Stable

4.4 The JT-60SA case study

In Section 4.2 the model-based control algorithms to be deployed in each of the components of the proposed architecture shown in Figure 4.2 have been introduced. As discussed in that section, the design of the various algorithms exploits the linear model (4.1); in particular the linear model corresponding to *Eq#2* in Table 4.2 has been used for the design of the controllers for the JT-60SA case study. The model has been obtained with the CREATE equilibrium codes tool-suite, which has been exploited to design and validate several magnetic control systems in the last two decades [122, 8, 9, 36, 31]. In what follows, we present the nonlinear assessment of the proposed magnetic control system, aimed at showing its effectiveness. In particular, the following two cases are considered:

- the rejection of Vertical Displacement Events (VDEs);
- the scenario similar to the one considered in [62], where a sudden increase of poloidal beta β_p due to the switch-on of the heating systems is included during the plasma current ramp-up from 1 MA to 2.5 MA. Such a β_p variation induces a HDE, which is rejected by the Plasma Fast Boundary Controller, as will be shown in Section 4.4.2.

All the results have been obtained by running nonlinear simulations with the CREATE-NL equilibrium code [8] in the Matlab environment. A

Table 4.3: Nominal growth rate (γ), growth rate obtained by removing the rows and columns of the matrices $\widehat{\mathbf{M}}$ and $\mathbf{R}$ that correspond to the PF coils (γ_{NC}), and the growth rate obtained by increasing the PF coil resistances (γ_R) are reported.

Growth rates comparison			
	γ [1/s]	γ_{NC} [1/s]	γ_R [1/s]
<i>Eq#1</i>	0.36	7.00	6.83
<i>Eq#2</i>	1.13	7.81	7.62
<i>Eq#3</i>	4.02	10.01	9.78
<i>Eq#4</i>	7.64	13.48	13.20
<i>Eq#5</i>	Stable	2.33	2.27

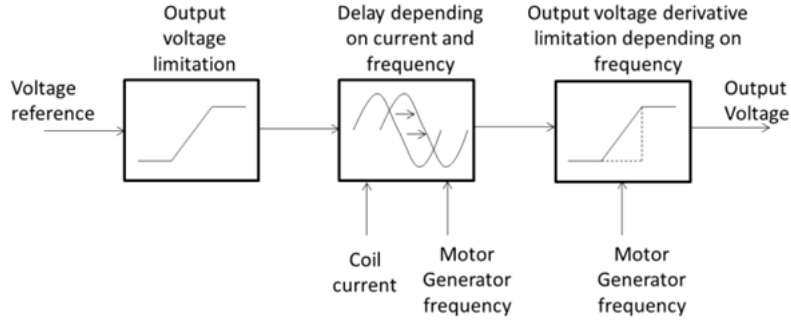


Figure 4.8: CS and EF power supplies model used for the simulation of the JT-60SA test cases considered in Section 4.4.

comparison between simulations obtained with the nonlinear equilibrium code and the linear model (4.1) used for controller design is also presented for the VDE case.

In all the cases considered, the model considered for the CS and PF power supplies is the series of a saturation, a rate limiter, and a time delay, as reported in Figure 4.8. With the only exception of the saturation, the values of the parameters for the power supply model change with the operative conditions, as reported in [97]. In Table 4.4 the chosen values are reported; the time delay has been chosen equal to the largest reported in [97], while the rate limit has been taken from [62]. Finally, an additional time delay of $350 \mu s$ has been introduced between the control output voltage and the PS model, to account for the delay due to both the real-time computation of the equilibrium values and the controller sampling time, as specified in [62].

Table 4.4: Maximum output voltages, rate limits, time delays and possible ranges chosen for the models of CS and EF power supplies as reported in [97].

Maximum output voltage, rate limit and time delay				
Coils	$V_{\max}$	$\frac{dV}{dt}_{\max}$	τ_d	Range τ_d
CS1-CS4-EF1-EF6	1 kV	100 V/ms	3.08 ms	1.07 – 3.08 ms
CS2-CS3	1.2 kV	100 V/ms	3.08 ms	1.07 – 3.08 ms
EF2-EF3-EF4-EF5	0.9 kV	100 V/ms	1.58 ms	0.53 – 1.58 ms

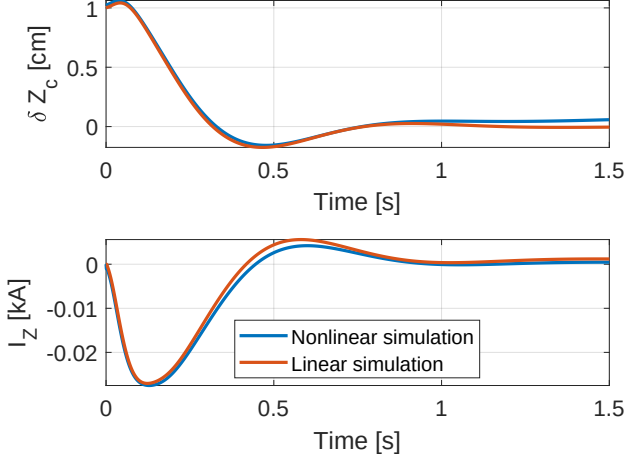


Figure 4.9: Time traces of the plasma centroid vertical position variation with respect to the equilibrium value δZ_c and of $I_Z = \mathcal{J}_z^T \cdot (\mathbf{I}_{PF} - \mathbf{I}_{scenario})$ in case of 1 cm VDE. The results of both linear and nonlinear simulations are reported.

4.4.1 VDE rejection

A 1 cm VDE, i.e. displacement of the vertical plasma centroid position along the unstable mode, is considered as a test case to show the reliability of the linear model used to design the controller by comparing its behaviour with the nonlinear equilibrium code used for the assessment when small perturbation are considered (see Figure 4.9). In particular, the results presented in this section have been obtained using the VS control direction (4.6).

Nonlinear assessment of VDE of 6 cm, which corresponds to a displacement normalized to the minor radius of $\Delta Z_c/a \sim 4\%$, has been performed to test a challenging scenario for the VS. Figure 4.10 reports the shape snapshots obtained during such a simulation, showing the capability of the control system to effectively reject even such a relatively big VDE. Moreover, Figure 4.11a reports the time traces of the error at the control points, while the voltages applied to the coils are shown in Figure 4.11b.

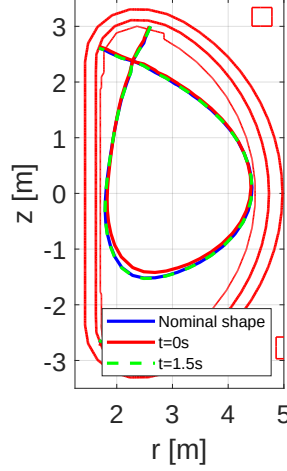


Figure 4.10: Snapshots of the plasma boundary at $t = 0$ s and $t = 1.5$ s for a nonlinear simulation of a 6 cm VDE applied at $t = 0$ s.

It should be noticed that in all considered cases, other than initial voltage saturation on the EF3 and EF4 coils, which are the only ones used by the VS system when the direction $\mathcal{J}_{z_1}$ is exploited, the voltage requests are within the prescribed ranges.

4.4.2 Plasma current ramp-up scenario

In this section, we consider the simulation of the ramp-up scenario similar to the one considered in [62]. In particular, a plasma current ramp-up from 1 MA to 2.5 MA in 6 s is considered, with a constant ramp-up rate equal to 0.25 MA/s. Similarly to what has been done in [62], during the ramp-up, a switch-on of the auxiliary heating at 2.0 s, and a switch-off at 2.5 s has been considered. When the heating system is turned on, the poloidal beta β_p is assumed to increase linearly from 0.14 to 0.5. Moreover, a minor disruption is considered at 3.5 s causing a 20 % β_p drop in 10 ms, followed by a linear recover to the nominal value in 0.15 s.

The simulation is performed considering a resistive flux consumption corresponding to $CEjima = 0.45$ [44].

The results presented in this section show that the proposed control

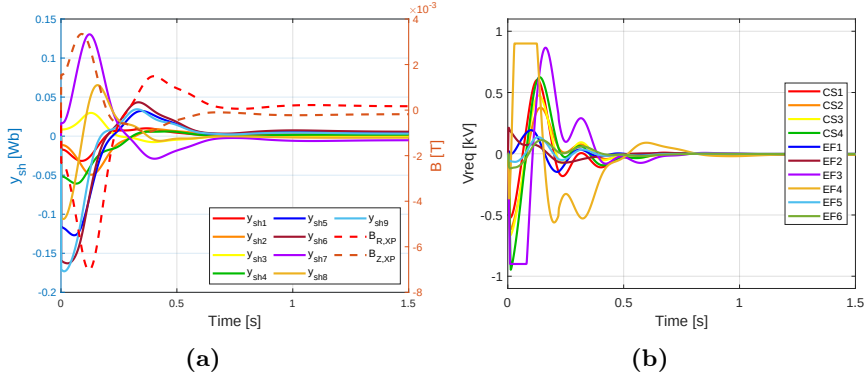


Figure 4.11: (a) Control errors at the control points shown in Figure 4.3a and (b) voltages applied to both CS and EF coils during a nonlinear simulation of a 6 *cm* VDE.

architecture is able to perform the I_p ramp-up – see Figure 4.12 – while maintaining the plasma shape as shown in Figures 4.13–4.14. The voltages remain within the prescribed limits as shown in Figure 4.15a. The total request to the PFC Decoupling Controller $\mathbf{I}_{P_{ref}}$ is reported in Figure 4.15b.

Moreover, a simulation without the Plasma Fast Boundary Controller has been performed to assess its necessity in the proposed control architecture. Note that this also means that the input to the Plasma Boundary Controller is not projected into the subspace orthogonal to $\mathcal{V}_r$. As shown in Figure 4.16a, without the Plasma Fast Boundary Controller, during the β_p ramp the plasma touches the first wall due to saturation of the the PF voltages reported in Figure 4.16b.

Finally, given the nominal value $\bar{\beta}_p = 0.5$, we performed nonlinear simulations including a poloidal beta variation $\Delta\beta_p$ spanning from $0.2 \cdot \bar{\beta}_p = 0.1$ to $0.6 \cdot \bar{\beta}_p = 0.3$, to assess the maximum perturbation that the closed-loop control can handle. The time traces of both radial and vertical positions of the plasma centroid are reported in Figure 4.17, showing that the proposed system is capable to reject a beta drop up to $0.5 \cdot \bar{\beta}_p = 0.25$, while when $\Delta\beta_p = 0.6 \cdot \bar{\beta}_p = 0.3$ the plasma is vertically lost. It is worth to remark that the capacity of the proposed architecture to reject a β_p drop is affected by the sharing of relatively slow actuators for both radial and vertical control. Indeed, when the disturbance amplitude becomes too

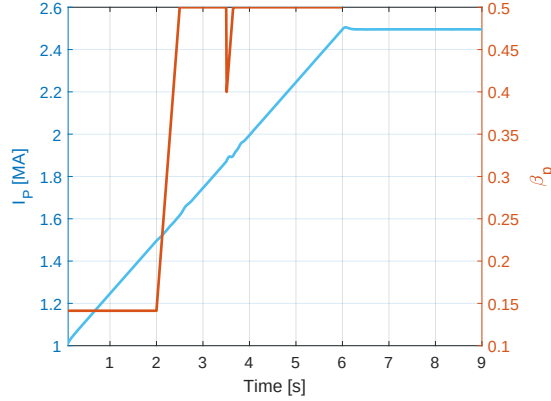


Figure 4.12: Time traces of the plasma current and poloidal beta for a nonlinear simulation of the I_p ramp-up scenario considered in Section 4.4.2. These time traces refer to the case of closed-loop compensation of the fast disturbance via the Plasma Fast Boundary Controller.

big, the plasma is practically vertically lost. This is not surprising, since the proposed architecture is aimed at carrying out the first operation of a superconductive tokamak. Dedicated in-vessel coils are usually envisaged to deal with more challenging plasma scenario, which, if equipped with independent actuators, allow to deploy a faster centroid position control by exploiting a *geometric* decoupling similar to the one described in Section 4.2.

Summarising, an architecture for magnetic control able to explicitly vertically stabilize the plasma column and to counteract fast plasma movements in the absence of a dedicated in-vessel actuator is presented in this chapter. The proposed architecture is suitable for the first operation of superconductive tokamaks, when not all in-vessel components are yet installed. In order to effectively share the available superconductive actuators among various magnetic control tasks, a model-based geometric control design approach has been adopted. However, the adoption of a current-driven control scheme increases the growth rate of the plasma vertical instability, requiring the presence of an efficient VS system also in the absence of a dedicated fast actuator. Therefore, in the proposed control architecture, the controller for the VS task is the first one designed: indeed,

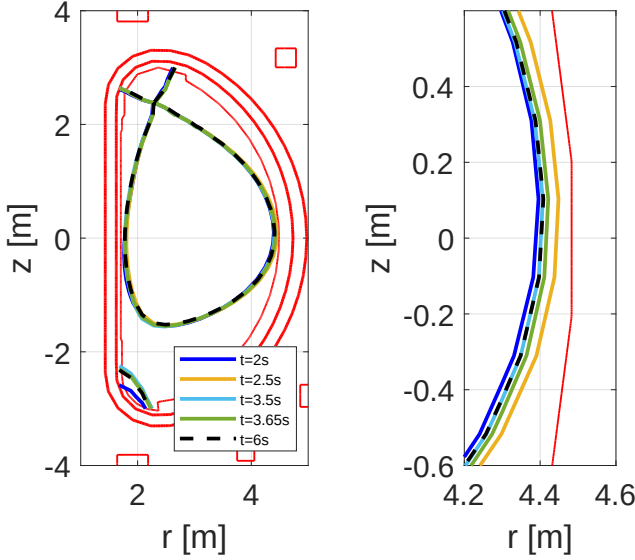


Figure 4.13: Plasma shapes obtained during a nonlinear simulations of the I_p ramp-up scenario considered in Section 4.4.2, when the fast disturbance are compensated by the Plasma Fast Boundary Controller.

since the stabilization of the plasma is paramount, this controller must act along an effective direction, without interference from the other magnetic control loops. To achieve such an objective, in the proposed architecture, the other magnetic control tasks act in directions orthogonal to the one chosen for the VS. Specifically, the Fast Boundary Controller acts along a direction that maximizes the vertical field, while the Plasma Boundary and Plasma Current Controller, since it is possible to obtain satisfactory performances even acting along sub-optimal directions, are designed to achieve effective actuator sharing with the previously designed controllers.

The effectiveness of the approach is proved by means of nonlinear validation against a possible operative scenario for the first JT-60SA operation, but the architecture and the corresponding algorithm can be applied to any superconductive tokamak, including ITER. Furthermore, the proposed model-based geometric decoupling is shown to be able to effectively deal with all the magnetic control tasks. Similar techniques will be ex-

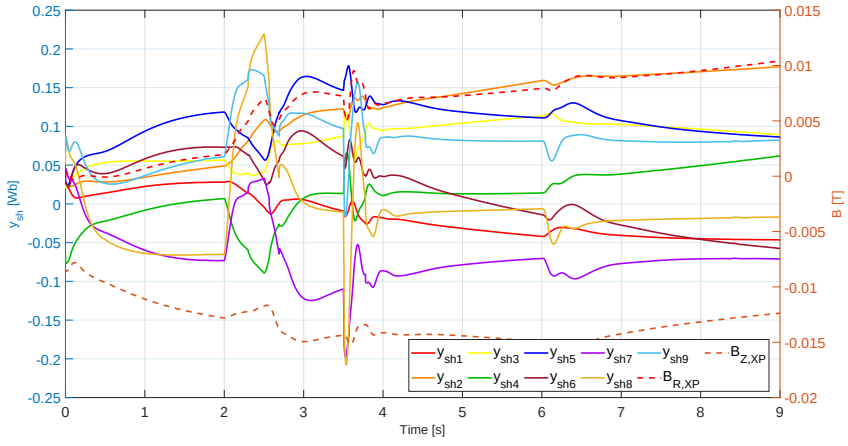


Figure 4.14: Control errors at the control points shown in Figure 4.3a for a nonlinear simulation of the I_p ramp-up scenario considered in Section 4.4.2. These time traces refer to the case of closed-loop compensation of the fast disturbance via the Plasma Fast Boundary Controller.

ploited in the next chapter as well, where a possible magnetic control architecture for the RFX-mod2 machine is proposed.

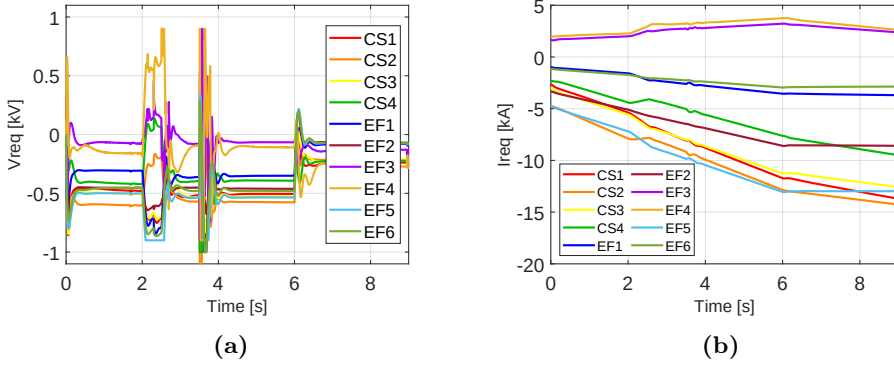


Figure 4.15: Time traces of the PF voltages (a) and currents (b) for a nonlinear simulation of the I_p ramp-up scenario considered in Section 4.4.2. These time traces refer to the case of closed-loop compensation of the fast disturbance via the Plasma Fast Boundary Controller.

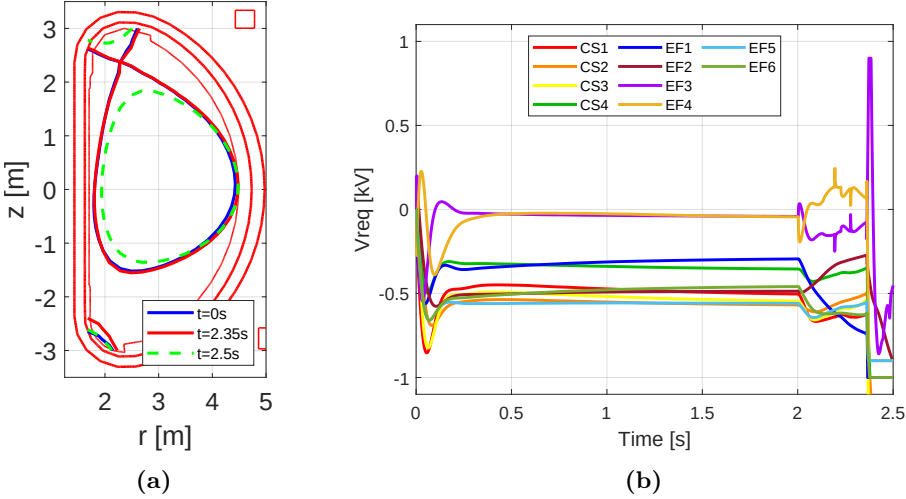


Figure 4.16: Plasma shapes snapshots (a) and PF voltages time traces (b) for a nonlinear simulation of the I_p ramp-up scenario considered in Section 4.4.2. These time traces refer to the case without the closed-loop compensation of the fast disturbance via the Plasma Fast Boundary Controller.

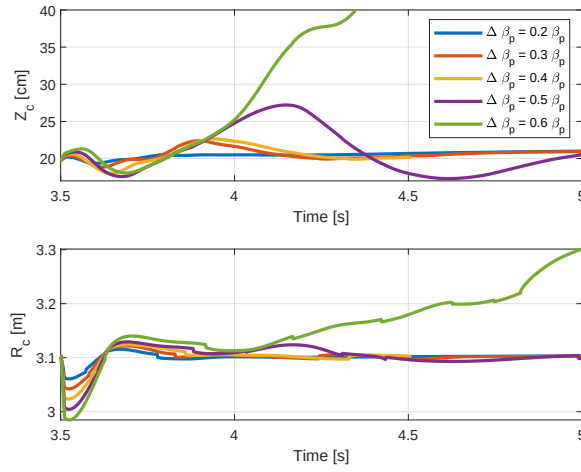


Figure 4.17: Time traces of the plasma centroid vertical and radial position for nonlinear simulations of the I_p ramp-up scenario considered in Section 4.4.2 with a poloidal beta variation $\Delta\beta_p$ spanning from $0.2 \cdot \bar{\beta}_p = 0.1$ to $0.6 \cdot \bar{\beta}_p = 0.3$. These time traces refer to the case of closed-loop compensation of the fast disturbance via the Plasma Fast Boundary Controller.

RFX-mod2 magnetic control system

5.1 RFX-mod upgrade: RFX-mod2

The RFX-mod toroidal device (major radius 2 [m], minor radius 0.5 [m]) has been exploited for more than ten years for nuclear fusion experiments in a reverse field pinch configuration, allowing to achieve the expected designed performances, i.e., a plasma current $I_p = 2$ MA and a temperature $T = 1.5$ KeV, and thus demonstrating the possibility to create a self-organized helical plasma discharges with increased confinement via a suitable saddle coil system [82]. Due to the high flexibility, the RFX-mod device has been thought for operating both in tokamak and reverse field pinch configuration [84, 149]. In tokamak configurations, even without systems for additional heating, RFX-mod could achieve H-mode plasmas thorough the operation of an elettrode [118]. In order to overcome some intrinsic limitations, a refurbishment of RFX-mod machine has brought to a new device referred to as RFX-mod2 [85], characterized by a reduced distance plasma-shell and the removal of the highly resistive Inconel vessel (see Figure 5.1), deemed responsible of a very low wall-locking threshold for tearing modes.

The assembly of RFX-mod2 is currently underway, with initial operations expected soon. Like its predecessor, the upgraded machine retains the capability to operate in both tokamak and RFP modes, owing to its

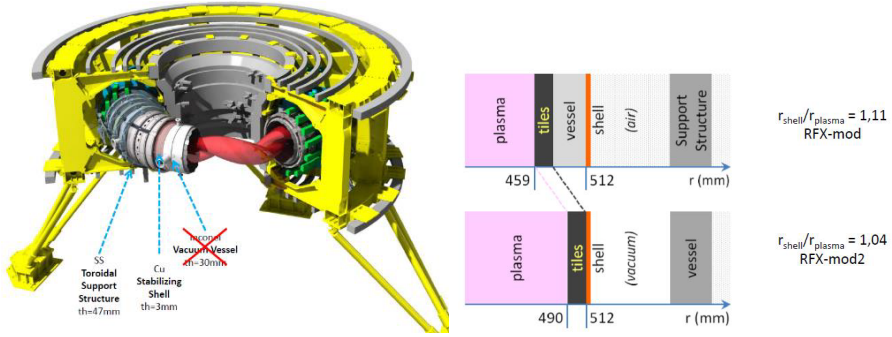


Figure 5.1: RFX-mod modification. The removal of the Inconel Vacuum Vessel gets the plasma closer to the PSS (Source [85]).

versatile power supply system [120]. For the initial operational phase, this system remains identical to that of RFX-mod and supports both symmetric and asymmetric configurations.

In both configurations, the Magnetizing Windings (MW) circuits are unchanged. In the symmetric configuration, referencing the $Z=0$ axis, each upper Field Shaping (FS) coil is connected in series with its corresponding lower coil. Conversely, in the asymmetric setup, the FS4 and FS5 coil pairs are separated—specifically, either FS4L–FS5L or FS4U–FS5U are disconnected. This setup allows the creation of upper or lower null-points within the vacuum vessel, enabling a wide range of plasma equilibrium configurations (see Figure 5.2). Thanks to this flexibility, many different plasma equilibria can be achieved. Some examples are reported in Figure 5.2. A key operational limitation arises from the use of one-quadrant AC-DC converters in the power supplies, which can only provide positive output voltages. This imposes strict constraints on the magnetic control strategies, which must explicitly account for these limitations. In addition, the RFX-mod2 operation will benefit from a largely improved and extended set of diagnostics [30], which will allow for a better analysis of the experiments in both tokamak and RFP configurations.

Moreover, to keep the graphite first-wall equipotential, an electromagnetic cage has been designed (see Figure 5.4). This cage is composed of both poloidal and toroidal conductors, which can support passive currents during plasma discharges, potentially affecting the plasma equilibrium.

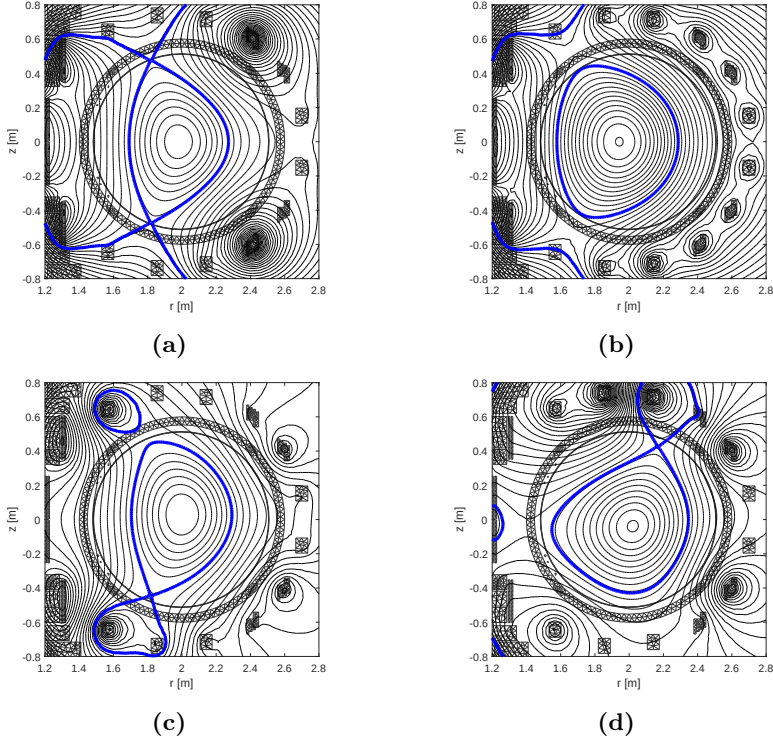


Figure 5.2: Some plasma configurations achievable thanks to the flexibility of the power supply system. The Double Null equilibrium is obtained with the symmetric power supply configuration. The Single Null equilibrium is obtained in the asymmetric configuration.

Therefore, they need to be explicitly considered in the new electromagnetic model of RFX-mod2.

In this chapter, we will focus on the design of the PF coil current controller, that can be used in plasmaless discharges, presented also in [1], and of a control system for RFX-mod2 tokamak plasma equilibrium control. The first is usually designed on the basis of a plasmaless model, i.e. of an electromagnetic model of the machine obtained without considering the presence of the plasma. Both proposed control system exploit the MPC approach, which is well-assessed in different engineering domains, ranging from energy systems [58] to intelligent transportation systems [23]. The

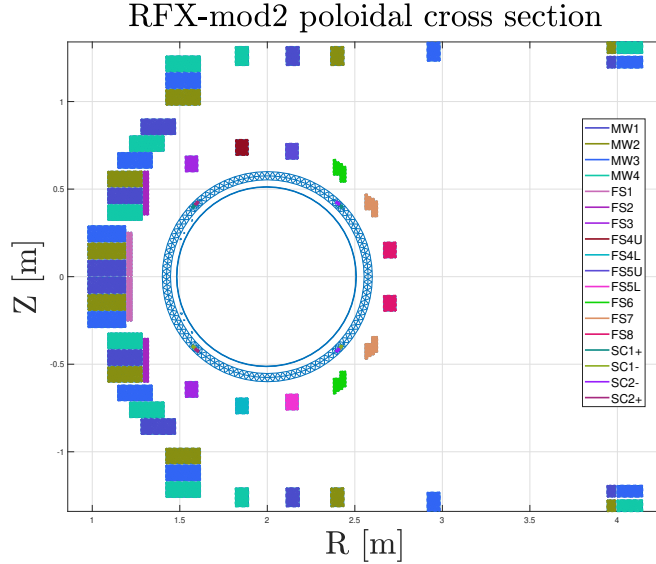


Figure 5.3: RFX-mod2 poloidal cross section. The poloidal field coils system and the new RFX-mod2 vessel are shown. The MW are the farthest from the vessel, while the FS coils are placed around it. Typically the MW are dedicated to the plasma current control while the FS coils control the plasma shape. The vertical stabilization is obtained with the Saddle Coils (SC) which are the closest to the plasma. Finally, it is possible to notice the new electromagnetic cage between the vessel and the support mechanical structure.

choice has been dictated by the great performances offered by this approach, together with the capability of pursuing the control objectives, simultaneously ensuring the fulfillment of state and control constraints. Note that, this last feature is crucial in this application, since there are strict physical constraints on the voltages (the control inputs) that can be requested to the one-quadrant converters feeding the PF coils, again making it hard to directly apply classical magnetic control schemes as the one shown in Figure 4.1. In a general context, MPC can be considered as a set of advanced control methods that explicitly exploit a mathematical model to predict the future behavior of the system over a time horizon, referred to as *prediction horizon*. According to this prediction, at each sampling period, the controller computes the optimal control action by solving a

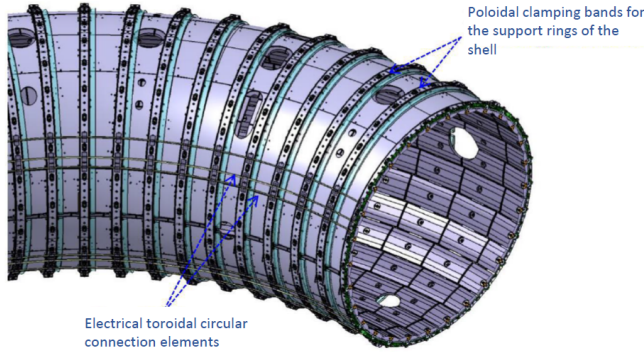


Figure 5.4: RFX-mod modification. Introduction of an electromagnetic cage to keep the first-wall equipotential. The circular connection elements are conductive and passive currents can flow inside, affecting the plasma equilibrium. (Source [85]).

constrained optimization problem. However, according to the most consolidated techniques, only the first sample of the optimal control sequence is applied, ignoring the rest. This method is referred to as *receding horizon* and ensures that only the most recent data are used.

5.2 Electromagnetic Model of RFX-mod2

The circuital connections of the PF coil of the RFX-mod2 device are shown in Figure 5.5. In particular RFX-mod2 has 4 MW coils, and 12 FS coils, whose intersection with the poloidal plane is color coded in Figure 5.3. These coils are connected to 12 one-quadrant a.c.-d.c. converters ($PCAT_1$ - $PCAT_4$, $PVAT_1$ - $PVAT_8$) according to the circuit drawn in Figure 5.5. Depending on the symmetric or asymmetric configuration, the circuit shown in Figure 5.5 needs to be consequently adapted.

The geometry (spatial positions, number of turns, electrical connections) of the coils and of the electrical conducting passive structures are provided as input to the CREATE-L code [3], which is a finite-element code able to process them and to provide as outputs inductance and resistance matrices, allowing to obtain a linear model in the form

$$\tilde{\mathbf{L}}\dot{\tilde{\mathbf{x}}}(t) + \tilde{\mathbf{R}}\tilde{\mathbf{x}}(t) = \tilde{\mathbf{S}}\mathbf{u}(t), \quad (5.1)$$

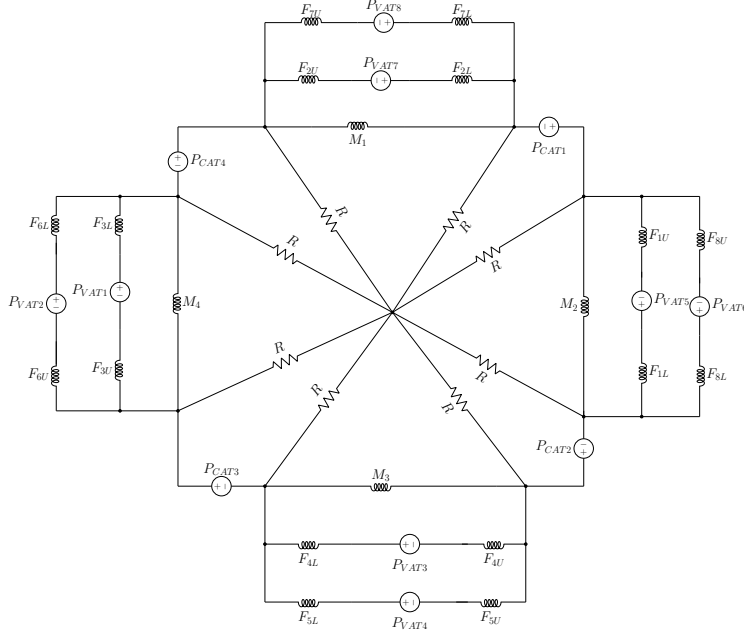


Figure 5.5: The circuitual connection of the Poloidal Field coils in symmetric configuration. In asymmetric configuration the F5U (F5L) and F4U (F4L) are short-circuited (Source [1]).

where the state vector $\tilde{\mathbf{x}}(t) = [\tilde{\mathbf{x}}_a^\top(t) \ \tilde{\mathbf{x}}_e^\top(t)]^\top \in \mathbb{R}^{138}$ contains a set of mesh currents, defined by the CREATE-L code, which allows to describe the currents flowing in the PF coils ($\tilde{\mathbf{x}}_a(t)$) and the passive structures ($\tilde{\mathbf{x}}_e(t)$), the input vector $\mathbf{u}(t) \in \mathbb{R}^{12}$ is the vector of the voltages requested to the PCAT and PVAT power supplies. The matrix $\tilde{\mathbf{L}} = [\tilde{\mathbf{L}}_{ij}] \in \mathbb{R}^{138 \times 138}$ is the inductance matrix, $\tilde{\mathbf{R}} = [\tilde{\mathbf{R}}_{ij}] \in \mathbb{R}^{138 \times 138}$ is the resistance matrix, and finally $\tilde{\mathbf{S}} = [\tilde{\mathbf{S}}_{ij}] \in \mathbb{R}^{138 \times 12}$ is the matrix describing the links among the input voltages and the mesh currents. Hence, $\tilde{\mathbf{S}}_{ij} \neq 0$ if voltage j acts on the mesh current i . Since the PF coil system in RFX-mod2 is unchanged with respect to RFX-mod, and since the eddy currents have a negligible effect on the active currents in plasmaless discharges, it is possible to validate the model in equation (5.1) by looking at the input-output behavior from the active voltages to the PF coil currents of RFX-mod discharges. The PF coil currents $\mathbf{x}_a(t)$ can be linked to the mesh

currents $\tilde{\mathbf{x}}_a(t)$ by an invertible linear transformation $\Phi \in \mathbb{R}^{12 \times 12}$, in such a way that $\mathbf{x}_a(t) = \Phi^{-1} \tilde{\mathbf{x}}_a(t)$, hence it is possible to introduce the full matrix transformation $\tilde{\mathbf{T}} = \text{diag}\{\Phi, I_{126}\}$, with I_{126} the identity matrix of dimensions expressed by the index, and obtain the model

$$\mathbf{L}\dot{\mathbf{x}}(t) + \mathbf{R}\mathbf{x}(t) = \mathbf{S}u(t), \quad (5.2)$$

where $\mathbf{L} = \tilde{\mathbf{T}}^\top \tilde{L} \tilde{\mathbf{T}}$, and $\mathbf{R} = \tilde{\mathbf{T}}^\top \tilde{R} \tilde{\mathbf{T}}$, maintain the physic meaning of inductance and resistance matrices, $\mathbf{S} = \tilde{\mathbf{T}}^\top \tilde{\mathbf{S}}$, and the PF coil currents appear as the first 12 state variables. Figure 5.6 shows the result of the validation of the model (5.2) made by using the experimental data of the RFX-mod discharge #19051. Although the agreement between experimental and simulated PF coil currents is not perfect, it is sufficiently accurate for control design purposes. Note also that a mismatch is anyway expected due to the difference in the machine configuration, as previously discussed. Furthermore, exact matching is not needed since the PF current controller should be sufficiently robust to deal with the uncertainties due to the presence of the plasma which is much larger than the one observed on the plasmaless model. Note that similar validation results are obtained on other RFX-mod plasmaless discharges.

Although there are some differences in the PF coil current behaviors, the agreement between the simulated and the experimental data seems to be sufficient to allow the use of the model to design the controller.

5.3 Constrained PF coil currents control via MPC

A PF coil current controller is useful not only for equilibrium control, as shown in the previous Chapter, but also for plasmaless discharges, in which it is used mostly to test the coil system and to calibrate magnetic sensors. For RFX-mod2, a nested control scheme like the one shown in Figure 4.2 is not favorable. In fact, its short pulses (0.5 s) call for very fast controllers. This is why the PC coils current controller will not be used for equilibrium control. The PF coil currents controller is designed on the basis of the model described in Section 5.2. This control produces the voltages for two actuator groups, i.e., the 4 power supplies named *PCAT*, and the 8 power supplies *PVAT* [49]. The most used approach to control

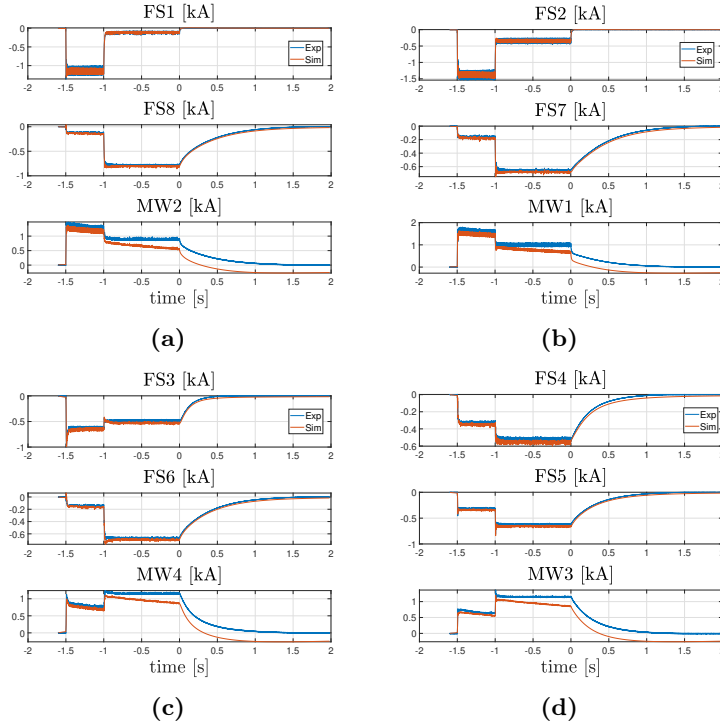


Figure 5.6: Open-loop comparative analysis of currents in FS and MW systems between the full model in (5.2) and the experimental data. The current time trajectories of full model are represented in red while the experimental currents in blue.

the PF coil currents is to neglect the effect of the eddy currents and to design a state feedback matrix which assigns a given closed loop behavior when considering the matrix transfer function from the current references to the current waveforms [21]. This approach is not applicable to RFX-mod2 due to the fact that the power supplies are one-quadrant thyristor bridges, which are not able to switch the polarity of the applied voltages. In other words, the control signals are constrained to be non-negative, this is a requirement that in general is difficult to account for in the design phase of the controller. Moreover, the maximum value for the power supply voltages is 1350 V.

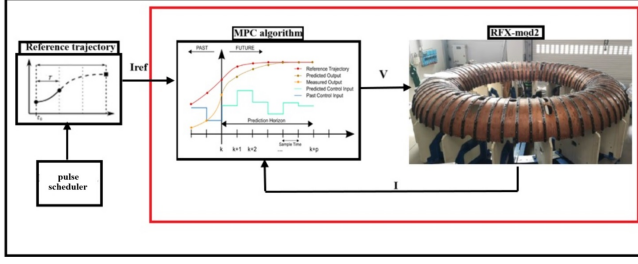


Figure 5.7: Overview of the MPC-based control for RFX-mod2 [24].

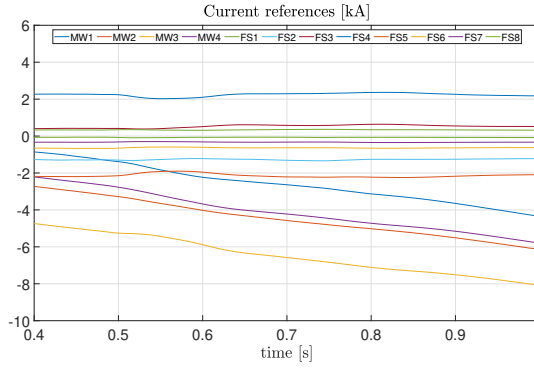


Figure 5.8: Time trajectories of current references obtained via experimental data

To face this issue, the powerful MPC approach is herein proposed. Accordingly, we first obtain a reduced order model by (5.2), neglecting the eddy currents then, from a suitable sampling time choice T_s [s], a discrete-time model is obtained in such a way that the behavior of the PF coil currents is described by the difference equation

$$\mathbf{x}_a(k+1) = \mathbf{A}\mathbf{x}_a(k) + \mathbf{B}\mathbf{u}(k). \quad (5.3)$$

The MPC-based control architecture is depicted in Figure 5.7. Accordingly, the current reference trajectories are obtained by the pulse scheduler (which is the tools used to define all the reference waveform and control parameters of a discharge) and the MPC-based control aims at ensuring the optimal tracking by evaluating the input voltages

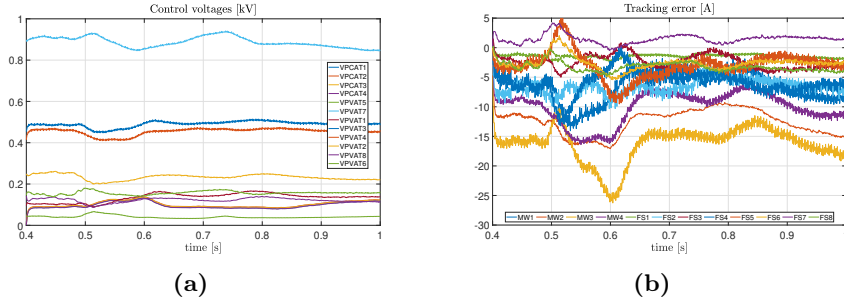


Figure 5.9: Time trajectories of: (a) optimal applied voltages obtained via the MPC-based architecture; (b) currents tracking errors.

$\mathbf{u}(t)$ to be required to the power supplies.

Specifically, exploiting the discrete reduced order mathematical model in (5.3), the first step in the design of an MPC system consists in evaluating the predicted trajectories of the state over the optimization time window $T_p \in \mathbb{N}_+$, obtaining the plant prediction model. After, given the current references $I_{ref}(t) \in \mathbb{R}^{12}$, the control system aim is to bring the predicted PF coil currents as close as possible to the reference signals, assuming that these remains constant in the optimization window. In doing so, we introduce the tracking error $\mathbf{e}_I(t) = \mathbf{x}_a(t) - I_{ref}(t) \in \mathbb{R}^{12}$ with the objective of minimizing it via an optimal strategy able to ensure at the same time the voltage constraints fulfillment. The control voltage trajectories are then evaluated over the time window namely *control horizon* $T_c \in \mathbb{N}_+$, with $T_c \leq T_p$, leveraging on the performance of numerical algorithms in solving quadratic programming problems. However, according to the receding horizon approach, only the first sample of the optimal control action is applied to the plant, while the other samples are discarded. In the next sampling period, other PF coil current measurements are available and the routine can start again.

In doing so, the optimal current references tracking problem can be for-

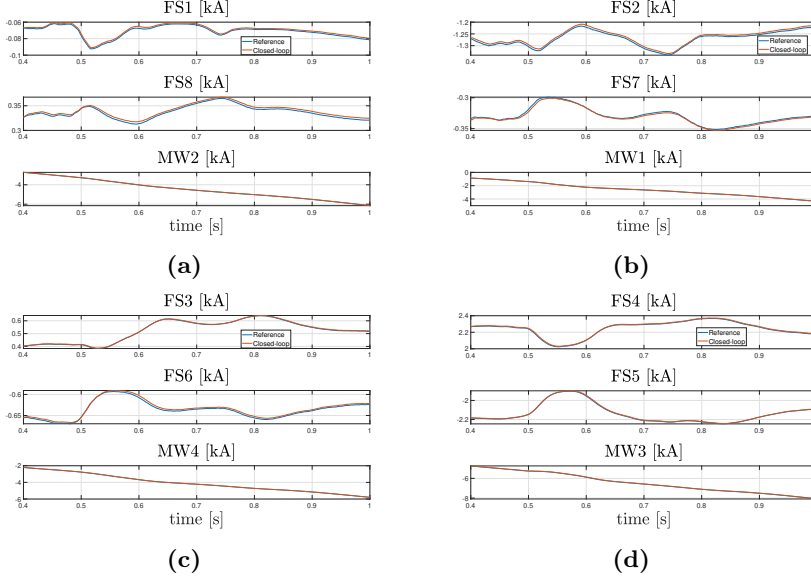


Figure 5.10: Closed-loop comparative analysis of currents in FS and MW systems between the full model in (5.2) and the experimental data. The current time trajectories of the full model are represented in red while the experimental currents in blue.

malized in the following manner:

$$\min_{\mathbf{U}_p} \sum_{i=0}^{T_p-1} \|\mathbf{e}_I(k+i)\|_Q^2 + \sum_{i=0}^{T_c-1} \|\mathbf{u}(k+i)\|_R^2 \quad (5.4)$$

$$\text{s.t. } \mathbf{x}_a(k+1) = \mathbf{A}\mathbf{x}_a(k) + \mathbf{B}\mathbf{u}(k), \quad (5.5)$$

$$\mathbf{u}(k) \in [\mathbf{u}_{\min}, \mathbf{u}_{\max}] \ V, \quad (5.6)$$

where $\|\xi\|_F^2 = \xi^\top F \xi$, F and ξ being a generic positive definite matrix and a vector of suitable dimensions, respectively, while Q and R are the positive definite weighting matrices of the optimization problem. Finally, $u_{\min} = 0$, $u_{\max} = 1350 \text{ V}$. Note that, the optimization problem in (5.3) can be solved by exploiting the prediction model, obtained from (5.3) and applying an

iterative procedure on the base of the equation:

$$\mathbf{X}_p = \mathbf{A}_p \mathbf{x}_a(k) + \mathbf{B}_p \mathbf{U}_p, \quad (5.7)$$

being $x_a(k)$ the current measurements at the step k ,

$$\mathbf{X}_p = [\mathbf{x}_a^\top(k+1) \quad \mathbf{x}_a^\top(k+2) \quad \cdots \quad \mathbf{x}_a^\top(k+T_p)]^\top,$$

$$\mathbf{U}_p = [\mathbf{u}^\top(k) \quad \mathbf{u}^\top(k+1) \quad \mathbf{u}^\top(k+2) \quad \cdots \quad \mathbf{u}^\top(k+T_c-1)]^\top$$

and with $\mathbf{A}_p$ and $\mathbf{B}_p$ defined as

$$\mathbf{A}_p = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^{T_p} \end{bmatrix}, \quad \mathbf{B}_p = \begin{bmatrix} \mathbf{B} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \cdots & \mathbf{0} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{A}^{T_p-1}\mathbf{B} & \mathbf{A}^{T_p-2}\mathbf{B} & \cdots & \mathbf{A}^{T_p-T_c}\mathbf{B} \end{bmatrix}. \quad (5.8)$$

Note that, the proposed formulation allows the optimal tracking of the current references, imposing at the same time the constraints on the voltages, such that the control inputs have a physical meaning.

The performance of the controller are assessed by means of numerical simulation in the MATLAB/Simulink[©] platform, by using the full model of Equation (5.2). The current references in the field shaping coils and the magnetizing windings are depicted in Figure 5.8. Figure 5.9 shows the results obtained by asking the MPC controller to track the PF coil currents of the discharge #39122 of RFX-mod in the time interval $t \in [0.4, 1] s$, whereas the voltages requested by the MPC controller are depicted in Figure 5.9a. It is possible to notice that they satisfy the imposed constraints, guaranteeing (see Figure 5.9b) an absolute tracking error smaller than 30 A (with an absolute relative error less than 5%). The results are further corroborated in Figure 5.10, in which the simulated time behavior of the FS and MWs coil currents are compared with their references.

5.4 RFX-mod2 equilibrium control

As previously anticipated, the proposed control system architecture for equilibrium control in RFX-mod2 is not *current-driven*, that is, it does not

rely on a dedicated PF current controller. In current-driven architectures, one of the main advantages lies in the ability to decouple and choose the coil dynamics and provide desired scenario currents directly as reference inputs. This approach tends to be more intuitive and accessible, particularly when current references can be followed without violating system constraints.

However, for RFX-mod2, the presence of stringent physical and operational limitations makes the implementation of a standard current controller, such as the one described in Section 4.2.1, unfeasible. Incorporating these constraints within classic control strategies is usually challenging. Therefore, the design choice was made to exclude the current controller from the control loop architecture. Instead, the plasma shape and plasma current controllers directly compute the reference voltages. This not only simplifies the control architecture but also improves its overall responsiveness, an important factor given the relatively short duration of RFX-mod2 pulses.

The proposed control architecture is composed of the following components:

- **VS Controller:** A fast controller that acts directly on the dedicated saddle coils to maintain vertical stability.
- **Plasma Current Controller:** Regulates the plasma current along a control direction specifically chosen to minimize interaction with the plasma shape.
- **MPC-based Plasma Shape Controller:** A controller that governs the plasma shape evolution without interfering with the current controller, while ensuring that the computed voltage references respect all power supply constraints.

The complete control architecture is illustrated in Figure 5.11.

For the design of this control architecture, let us refer again to the same linear model (4.1a), where the system's state has the following structure,

$$\mathbf{x}(t) = [\delta \mathbf{I}_{PF} \quad \delta \mathbf{I}_{SC} \quad \delta \mathbf{I}_e \quad \delta I_p]^T,$$

where $\delta I_{PF} = [\delta \mathbf{I}_{MW} \quad \delta \mathbf{I}_{FS}]^T$ is the vector representative of the currents flowing in the MW and FS circuits. On the other hand, the input vector

integration of the VS control action within the MPC loop used for plasma shape control. As a result, the effects of vertical stabilization can be more easily incorporated into the overall plasma control strategy.

To formalize the vertical stabilization controller, consider the output equation

$$z_c = \mathbf{C}_z \mathbf{x}(t),$$

where z_c represents the vertical position of the plasma. The VS control input is then defined as

$$\mathbf{u}_{SC}(t) = -K_{SC} \mathbf{C}_z \mathbf{x}(t),$$

such that the closed-loop matrix $\mathbf{A}_z = \mathbf{A} - \mathbf{B}_{SC} K_{SC} \mathbf{C}_z$ has all eigenvalues with negative real parts, ensuring system stability.

5.4.2 Plasma Current Control

The plasma current controller is designed based on the stabilized system model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_z \mathbf{x}(t) + \mathbf{B} (\mathbf{u}_{I_p}(t) + \mathbf{u}_{sh}(t)), \quad (5.9)$$

where $u_{I_p}(t)$ is the control input generated by the PI controller for plasma current regulation, and $u_{sh}(t)$ is the control input from the shape controller.

In the original RFX-mod system, plasma current control was implemented via a simple SISO loop, which applied the same voltage signal to all MW. However, this approach can introduce undesired coupling between current and shape control actions.

To reduce this coupling, we exploit the fact that the shape output is given by:

$$\mathbf{y}_{sh}(t) = \mathbf{C}_{sh} \mathbf{x}(t).$$

Restricting $\mathbf{C}_{sh}$ to the PF (Poloidal Field) coils, which include both MW and FS circuits, we define the shape-relevant output due to PF currents as:

$$\mathbf{y}_{sh}^{\text{PF}}(t) = \mathbf{C}_{sh}^{\text{PF}} \delta \mathbf{I}_{\text{PF}}(t),$$

where $\mathbf{C}_{sh}^{\text{PF}} \in \mathbb{R}^{n_{sh} \times n_{\text{PF}}}$, and $\delta \mathbf{I}_{\text{PF}}(t)$ denotes the variation in PF coil currents.

Taking the derivative with respect to time:

$$\dot{\mathbf{y}}_{\text{sh}}^{\text{PF}}(t) = \mathbf{C}_{\text{sh}}^{\text{PF}} \delta \dot{I}_{\text{PF}}(t) = \mathbf{C}_{\text{sh}}^{\text{PF}} \mathbf{A}^{\text{PF}} \delta \mathbf{I}_{\text{PF}}(t) + \mathbf{C}_{\text{sh}}^{\text{PF}} \mathbf{B}^{\text{PF}} (\mathbf{u}_{I_p}(t) + \mathbf{u}_{\text{sh}}(t)),$$

where $\mathbf{A}^{\text{PF}}$ and $\mathbf{B}^{\text{PF}} \in \mathbb{R}^{n_{\text{PF}} \times n_{\text{PF}}}$ are the components of the system matrices restricted to the PF subspace.

To identify control directions that do not influence the shape, we compute an orthonormal basis $\mathcal{E} \in \mathbb{R}^{n_{\text{PF}} \times (n_{\text{PF}} - n_{\text{sh}})}$ for the null space of $\mathbf{C}_{\text{sh}}^{\text{PF}} \mathbf{B}^{\text{PF}}$, such that:

$$\mathbf{C}_{\text{sh}}^{\text{PF}} \mathbf{B}^{\text{PF}} \mathcal{E} = 0.$$

The basis $\mathcal{E}$ spans directions in PF current space that do not affect plasma shape.

To design a decoupled SISO plasma current controller, we select a vector $\mathcal{V}_{I_p} \in \mathbb{R}^{n_{\text{PF}}}$ from the subspace spanned by $\mathcal{E}$, such that the coupling between the corresponding active currents and the plasma current is maximized. The controller is then defined in the Laplace domain as:

$$U_{I_p}(s) = \mathcal{V}_{I_p} \cdot K_{I_p}(s) (I_{p\text{ref}}(s) - I_p(s)), \quad (5.10)$$

where $K_{I_p}(s)$ is a standard PI controller. This approach ensures effective plasma current regulation and, thanks to the integral action, constant disturbance rejection while minimizing influence on the shape control loop.

5.4.3 Decoupled MPC Shape Control

To ensure decoupling between current and shape control, the MPC shape controller is designed to act along directions orthogonal to those used by the current controller. Let $\mathcal{V}^\perp \in \mathbb{R}^{n_{\text{PF}} \times (n_{\text{PF}} - 1)}$ be an orthonormal matrix such that:

$$\mathcal{V}_{I_p}^\top \mathcal{V}^\perp = 0.$$

The shape control input can then be expressed as:

$$\mathbf{u}_{\text{sh}}(t) = \mathcal{V}^\perp \mathbf{u}_{\mathcal{V}_{\text{sh}}}(t),$$

where $u_{\mathcal{V}_{\text{sh}}}(t)$ is the control action in the orthogonal subspace.

From the perspective of shape control, the plasma current controller introduces a disturbance that must be accounted for. Incorporating the

control actions into the system dynamics, the model in Equation (5.9) becomes:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_z \mathbf{x}(t) + \mathbf{B} \mathcal{V}_{I_p} \mathbf{u}_{\mathcal{V}_{I_p}}(t) + \mathbf{B} \mathcal{V}^\perp \mathbf{u}_{\mathcal{V}_{sh}}(t),$$

where $\mathbf{u}_{\mathcal{V}_{I_p}}(t) = d(t)$ is given by the PI controller in Equation (5.10) and is treated as an acting known constant disturbance over the prediction time horizon.

Following the approach described in Section 5.3, we derive a discrete prediction model that accounts for both shape control inputs and the disturbance introduced by the current controller:

$$\mathbf{Y}_{sh,p} = \mathbf{C}_{sh,p} \mathbf{A}_{z_p} \mathbf{x}(k) + \mathbf{C}_{sh,p} \mathbf{B}_p \mathcal{V}^\perp \mathbf{U}_p \mathcal{V}^\perp + \mathbf{C}_{sh,p} \mathbf{B}_p \mathcal{V}_{I_p} d_p, \quad (5.11)$$

where $\mathbf{Y}_{sh,p}$ are the predicted shape outputs (e.g., gap variations), $\mathbf{U}_p \mathcal{V}^\perp$ is the future sequence of shape control inputs, $\mathbf{x}(k)$ is the current state, and d_p is the disturbance replicated over the optimization horizon.

The optimization problem to be solved by the MPC controller is:

$$\min_{\mathbf{U}_p} \sum_{i=0}^{T_p-1} \|\mathbf{e}_{sh}(k+i)\|_Q^2 + \sum_{i=0}^{T_c-1} \|\mathbf{u}_{\mathcal{V}_{sh}}(k+i)\|_R^2 \quad (5.12)$$

$$\text{s.t. } \mathcal{V}^\perp \mathbf{u}_{\mathcal{V}_{sh}}(k) + \mathcal{V}_{I_p} d + \mathbf{u}_0 \in [\mathbf{u}_{\min}, \mathbf{u}_{\max}] \quad V, \quad (5.13)$$

where $\mathbf{u}_0$ is a feedforward input, $\mathbf{e}_{sh}$ represents the shape tracking error, and prediction model (5.11) can be used in the minimization of the objective function (5.12).

Finally, to effectively predict the gap variations, the MPC uses the full state of the linearized model, which is not directly measured. This is why an observer is introduced. It can be easily designed using a reduced-order model with the classic Luenberger or Kalman filtering approaches.

5.4.4 Simulation Results

This section presents the simulation results of the plasma control system introduced earlier. The control objectives are twofold: first, to maintain a desired plasma equilibrium, and subsequently, to reach a new target equilibrium. The plasma configurations considered are shown in Figures 5.12a and 5.12b. Key equilibrium parameters for the Positive Triangu-

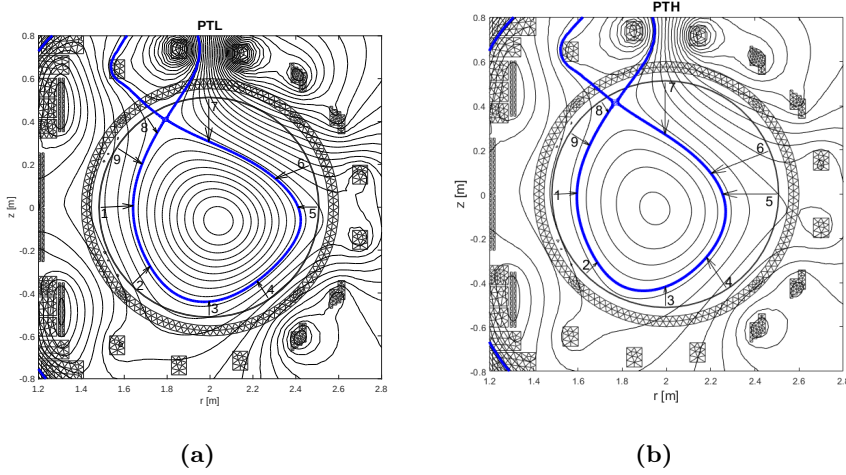


Figure 5.12: Equilibrium configurations for Positive Triangularity L-mode (PTL) and H-mode (PTH). The controlled plasma-wall gaps are indicated by black arrows.

larity L-mode (PTL) and Positive Triangularity H-mode (PTH) scenarios are summarized in Table 5.1.

Simulation setup

To assess the controller's performance, the linearized plasma model given by the CREATE-L code has been used as been used to approximate the plasma behaviour. Moreover, to model the voltage drop in the plasma circuit, a plasma resistance of $R_p = 10^{-7} \Omega$ is assumed for both configurations (with $I_p \simeq 60\text{kA}$). A worst-case scenario is considered where the voltage drop in the plasma circuit appears as a step-like function from the beginning of the pulse.

The control system is designed based on the PTL equilibrium. The vertical stabilization (VS) controller uses a proportional gain of $K_{SC} = 14 \times 10^5$, while the plasma current PI controller has the following transfer function:

$$K_{I_p}(s) = 3.62 \cdot \frac{1}{s} + 0.05.$$

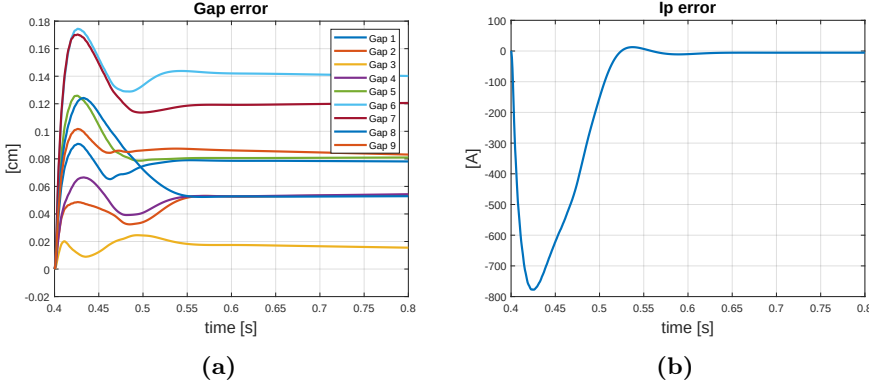


Figure 5.13: (a) Plasma-wall gap errors; (b) Plasma current tracking error. All errors are with respect to the equilibrium references.

For the MPC controller, the prediction horizon is set to $T_p = 25$, and the control horizon to $T_c = 10$, resulting in 120 optimization variables for the constrained QP problem solved at each time step. The prediction matrices used in the QP problem are derived from system (5.15), which is of order 139. Additionally, a system delay of $600 \mu\text{s}$ is included between controller output and system input. As in Section 5.3, the voltage limits on the PVAT and PCAT are $\mathbf{u}_{min} = 0 \text{ V}$ and $\mathbf{u}_{max} = 1350 \text{ V}$. Considering the SC power supply, their delay is negligible since they are much faster and the saturation limit is $\pm 600 \text{ V}$ [84, 116]. Finally, to estimate the state of the plant needed for the MPC, a reduced-order observer with 40 states is designed.

The PTL and PTH configurations have also been experimentally

Table 5.1: Main equilibrium parameters for PTL and PTH configurations.

Parameter	PTL	PTH
Plasma current [kA]	61	62
β_p	0.68	0.86
α_m	0.50	1.56
α_n	1.09	1.08
$\gamma [s^{-1}]$	1.70	3.21

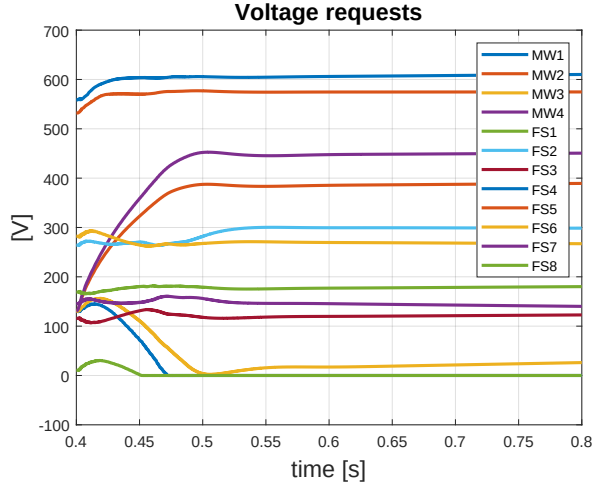


Figure 5.14: Total voltage commands applied to the PF power supplies. The MPC ensures input limits are not exceeded.

achieved in RFX-mod (pulse #39122). Therefore, the feedforward voltage $\mathbf{u}_0$ is initialized using the experimental voltage measured at $t = 0.4$ s, at which the PTL has been achieved, and subsequently the PTH. The ramp-down starts at $t = 0.8$ s

Disturbance rejection

Figure 5.13 shows the system's response to this disturbance. The corresponding gap errors are shown in Figure 5.13a, while Figure 5.13b displays the plasma current tracking error. In steady state, the maximum gap error remains below 1.5 mm, which is negligible from an experimental standpoint. Similarly, the plasma current error is minimal and remains bounded. To maintain zero steady-state error in the presence of plasma resistance, the voltage commands must follow a ramp profile.

Figure 5.14 illustrates the total voltage requests applied to the PF coils. The MPC guarantees input constraint satisfaction by appropriately adapting the voltages required for shape control, considering the contribution from the PI current controller.

PTL-PTH transition

A second test evaluates the controller's performance during a transition from the PTL to the PTH equilibrium. In this case, the controller is provided with a reduced shape variation reference, shown in Figure 5.15a, while the resulting shape tracking errors are reported in Figure 5.15b.

Although the absolute errors are small, performance is suboptimal. The MPC is constrained by the voltage commands of the plasma current PI controller, which limits its ability to track the desired shape change. These limitations prevent a full PTL-PTH transition under the current architecture.

Therefore, despite its effectiveness as an equilibrium controller, this configuration cannot ensure full shape transitions. A more advanced magnetic control architecture will be introduced in the next section to address this limitation.

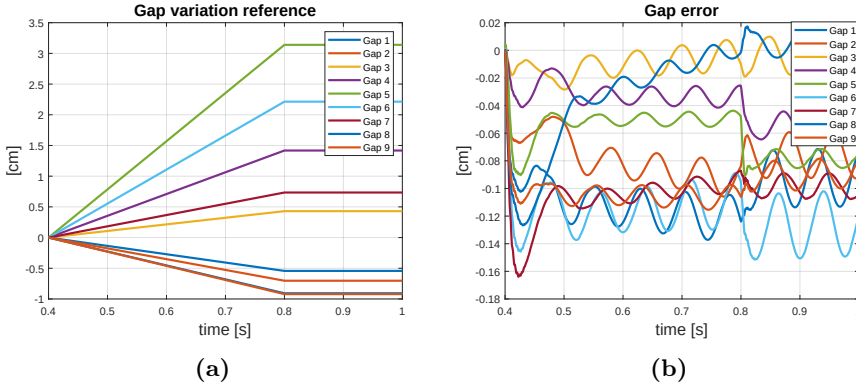


Figure 5.15: (a) Reduced gap variation reference for PTH transition; (b) Corresponding shape tracking errors.

5.5 Plasma current and shape control via Velocity-Form MPC

To improve the previous control architecture, the MPC controller must be able to regulate both plasma current and shape, while simultaneously rejecting constant disturbances on the plasma current. This can be achieved

by designing an MPC in *velocity form*, where the optimization is performed over the future incremental control inputs $\Delta \mathbf{u}$.

With a slight abuse of notation, let us consider the discretized version of system (5.9) as

$$\mathbf{x}(k+1) = \mathbf{A}_z \mathbf{x}(k) + \mathbf{B} (\mathbf{u}_{I_p}(k) + \mathbf{u}_{sh}(k)) , \quad (5.14)$$

hence using the same proportional VS controller presented in the previous section. It is still possible to decouple the control components by considering the same control directions $\mathcal{V}_{I_p}$ and $\mathcal{V}^\perp$. However, since the MPC now directly computes the entire control action, this decoupling becomes less relevant. Nevertheless, this structure can still be useful for imposing additional constraints. For example, since the MW coils are mainly used to drive the plasma current, in all RFX-mod pulses, the voltage applied to the MW coils was identical. Therefore, the same constraint can be applied in the MPC formulation to reduce the degrees of freedom. Hence, we can rewrite system (5.14) as:

$$\mathbf{x}(k+1) = \mathbf{A}_z \mathbf{x}(k) + \mathbf{B} (\mathcal{V}_{MW} \mathbf{u}_{MW}(k) + \mathcal{V}_{FS} \mathbf{u}_{FS}(k)) , \quad (5.15)$$

where $\mathcal{V}_{MW} \in \mathbb{R}^{n_{PF}}$ is defined as

$$\mathcal{V}_{MW} = [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad \cdots \quad 0]^\top ,$$

and $\mathcal{V}_{FS} \in \mathbb{R}^{n_{PF} \times (n_{PF}-1)}$ is given by

$$\mathcal{V}_{FS} = \begin{bmatrix} 0_3 & 0 \\ 0 & I_8 \end{bmatrix}^\top .$$

We define the modified input and input matrix as:

$$\tilde{\mathbf{B}} = \mathbf{B} [\mathcal{V}_{MW} \quad \mathcal{V}_{FS}] = \mathbf{B} \mathbf{T}, \quad \tilde{\mathbf{u}}(k) = \begin{bmatrix} \mathbf{u}_{MW}(k) \\ \mathbf{u}_{FS}(k) \end{bmatrix} .$$

To use $\Delta \tilde{\mathbf{u}}$ as the optimization variable, we reformulate the state-space model (5.15) in velocity form. Define

$$\Delta \mathbf{x}(k) = \mathbf{x}(k) - \mathbf{x}(k-1),$$

and the output equation as

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{C}_{I_p} \\ \mathbf{C}_{sh} \end{bmatrix} \mathbf{x}(k) + d = \mathbf{C}\mathbf{x}(k) + d,$$

where d is a constant output disturbance (e.g. a constant drop voltage on the plasma circuit). Note that constant state disturbances are canceled when using $\Delta\mathbf{x}(k)$.

Let us define the new state vector:

$$\zeta(k) = \begin{bmatrix} \Delta\mathbf{x}(k) \\ \bar{\mathbf{y}}(k) - \mathbf{y}(k) \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{x}(k) \\ \mathbf{e}(k) \end{bmatrix},$$

where $e(k)$ is the tracking error.

The system dynamics in velocity form then becomes:

$$\zeta(k+1) = \begin{bmatrix} \mathbf{A}_z & 0 \\ \mathbf{C}\mathbf{A}_z & I \end{bmatrix} \zeta(k) + \begin{bmatrix} \tilde{\mathbf{B}} \\ \mathbf{C}\tilde{\mathbf{B}} \end{bmatrix} \Delta\tilde{\mathbf{u}}(k), \quad (5.16)$$

$$\mathbf{y}_\zeta(k) = \begin{bmatrix} 0 & \mathbf{I} \end{bmatrix} \zeta(k). \quad (5.17)$$

It follows that $\mathbf{e}(k)$ is the *actual* tracking error, and the MPC control action is effectively of the form $K\zeta(k)$. This is why the velocity form MPC does not require an explicit disturbance model to reject constant disturbances. As done in the previous sections, model (5.16) can be used to construct the prediction model for the optimization of the quadratic cost function:

$$\min_{\Delta\tilde{\mathbf{U}}} \sum_{i=0}^{T_p-1} \|\mathbf{e}(k+i)\|_Q^2 + \sum_{i=0}^{T_c-1} \|\Delta\tilde{\mathbf{u}}(k+i)\|_R^2. \quad (5.18)$$

To include hard input constraints, they must be expressed in terms of

control increments:

$$\begin{bmatrix} \mathbf{u}(k+1) \\ \mathbf{u}(k+2) \\ \mathbf{u}(k+3) \\ \vdots \\ \mathbf{u}(k+T_c) \end{bmatrix} = \begin{bmatrix} I \\ I \\ I \\ \vdots \\ I \end{bmatrix} \mathbf{u}_{\text{act}}(k) + \begin{bmatrix} \mathbf{T} & 0 & 0 & \cdots & 0 \\ \mathbf{T} & \mathbf{T} & 0 & \cdots & 0 \\ \mathbf{T} & \mathbf{T} & \mathbf{T} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{T} & \mathbf{T} & \mathbf{T} & \cdots & \mathbf{T} \end{bmatrix} \begin{bmatrix} \Delta \tilde{\mathbf{u}}(k) \\ \Delta \tilde{\mathbf{u}}(k+1) \\ \Delta \tilde{\mathbf{u}}(k+2) \\ \vdots \\ \Delta \tilde{\mathbf{u}}(k+T_c-1) \end{bmatrix}, \quad (5.19)$$

where $\mathbf{u}_{\text{act}}(k)$ is the measured input applied to the system, with $\mathbf{u}_{\text{act}}(0) = \mathbf{u}_0$.

From equation (5.19), the constraints for the QP problem can be written as:

$$-(L_1 \mathbf{u}_{\text{act}}(k) + L_2 \Delta \tilde{U}) \leq -\mathbf{u}_{\min}, \quad (5.20)$$

$$L_1 \mathbf{u}_{\text{act}}(k) + L_2 \Delta \tilde{U} \leq \mathbf{u}_{\max}, \quad (5.21)$$

where

$$\Delta \tilde{U} = \begin{bmatrix} \Delta \tilde{\mathbf{u}}(k) \\ \Delta \tilde{\mathbf{u}}(k+1) \\ \Delta \tilde{\mathbf{u}}(k+2) \\ \vdots \\ \Delta \tilde{\mathbf{u}}(k+T_c-1) \end{bmatrix}$$

is the optimization variable. With this formulation, it is also possible to impose explicit bounds on the control variations, if needed. Finally, note that it is still required an observer to estimate $\Delta \mathbf{x}(k)$.

5.5.1 Simulation results

In this section, the simulation results obtained with the MPC-velocity form control are shown. The simulation setup is the same as in Section 5.4.4. The only difference is that the plasma current control is done by the MPC and the PI is eliminated from the control architecture. The same observer is employed and the same model is used to construct the prediction of the output.

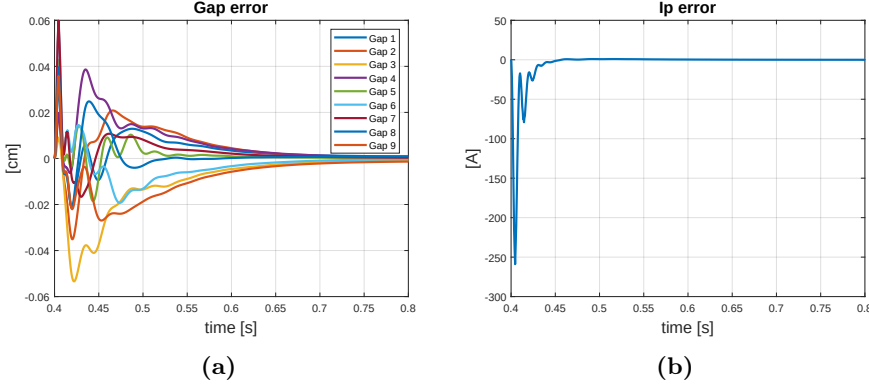


Figure 5.16: (a) Plasma-wall gap errors; (b) Plasma current error. All errors are with respect to the equilibrium references.

Disturbance rejection

The new control scheme is evaluated under the same disturbance conditions: a step-like voltage drop in the plasma circuit, with $R_p = 10^{-7} \Omega$ and $I_p \simeq 60 \text{ kA}$, applied at $t = 0.4 \text{ s}$. Figure 5.16a shows the deviations of the plasma gaps and current from their equilibrium values. Notably, the gap deviations are reduced by an order of magnitude compared to those in Figure 5.13a, and they converge to zero. This demonstrates the capability of the MPC in velocity form to effectively reject constant disturbances.

Regarding the plasma current, although some oscillations are present, the maximum deviation is limited to 250 A—significantly lower than the 700 A observed in Figure 5.13b. Finally, Figure 5.17 shows that the actuator voltage saturation limits are only reached at the end of the pulse. This behavior is due to the MPC being free to determine the full voltage reference, unlike in the previous architecture where it was constrained by the voltage demands of the PI plasma current controller.

Disturbance rejection with ELM reconstruction in the loop

To better assess the realistic performance of the proposed architecture, the ELM-based boundary reconstruction method described in Section 3.5.2 is integrated into the control loop, as illustrated in Figure 5.18. In this configuration, the ELM employs 80 neurons. Hard constraints on the ho-

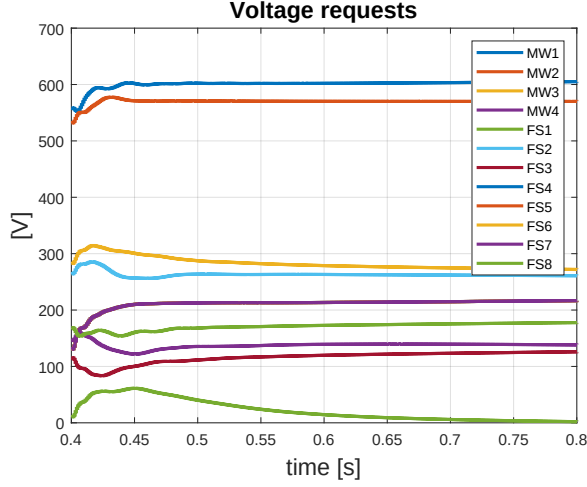


Figure 5.17: Total voltage commands applied to the PF power supplies. The MPC ensures input limits are not exceeded.

homogeneous GS equation are enforced at 24 points along the first wall, resulting in 56 degrees of freedom for the network. The available measurements consist of 56 magnetic field sensors and 8 flux loops.

The accuracy of the reconstruction algorithm is first evaluated by comparing the estimated plasma-wall gaps to the reference equilibrium provided by CREATE-L, as shown in Figure 5.12a. The corresponding errors are reported in Table 5.2, and are consistent with the reconstruction performance reported in [77]. During the pulse, the reconstructed boundary is treated as the ground truth, meaning that any reconstruction error is hidden from the controller.

Once integrated, the reconstruction module estimates the gap variations to be regulated by the MPC. This is feasible because the CREATE-L linear model links both active and passive circuit currents to the magnetic field and flux measurements. As a result, near the equilibrium point, the model provides variations in magnetic field and flux at sensor locations, which are then used to reconstruct the plasma boundary.

The final results are shown in Figure 5.19. Figure 5.19a demonstrates that the performance closely matches that in Figure 5.16a, with minimal differences. Additionally, Figure 5.19b confirms that the reconstructed

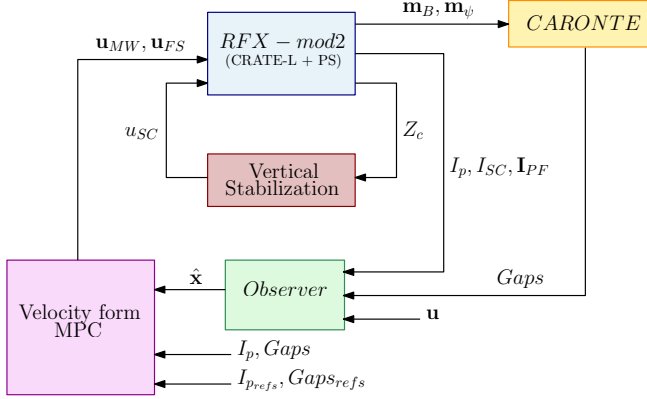


Figure 5.18: Block scheme of the velocity form MPC-based equilibrium controller for RFX-mod2. The velocity form MPC controller is capable of tracking references on both plasma current and gaps. The ELM reconstruction in the loop gives an estimation of the controlled plasma-wall gaps from the magnetic measurements. (Courtesy of Dott. Sara Dubbioso)

plasma boundary at the time of maximum error remains very close to the equilibrium shape.

PTL-PTH transition

Finally, in the previous section, a gap variation reference corresponding to a configuration transition from PTL to PTH was introduced, and the control architecture could not effectively track the reference associated with the full PTH configuration. In this section, it is demonstrated how the MPC in velocity form is not only capable of accurately tracking the desired references, but also exhibits robustness with respect to model variations.

To this end, given the linearized models of the PTL and PTH configurations, a Linear Time-Varying (LTV) model has been constructed as

Table 5.2: CARONTE gap reconstruction error with respect to equilibrium values

	G1	G2	G3	G4	G5	G6	G7	G8	G9
Error [cm]	0.13	-0.07	0.10	-0.14	0.43	0.27	0.49	0.22	-0.06

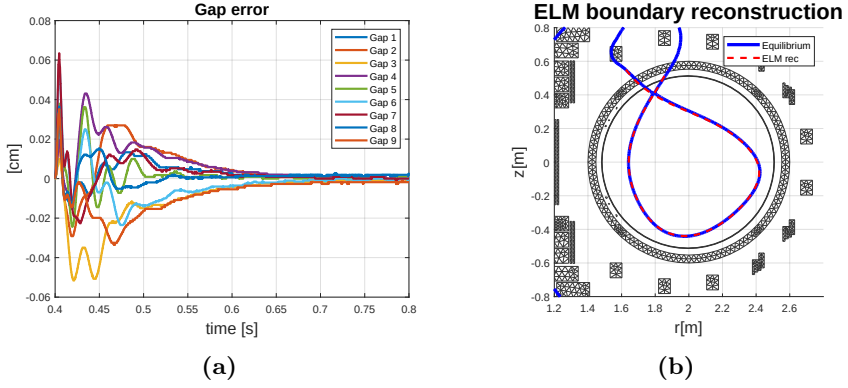


Figure 5.19: Gap errors computed from the ELM boundary reconstruction algorithm CARONTE (a). Reconstructed plasma shape at time of maximum error ($t = 0.403$ s) compared to the desired equilibrium shape (b).

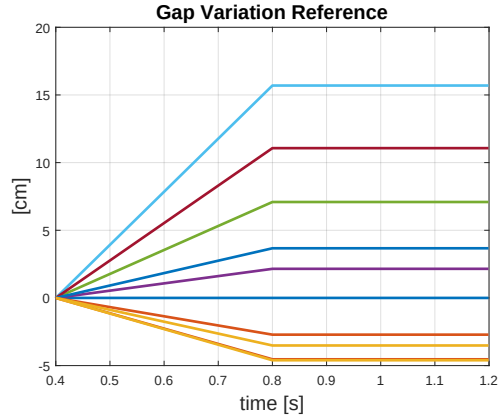


Figure 5.20: Gap variation references for a transition from PTL to PTH.

follows:

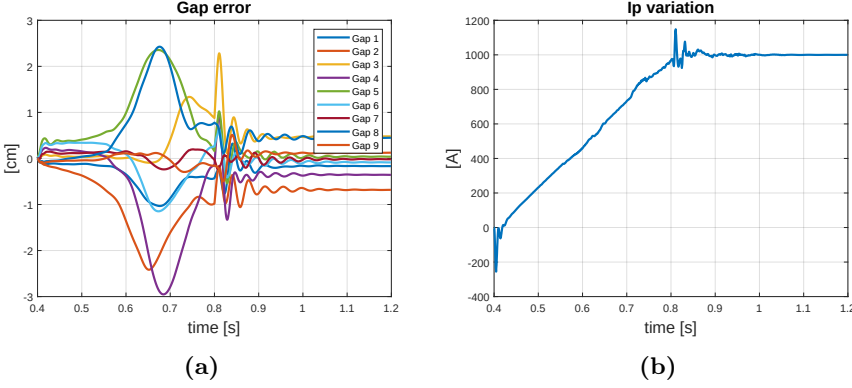


Figure 5.21: (a) Plasma-wall gap errors; (b) Plasma current variation. The gap errors are with respect to the references shown in Figure 5.20, while the plasma current reference increases of 1 kA.

$$\begin{aligned}
 \dot{\mathbf{x}}(t) &= \left(\mathbf{A}_{\text{PTL}} \left(\frac{t_f - t}{t_f - t_0} \right) + \mathbf{A}_{\text{PTH}} \left(\frac{t - t_0}{t_f - t_0} \right) \right) \mathbf{x}(t) \\
 &\quad + \left(\mathbf{B}_{\text{PTL}} \left(\frac{t_f - t}{t_f - t_0} \right) + \mathbf{B}_{\text{PTH}} \left(\frac{t - t_0}{t_f - t_0} \right) \right) \mathbf{u}(t) \\
 \mathbf{y}(t) &= \left(\mathbf{C}_{\text{PTL}} \left(\frac{t_f - t}{t_f - t_0} \right) + \mathbf{C}_{\text{PTH}} \left(\frac{t - t_0}{t_f - t_0} \right) \right) \mathbf{x}(t)
 \end{aligned} \tag{5.22}$$

At $t = t_0 = 0.4$ s, the system corresponds exactly to the PTL linearized model, while at $t = t_f = 0.8$ s, it transitions fully to the PTH model. It is important to note that simulations based on the LTV model (5.22) are not intended to be accurate representations of the physical system, as the PTL and PTH models differ significantly. Consequently, the interpolated model may not reliably capture plasma dynamics, particularly during the transition phase.

However, since the controller design is based on the PTL model, these simulations provide a meaningful test to evaluate the controller's ability to drive the system to the desired reference, even in the presence of significant and time-varying model discrepancies, as typically encountered during real pulse operations.

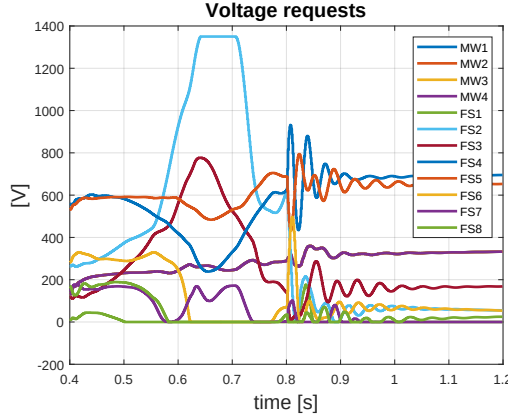


Figure 5.22: Total voltage commands applied to the PF power supplies. The MPC ensures input limits are not exceeded.

In Figure 5.20, the gap variations needed for a transition from the PTL to PTH configuration are shown. It is supposed that after 0.4 s the plasma transitions to PTH and that this equilibrium is then kept for another 0.4 s. The shape change, and therefore the plasma behavior, is considerable; this is why robustness of the controller is particularly important in this context.

Figure 5.21 summarises the results. In Figure 5.21a the gaps tracking error is reported. At the beginning of the references ramp, the error is small, then reaches a maximum of 3 cm during the transient, and then it stabilizes in the 0.6 cm range at steady-state. This obtained maximum gap error is due to the strict saturation limits being reached, as shown in Figure 5.22. Tracking these references is a challenging task for the control system, especially as the plant's behavior changes rapidly. Nevertheless, the MPC is capable of tracking the reference, adjusting the voltage requests at each time step. Moreover, the highest gap error is on Gap 1, 2, 4 and 5 which are well distant from the first wall (see Figure 5.12). Finally, the plasma current reference of a 1 kA variation, needed for the PTH transition (see Table 5.1), is also successfully tracked throughout the simulated pulse, apart from small and negligible oscillations.

QP problem optimization for improved computational performances

To enhance the computational efficiency of the control algorithm, a reduced-order model can be employed in the design of the MPC controller. Specifically, the prediction model matrices can be derived from a reduced-order approximation of system (5.15). Consequently, the state observer must also be designed based on this reduced-order model to provide the estimated reduced states required by the MPC.

Further reduction in computational burden can be achieved by decreasing the number of optimization variables. This is accomplished by constraining the control input to remain constant over certain time intervals. When the MPC is formulated in velocity form, such constraints can be easily incorporated as follows:

$$\begin{aligned}\Delta \tilde{\mathbf{u}}(k+1) &= 0, \\ \Delta \tilde{\mathbf{u}}(k+3) &= 0, \\ &\vdots \\ \Delta \tilde{\mathbf{u}}(k+T_c-1) &= 0,\end{aligned}\tag{5.23}$$

where T_c denotes the control horizon over which the input is held constant.

Figure 5.23 summarizes the results obtained using a reduced-order model of order 40 for both the prediction and observer models and, thanks to constraints (5.23), 60 optimization variables (half of the ones used in the previous sections). While the performance in gap control remains comparable to that of the full-order implementation, a slightly more oscillatory behavior is observed in the plasma current control.

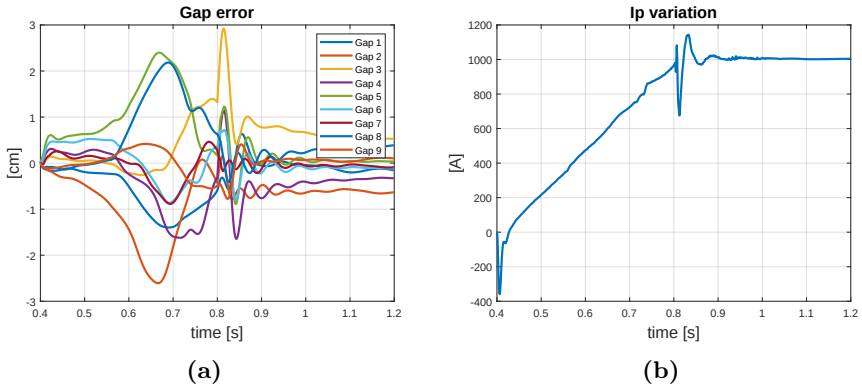


Figure 5.23: (a) Plasma-wall gap errors; (b) Plasma current variation. The gap errors are with respect to the references shown in Figure 5.20, while the plasma current reference increases of 1 kA.

Conclusions and future work

In this work, the performance of the official ITER reconstruction algorithms has been analyzed using experimental data from JT-60SA IC discharges.

First, it was demonstrated that the measurement noise of the magnetic sensors does not significantly affect the algorithms' performance. The reconstructed plasma parameters were compared with CREATE-NL equilibria, used as a ground truth reference. The results show that the algorithms' performances do not always satisfy ITER requirements. Although the CCS algorithm generally exhibits superior performance, both algorithms successfully meet the requirement for plasma current estimation.

Regarding plasma position estimation, the ITER reconstruction algorithm is more reliable due to its robustness, even though the requirements are not met. In fact, the CCS method assumes a single filament current source that does not always fit well with broadened current distributions. Note also that both algorithms, and in particular LIUQE, present higher shape reconstruction error during the ramp-up phase, as expected.

In terms of β_p estimation, CCS provides more accurate results, as it directly computes the Shafranov integrals. In contrast, Equation (3.8) employs a simplified formula designed to reduce computational cost during quench scenarios. Nevertheless, considering the limited number of available magnetic measurements, the obtained results are encouraging for future ITER and JT-60SA operations, where an extended magnetic sensor set will be available. These findings were further validated through

a benchmark comparison using CCS-based estimations. During JT-60SA OP2, the operational space will be expanded, allowing this same analysis to be extended to high- β , H-mode, and lower-divertor configurations.

A fast neural-network-based plasma boundary reconstruction algorithm has also been proposed. The specific network architecture used, the ELM, is particularly suitable for real-time training. Each time a new set of measurements becomes available, the network's weights are retrained to optimally fit the updated data.

The main advantage of this approach lies in its simplicity and computational efficiency. Minimizing the loss function (3.19) requires only an offline pseudo-inverse computation. Then a single online matrix multiplication suffices to reconstruct the entire flux map and poloidal magnetic field. The algorithm tuning is also simplified, as the number of neurons, which is the main network parameter, is directly constrained by the number of measurements.

In contrast, other reconstruction techniques typically involve local approximations of the poloidal flux function or explicit identification of current sources. These approaches often require handling eddy currents by introducing distributed current sources throughout the vessel, which increases complexity and computational cost. Furthermore, many existing methods rely on a fixed spatial grid, whereas the proposed ELM-based method allows evaluation of the poloidal flux function at arbitrary points once the model parameters are determined.

Compared to other neural-network-based reconstruction techniques proposed for equilibrium or real-time LCFS estimation, the proposed method is significantly more efficient. Unlike classical deep neural networks, which require extensive pre-training, preprocessing, and large datasets, the ELM retraining its weights online with each new set of measurements, eliminating the heavy computational burden associated with deep network training.

Regarding magnetic control, an architecture has been developed to explicitly stabilize the plasma column vertically and counteract rapid plasma displacements in the absence of a dedicated in-vessel actuator. This control structure is particularly suited to the initial operation phases of superconducting tokamaks, when not all in-vessel components are yet installed.

To efficiently share the limited number of superconducting actuators among multiple magnetic control tasks, a model-based geometric control

design has been adopted. However, using a current-driven control scheme inherently increases the plasma's vertical instability growth rate. Consequently, the architecture requires an effective VS controller, even when no dedicated fast actuator is available.

In the proposed scheme, the VS controller is designed first, ensuring it acts along the most effective direction with minimal interference from other control loops. Subsequent controllers operate in directions orthogonal to that used for VS. Specifically, the Fast Boundary Controller acts along a direction that maximizes the vertical field, while the Plasma Boundary and Plasma Current Controllers operate along suboptimal directions, enabling effective actuator sharing and decoupled control actions.

The effectiveness of this architecture has been validated through nonlinear simulations reproducing realistic scenarios for the first JT-60SA operations. The same approach can be readily applied to any superconducting tokamak, including ITER. Moreover, the model-based geometric decoupling technique demonstrated here effectively manages all key magnetic control tasks.

Finally, an architecture for the magnetic control of RFX-mod2 has been proposed. Due to asymmetric constraints on the power supplies, a MPC strategy was adopted. Several tests were performed: initially, a PI controller was used to regulate the plasma current and compensate for constant disturbances such as voltage drops in the plasma circuit, while the MPC only controlled the plasma shape. While this controller provided satisfactory equilibrium regulation, it could not accurately track a gap reference necessary for transitions from PTL to PTH configurations.

To address this limitation, a velocity-form MPC controller was developed. This controller proved robust against constant disturbances and was extensively tested, showing excellent disturbance rejection and reference tracking capabilities. Further tests were conducted with the ELM-based reconstruction algorithm integrated into the control loop, as well as with a reduced-order model used for output prediction. Additional constraints were introduced to simplify the QP problem, enabling real-time feasibility.

Future work will focus on several directions:

- Reassessing the ITER reconstruction algorithms on JT-60SA OP2 data, once the full magnetic sensor set becomes available, to evaluate

the expected improvements in reconstruction accuracy.

- Experimentally testing the ELM-based reconstruction method, particularly by imposing Grad-Shafranov (GS) constraints on the estimated first open flux surface.
- Implementing the MPC controller on dedicated real-time hardware to evaluate its computational feasibility and performance in an operational environment.

These developments will further advance the readiness of magnetic reconstruction and control strategies for tokamak experiments.

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