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Star Tracking Control Architectures for Ground-based Telescopes: the Telescopio Nazionale Galileo Case

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SCUOLA POLITECNICA E DELLE SCIENZE DI BASE
DIPARTIMENTO DI INGEGNERIA ELETTRICA E DELLE TECNOLOGIE DELL'INFORMAZIONE

*The future belongs to those who believe in the beauty
of their dreams.*
Eleanor Roosevelt

STAR TRACKING CONTROL ARCHITECTURES FOR GROUND-BASED TELESCOPES: THE TELESCOPIO NAZIONALE GALILEO CASE

Ph.D. Thesis presented
for the fulfillment of the Degree of Doctor of Philosophy
in Information Technology and Electrical Engineering
by

GIACOMO BASILE

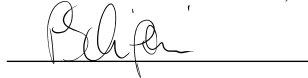
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Candidate's declaration

I hereby declare that this thesis submitted to obtain the academic degree of Philosophiæ Doctor (Ph.D.) in Information Technology and Electrical Engineering is my own unaided work, that I have not used other than the sources indicated, and that all direct and indirect sources are acknowledged as references.

Parts of this dissertation have been published in international journals and/or conference articles (see list of the author's publications at the end of the thesis).

Napoli, January 2, 2025

A handwritten signature in black ink, reading "Giacomo Basile", is written over a horizontal line.

Giacomo Basile

Abstract

Ground-based telescopes play a key role in astronomical research by observing celestial objects from the Earth's surface. These telescopes are equipped with large mirrors or lenses to collect and focus light from distant stars, planets, galaxies, and other cosmic phenomena. They are essential for a wide range of studies, including stellar evolution, planetary formation, and the detection of exoplanets. Indeed, compared to space-based telescopes, ground-based telescopes offer the advantage of being more easily maintained and upgraded making them invaluable tools for long-term astronomical surveys and real-time observation of transient events, such as supernovae and gamma-ray bursts. However, even if they are limited by atmospheric distortion, optimal seeing and light pollution, ground-based telescopes are expected to remain a cornerstone of observational astronomy thanks to continued technological innovations. Indeed, several innovations have been provided over the past year to overcome the mentioned limitations and improve astronomical observations. In particular, new adaptive optics systems are being deployed to address atmospheric distortion, while more suitable sites have been proposed and protected in order to mitigate light pollution and improve seeing. Based on this, and thanks to the improvement of industrial technologies, ever larger telescopes are being designed so to allow the study of farther celestial bodies. In this framework, the novel ground-based telescopes require a tracking control system more accurate in terms of both hardware and control algorithm.

The aim of the thesis is to provide new control solutions based on optimal approaches which aim at improving the ground-based telescope tracking control systems by leading them to achieve more accurate performance.

Keywords: Ground-Based Telescope, Telescope Tracking Control, Optimal Control, LQG, Model-Predictive Control, TNG.

Sintesi in lingua italiana

I telescopi terrestri giocano un ruolo chiave nella ricerca astronomica permettendo l'osservazione gli corpi oggetti direttamente dalla superficie terrestre. Questi sono dotati di grandi specchi o lenti per raccogliere e mettere a fuoco la luce proveniente da stelle, pianeti, galassie e altri fenomeni cosmici distanti. Sono essenziali per una vasta gamma di studi, tra cui l'evoluzione stellare, la formazione planetaria e la rilevazione di esopianeti. Infatti, rispetto ai telescopi spaziali, i telescopi terrestri offrono il vantaggio di essere più facilmente mantenuti e aggiornati, rendendoli strumenti preziosi per indagini astronomiche a lungo termine e osservazioni in tempo reale di eventi transitori, come supernove e raggi gamma. Tuttavia, anche se sono limitati dalla distorsione atmosferica, dall'inquinamento luminoso, ci si aspetta che i telescopi terrestri rimangano una pietra miliare dell'astronomia grazie alle continue innovazioni tecnologiche. Infatti, negli ultimi anni sono state introdotte diverse innovazioni per migliorare la qualità delle osservazioni astronomiche. In particolare, nuovi sistemi di ottica adattiva vengono implementati per far fronte alla distorsione atmosferica, mentre sono stati proposti siti più adatti così da mitigare gli effetti dovuti all'inquinamento luminoso e migliorare il seeing. Quindi, basandosi su ciò e grazie al miglioramento delle tecnologie industriali, si stanno progettando telescopi sempre più grandi per consentire lo studio di corpi celesti più lontani. In questo contesto, i nuovi telescopi terrestri richiedono un sistema di controllo del puntamento sempre più accurato sia in termini di hardware che di algoritmi di controllo. A tal scopo, l'obiettivo di questa tesi è fornire nuove soluzioni di controllo basate su approcci ottimali che mirano a migliorare i sistemi di controllo di puntamento dei telescopi terrestri, portandoli a ottenere prestazioni più accurate ed efficienti.

Parole chiave: Telescopi Terrestri, Sistemi Di Puntamento, Controllo Ottimo, LQG, Model-Predictive-Control, TNG.

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List of Acronyms

The following acronyms are used throughout the thesis.

AO Adaptive Optics

CPP Command Pre-Processor

CU Control Unit

DEC Declination

DFS Data Flow System

DSN Deep Space Network

E-ELT European Extremely Large Telescope

ELT Extremely Large Telescope

ExAO Extreme Adaptive Optics

FOV Field of View

FPGA Field Programmable Gate Array

GLAO Ground Layer Adaptive Optics

GPS Global Positioning System

GRBT Green Bank Radio Telescope

GTC Gran Telescopio Canarias

GUI Graphical User Interface

HBS Hydrostatic Bearing System

LGS Laser Guide Star

LGSAO Laser Guide Star Adaptive Optics

LGSF Laser Guide Star Facility

LQG Linear Quadratic Gaussian

LTAO Laser Taxometry Adaptive Optics

LTI Linear Time Invariant

LZT Large Zenith Telescope

MCAO Multi Conjugate Adaptive Optics

MMT Multiple Mirror Telescope

MOAO Multi Object Adaptive Optics

MPC Model Predictive Control

NGS Natural Guide Star

OS Operating System

PI Proportional-Integral

PID Proportional - Integral - Derivative

PM Prediction Model

PSD Power Spectrum Density

RA Right Ascension

RMSE Root - Mean - Square - Error

SCAO Single Conjugate Adaptive Optics

TCS Telescope Control System

TG Trajectory Generator

TNG Telescopio Nazionale Galileo

VLT Very Large Telescope

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Nomenclature

The following symbols are used within the thesis

0_{nm} Null matrix of $n \times m$ dimensions

$\dot{x}(t)$ First order derivatives of x with respect to time t

$\mathbb{N}$ The set of natural numbers

$\mathbb{R}$ The set of real numbers

$\mathbb{R}^{m \times n}$ The set of real matrices of dimensions $m \times n$

$\mathbb{S}^n$ The set of symmetric matrices of order n

saturation (x_{min}, x_{max}) refers to the following operation

$$x(t) = saturation(x_{min}, x_{max}) = \begin{cases} x_{max} & \Longleftrightarrow x(t) \geq x_{max} \\ x(t) & \Longleftrightarrow x_{min} \leq x(t) \leq x_{max} \\ x_{min} & \Longleftrightarrow x(t) \leq x_{min} \end{cases} \quad (1)$$

$A^\top$ Transpose matrix of A

I Identity matrix

I_n Identity matrix of dimension n

$M > 0$ Positive definite matrix $M \in \mathbb{S}^n$

$M \geq 0$ Positive semidefinite matrix $M \in \mathbb{S}^n$

x_{max} The upper-bound of the trajectory $x(t)$

x_{min} The lower-bound of the trajectory $x(t)$

Chapter 1

Introduction

“Nelle mie scoperte scientifiche ho appreso più col concorso della divina grazia che con i telescopi.”

Galielo Galieli

Humans have always been fascinated by the surroundings, even more, so when it comes to the cosmos. Indeed, since the 14th century, the human population has started growing rapidly thanks to technological innovations involving sanity and the food industry (for instance agriculture innovations). In this view, the human species has spread all over the world leaving unknown just some inaccessible sites of the Earth such as the ocean seabed and so on.

In doing so, attention has been captured by the surrounding cosmos which has also been driven by the fundamental desire to understand our place in the universe.

Indeed, from ancient civilizations mapping the stars to modern-day astronomers probing the depths of space, the quest to explore the cosmos is fueled by profound questions: Where did we come from? How did the universe begin? Are we alone?

Humans seek to study the cosmos to unlock the mysteries of existence, both scientific and philosophical. Moreover, by searching for the answer to those existential questions, humanity could understand how the stars and planets are a book of knowledge, indeed, they offer insights into the fundamental laws of physics, the origins of galaxies [11], and the potential

for life beyond Earth [12].

In this regard, it is worth highlighting some scientific discoveries led by the study of the cosmos, for instance, the new galaxy NGC 1599 [13], obtained via the space telescope Hubble, or the new discoveries about the possible habitable extra-solar planets (see [14]), obtained via the ground-based telescope TNG, and last but not least, the Gibbs particles [15].

It is easy to understand that, as human curiosity is driving the study of the surrounding cosmos, this latter is pushing the engineering in researching more advanced technologies which allows obtaining more accurate and meaningful data and, at the same time, study the same cosmos ever more deeply.

In this view, considering the first telescope built in the Netherlands during the 17th century by Hans Lippershey and then exploited by Galileo Galilei for the first time for astronomical purposes [16], we can claim that in less than three centuries we have obtained innovations such as the radio telescopes, infrared telescopes, and even space telescopes (at the end of the 20th century). Moreover, w.r.t. the first telescope, which could allow to observe and study the moon, nowadays, the telescopes are becoming bigger and more complex (see for instance the E-ELT [17] and the TMT [18]) allowing the study of the depth of the space.

But what does mean bigger and more complex telescope?

Unlike the first telescopes whose main mirror diameter is about $\phi 3[m]$, the novel telescopes such as TMT and E-ELT consist of several mirrors/lenses with a main mirror of $\phi 30.0[m]$ and $\phi 40.0[m]$, respectively, which improve the optics power of the instruments. Indeed, by increasing the telescope's mirror dimension, the telescope improves in terms of: *i*) Light-Gathering Power: A larger mirror can collect more light from distant stars, galaxies, and other celestial objects. The more light a telescope gathers, the better it can observe faint objects at greater distances. This is especially important for studying the early universe, distant galaxies, and exoplanets that emit faint light; *ii*) Resolution: Larger mirrors enable telescopes to achieve higher resolution, meaning they can distinguish between objects that are close together in the sky. This helps astronomers capture more detailed images and make precise measurements of celestial bodies, such as the structure of distant galaxies or the atmospheres of exoplanets; *iii*) Better Sensitivity: As the mirror size increases, telescopes become more sensi-

tive to a wider range of wavelengths, including visible, infrared, and radio waves. This broadens the scope of observations, allowing astronomers to study phenomena across the electromagnetic spectrum, such as the formation of stars, the motion of galaxies, and the detection of gravitational waves; *iv*) Imperceptible objects and deeper universe exploration: Bigger mirrors enable astronomers to observe fainter objects, which are often located at the farthest edges of the observable universe. This helps scientists study the formation of the earliest galaxies, the nature of dark matter, and the expansion of the universe, providing insights into cosmology and the origins of the cosmos.

In summary, the trend toward larger mirrors in telescopes is driven by the need to observe more distant and faint objects, improve image clarity, and enhance the overall capability of telescopes to explore inaccessible areas of the universe.

This is a crucial aspect of taking astronomical research one step further. In doing so, ground-based telescopes are a promising solution since, unlike space telescopes, they are not limited by issues such as installation and maintenance, which would be extremely complicated, especially if they had to be performed in orbit. Nevertheless, ground-based telescopes suffer optic distortions due to the presence of the atmosphere. Note that the atmosphere, as it is composed of different means, distorts the wavefront coming from the light source.

To this end, in the last years, many new active optics systems are been designed to overcome this limitation. These novel instruments are called Adaptive Optics (AO) systems and, by leveraging the so-called deformable mirrors (DM), i.e. mirrors whose shape is controllable, these systems aim to properly adjust the mirrors' shape to correct the captured wavefront of the light beam.

These instruments can further improve the observation even if they are intrinsically restricted by physical and design issues such as: *i*) the DMs cannot be reshaped in a continuous fashion, namely, the DMs consist of a finite number of segmented mirrors, hence, they can just adjust their shape in a discrete fashion; *ii*) the AO systems require the estimation of the wavefront distortion to properly drive the mirrors' shape. Moreover, the estimation and correction can be performed only for a specific atmospheric altitude.

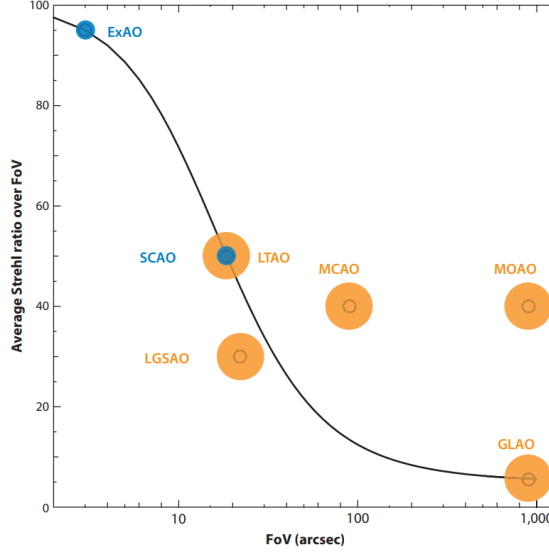


Figure 1.1. Taxometry of the designed AO system over the years. From [1].

In this view, as reported in Figure 1.1, we can define the different AO systems based on their complexity and objective to achieve. Hence, we have:

- i. **Single Conjugate Adaptive Optics (SCAO):** it is the first AO solution. Thanks to the estimated distortion obtained through a near-known star, the SCAO adjust the shape of the DMs to obtain a more accurate image, for instance, see Figure 1.2(a) that discloses the improved observation performance. However, of all the atmospheric volume, it is able to manage only on layer of it.
- ii. **Laser Guide Star Adaptive Optics (LGSAO):** Since the cosmos is not full of stars, during the observation it can occur that there are not any stars near the scientific target that allows estimating the wavefront distortion. Therefore, in the lack of natural stars, to properly estimate the atmospheric distortions, the novel telescope observatory includes the so-called Laser Guide Star Facility (LGSF), namely, lasers which are shot in the atmosphere in order to estimate

the optic distortions. In this regard, those systems present their own AO system. The exemplary LGSF of the VLT observatory is depicted in Figure 1.2(b).

- iii. **Ground Layer Adaptive Optics (GLAO):** The GLAO system is the optics system which aims to mitigate the atmospheric distortions at the so-called ground layer, namely the lowest layer of the atmosphere. They are a specialisation of the SCAO systems. Specifically, with a uniform correction quality over the entire FoV, it aims at improving the image over the FoV. An exemplary image is reported in Figure 1.2(c).
 - iv. **Multi Conjugate Adaptive Optics (MCAO):** it is the combination of several multiple SCAO systems. Indeed, its advantage is to properly correct the atmospheric distortions to different altitudes at the same time, hence, mitigating the distortion due to the entire volume. An exemplary MCAO system is MAORY/MORFEO which will be equipped on the new E-ELT [19].
 - v. **Multi Object Adaptive Optics (MOAO):** Different from the MCAO systems, the MOAO aims at achieving the same image quality but with a larger FoV. This is possible by correcting each image area via separate DMs. However, to apply those corrections, the MOAO systems require the estimation of the entire atmospheric tomography which can be ensured by exploiting several LGS and NGS in order to properly manage the several DMs demands. Finally, the system requires the solution of huge mathematical systems in a short time to fit the required compensation quality and speed.
 - vi. **Laser Tomography Adaptive Optics (LTAO):** as mentioned, the exploitation of the several LGS and NGS to reshape the DMs allows to adoption of the MOAO system. To this end, the instruments allowing this task are called LTAO systems and aim to estimate the atmospheric tomography.
 - vii. **Extreme Adaptive Optics (ExAO):** it is a specific solution among the AO system. Specifically, it allows to capture of high-resolution images of exoplanets by correcting the distortion thanks to the es-
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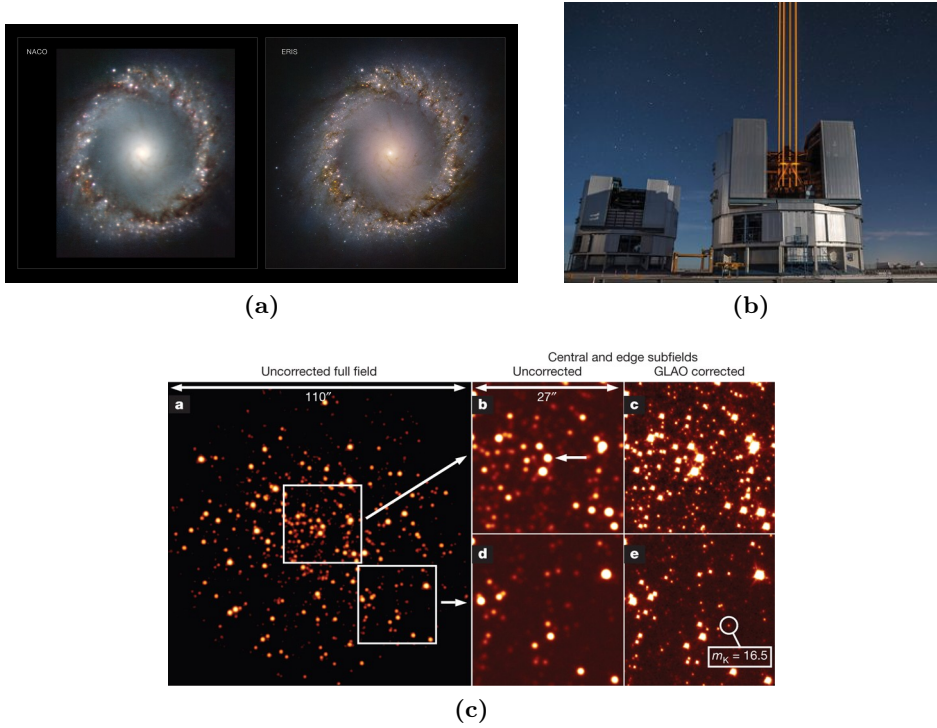


Figure 1.2. Exemplary AO systems and their performances: (a) NGC 1097 galaxy from the VLT telescope, left side the "uncorrected", right side the corrected image via ERIS SCAO instrument [2]; (b) VLT Laser Guide Facility [3]; (c) depth space image from the GLAO system equipped on the MMT telescope [4].

timination of the distortion carried by leveraging the near star to the planet.

Note that, Figure 1.1 represents how much the AO system improves the quality of the image, i.e. the Strehl ratio, w.r.t. the FOV. To sum up, as the literature discloses, the research is moving toward the improvement of the optics system to deploy more complex and bigger ground-based telescopes which also implies having efficient OA systems so to obtain more accurate and clear images.

However, as far as the scientific target is far, the more accurate the motion

control of the telescopes has to be.

To this end, in recent years, the research has stopped to investigate and innovate the motion system as well as the tracking control system for telescopes. Therefore, from the oldest [20] to the newest [21], the telescope tracking system leverages PID-based or frequency-based and there are several reasons for these choices. Firstly, the time designated for telescope installation. Indeed, from the telescope concept to the installation, just a few years are assigned to install the new facility/observatory limiting the investigation of novel control architectures or tracking approaches. Another important factor is the absence of a nonlinear telescope model or the lack of a very established identification procedure tailored for the telescopes. Indeed, due to the short time available, the dynamic models of the telescopes are always identified via approaches as frequency-based which, until today, are the most assessed in the control technical literature.

Nevertheless, these strategies do not allow the investigation of new approaches such as MPC, robust and adaptive approaches since they are depicted to derive dynamical models which allow the designing of the frequency-based approaches.

In this view, this thesis aims to provide a novel solution for the tracking control of ground-based telescopes able to improve tracking accuracy. Specifically, the proposed tracking architectures are tailored for the specific case of study Telescopio Nazionale Galileo (TNG) whose dynamics are identified by investigating different identification approaches. Finally, in the view of future experimental testing, herein, simulation results, obtained through an own-made advanced simulation platform (realized in *Matlab&Simulink*), confirm the capability of the proposed solutions in ensuring optimal star tracking while mitigating the wind external disturbance forces. Finally, to better highlight the drawbacks and benefits of the proposed architectures, an extensive comparison analysis including both the solutions and the traditional tracking algorithm in [10] is reported.

1.1 Thesis Outline

The thesis is structured as follows:

- in Chapter 2 the emerging ground-based telescope systems are introduced by highlighting the main novel technologies allowing the

improved performance. Moreover, to better highlight the limits of the control approaches exploited in literature, the traditional hardware control set-up for the emerging ELT program is presented.

- in Chapter 3 the tracking control problem for the ground-based telescope is formulated including an assessment of the state-of-the-art which highlights the limitations of the existing control architectures. Moreover, the main contributions of the proposed control architectures w.r.t. the technical literature are drawn.
 - in Chapter 4 is devoted to presenting the designing procedure of the proposed control architectures. Firstly, since both solutions embed the same trajectory generator, to better understand the capability of the proposed architectures in ensuring the physical telescope's trajectories, the trajectory generator's details are provided. Then, the design procedure leading to the first solution is presented. Herein, with the aim of highlighting all the architectural features, its closed-loop stability along with the robustness features are provided. Finally, with the aim of providing a robust tracking control w.r.t. the wind disturbance and the noise measurements, the second solution is designed by leveraging an MPC supported by a Kalman Filter and herein its designing procedure along with the closed-loop stability is derived.
 - in Chapter 5, moving toward the TNG tracking upgrading project, herein, both solutions are tailored for the specific case of study. Specifically, with the aim of carrying out a thorough numerical validation, we first present the identification procedure tailored for the TNG and its results. Then, to disclose the effectiveness of the proposed solutions, two tracking scenarios are performed, namely, the first consists of the tracking of a pseudo star near the singularity condition (the telescope is almost upright) while the former is the tracking of the real star TYC 1731-916-1. Finally, a comparison analysis of both proposed solutions w.r.t. the control architecture provided in [22] discloses the improved tracking performance despite the presence of input wind torque disturbance and noise measurements.
-

- in Chapter 6.
- in Chapter 7 conclusion are drawn.

Chapter 2

The Groud-based Telescope

A telescope¹ is an instrument that, exploiting the property of the light, allow to observe distant objects. Indeed, when a photon, i.e. the quantum of the electromagnetic field, hits an object it can, with respect to the impacted surface, react with the following phenomena [23]: *i*) **Transmission**: occurring when light waves pass through an object without a change of direction; *ii*) **Absorption**: the photons, incoming from a light source, hit the object's atoms and molecules (e.g., pavement) and make them vibrate. The more an object's molecules move and vibrate, the hotter the object becomes. If enough energy is absorbed, it may then be remitted from the object as infrared, or thermal energy; *iii*) **Scattering**: occurs when a photon hits an object and bounces off it in one or many directions. The scattering can change the wavelength of the light or leave it unchanged; *iv*) **Reflection**: when incoming light hits an object and is redirected in a new and predictable direction. Smooth surfaces, like mirrors, or the still surface of a pond, reflect almost all the light; *v*) **Refraction**: the light slows down very slightly and changes direction as it passes from one medium to another with a different density. This occurs, for example, as light travels between air and water; *vi*) **Diffraction**: the incoming light, hitting an object, breaking up into many pieces and going

¹**Etymology**: The word telescope was coined in 1611 by the Greek mathematician Giovanni Demisiani for one of Galileo Galilei's instruments presented at a banquet at the Accademia dei Lincei. In the Starry Messenger, Galileo had used the Latin term *perspiciillum*. The root of the word is from the Ancient Greek $\tau\eta\lambda\epsilon$, romanized *tele* 'far' and $\sigma\kappa\omicron\pi\epsilon\tilde{\iota}\nu$, *skopein* 'to look or see'; $\tau\eta\lambda\epsilon\sigma\kappa\omicron\pi\zeta$, *teleskopos* 'far-seeing'.

in many directions.

Originally the telescope was made as an optical instrument which, based on the absorbing and refraction properties of the lights and thanks to lenses and curved mirrors, lead a beam of light into a sensor such as a human eyes or camera, allowing the observation and study of distant objects. Nowadays, the word "*telescope*" defines a wide range of instruments capable of detecting wide bands of the electromagnetic spectrum and are leveraged to study and understand the surrounding galaxy.

In this way, the first known telescope, which belongs to the refracting telescopes, was invented in the Netherlands at the beginning of the 17th century. Thanks to glass lenses, this tool allows studying distant objects in the visible spectrum, e.g. the TNG (Figure 2.1(a)), the Magellan telescopes facility and the LZT.

Some century later, namely in the 20th century, many new types of telescopes were invented, including radio telescopes [24] (1930) and infrared telescopes (1960)[25]. Those latter, by leveraging new instruments such as spectrometers, sensors able to detect the density of the absorbed photons, or the infrared sensors, which allow to detect and capture the $750 [nm]$ to $50 [\mu m]$ wavelength, have opened up the study of the no-visible electromagnetic spectrum. Exemplary radio and infrared are the FAST (Figure 2.1(b)) and TAO telescope (Figure 2.1(c)), respectively.

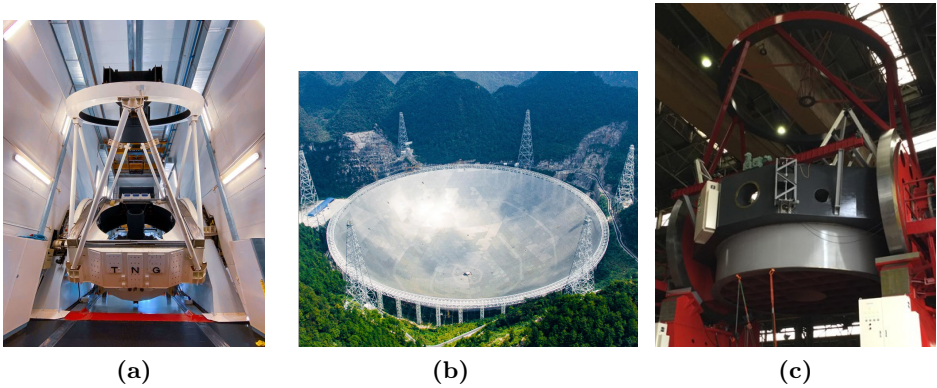


Figure 2.1. Ground-based Telescope: (a) The TNG optical telescope; (b) FAST the radio telescope; (c) TAO telescope infrared telescope.

Nevertheless, all telescopes mentioned above suffer from big limitations due to the presence of the atmosphere, i.e. a layer of gasses whose particles, depending on the gas property, are able to distort the light beam that comes from the sky. Indeed, due to this latter, some bands of the electromagnetic spectrum such as X-ray or far-infrared bands cannot reach the ground of the Earth [26] limiting the operativity of the ground-based telescopes, while others, such as the visible spectrum, are distorted, hence, deteriorating the observation.

To this end, to overcome these limitations, in the last century (specifically, 1970 – 1990), the first space telescopes were launched into orbit. Among them, play a crucial role, the first X-ray space telescopes HEAO-2 [27] (1978) and IRAS [28] (1983) and the first optic space telescope Hubble [29] (1990).

Thanks to his features, space telescopes such as Hubble earned a lot of attention since, for the first time, astrophysics had access to the non-visible light bands from the Earth. Furthermore, the deployment of a space telescope is advantageous not only to overcome the limitation of the observable spectrum but also due to issues such as clouds, light pollution, low humidity and atmospheric turbulence which are denoted by the astronomical seeing² parameter [30].

Conversely, it is easy to understand that space telescopes, such as Hubble or James Webb [31], with respect to the ground-based one, are hard to maintain, upgrade and require a big project budget to be deployed [32].

To this end, in the last years, several novel ground-based telescope projects and upgrade telescope system projects have been made to mitigate the limitations of the ground-based telescope.

Firstly, thanks also to the industry development, the novel ground-based telescope programs aim to build bigger and more complex astrophysics telescopes. Therefore, the novel telescopes are built in places where the seeing is improved (lower), i.e.: 1) the clouds are not always present, i.e. in the higher place; 2) the light pollution is almost absent; 3) where the atmosphere is very dry.

Based on these features, the best places on the Earth are the Canary Is-

²In astronomy, seeing is the degradation of the image of an astronomical object due to turbulence in the atmosphere of Earth that may become visible as blurring, twinkling or variable distortion.

lands (Spain), the Nevada desert (USA), Haway Island (USA) and the Atacama Desert (Cile).

Nevertheless, even if the mentioned location can further improve the operativity of the ground-based telescope, all the new telescope programs include novel instruments aiming at mitigating atmospheric turbulence, the so-called AO system. Specifically, the AO systems, by leveraging one or several DM, whose shape is controllable, are able to correct the shape of the wavefront coming from a light source, i.e. the celestial body.

Then, among the newest ground-based telescope programs, it is worth noting that, related to the main mirror diameter dimension, we can define two main telescope classes, namely, the VLT, whose first mirror diameter belongs to $(6, 15)$ [m], and the ELT, with a diameter of $\simeq 30$ [m].

The new VLT and ELT programs along with the AO instruments, are a promising solution for studying the visible and non-visible bands of the electromagnetic spectrum from the Earth.

Finally, in what follows, the discussion will focus on the general control system architecture for optical ground-based telescopes, more in detail, on the motion control.

2.1 Groud-Based Telescope Control Architectures

The control system tailored for a ground-based telescope includes several controllers which aim to fulfil different control tasks simultaneously. Specifically, as shown in the control system developed for the VLT telescope depicted in Figure 2.2, in order to guarantee a good observation of the target celestial body, the control system includes the following macro-modules:

- **Motion Module:** consists of the Astronomical Computation module, which provides the trajectory of the celestial understudy, the Axis Control module, aiming at driving the telescope in manual and automatic modes to reach a specific position (e.g. the slewing motion), the Tracking module [33], aiming at governs the telescope motion in order to track a specific scientific target, the autoguiding module [34], depicted to detect celestial bodies in the FOV so to adjust the tracking command to properly centring the camera on the scientific target, and the pointing modelling module [35] which,

based on the computed references, provides more dense and precise references for improving the tracking performance;

- **Active Optics Modules:** are all the modules which aim to improve the observation by mitigating the atmospheric turbulence. To this end, the Active Optics module [34] is the main one and, by determining the magnitude and type of aberrations affecting the observation, it provides the proper corrected shape to the DM. Then, based on the correct shape reference, the Optical Control module computes the single control torques in order to properly reshape the DMs accordingly;
- **Control Support Modules:** as every complex control system, the TCS is supported by other systems such as the HBS [36], aiming at controlling all the devices present in the HBS pumping station and at monitoring the pressure values in the different points of the plant, the cooling system [37], which manage the temperature inside the

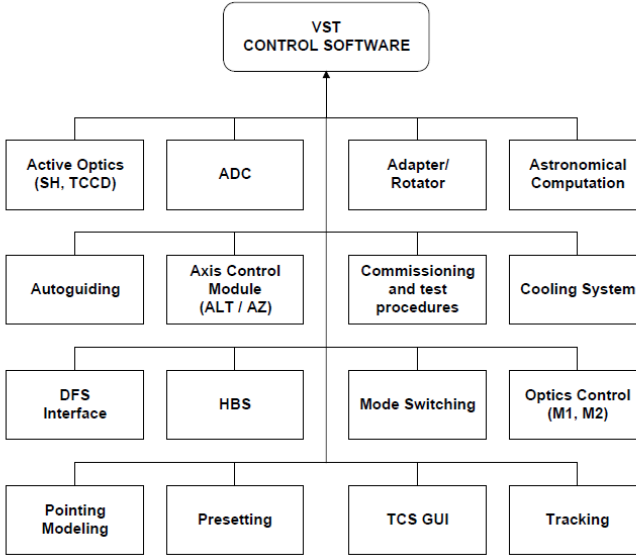


Figure 2.2. Exemplary ground-based telescope control system. VLT Control Software Module Distribution Layout from [5].

observatory dome so to avoid the dilatation of the mirrors, and the DFS [38, 39, 40] which guarantees the highest data quality.

- **Modules Interfaces:** within this macro module belong all the modules allowing the operator to check and manage all the VLT facility features, i.e. the DFS interface, TCS GUI and the external astronomical instrumentation interfaces.

It is worth noting that, while the Motion modules, the Control Support module and the Modules Interfaces are deployed directly on the main telescope CU, the AO systems are usually implemented on the local CU of astronomical instruments since they are designed according to the distorted wavelength which the instrument intends to study, e.g. infrared wave correction as MATIS [41] or the visible band correction MORFEO (well known also as MAORY) [42].

Finally, this thesis focuses on the macro-module *Motion Modules*. Specifically, the tracking telescope control problem is faced and solved via different approaches. In this view, in what follows, the technologies supporting the tracking telescope control problem are presented.

2.2 Tracking Control Technology

Herein, to better highlight the limitation and the complexity of the tracking control system for ground-based telescopes, its general control scheme is detailed. Specifically, by neglecting the coupling effect, as depicted in Figure 2.3, the ground-based telescope can be driven along two angular directions, the altitude and the azimuth angles, by imposing two independent torque commands acting on the respective axis. In this view, the tracking control architecture consists of:

- **Astronomical Computation:** this module is depicted for the computation of the scientific target trajectories. Specifically, based on the geographical telescope location in the GPS coordinates and the sidereal time [43], the module computes the scientific target trajectories to be tracked in the horizontal coordinate system, i.e. the altitude and azimuth coordinates.

- **Telescope Trajectory Generator:** Based on the scientific target reference trajectories, computed via the astronomical computation, the trajectory generator module aims to compute the physically admissible telescope trajectory to be tracked for both axes. In this view, by constraining the telescope positions and velocities, this module ensures the tracking of the scientific target in the safest manner.
- **Telescope Tracking Control Algorithm:** it is the main module. Specifically, based on the provided desired trajectories, the tracking control module computes the suitable torque profiles in order to properly drive the ground-based telescope in tracking the desired target star.

In what follows, we detail the astronomical computation, the TG and the main tracking control algorithms, along with its hardware details, for one of the telescope axes (the other one presents the same structure).

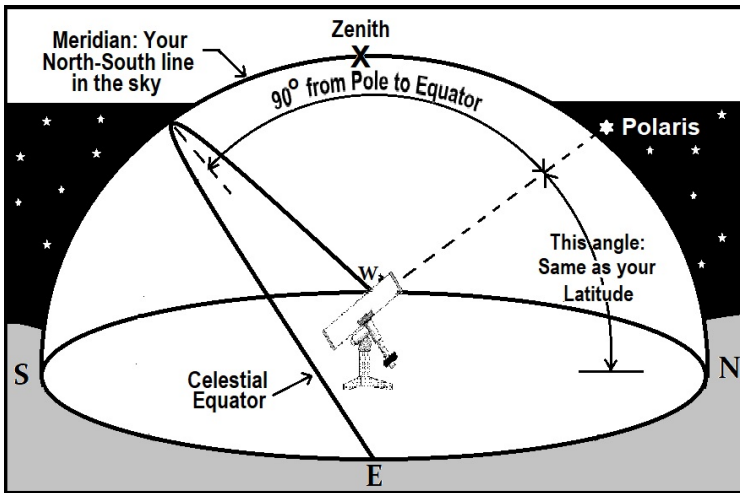


Figure 2.3. Overview of the general tracking control problem for ground-based telescope.

2.2.1 Astronomical Computation

Considering a scientific target, in the Equatorial coordinate system [44, 45], depicted in Figure 2.4, and assuming the scientific target moves with elliptic orbit over time, we want to compute its coordinates in the horizontal system, namely, the fixed telescope body-frame coordinates. In this view, considering the Equatorial coordinates system, i.e.: *i*) the RA $\bar{\alpha}$ [deg], namely the angular distance measured eastward along the celestial equator³ from the Sun at the March equinox; *ii*) the DEC $\bar{\delta}$ [deg], i.e. one of the two angles that locate a point on the celestial sphere in the equatorial coordinate system; *iii*) the hour angle $\bar{\tau}(t)$ [deg], which is the dihedral angle between the meridian plane and the hour circle [46], it is usually computed w.r.t. the sidereal time, i.e. the time w.r.t. a far star such that the hour time can be assumed equal wherever the telescope is geolocated.

To properly compute the scientific target in the horizontal coordinate, namely the altitude angle $\vartheta^*(t)$ and azimuth angle $\varphi^*(t)$, the rotation matrix is given by adopting the classical equations derived from spherical trigonometry [44], i.e.:

$$\begin{bmatrix} \cos \vartheta^*(t) \cos \varphi^*(t) \\ \cos \vartheta^*(t) \sin \varphi^*(t) \\ \sin \vartheta^*(t) \end{bmatrix} = \begin{bmatrix} \sin \bar{\lambda} & 0 & -\cos \bar{\lambda} \\ 0 & 1 & 0 \\ \cos \bar{\lambda} & 0 & -\sin \bar{\lambda} \end{bmatrix} \begin{bmatrix} \cos \bar{\delta} \cos \bar{\tau}(t) \\ \cos \bar{\delta} \sin \bar{\tau}(t) \\ \sin \bar{\delta} \end{bmatrix}. \quad (2.1)$$

Then, by dividing the first and second equation it holds that:

$$\tan \varphi^*(t) = \frac{\sin \bar{\tau}(t)}{\cos \bar{\tau}(t) \sin \bar{\lambda} - \tan \bar{\delta} \cos \bar{\lambda}}, \quad (2.2a)$$

$$\sin \vartheta^*(t) = \sin \bar{\lambda} \sin \bar{\delta} + \cos \bar{\lambda} \cos \bar{\delta} \cos \bar{\tau}(t), \quad (2.2b)$$

being $\bar{\lambda}$ [deg] the latitude.

Note that, while the RA $\bar{\alpha}$ and the DEC $\bar{\delta}$ are provided by the astrophysics, the hour angle $\bar{\tau}$ is computing according to:

$$\bar{\tau}(t) = \tilde{\tau} - \bar{\alpha} \quad (2.3)$$

³https://en.wikipedia.org/wiki/Celestial_equator

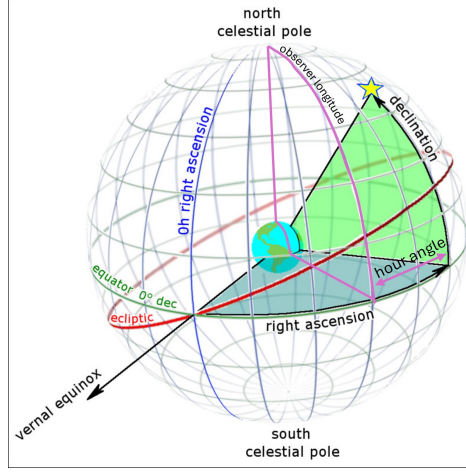


Figure 2.4. Overview of the equatorial coordination system.

being $\tilde{\tau} [deg] = 360/24 (t + \bar{\lambda}/15)$ the local sidereal time; t the current universal time, while, $360/24$ and $1/15$ are conversion factors which recast the physical quantities from second to degree. Finally, in what follows, the star positions computed via the astronomic computation are denoted with $\tilde{p}^*(t) = [\varphi^*(t), \vartheta^*(t)]$.

2.2.2 Telescope Trajectory Generator

In this section, the most exemplary and used TGs for the planning of the motion of the ground-based telescope are presented. Specifically, among all, we can find, as the baseline, these three TGs:

- i. **Variable Gain based TG:** by exploiting an adaptive gain mechanism it provides smooth trajectories by adapting itself w.r.t. the tracking error.
- ii. **Trapezoidal Speed Profile TG:** computes the suitable position trajectory by constraining this latter via a trapezoidal speed profile.
- iii. **Polynomial TG:** based on the polynomial cam design [47], this approach allows to compute a very smooth and accurate position

trajectory. However, it is the heaviest TG in terms of the computational burden.

Variable Gain Command Pre-Processor

Introduced for the NASA DSN antennas [48], the variable gain CPP has been designed in order to improve the pointing accuracy. Specifically, the variable gain CPP, shown in Figure ??(a), consists of a variable integral gain action limited via acceleration and rate limiters so to ensure the telescope's physical constraint.

To this end, its pre-processed command can be formalized as:

$$\hat{u}(k) = \hat{\zeta}(\epsilon(k)) \epsilon(k) + v(k) \quad (2.4a)$$

$$v(k) = \frac{r(k) - r(k-1)}{\Delta t}, \quad (2.4b)$$

where $\hat{u}(k)$ is the error to be integrated; $\hat{\zeta}(\epsilon(k))$ is the variable gain; $\epsilon(k) = p(k) - r(k)$ is the tracking error; $r(k)$ is the desired trajectory; $p(k)$ is pre-process command provided by the TG; Δt is the chosen sample time.

Then, the adaptive gain mechanism, i.e. $\hat{\zeta}(k)$, is formalized as:

$$\hat{\zeta}(\epsilon(k)) = \hat{\zeta}_0 + \hat{\zeta}_v e^{-\beta |\epsilon(k)|}, \quad (2.5)$$

Trapezoidal Speed Profile Command Pre-Processor

According to [49, 50], the trapezoidal speed profile CPP generates the telescope trapezoidal speed profile and the physical admissible position trajectories.

Accordingly, telescope velocities keep a constant maximum value v_{max} or increase and decrease at the fixed maximum allowable acceleration $\pm a_{max}$. Thus, according to [6], the desired position trajectories $s(k)$ and speed $v(k)$ are computed as:

$$\begin{aligned} s(k+1) &= v(k) \\ v(k) &= \text{saturation}(v_a(k), \min(v_d(k), v_{max})) \end{aligned} \quad (2.6)$$

where $v_a(k)$ is the speed during the acceleration phase while $v_d(k)$ is the one during the deceleration phase. The acceleration ramp can be expressed

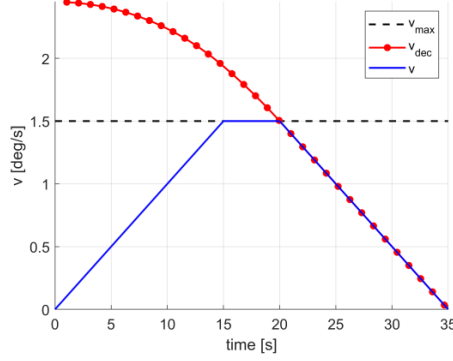


Figure 2.5. Trapezoidal speed profile CPP. Computed speed (blue line); maximum admissible speed (back dashed line); the deceleration profile computed as in (2.8) (red dotted line). From [6]

as

$$v_a(k) = v(k-1) + a_{max} \Delta t \quad (2.7)$$

where $v(k-1)$ is the measured telescope speed at time instant $t - \Delta t$, being Δt the sample time. The deceleration ramp can be computed as [49], i.e.:

$$v_d(k) = \dot{p}^*(k) + \text{sign}(\varepsilon(k)) \sqrt{2 a_{max} |\varepsilon(k)|} \quad (2.8)$$

where $\dot{p}^*(k)$ is the reference velocity of the target star at step time k , computed via the astronomical computation; $\varepsilon(k) = s^*(k) - s(k)$ represents the error between the planned trajectories and the star positions.

Note that (4.3) is valid for an arbitrary instant and represents the initial velocity of an ideal decelerating motion which would drive ε to 0 with a final velocity equal to $v^*(k)$. Therefore, $v_d(k)$ is the velocity limit that determines the moment at which the deceleration phase starts.

In doing so, the velocity trajectories can initially increase at the fixed maximum acceleration value a_{max} until reaching, or the maximum admissible speed v_{max} -if this latter is still less than $v_d(k)$ - or start decelerating according to (4.3).

Polynomial Command Pre-Processor

According to [7], the polynomial CPP aims at further improving the tracking accuracy while ensuring smoother behaviour. Specifically, as depicted in Figure 2.6, consider a telescope moves from a position $s(0)$ to $s(1)$ with a speed variation from $v(0)$ to $v(1)$ and acceleration from $a(0)$ to $a(1)$, the polynomial CPP computes the suitable position trajectory constrained to this six points. In doing so, the TG has to match the initial condition $(s(0), v(0), a(0))$ along with the terminal one $(s(1), v(1), a(1))$. To this end, to properly satisfy these conditions, the computed position trajectory can be formulated via a six-order polynomial equation, i.e.:

$$s(k) = C_0 + C_1 k + C_2 k^2 + C_3 k^3 + C_4 k^4 + C_5 k^5, \quad (2.9)$$

being $k = (t_0 - t_1)/\Delta t$ the sample time which relates the starting time t_0 with the next time instant $t_1 = t_0 + \Delta t$, being Δt the sample time. Then, by leveraging the differential relation between the position and speed the speed trajectory can be derived, i.e.:

$$v(t) = \frac{ds}{dt} = \frac{ds}{dk} \frac{dk}{dt} = \frac{ds}{dk} \frac{1}{\Delta t} = \frac{v(k)}{\Delta t} \quad (2.10)$$

with

$$v(k) = C_1 + 2 C_2 k + 3 C_3 k^2 + 4 C_4 k^3 + 5 C_5 k^4. \quad (2.11)$$

Similarly for the acceleration trajectory:

$$a(t) = \frac{a(k)}{\Delta t^2} = \frac{2 C_2 + 6 C_3 k + 12 C_4 k^2 + 20 C_5 k^3}{\Delta t^2}. \quad (2.12)$$

These lead to the following system of equations:

$$\begin{aligned} s(0) &= C_0 \\ v(0) &= C_1/\Delta t \\ a(0) &= 2 C_2/\Delta t^2 \\ s(1) &= C_0 + C_1 + C_2 + C_3 + C_4 + C_5 \\ v(1) &= (C_1 + 2 C_2 + 3 C_3 + 4 C_4 + 5 C_5)/\Delta t \\ a(1) &= (2 C_2 + 6 C_3 + 12 C_4 + 20 C_5)/\Delta t^2. \end{aligned} \quad (2.13)$$

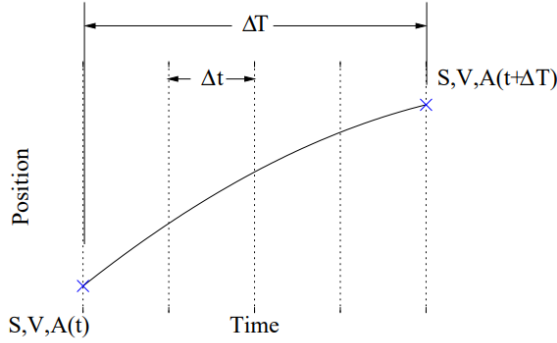


Figure 2.6. Overview of the Polynomial Command Pre-Processor. From [7].

Finally, since the starting condition $(s(0), v(0), a(0))$ and terminal condition $(s(1), v(1), a(1))$ are known, the system equations is readily solvable. Note that the starting condition is telescope position, speed and acceleration at the current time, while the terminal conditions are provided by the astronomical computation.

2.2.3 Telescope Tracking Control Algorithm

In this section, we provide more details about the common tracking control architecture for ground-based telescopes.

To this end, considering the reference position trajectories, computed via the astronomical computation and then reshaped by the TG, the tracking controller aims to compute the suitable torque commands for both axes, i.e. the altitude and azimuth axis. Specifically, as depicted in Figure 2.7 and as reported in the technical literature, see for instance [51, 10], the most common control architecture presents a cascaded control loop where the outer loop, based on the position tracking error, computes the suitable speed reference command which is provided to the inner loop whose aim is to provide to the telescope the torque command able to guarantee the tracking of the scientific target.

Within this framework, to properly solve the tracking control problem, the control architecture exploits the following tools:

- *Control unit:* the CU must be able to ensure the real-time feasibility

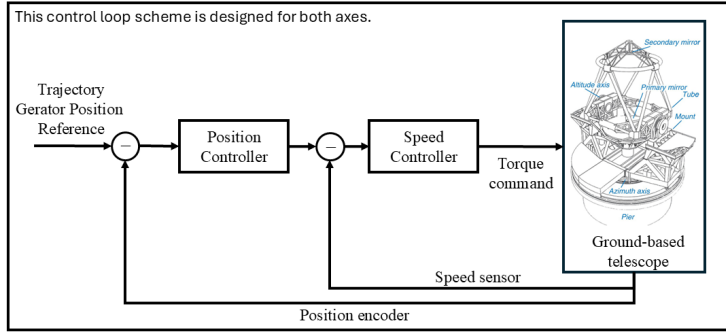


Figure 2.7. Overview of the Altitude and Azimuth control architecture.

of the control tasks. Typically implemented by exploiting a Linux-based real-time OS.

- *Drive system:* to properly drive the telescope, the control architecture has to present at least a motor for each axis. However, due to the telescope's size and the mechanical transmissions, typically four motors are used on each axes. Moreover, it is worth noting that, once the torque commands are provided, the drive system has to guarantee that each motor imposes the desired torque to the telescope axis.
- *Equipped sensors:* the position, speed and acceleration sensors for both axes. Those latter play a crucial role in the tracking performance since to ensure the requirements they must have a high resolution.

Finally, to better understand the limitations of the tracking control system, while the chosen CU is a proprietary solution tailored for the specific telescope program, e.g. the TNG control system is Labview-based [52] while the novel E-ELT is Linux-based OS [17], in what follows, by referring to the E-ELT program (see Figure 2.8), its drive system and sensors system are provided.

Telescope Drive System To clearly understand the drive system, we first present the mechanical structure of the axis. Indeed, referring to the

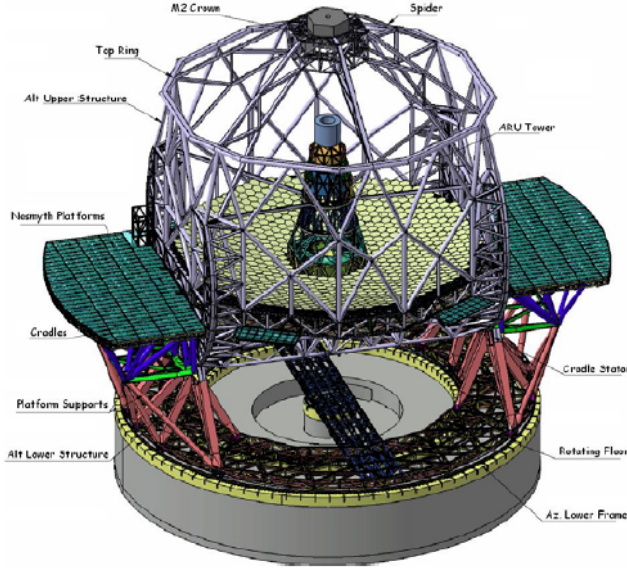


Figure 2.8. Overview of the E-ELT main structure.

E-ELT program, whose main structure is shown in Figure 2.8, the telescope, which is built on a rotating base, presents the altitude axis (the horizontal one) perpendicular to the azimuth axis (the vertical one).

Specifically, the altitude mechanical structure (Figure 2.9(a)) is about $1500 [ton]$ and it is driven via 2 cradles (Figure 2.9(b)).

The two cradles are supported by radial and lateral hydrostatic bearings, each of which has ten locations of identical supports encompassing one radial and two lateral hydraulic pads. The pads are mounted on a common structure and in proximity to the motor segments. The altitude cradle section facilitates integrating the radial and axial bearings in the main structure.

Concerning the azimuth mechanical structure (Figure 2.10(a)), it weights about $1300 [ton]$ and via three azimuth rings (Figure 2.10 it supports the altitude structure and the instrument platforms, i.e. the Nasmyth platforms. The two largest rings, the outer and the intermediate ones, of $\phi 51.5 [m]$ and $\phi 34 [m]$ of diameter, respectively, support the main vertical reactions during all operational modes. Besides, the inner ring of $\phi 4.5 [m]$,

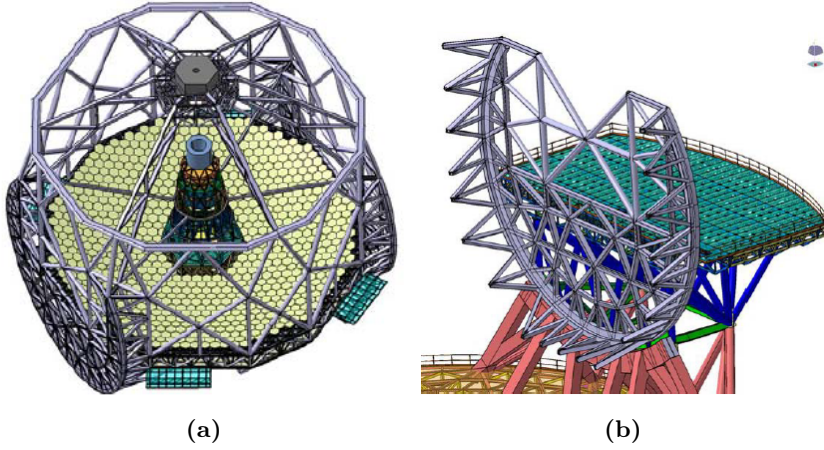


Figure 2.9. E-ELT altitude structure overview: (a) altitude main structure; (b) altitude candles structure.

supports part of the rotating floor.

To govern the telescope along the azimuth coordinate, only the outer azimuth ring is exploited. Indeed, this latter is depicted for the allocation of the azimuth main drives and the azimuth brake system while the inter-

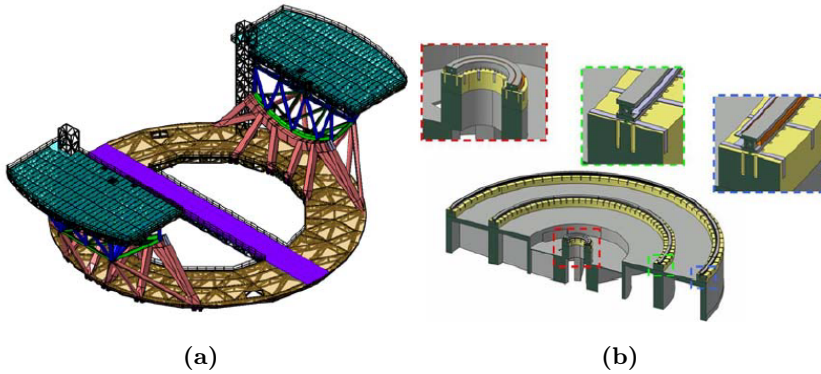


Figure 2.10. E-ELT azimuth structure overview: (a) azimuth main structure; (b) azimuth rings structure.

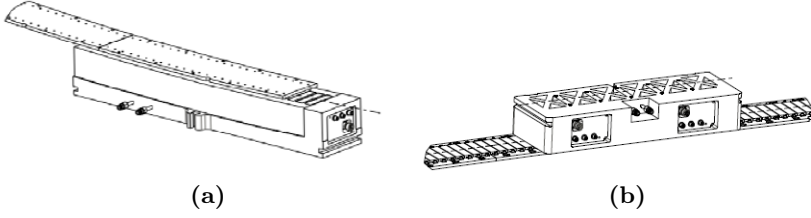


Figure 2.11. E-ELT drive system overview: (a) altitude drive system; (b) azimuth drive system.

mediate azimuth ring supports the azimuth encoder tapes.

Note that, the azimuth main structure refers only to the telescope floor and Nasmyth platforms, this is why it is lighter than the altitude one. Nevertheless, from a control point of view, while the altitude control system has just to consider the altitude structure's weight, the azimuth one has to be designed according to the whole telescope structure's weight.

Within this framework, to deal with the telescope structure's weight and dimension, both axes are driven via distributed segmented permanent magnet linear motors (Figure 2.11(a)-(b)) placed on the candles and the ring. For the sake of completeness, their requirements are reported in Table 2.1.

Table 2.1. Drive system requirements for Altitude and Azimuth axis.

Parameter	Altitude	Azimuth
Peak Torque SIR 3% (max. 30 [s])	3.769 [kNm]	2,104 [kNm]
Number of single gap stator segments	24 (48 drives)	16
Single gap section mass	76 [kg]	169 [kg]
Air gap, mechanical	2.9 [mm]	2.9 [mm]
Average rotor diameter	50,760 [mm]	28,200 [mm]
Expected cogging torque	± 5000 [Nm]	± 2500 [Nm]
Expected torque ripple	37,692 [Nm]	21,040 [Nm]

Table 2.2. Motion sensors requirements for Altitude and Azimuth axis.

Parameter	Altitude	Azimuth
Number of tapes	One per side	full circle
Slot diameter	34, 507.94/1.725 [mm]	26, 915.04/1.346 [mm]
Tape mounting	Outside	Outside (tensioned)
Number of scanning heads	Two per side	2 pairs separated by 60°
Nominal reference marks spacing	2000 Signal Periods	5000 Signal Periods
Number of signal periods	2, 113, 700	2, 710, 000
Signal period	0.613 [arcsec]	0.4782 [arcsec]
Measuring step	0.150 [marsec]	0.1168 [marsec]

Motion Sensors As reported in several works [52, 17], the ground-based telescope equips high-resolution encoders as position sensors and speed sensors. Moreover, some telescopes equip also tachometers for speed measurements.

In this view, considering again the E-ELT as exemplary solution, its main axes equip high-accuracy incremental optical encoders in order to fulfil the performance requirements. Indeed, the encoder system tailored for the E-ELT aims at ensuring a jitter of ≤ 0.02 [arcsec] without any disturbance and ≤ 0.3 [arcsec] in the presence of wind disturbance.

Specifically, due to the mechanical azimuth structure, the encoder equipped on the respective axis is a 360° tape placed on the external radial surface of the intermediate azimuth ring.

The dimensions of the azimuth ring require the azimuth tape along the external cylindrical surface of the ring. Indeed, the tape shall be pulled tight by a dedicated system to ensure equal tension along the complete length of the tape.

Concerning the altitude encoder system, since they are equipped on each candle, a 110° tape segment is chosen per side and is located in the external cylindrical surface of the altitude cradle. For the sake of completeness, in Table 2.2 their requirements are reported.

2.2.4 Final Remark

In conclusion, based on the mechanical structure of the telescope and on the involved designed drive system and sensors system, we can state

that the main challenge of the telescope tracking control problem is the identification of the telescope model which, referring to the specific telescope program, can involve more or less issues. Indeed, it is clear how the transmission system, which involves elastic phenomena due to the huge dimension of the structure, along with the telescope structure's weight, affects the telescope dynamics by introducing non-linearity. From a control point of view, this limits the choice of the control approach which, due to the planned Gantt project, forces to adopt approaches based directly on the tracking errors such as PID-based or frequency-base, hence, reducing the need to find a very accurate telescope dynamical model. Note that, due to the telescope model identification limitations, the developed control approaches for solving the telescope tracking control problem are typically PID-based or frequency-based, indeed, the telescope model identification is still an open challenge.

Tracking Control Problem For Ground-Based Telescope

Over the last few years, the increasing demand of high-performance telescopes in the study of the universe has led to the investigation of improved control architectures for the advanced telescope axes control [53]. Indeed, the problem has been deeply investigated with different hardware in previous telescopes and antenna projects [33]. More specifically, for the class of ground-based telescopes, the problem of axes control can be divided into several tasks:

- i. The pointing system control (i.e. the core of the tracking controller) which provides the reference trajectories to be tracked [29].
- ii. The servo drive task which ensures the correct fitting of the motor engines [54].
- iii. The axes control [task](#) which aims to guarantee the basic telescope motion tasks such as manual movement, to enable the operator to control the telescope, or the slewing of the telescope in a specific position [55].
- iv. The tracking controller has to ensure that the telescope, over the night, tracks the desired scientific target.

Among the several control tasks, which the architectures aim at [executing](#), the tracking control of the telescope is one of the most critical [56].

Specifically, this latter aims to stabilize the position of the telescope on the scientific target to guarantee high precision and tracking performance. In this view, this thesis aims to propose some solutions to achieve the tracking control problem for the class of ground-based telescopes.

Therefore, to satisfy the demand for even more powerful telescopes, the novel tracking controllers (as discussed in chapter 1) have to ensure very high tracking performance. Moreover, when designing tracking control for radio and optical ground-based telescopes, it is necessary to handle the presence of unavoidable external disturbances and noise measurements, which have to be attenuated and reduced, respectively, as well as to compensate the gravity effects [57]. To this end, as the technical literature reports, over the years, only the traditional tracking control scheme via the more assested control algorithms has been investigated.

Therefore, as discussed in section 2.1, the tracking control system is usually designed according to the double-layer approach where the inner layer aims to control the speed behaviour of the telescope, while the outer one ensures the correct tracking of the telescope's position. In doing so, the tracking system is designed by defining two different controllers, namely, the one for the speed and the other for the position. Those systems have been developed by leveraging mainly approaches as PID-based [17, 58, 10], frequency-based (for instance loop shaping or IIR filter [52]) or LQG-based [48].

In this regard, as also claimed in the remark 2.2.4, it is possible to note that most of the works mainly focus on the development of the control system rather than exploring novel control architecture solutions. Therefore, with the aim to move toward a fully optimal control architecture –note that, in literature, some tracking controllers are based on optimal control strategies [51], however, these latter are only exploited for controlling one of the telescope dynamics – we propose fully-optimal solutions which, by satisfy an optimality criteria, guaranteeing an optimal and robust tracking performance despite the presence of the unavoidable wind disturbance and noise measurements. Moreover, thanks to the embedded trajectory generator, our solutions also ensure the physical telescope's trajectory.

Note that, the proposed architectures are robust in the sense of performance as they are able to ensure good tracking despite the presence of measurement noise while counteracting the wind disturbance.

Finally, in what follows, section 3.1 reports the related works and highlights the contributions/advantages of the proposed solutions, while, in section 3.2.1 the telescope dynamical model is described, then, in section 3.2.2 the tracking control problem for the ground-based telescope is formalized.

3.1 Related Works and Contributions

As reported in the technical literature, there is a significant amount of works related to the development of ground-based telescopes. Indeed, among all those works, which mainly focus on a development point of view, only a few aim to report in detail the control approach exploited to solve the tracking problem.

In this regard, by considering the most important ground-based telescope programs along with the more innovative solutions, we can retrace the evolution of the tracking control algorithms for ground-based telescopes.

Consequently, taking into account the tracking control problem and its robustness requirements related to the wind and noise measurement, the first attempt is reported in [59], where the authors design a state-feedback control plus a feed-forward disturbance compensation gain action for solving the axes control problem of the 8.2[m] VLT while facing the presence of torque wind disturbance. By considering again the VLT, with the aim at improving the tracking performance, [60] has proposed the same control strategy for a novel servo system. Regarding the 10[m] GTC, [54] has suggested an output integral feedback position controller for the solely drive motor dynamics. Conversely, for the specific case of the 3.5[m] TNG, [61, 62] has proposed a double layer control system which consists of a speed filter control loop and a PI-based position control loop, both tuned to mitigate the external wind disturbance. Another exemplary tracking controller has been designed in [63] for the 17[m] major atmospheric gamma-ray imaging Cherenkov telescopes, where the controller, inspired by [61], embeds a trajectory generator to deal with the physical limitation of the telescope. For the specific case of the 34[m] and 70[m] antennas of the NASA DSN, a cascaded control loop, combining a LQG regulator for the position control and a PID controller for the speed dynamics, has been designed in [51]. However, the design does not consider the presence of external disturbance, which is an unrealistic operating condition.

Furthermore, with respect to the novel ELT projects, i.e. the 30 [m] Thirty Meter Telescope (TMT) and the 40 [m] E-ELT, [10, 64] have suggested a tracking controller, where a double PI-based control loop, combined with a low-pass filter, deal with the noise measurements, high-frequency input disturbance and anti-gravity compensation. More recently, the tracking control problem for the GRBT telescope has been addressed and solved in [65] via a control strategy which exploits the pointing model parametric function approach.

3.1.1 Contributions

From the literature review of the tracking control problem for ground-based telescopes, it is possible to observe a lack of an architecture that, by exploiting an optimality criteria, guarantees the tracking of a scientific target in an optimal fashion while also ensuring robustness with respect to wind disturbance and noise measurements. Along this direction, this thesis fills this gap by proposing two optimal-based control architectures. The solutions consist of: *i*) the trapezoidal speed profile trajectory generator which, based on the astronomical computation, determines the position and speed reference to be tracked while ensuring that all the telescope physical constraints are always not violated; *ii*) an optimal-based control strategy, which, based on the pre-processed trajectories and the estimation of the telescope dynamics, provided by an infinite horizon Kalman filter, computes the optimal telescope torque profiles for driving the telescope's axes towards the scientific target, this is done despite the presence of input external disturbance such as the wind force. Specifically, both solutions exploit the same trajectory generator while, concerning the tracking algorithm, the first solution preserves the typical control scheme (i.e. the double-layer tracking scheme) and leverages an optimal output-feedback controller for the speed loop, while an LQG plus PI action supported also by an infinite Kalman Filter Observer for the position one. Differently from the telescope tracking state-of-the-art and the first solution, the proposed novel control solution aims at controlling both telescope axes in terms of position and speed trajectories in a unified framework while ensuring optimal performance criteria. To this end, it leverages an MPC controller, to ensure the tracking requirements, plus a Kalman Filter Observer, to properly mitigate the measurement noise.

Table 3.1. Overview of the ground-based telescope tracking architecture Related Works and Comparison with our approach.

Reference	Tracking Control Scheme	Control Protocol	Knowledge of telescope dynamics	Mitigation of Measurement Noise	Resilience to wind disturbance
[59]	Double-Layer	State FB + FF Compensator	✓	—	✓
[60]	Double-Layer	State FB + FF Compensator	✓	—	—
[54]	Double-Layer	Output Feedback Integrator	✓	✓	—
[62]	Double-Layer	PID-based Controller	✓	—	—
[63]	Double-Layer	PID-based Controllers	✓	—	✓
[51]	Double-Layer	LQG + IIR filter	✓	—	—
[10]	Double-Layer	PID-Based Controllers	✓	—	✓
[64]	Double-Layer	PID-Based Controllers	✓	✓	✓
[65]	Double-Layer	Pointing Model Parametric Function	✓	—	✓
[9]	Double-Layer	Output FB + LQG-PI + Kalman Filter	✓	✓	✓
[22]	Single-Layer	MPC-based + Kalman Filter	✓	✓	✓

Most notably, the proposed solutions are tailored for the specific case of study of TNG. Finally, Table 3.1 offers a prompt comparison of the proposed control solutions with respect to existing ground-based telescope tracking architecture.

3.2 Tracking Control For Ground-Based Telescope: Problem Statement

Consider a ground-based telescope equipped with on-board sensors, such as the tachometers and encoders for the measurement of its altitude and azimuth velocities and positions, and a scientific target star to be tracked, whose features are expressed, via the astronomical computation, with respect to the azimuth coordination system, i.e. the altitude angle $\vartheta^*(t)$ [deg] and the azimuth angle $\varphi^*(t)$ [deg].

The aim of this work is to design a novel MPC-based control architecture able to drive the telescope axes for the optimal achievement of the star tracking. The control architecture is designed to fulfill the following requirements: *i*) computation of the appropriate telescope axes position and speed trajectories such that all the telescope physical constraints, in terms of speed and acceleration, are always not violated; *ii*) computation of the optimal telescope control torque which ensures the high tracking performance for both the altitude and the azimuth axes, while guaranteeing robustness w.r.t. measurement noise and external disturbance affecting the telescope dynamics.

3.2.1 Ground-Based Telescope Dynamics

The behavior of a ground-based telescope is mainly described by its measurable velocities $y(t) = [\dot{\vartheta}(t), \dot{\varphi}(t)]^\top \in \mathbb{R}^2$ in the azimuth coordination system, being $\dot{\vartheta}(t)$ [deg/sec] the altitude angular speed and $\dot{\varphi}(t)$ [deg/s] the azimuth angular speed. Within this framework, according to [66], the ground-telescope dynamical behavior can be described by the following linear model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) - \delta(t)) \\ y(t) &= Cx(t) + \omega \end{aligned} \quad \text{with} \quad (3.1)$$

$$A = \begin{bmatrix} A_e & 0_{6 \times 6} \\ 0_{6 \times 6} & A_a \end{bmatrix} \quad B = \begin{bmatrix} B_e & 0_{6 \times 1} \\ 0_{6 \times 1} & B_a \end{bmatrix} \quad C = \begin{bmatrix} C_e & 0_{1 \times 6} \\ 0_{1 \times 6} & C_a \end{bmatrix}$$

where $x(t) = [x_e(t) \ x_a(t)]^\top \in \mathbb{R}^{12}$ is the flexible dynamics of the telescope, being $x_e(t)$ the state vector related to altitude dynamics while $x_a(t)$ is the one related to azimuth dynamics; $u(t) = [\tau_e(t), \tau_a(t)]^\top [Nm] \in \mathbb{R}^2$ is the control input vector representing the torque commands along the altitude axis, i.e. $\tau_e [Nm]$, and the azimuth axis, i.e. $\tau_a [Nm]$; $\delta(t) [Nm] \in \mathbb{R}^2$ is the torque input wind external disturbance; $\omega \in \mathbb{R}^{2 \times 1}$ is the measurements white noise affecting the output vector; $A \in \mathbb{R}^{12 \times 12}$ is the overall dynamical matrix of the telescope being $A_e \in \mathbb{R}^{6 \times 6}$ and $A_a \in \mathbb{R}^{6 \times 6}$ the dynamical matrices related to altitude and azimuth axes dynamics, respectively; $B \in \mathbb{R}^{12 \times 2}$ is the input telescope matrix where $B_e \in \mathbb{R}^{6 \times 1}$ and $B_a \in \mathbb{R}^{6 \times 1}$ map the relation between the torque commands along the altitude and azimuth axes with the related states $x(t)$, respectively; $C \in \mathbb{R}^{2 \times 12}$ is the output telescope matrix whose blocks $C_e \in \mathbb{R}^{1 \times 6}$ and $C_a \in \mathbb{R}^{1 \times 6}$ represent the altitude and the azimuth outputs matrices, respectively.

Remark 1 The value of the matrices A, B, C strongly depends on the specific telescope and they can be tailored by exploiting the identification procedure suggested in [66]. The specific matrices for TNG case of study can be found in Section 5.0.1.

The matrices describing the telescopes dynamics are such that the following assumption hold.

Assumption 1 *The pair (A, B) is stabilizable.*

Assumption 2 *The pair (A, C) is observable.*

Assumption 3 *C is a full row rank matrix.*

According to [67], the wind external disturbance $\delta(t)$ is modeled via the Von-Karman wind PSD as:

$$\delta(t) = \sum_{j=1}^D \sqrt{2\alpha S_\tau(f_j)\Delta f} \cos(2\pi f_j t + \phi_j), \quad (3.2)$$

with $D \in \mathbb{N}$ the number of the contributed frequency samples; α is a scale intensity factor; $\Delta f [1/s]$ is the frequency resolution; f_j is the j -th element of the numerical sequences $f(j) = \{j\Delta f\}$ being $j \in (1, D)$;

$\phi_j = \mathcal{N}(\mu, \sigma) [deg]$ ($\forall j = 1, \dots, D$) is the initial phase computed through a random uniform distribution with mean μ and standard deviation σ . The torque Von-Karman PSD $S_\tau(f_j)$ is described by:

$$S_\tau(f_j) = 4 \left(\frac{\bar{\tau}}{\bar{v}} \right)^2 S_v(f_j) \chi(f_j)^2, \quad (3.3)$$

where $\bar{\tau} [Nm]$ is the wind mean torque disturbance; $\bar{v} [m/s^2]$ is the mean wind speed; $S_v(f_j)$ is the Von-Karman speed wind PSD while $\chi(f_j)$ is the aerodynamic correction factor.

In detail, the $S_v(f_j)$ is computed as:

$$S_v(f_j) = 4 (\mathcal{J} \bar{v})^2 \frac{\Gamma}{\bar{v}} \frac{1}{\left(1 + 70.78 \left(f_j \frac{\Gamma}{\bar{v}} \right)^2 \right)^{5/6}}, \quad (3.4)$$

being $\mathcal{J}$ the wind disturbance intensity and Γ the outer scale of the wind disturbance.

Besides, $\chi(f_j)$ can be derived as:

$$\chi(f_j) = \frac{1}{1 + \left(2 f_j \frac{\sqrt{\Lambda}}{\bar{v}} \right)^{4/3}} \quad (3.5)$$

where $\Lambda [m^2]$ is the telescope area exposed to the wind disturbance.

3.2.2 Problem Statement

Now it is possible to formulate the tracking control problem as follows.

Problem $\mathcal{C}$ Consider the ground-based telescope dynamics as in (3.1). Let the scientific target star be expressed, within the equatorial coordination system, hence, in terms of the vector $y^*(t) = [\vartheta^*(t), \varphi^*(t)]^\top$, being $\vartheta^*(t)$ the altitude and $\varphi^*(t)$ the azimuth star reference trajectories. Design an optimal-based control tracking architecture such that the telescope torques $u(t) = [\tau_e, \tau_a]^\top$ can properly drive the altitude and azimuth axes toward the star scientific target by avoiding violating the admissible physical constraints and fulfilling the following control objective:

$$\lim_{t \rightarrow \infty} \left\| y^*(t) - \int_0^t y(\rho) d\rho \right\|_2 = 0 \quad (3.6a)$$

$$\lim_{t \rightarrow \infty} \|\dot{y}^*(t) - y(t)\|_2 = 0, \quad (3.6b)$$

despite the presence of the input wind disturbance $\delta(t)$ and noise measurements ω .

Ground-based telescope Tracking Control Architecture

In this chapter, the two proposed tracking architectures solving the control problem $\mathcal{C}$ are presented. Specifically, both solutions embed a telescope trajectory generator to guarantee the compliance of the designed control inputs to the telescope's physical constraints. In detail, the control architectures can be generalized by considering the following components:

- **Trajectory generator:** Given the scientific star target to be tracked, i.e. $y^*(t) = [\vartheta^*(t), \varphi^*(t)]^\top$, computes the safe and physical admissible reference position and speed trajectories for telescope latitude and azimuth axes, i.e. $\bar{y}(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]^\top$ and $\dot{\bar{y}}(t) = [\dot{\bar{\vartheta}}(t), \dot{\bar{\varphi}}(t)]^\top$, respectively;
- **Tracking control algorithm:** Based on the constrained telescope trajectory $\bar{y}(t)$ and $\dot{\bar{y}}(t)$, computed via the trajectory generator, the tracking control algorithm aims to provide the suitable torque commands $u(t)$ able to drive the telescope axes motion in order to ensure that the scientific target is pursued despite the presence of the wind disturbance and white noise measurements.

It is worth noting that, both architectural solutions embed the same trajectory generator.

Moreover, the first solution [9] is inspired by the telescope technical literature and consists of an optimal-based cascaded control loop approach

which leverages an output feedback H_∞ algorithm for the inner loop and an LQG-PI controller for the outer one.

Conversely, to fill the gap of the telescope tracking state-of-the-art w.r.t. to the control one, the second solution [22] exploits an MPC plus steady-state Kalman filter observer to fulfil the control objectives in (3.6a)-(3.6b). Finally, in what follows, we first detail the trajectory generator features since it is embedded in both the architectural solution, then, the two different tracking control algorithms are presented along with their closed-loop stability proof.

4.1 Trajectory-Generator

According to [49, 50, 6] the trajectory generator is designed by leveraging the trapezoidal speed profile approach. Specifically, given the star position $y^*(t)$, the trajectory generator computes: *i*) the telescope trapezoidal speed profile $\bar{y}_s(t) = [\dot{\bar{\vartheta}}(t), \dot{\bar{\varphi}}(t)]^\top$; *ii*) the physical admissible position trajectories $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]^\top$.

Accordingly, telescope velocities keep a constant maximum value v_{max} or increase and decrease at the fixed maximum allowable acceleration $\pm a_{max}$. Thus, according to [6], the desired trajectories $\bar{y}_p(t)$ and $\bar{y}_s(t)$ are computed as:

$$\dot{\bar{y}}_p(t) = \bar{y}_s(t) \quad (4.1a)$$

$$\bar{y}_s(t) = \text{saturation}(v_a(t), \min(v_d(t), v_{max})) \quad (4.1b)$$

where $v_a(t)$ and $v_d(t)$ are the speeds during the acceleration and deceleration phases, respectively. The acceleration ramp can be expressed as

$$v_a(t) = v(t - \Delta t) + a_{max} \Delta t \quad (4.2)$$

where Δt is the sampling time while $v(t - \Delta t)$ is the measured telescope speed at time instant $t - \Delta t$. The deceleration ramp can be computed as [49]:

$$v_d(t) = \dot{y}^*(t) + \text{sign}(\varepsilon(t)) \sqrt{2 a_{max} |\varepsilon(t)|} \quad (4.3)$$

where $\dot{y}^*(t)$ is the reference velocity of the target star, computed via the astronomical computation; $\varepsilon(t) = y^*(t) - \bar{y}(t)$ represents the error between

the planned trajectories and the star positions. Note that (4.3) is valid for an arbitrary instant and represents the initial velocity of an ideal decelerating motion which would drive ε to 0 with a final velocity equal to $v^*(t)$. Therefore, $v_d(t)$ is the velocity limit which determines the start of the deceleration phase.

In doing so, the velocity trajectories can initially increase at the fixed maximum acceleration value a_{max} until reaching, either the maximum admissible speed v_{max} –if this latter is still less than $v_d(t)$ – or start decelerating according to (4.3).

The output $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]^\top$ and $\bar{y}_s(t) = [\dot{\bar{\vartheta}}(t), \dot{\bar{\varphi}}(t)]^\top$ of the trajectory generator, i.e. the planned altitude $\bar{\vartheta}(t)$, azimuth $\bar{\varphi}(t)$ positions and its computed trapezoidal speed profiles, i.e. $\dot{\bar{\vartheta}}(t)$ and $\dot{\bar{\varphi}}(t)$, represent the desired behaviour to be tracked by the MPC-based tracking controller.

Remark 2 *It is worth noting that, the proposed trajectory generator does not take into account the telescope dynamics, hence, solving (3.1). However, thanks to (4.1b), the proposed algorithm is able to ensure that the telescope does not violate its maximum speed while, with (4.2) and (4.3), it guarantees that the maximum admissible telescope acceleration, i.e. a_{max} , is not violated during the acceleration and deceleration manouvers, respectively.*

4.2 Optimal Tracking Control Architecture Design

The first tracking control algorithm designed to solve the Problem $\mathcal{C}$ is a robust optimal cascade loop controller as in Fig.4.2.

Specifically, based on the constrained position trajectory $\bar{y}(t)$ computed by the trajectory generator, it computes the optimal torque profiles which guarantee the tracking of the scientific target despite the presence of the external wind disturbance and measurement noise. In detail, the proposed algorithm consists of:

- *The position control layer*, i.e. the outer control loop, which allows, via LQG-PID regulator, the computation of the optimal speed profile $s(t)$ such that a reliable tracking of the desired pre-processed position trajectories $\bar{y}_p(t)$ is achieved;
- *The speed control layer*, i.e. the inner control loop, which, given the optimal speed profile $s(t)$ and by leveraging an Output-feedback (OPFB) H_∞ controller, provides the optimal telescope control torques

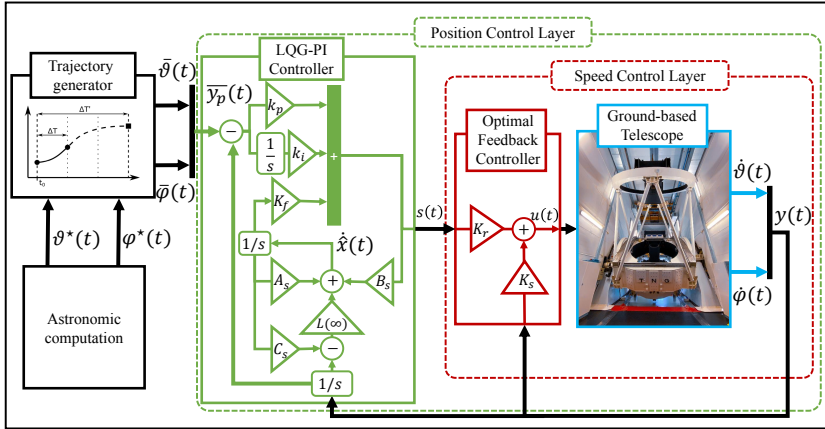


Figure 4.1. Overview of the optimal tracking architecture. On the left, there is the trajectory generator while the red and green dashed-dot squares highlight the speed and the position control loop, respectively.

$u(t)$ in (3.1) so to fulfil the control objective (3.6a) and (3.6b).

In what follows, we detail the design of each loop composing the robust optimal tracking control architecture in Fig.4.2 along with its closed-loop stability proof. Note that, for the design of the control layers, as best practice in control theory, we start from the inner loop which will represent the novel plant to be controlled by the outer one.

4.2.1 Speed Control Layer Design

The speed layer controller, i.e. the red square block in Fig.4.2, aims at imposing to the telescope axes the appropriate input torques $u(t)$ so to drive the telescope speeds $y(t)$ towards the desired reference behaviour $s(t) \in \mathbb{R}^2$. This latter is such that the following assumption holds:

Assumption 4 *The reference speed profile $s(t)$ is provided by the following reference generator system:*

$$\dot{s}(t) = F s(t). \quad (4.4)$$

Note that the matrix F will be designed as part of position control layer. Now, considering the telescope LTI model (3.1), the speed controller is designed according to the OPFB H_∞ optimal tracking control strategy [?] as:

$$u(t) = K_s y(t) + K_r s(t), \quad (4.5)$$

where $K_s \in \mathbb{R}^{2 \times 2}$ and $K_r \in \mathbb{R}^{2 \times 2}$ are the feedback and feed-forward gains to be properly chosen for guaranteeing the optimization of the functional cost $z(t)$ and the external disturbance attenuation. To this aim, control gains are designed as the solution of the following optimization problem.

Problem $\mathcal{T}$ *Consider the LTI dynamics in (3.1) under the action of the control strategy (4.5). Let Assumptions 1, 2 and 4 hold. Find the OPFB H_∞ optimal control gains such to:*

➤ *minimise the output performance cost function*

$$\|z(t)\|_2 = (s(t) - y(t))^\top Q_s (s(t) - y(t)) + u(t)^\top R_s u(t), \quad (4.6)$$

where $Q_s \geq 0 \in \mathbb{R}^{2 \times 2}$ is a diagonal matrix weighting the tracking error while $R_s > 0 \in \mathbb{R}^{2 \times 2}$ is the input diagonal weighing matrix.

- ensure the disturbance attenuation according to the following L_2 -gain condition:

$$\frac{\int_0^\infty e^{-v(\rho-t)} \|z(t)\|_2 d\rho}{\int_0^\infty e^{-v(\rho-t)} \|\delta(t)\|_2 d\rho} \leq \eta^2, \quad (4.7)$$

being $v > 0$ the discount factor and η the amount of attenuation from the disturbance input $\delta(t)$ to the defined performance output $z(t)$.

Remark 3 Problem $\mathcal{T}$ is solved by exploiting Matlab/Simulink platform. Specifically, with a proper selection of the matrix Q_s R_s , as well as of v and η , the control gains K_s and K_r are tuned via Yalmip toolbox with Mosek optimizer.

Finally, we derive the closed-loop speed layer dynamics, which will be helpful for the design of the position layer controller. Accordingly, introduce the augmented state vector $x_s(t) = [y_p(t), x(t)]^\top$ with $y_p(t) = \int_0^t y(\rho) d\rho$, i.e. the position dynamics derived by integrating the tachometers measurements. Hence, given (3.1) and the control input (4.5), the speed layer closed-loop can be derived as:

$$\begin{aligned} \dot{x}_s(t) &= A_s x_s(t) + B_s s(t) + \Omega \omega \\ y_p(t) &= C_s x_s(t) \end{aligned} \quad (4.8)$$

with

$$\begin{aligned} A_s &= \begin{bmatrix} 0_{2 \times 2} & C \\ 0_{12 \times 12} & A + B K_s C \end{bmatrix} & B_s &= \begin{bmatrix} 0_{2 \times 2} \\ B K_r \end{bmatrix} \\ \Omega &= \begin{bmatrix} I_2 \\ B K_s \end{bmatrix} & C_s &= [I_2 \quad 0_{2 \times 12}], \end{aligned}$$

where $A_s \in \mathbb{R}^{14 \times 14}$ is the dynamical matrix; $B_s \in \mathbb{R}^{14 \times 2}$ is the input mapping matrix; $\Omega \in \mathbb{R}^{14 \times 2}$ is the noise mapping matrix; $C_s \in \mathbb{R}^{2 \times 14}$ is the output matrix used for the derivation of the telescope position dynamics, i.e. the altitude $\vartheta(t)$ and azimuth $\varphi(t)$ angles.

4.2.2 The Position Control Layer Design: The optimal LQG-PI controller

The position control layer, namely the green square in Fig.4.2, is the outer loop. Based on the pre-processed position trajectories $\bar{y}_p(t) = [\bar{\theta}(t), \bar{\varphi}(t)]$, it is devoted to the fulfilment of the control goals (3.6a)-(3.6b) by computing the optimal speed profiles $s(t)$ via a LQG-PI regulator which consists of a steady-state Kalman filter observer plus a LQR-PI regulator. Note that the Kalman Filter is used for the estimation of the enlarged states $\hat{x}_s(t)$, crucial for the proper control design of the full-state feedback LQR-PI regulator.

The overall design is based on the following assumptions.

Assumption 5 *The couple (A_s, B_s) and (A_s, C_s) in (4.8) is reachable and observable, respectively.*

Kalman filter observer dynamics can be formalized as:

$$\begin{aligned}\dot{\hat{x}}_s(t) &= A_s \hat{x}_s(t) + B_s s(t) + L_\infty (y_p(t) - \hat{y}_p(t)) \\ \hat{y}_p(t) &= C_s \hat{x}_s(t)\end{aligned}\tag{4.9}$$

where $L_\infty \in \mathbb{R}^{14 \times 2}$ is the gain observer matrix to be computed according to optimality criteria for the expected value of the estimation error [68].

In doing so, define the estimation error $\hat{e}(t) = x_s(t) - \hat{x}_s(t) \in \mathbb{R}^{14}$ and the covariance matrix of the expected error $\tilde{P}(t) = \mathbb{E}[\hat{e}(t) \hat{e}(t)^T] \in \mathbb{R}^{14 \times 14}$. Moreover, we define covariance matrix of systems noises V as

$$V = \begin{bmatrix} \tilde{Q} & 0_{14 \times 2} \\ 0_{2 \times 14} & \tilde{R} \end{bmatrix}$$

being, $\tilde{Q} \geq 0$ the diagonal matrices weighting the estimation error states and $\tilde{R} > 0$ weighting the output measurement. Under Assumption 5, according to the well-known procedure [68], the matrix gains observer L_∞ is hence computed as follows:

$$L_\infty = \tilde{P}_\infty C_s^\top R^{-1}\tag{4.10}$$

being $\tilde{P}_\infty$ the unique solution of the stationary Riccati equation:

$$0 = A_s \tilde{P}_\infty + \tilde{P}_\infty A_s^T + \tilde{Q} - \tilde{P}_\infty C_s^T \tilde{R} C_s \tilde{P}_\infty. \quad (4.11)$$

Now, for the design of LQR-PI control, introduce the tracking state vector $x_T(t) = [e_i(t), e_p(t), \hat{x}_s(t)]^T$ where $e_p(t) = \bar{y}_p(t) - y_p(t)$ is the tracking error and $e_i(t) = \int_0^t \bar{y}_p(\rho) - y_p(\rho) d\rho$ is the integral tracking error. Accordingly, the augmented state dynamics are derived as

$$\dot{x}_T(t) = A_T x_T(t) + B_T s(t)$$

with

$$A_T = \left[\begin{array}{c|c} & -C_s \\ \hline 0_{18 \times 4} & -C_s A_s \\ & A_s \end{array} \right], \quad B_T = \left[\begin{array}{c} 0_{2 \times 2} \\ -C_s B_s \\ B_s \end{array} \right]. \quad (4.12)$$

Let the following assumption hold.

Assumption 6 *The pair (A_T, B_T) is completely reachable and controllable.*

Under this assumption, according to [56], the LQR-PI control $s(t)$ is defining as

$$s(t) = K_c x_T(t), \quad (4.13)$$

where $K_c = [k_i \ k_p \ K_f] \in \mathbb{R}^{2 \times 18}$ is the optimal LQR-PI gain controller being $k_i \in \mathbb{R}^{2 \times 2}$ the integral gain related to $e_i(t)$, $k_p \in \mathbb{R}^{2 \times 2}$ the proportional gain weighting $e_p(t)$ and $K_f \in \mathbb{R}^{14 \times 14}$ the feedback control gain related to the estimated states.

The optimal control gain K_c is found by solving the following optimal control problem:

Problem $\mathcal{L}$ *Consider the LTI model (4.12). Under assumption 6, the proportional-integral-feedback control input gain K_c is chosen by minimizing the following cost function:*

$$J_c(t) = \int_0^\infty x_T(t) Q_c x_T(t)^T + s(t) R_c s(t)^T dt. \quad (4.14)$$

where $Q_c \geq 0 \in \mathbb{R}^{18 \times 18}$ is a diagonal matrix weighting the contribution

of each state $x_T(t)$ while $R_c > 0 \in \mathbb{R}^{2 \times 2}$ is the input diagonal weighing matrix.

Remark 4 Problem $\mathcal{L}$ is solved by exploiting Matlab/Simulink platform. Specifically, the observer gain $L(\infty)$ and the LQR-PI gain K_c is selected via Yalmip toolbox and Mosek optimizer by properly choosing the degree of freedom Q_c and R_c .

The final step of our design consists in defining the matrix F in (4.4). Given the position control $s(t)$ in (4.13) and the augmented state $x_T(t)$ in (4.12) it follows that the matrix F is such that $F = K_c(A_T + B_T K_c)$.

4.3 MPC-based Tracking Control Architecture Design

Herein, as depicted in Figure 4.2, the second architectural tracking solution is presented. Specifically, to solve the Problem $\mathcal{C}$, the proposed algorithm consists of an MPC plus a steady-state Kalman observer. Therefore, based on the constrained position and speed trajectories carried out via the trajectory generator, i.e. $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]^\top$ and $\bar{y}_s(t) = [\dot{\bar{\vartheta}}(t), \dot{\bar{\varphi}}(t)]^\top$ respectively, the MPC-based tracking algorithm ensures the computation of the optimal torque profile $u(t)$ that guarantees the optimal and robust tracking of the $\bar{y}_p(t)$ and $\bar{y}_s(t)$, despite the presence of input external disturbance $\delta(t)$ and output measurement noise ω . In doing so, the control objective (3.6a)-(3.6b) are achieved. In what follows, we detail the design of the steady-state Kalman filter and the MPC's local optimal control problem along with its closed-loop algorithm through which its stability proof is derived.

4.3.1 MPC-Based Tracking Controller

Given the reference behaviour to be tracked, the Tracking controller layer (see the red box in Figure 4.2) aims at ensuring an optimal, robust and highly-performance tracking of $\bar{y}_p(t)$ and $\bar{y}_s(t)$ by computing the tele-

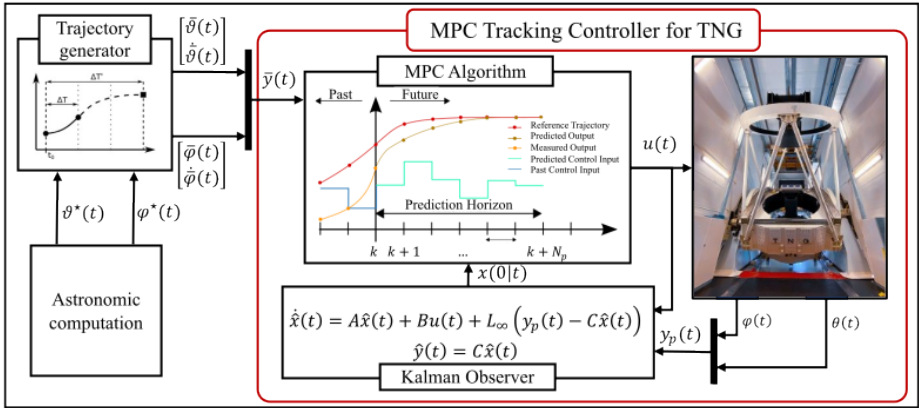


Figure 4.2. Overview of the MPC-based tracking architecture.

scope control $u(t)$ according to MPC method, hence achieving the control goals (3.6a)-(3.6b).

To this end, a steady-state Kalman filter observer is also exploited for the estimation of the whole telescope dynamics $\hat{x}(t)$ in order to let the MPC algorithm properly compute the optimal torque action $u(t)$ in (3.1) to be imposed to the ground-based telescope.

Steady-State Kalman Filter

Since the ground-based telescope model in (3.1) is not affected by any process noise and, hence, its dynamics are such that the measurement noise is uncorrelated to process one, a steady-state Kalman state observer is designed in order to provide the estimation of the telescope state variables, i.e. $\hat{x}(t) \in \mathbb{R}^{12}$.

Specifically, the Kalman filter is described by the following dynamical system:

$$\begin{aligned}\dot{\hat{x}}(t) &= A \hat{x}(t) + B u(t) + L_{\infty} (y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C \hat{x}(t),\end{aligned}\tag{4.15}$$

where $L_{\infty} \in \mathbb{R}^{12 \times 2}$ is the gain observer matrix to be computed according to the optimality criteria for the expected value of the estimation error [68].

In doing so, let consider the estimation error $\hat{e}(t) = x(t) - \hat{x}(t) \in \mathbb{R}^{12}$, its expected covariance matrix $\tilde{P}(t) = \mathbb{E} [\hat{e}(t) \hat{e}(t)^{\top}] \in \mathbb{R}^{12 \times 12}$ and the covariance matrix V as

$$V = \begin{bmatrix} \tilde{Q} & 0_{14 \times 2} \\ 0_{2 \times 14} & \tilde{R} \end{bmatrix},$$

being $\tilde{Q} \geq 0 \in \mathbb{S}^{12}$ the diagonal matrices weighting the estimation error states and $\tilde{R} > 0 \in \mathbb{S}^2$ the one weighting the output measurements.

Under the Assumptions 2-3, according to the well-known procedure [68], the matrix gains observer L_{∞} is computed as follows:

$$L_{\infty} = \tilde{P}_{\infty} C^{\top} R^{-1},\tag{4.16}$$

being $\tilde{P}_\infty$ the unique solution of the stationary Riccati equation:

$$0 = A \tilde{P}_\infty + \tilde{P}_\infty A^\top + \tilde{Q} - \tilde{P}_\infty C^\top \tilde{R} C \tilde{P}_\infty. \quad (4.17)$$

Note that, as the altitude and azimuth dynamics are decoupled, the Kalman optimal gain is decomposed as

$$L_\infty = \begin{bmatrix} L_e & 0 \\ 0 & L_a \end{bmatrix}$$

where the matrices L_e and L_a are related to the altitude and azimuth dynamics, respectively.

MPC Control Design

Considering the desired set point $\bar{y}(t) = [\bar{\vartheta}(t), \bar{\varphi}(t), \dot{\bar{\vartheta}}(t), \dot{\bar{\varphi}}(t)]^\top$ as imposed by the trajectory generator and the telescope state estimation $\hat{x}(t)$, the MPC iteratively computes, over a time horizon $(t, t+T_p]$ (being T_p the prediction horizon), the optimal input torque $u(t)$ driving the telescope axes at time t . More specifically, in order to track the reference set-point $\bar{y}(t)$, the torque input is computed based on an optimality criteria (e.g. a quadratic cost function) and by leveraging the prediction model in (3.1) over $(t, t+T_p]$. In doing so, the optimal MPC control problem is first formulated and solved over the time horizon $(t, t+T_p]$, namely the so-called MPC open control loop. Then, we present the iterative mechanism which enables the control input to adapt to the actual prediction, i.e. MPC closed-loop [69].

MPC Open Control loop Design: With the aim of predicting the position and velocity dynamics of the telescope, we consider the following dynamical system:

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m (u(t) - \delta(t)) \\ y_m(t) &= C_m x_m(t), \end{aligned} \quad (4.18)$$

being $x_m(t) = [y(t), x(t)]^\top \in \mathbb{R}^{14}$ and

$$A_m = \left[\begin{array}{c|c} 0_{14 \times 2} & C \end{array} \right] \quad B_m = \begin{bmatrix} 0_{2 \times 2} \\ B \end{bmatrix} \quad C_m = \begin{bmatrix} I_2 & 0_{2 \times 12} \\ 0_{2 \times 2} & C \end{bmatrix}. \quad (4.19)$$

Denote with $T_p \in \mathbb{R}$ the prediction horizon used for solving the optimal local control problem in the time interval $(t, t + T_p]$. Instead, the control horizon $T_c \in \mathbb{R}$ (being $T_c \leq T_p$) determines the computed control command to be imposed to the telescope axes over the time interval $(t, t + T_c]$. The notation $x_m(\cdot|t)$ stands for the trajectory of x_m over the prediction horizon $(t, t + T_p]$ at the time instant t , while, $x_m(\rho|t)$ with $\rho \in (0, T_p]$ indicates the computed value of x_m at time $t + \rho$. Moreover, we define three different telescope trajectories, namely: $(\cdot)^p$ for the one predicted via (4.18); $(\cdot)^\varnothing$ for the optimal desired trajectory; $(\cdot)^a$ for the assumed trajectory. More specifically, since the desired trajectory is known over the prediction horizon $(t, t + T_p]$, we assume this latter to be computed via the trajectory generator as in (4.1), i.e. $y^\varnothing(\cdot|t) = \bar{y}(\cdot|t)$.

Hence, the local open-loop optimal control problem for the ground-based telescope can be formalized as follows.

Problem $\mathcal{F}$ *At time t , find*

$$\min_{u^p(\cdot|t)} \int_t^{t+T_p} l(y_m^p(\rho|t), u^p(\rho|t), y_m^a(\rho|t), \bar{y}^a(\rho|t)) d\rho \quad (4.20a)$$

s.t.

$$x_m^p(\rho|t) = A_m x_m^p(\rho|t) + B_m (u^p(\rho|t) - \delta^p(\rho)) \quad (4.20b)$$

$$y_m^p(\rho|t) = C_m x_m^p(\rho|t) \quad (4.20c)$$

$$\delta^p(\rho) = \sum_{j=1}^D \sqrt{2\alpha S_\tau(f_j)\Delta f} \cos(2\pi f_j \rho + \phi_j), \quad (4.20d)$$

$$x_m^p(0|t) = \hat{x}_m(t) \quad (4.20e)$$

$$u^p(\rho|t) \in [u_{min} \quad u_{max}] \quad (4.20f)$$

$$y_m^p(T_p|t) = \bar{y}^a(T_p|t) - y^a(T_p|t) \quad (4.20g)$$

$$u(T_p|t) = \tilde{h}(T_p|t) \quad (4.20h)$$

where $u^p(\cdot|t)$ denotes the unknown control trajectory to be optimized; u_{min} and u_{max} represents the maximum admissible values for the control input. Note that additional constraints related to the azimuth and altitude position and speed are not considered in the MPC formulation since the designed trajectory generator always computes admissible reference trajectories for the telescope dynamics. $\tilde{h}(\cdot)$ is the desired final value for the control input for the counteraction of the resistance forces, i.e. the wind disturbance, at time instant T_p . Note that, it is computed from the prediction model (3.1) by imposing the tracking equilibrium condition, i.e. $\tilde{h}(T_p|t) = K_r \bar{y}^a(T_p|t) + K_f y_m^a(T_p|t) + \delta(T_p)$ being $K_r \in \mathbb{R}^4$ and $K_f \in \mathbb{R}^4$ the feed-forward and the feedback gains, respectively, which should be properly selected.

The cost function $l(\cdot)$ in (4.20a) at time instant ρ and evaluated from the instant t is defined as:

$$l(\cdot) = \|y_m^p(\rho|t) - \bar{y}(\rho|t)\|_Q + \|u^p - \tilde{h}(x_m^p(T_p|t))\|_R + \|y_m^p(T_p|t) - \bar{y}^a(T_p|t)\|_F \quad (4.21)$$

where $Q \geq 0 \in \mathbb{S}^4$ penalizes the output error from the desired equilibrium; $R > 0 \in \mathbb{S}^2$ penalizes the input error; $F \geq 0 \in \mathbb{S}^2$ represents the strength at the instant T_p to maintain the telescope output as close as possible to the assumed desired trajectories.

Tracking Model Predictive Control Closed Loop Algorithm: The Tracking MPC algorithm, allowing the derivation of the optimal torque trajectory $u(\cdot|t)$ over the control horizon $(t, t + T_c]$, for the ground-based optical telescope is developed according to the following iterative procedure.

- i. *Initialization:* At $t = 0$, based on the target star trajectory $y^*(\cdot|t)$, and the pre-processed trajectories $\bar{y}(\cdot|t)$, initialize the assumed variables as:

$$\begin{cases} u^a(\cdot|t) = \tilde{h}(0|t) \\ y^a(\cdot|t) = C_m x_m^p(0|t), \end{cases} \quad (4.22)$$

where x_m^p is the trajectory predicted via (4.18).

- ii. *First step:* At any $t > 0$, based on the estimated current Kalman filter state $\hat{x}(t)$, the assumed state $x_m^a(\rho|t)$ and the assumed star trajectory $\bar{y}^a(\rho|t)$, solve the optimal control problem $\mathcal{F}$, hence obtaining the optimal control effort $u^\heartsuit(\cdot|t)$. Based on this latter, the optimal state trajectory $x_m^\heartsuit(\cdot|t)$ is computed via the telescope model in eq.(4.18).
- iii. *Second step:* Compute the assumed control input for the next open loop control problem, namely the ones over the horizon $[t + \Delta t, t + \Delta t + T_p]$, being $\Delta t \in \mathbb{R}$ an arbitrary samples time, by disposing the optimal terms within $T_p - \Delta t$ and adding an additional one for the instant T_p , i.e.:

$$u^a(\rho|t + \Delta t) = \begin{cases} u^\heartsuit(\rho|t) & \rho \in (0, T_p - \Delta t] \\ \tilde{h}(T_p|t). \end{cases} \quad (4.23)$$

Then, compute the corresponding assumed state and output for $\rho \in (0, T_p - \Delta t]$, as:

$$\begin{aligned} \dot{x}_m^a(\rho|t + \Delta t) &= A_m x_m^a(\rho|t + \Delta t) + B_m (u^a(\rho|t + \Delta t) - \delta^a(k)) \\ y_m^a(\rho|t + \Delta t) &= C_m x_m^a(\rho|t + \Delta t) \\ x_m^a(0|t + \Delta t) &= x_m^\heartsuit(1|t). \end{aligned} \quad (4.24)$$

- iv. *Third step:* receive $\bar{y}^a(\rho|t + \Delta t)$ from the trajectory generator.
- v. *Fourth step:* Impose the optimal control input within the time interval $(t, t + T_c]$, i.e. $u(t) = u^\heartsuit(\rho|t)$ with $\rho \in (0, T_c]$.
- vi. *Fifth step:* increment t until $t + T_c$ and go to *First step*.

Remark 5 [MPC Closed-loop Stability] The stability of the telescope closed-loop system under the action of the MPC control in (4.20a) can be derived by following well-known results in the control theory literature (e.g., see [70, 71, 72, 69]). Namely, the MPC stability is mathematically ensured if the plant satisfies the following assumptions: A1) the nonlinear vector field in the plant model is twice continuously differentiable; A2) the control input region is compact, convex

and contains the origin; A3) the Jacobian linearization of the nonlinear system is stabilizable; A4) the optimal control problem is feasible at time $t = 0$. Note that, since the telescope dynamical model (3.1) is linear and satisfies Assumption 1, A1 and A3 surely hold. Moreover, the assumed control input in (4.23), thanks to the terminal input constraint (4.20h), which compensates the wind turbulence effect $\delta(t)$, satisfies A2 by construction since its admissible region is defined as $\mathcal{U} : \{u(\cdot|t) \in \mathbb{R} : u_{\min} \leq u(\cdot|t) \leq u_{\max}\}$. Finally, A4 holds for the selection of the telescope initial value for the torque within the telescope physical limits.

Remark 6 Since the proposed MPC approach belongs to the class of terminal constraint MPC [69], constraint (4.20h) plays a crucial role in the stability of the MPC control algorithm in a twofold way [70, 69]: *i*) it aims at compensating the wind disturbance $\delta(t)$ under the assumption that the Von Karman PSD model fits the real wind data, hence ensuring the robustness of telescope w.r.t. the wind disturbance; *ii*) the proper choice of K_r and K_f (tuned for instance via pole-placement and LQR techniques [9, 73]) guarantees the telescope tracking error $e(t) = \bar{y}(t) - y_m(t)$ goes toward zero as $t \rightarrow \infty$ as also explicitly imposed by the constraint (4.20g). Finally, note that this latter is also necessary to formally guarantee that the functional cost is convex and optimizable at each iteration step [70].

The Case of Telescopio Nazionale Galileo

In this section, we tailor the design of the proposed control architectures (see section 4.2 and 4.3.1) to the Telescopio Nazionale Galileo, located at “*Roque de los Muchacho*”, La Palma (Spain). To this aim, an identification procedure is firstly carried out so to obtain the TNG dynamics as in (3.1) which are exploited for simulation purposes in the Matlab platform. Then, this latter is used for the validation of the optimal tracking controller and the MPC-based control architecture.

An extensive simulation analysis for both the proposed approaches is provided. Specifically, the tracking scenario of a pseudo-star close to the singularity position, i.e. $\vartheta(t) = 90^\circ$, is performed to disclose the tracking performance of both, while, concerning the MPC-based approach, its analysis also involves the tracking of the real scientific target *TYC 1731-916-1*. Moreover, the simulation analysis includes a comparison w.r.t. other state-of-the-art control architectures, while, to better highlight the achieved performance via the MPC-based architecture, its performance analysis also involves a robustness analysis w.r.t. model mismatch, disclose the benefits and the advantages of the proposed solution.

5.0.1 TNG Dynamics Identification

By leveraging the real data in [55], we derive the control-oriented model for TNG able to reproduce its Input/Output (I/O) behaviour as in (3.1)

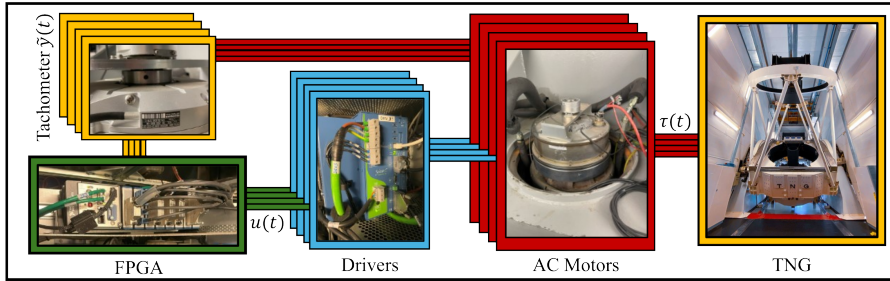


Figure 5.1. Schematic overview for data acquisition experimental set-up for both altitude and azimuth axes.

such that it can be exploited for the designing of proposed optimal tracking controller and MPC-based tracking control solution.

It is worth noting that, due to the morphology of the area of "Roque de lo Muchachos" and the TNG observatory structure the Von Kaman PSD model can represent the wind disturbance [74]. For this purpose, data are recorded via the experimental set-up depicted in Figure 5.1. It consists of: *i*) 4 Heidenhain RPN 886 tachometers (yellow squares) for the measurement of TNG velocity and position trajectories; *ii*) 4 DC brush-less permanent magnets electric motors with 32 pole pair stator, 3 phase at 180 [V] Voltage Alternative Current (VAC) (red squares) for the imposition of the real torque to the TNG; *iii*) 4 Aerotech Soloist Hpe-50 drivers (blue squares) for the providing of the suitable voltages to the actuators such that the desired torques, computed by the CU, are ensured; *iv*) the FPGA (green square), i.e. the CU, of the National Instrument compactRIO (NI cRio) 9039 for the controlling of the TNG telescope. Note that, the presented hardware is replicated for both axes unless the FPGA aims at controlling both simultaneously.

The exploited data have been acquired by exciting the TNG real system via a mean zero white noise torque, at a sample time of 0.002 (namely the control unit sample time) and recording the altitude and azimuth speed trends $\tilde{y}(t) = [\dot{\hat{v}}(t), \dot{\hat{\phi}}(t)]$ via the on-board equipped tachometers. Note that, the ground telescope presents different operative scenarios depending on the altitude dynamics. In this work, data measurements refer to the altitude operative range of $[80, 90] [deg]$ since it is the challenging op-

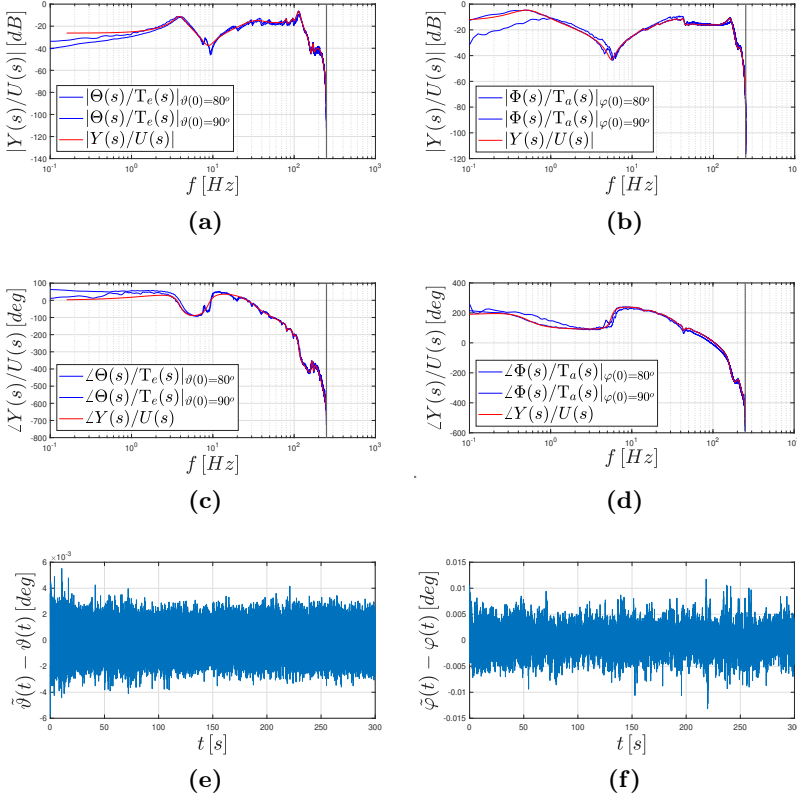


Figure 5.2. Identification results within the operative range $\vartheta \in (80, 90) [deg]$: (a) comparison of the magnitude measured data (blue lines) and the frequency response of the identified altitude axes model (red line); (b) comparison of the magnitude measured data (blue lines) and the frequency response of the identified azimuth axes model (red line); (c) comparison of the phase measured data (blue lines) and the phase diagram of the identified altitude axes model (red line); (d) comparison of the phase measured data (blue lines) and the phase diagram of the identified azimuth axes model (red line); (e) time history of the prediction error $\tilde{\vartheta}(t) - \vartheta(t)$; (f) time history of the prediction error $\tilde{\varphi}(t) - \varphi(t)$.

Table 5.1. Ground-based Telescope model parameter set-up

(a) TNG Linear State Space Matrices

$A_e = \begin{bmatrix} -21.59 & 374.92 & 522.37 & 174.12 & 85.41 & -448.44 \\ -363.16 & -4.54 & -360.01 & -60.97 & 47.62 & 193.88 \\ -321.21 & 331.22 & -127.72 & -588.27 & 7.52 & 497.92 \\ -414.77 & -64.38 & 1353.24 & -519.62 & 232.46 & 1087.37 \\ -39.80 & -33.56 & -70.82 & 2.58 & -4.57 & -371.65 \\ 180.89 & -140.82 & -15.70 & -682.73 & 311.77 & 47.47 \end{bmatrix}$

$A_a = \begin{bmatrix} 226.76 & 959.76 & 478.91 & 174.65 & 1114.18 & -942.69 \\ -328.81 & 328.57 & 679.15 & 130.25 & 145.20 & -132.75 \\ 537.10 & -481.54 & 114.51 & 153.75 & 470.65 & -234.11 \\ -306.69 & -133.06 & -209.26 & 61.65 & -94.12 & -14.57 \\ -595.34 & -1032.85 & -2433.71 & 183.40 & -1354.58 & 1483.91 \\ 250.63 & 321.98 & 945.13 & 4.65 & 396.46 & -458.34 \end{bmatrix}$

$B_e = \begin{bmatrix} 0.0072 \\ -0.0048 \\ -0.0048 \\ -0.0162 \\ 0.0028 \\ -0.0131 \end{bmatrix}$

$B_a = \begin{bmatrix} 0.0011 \\ 0.0004 \\ -0.0006 \\ -0.0013 \\ -0.0020 \\ 0.00057 \end{bmatrix}$

$C_e = \begin{bmatrix} 10614.83 \\ 6577.62 \\ 3222.93 \\ 334.27 \\ 846.50 \\ 292.02 \end{bmatrix}^\top$

$C_a = \begin{bmatrix} -17490.42 \\ -13275.20 \\ 9480.43 \\ -3518.08 \\ -1713.89 \\ 24.57 \end{bmatrix}^\top$

(b) Von-Karman model parameters and White Noise measurement features

$f(j) = j\Delta f, j \in (0, D)$
 $\bar{\tau} = 1 [kN]$
 $\mathcal{J} = 0.15$
 $\Lambda = 10 [m^2]$

$D = 10000$
 $\alpha = 0.35$
 $\Gamma = 3.2$
 $\phi_j = \mathcal{N}(0, 0.5)$

$\Delta f = 0.01 [1/s]$
 $\omega = \mathcal{W}(0, 1.16) [deg]$
 $\bar{v} = 3.2 [m/s^2]$

erative range. Note that, within this range, the telescope azimuth speed trajectory is the fastest required.

The system identification problem is here formulated as an optimization task where the objective is to find the set of linear state space matrices A ,

B , C and D in (3.1) such that the prediction error between the plant outputs, i.e. the measured data $\tilde{y}(t)$, and the model output, computed based on the known inputs, i.e. $y(t)$, is minimised. Specifically, by assuming the TNG model as a black-box system, the exploited identification procedure leverages the non-iterative subspace method, to properly compute the TNG linear state space model, and the prediction error minimisation approach, to improve the accuracy of the identified matrices [75].

The resulting TNG matrices, being D a null matrix, are identified as in Table 5.1. Moreover, by leveraging the well-known procedure as in [76], is easy to find that the derived observability and controllability matrices are full-row rank, hence, the identified model is such that assumption 1 and 2 hold allowing the design of the proposed control approach. Concerning the assumption 3, is clear that the provided TNG output matrix C reported in Table 5.1 is full row rank.

The identified TNG linear model presents an RMSE of $6e^{-4}$ [deg] and of 0.002 [deg] for the altitude and azimuth axes, respectively, hence highlighting a good prediction capability.

The good prediction capability is also confirmed by Figure 5.2(e)-(f) where the time history of the altitude prediction error $\hat{\vartheta}(t) - \vartheta(t)$ [deg] and azimuth prediction error $\hat{\varphi}(t) - \varphi(t)$ [deg] are reported, respectively.

The reliability of the identified model in correctly predicting the TNG behaviour is also disclosed in Figure 5.2(a)-(b)-(c)-(d). Specifically, the magnitude of the input-output measurements w.r.t. magnitude of the frequency response of the identified altitude TNG dynamics is reported in Figure 5.2(a), while their phase diagrams are reported in Figure 5.2(c). The same behaviour can be appreciated for the azimuth TNG dynamics in Figure 5.2(b)-(d).

Finally, for simulation purposes, the Von-Karman PSD features are reported in Table 5.1(b).

Remark 7 From the experimental analysis of the TNG behaviour, we noted, at the low-frequency range, a mismatching between the frequency responses of measurements and the identified model (see Figure 5.2(a)-(b)-(c)-(d)). This is because TNG dynamics are affected by nonlinearities, such as balancing issues (caused by the TNG ageing) and its strong static friction when passing from a specific position to another one with non-zero speed. Moreover, the frequency mismatches are also justified by the exploited identification algorithm whose aim is the minimisation of the prediction error trend over the time domain.

5.1 Optimal Tracking Control Architecture: Performance Analysis

In this section, the effectiveness of the optimal tracking control architecture –presented in section 4.2– for the case study of TNG whose dynamics in (3.1) are identified as in section 5.0.1.

With the aim of implementing the control architecture in Section 4.2, the derived TNG state space representation is then decomposed into its flexible dynamics as in [66]. By solving the optimal problem $\mathcal{J}$, according to the procedure reported in Remark 3, we obtain the speed layer control gains K_r , K_s as in Table 5.4(a) with a proper choice of the degree of freedom Q_s , R_s , ν and η (omitted for the sake of brevity). The Kalman Filter matrix L_∞ is computed according to (4.16). For the sake of readability, the Kalman optimal gain is decomposed as

$$L_\infty = \begin{bmatrix} L_e & 0 \\ 0 & L_a \end{bmatrix}.$$

where the matrices L_e and L_a , reported in Table 5.2(a), are related to the altitude and azimuth dynamics, respectively.

Regarding the selection of the control gains K_c in the position layer, it is worth noting that the inner loop –i.e. the closed-loop system comprehensive of the identified TNG dynamics in flexible states and speed controller– is such that Assumption 6 does not hold. To this end, since the overall closed-loop speed layer is asymptotically stable, we apply the decomposition of their dynamics in controllable and uncontrollable sets. The design of K_c only involves the controllable part of the inner loop (i.e. $x_T \in \mathbb{R}^8$) and by exploiting the procedure in Remark 4, we derive the control gain as in (4.13) (with a proper selection of the matrices Q_c and R_c , omitted for the sake of brevity). Finally, the physical constraints involved in the parameter set-up of the trajectory generator are set as in [6] and reported in Table 5.2(c).

The validation is performed by exploiting an own-implemented *Matlab* & *Simulink*-based simulation platform with an integration time step of 0.002 [s] and trapezoidal stiffness algorithm solver (ode23t). Moreover, different telescope configurations are emulated so to show the effectiveness of the optimal tracking architecture in tracking the scientific target as well

Table 5.2. Control algorithm Set-Up

(a) Control Gains of the Optimal Tracking Architecture		
$K_r = [122.45 \quad 89.74]^\top$	$K_s = [-471.28 \quad 190.56]^\top$	
$k_i = \begin{bmatrix} 70.71 & 0 \\ 0 & 10.20 \end{bmatrix}$	$k_p = \begin{bmatrix} 108.87 & 0 \\ 0 & 88.25 \end{bmatrix}$	
$K_f = \left[\begin{array}{c c} \begin{matrix} 4.5e^{-4} \\ 15.97 \\ 4.95e^4 \\ 3.65e^3 \end{matrix} & 0_{4 \times 1} \\ \hline \begin{matrix} 52.28 \\ 8.40 \\ 11.61 \\ 2.52 \end{matrix} & 0_{4 \times 1} \end{array} \right]^\top$	$L_e = \begin{bmatrix} -2.71e^{-14} \\ 7.69e^{-13} \\ -6.77e^{-13} \\ -1.28e^{-12} \\ 4.77e^{-11} \\ -1.21e^{-10} \\ -0.99 \end{bmatrix}$	$L_a = \begin{bmatrix} 3.16 \\ -2.84e^{-15} \\ -1.38e^{-16} \\ -1.25e^{-15} \\ 4.32e^{-15} \\ -1.86e^{-15} \\ -3.04e^{-5} \end{bmatrix}$
(b) Trajectory generator parameters		
$Tc = 0.001 [s]$	$v_{max} = 1.5 [deg/s]$	$a_{max} = 0.1 [deg/s^2]$

as its robustness w.r.t. input external disturbances and white noises in all the operative conditions. A comparison analysis w.r.t. a similar control architecture which, however, embeds a PID controller in the position layer, is also provided to highlight the advantages of the proposed control architecture.

5.1.1 Validation Results

The effectiveness of the optimal control architecture is disclosed by considering the more challenging tracking scenario for a ground-based telescope, namely, the one close to the singularity condition $\vartheta(t) \simeq 90^\circ$. Considering the TNG observatory, located at latitude $28^\circ 45' 14'' N$ and longitude $17^\circ 53' 17'' W$, the scientific star target trajectories are emulated via the astronomical computation with RA of $21 : 25 : 00 [hh : mm : ss]$, DEC of $27 : 60 : 00 [dd : mm : ss]$ and observation date $9 : 10 : 2023 [dd : mm :$

$yy]$ at time $20 : 53 : 52.72$ [$hh : mm : ss$]. In doing so, the altitude of the pseudo star is about 89° while the telescope's initial condition is set at zero state.

According to the initial setting for the astronomical instrument and the scientific target, the trajectory generator in Section 4.1, computes the admissible telescope trajectories to reach the star in less than 100 [s]. Results are depicted in Figure 5.3 - 5.4 - 5.5 - 5.6 and disclose the high-tracking performance ensured by the proposed optimal control architecture despite the presence of the external disturbance $\delta(t)$ and the noise measurements ω . More specifically, Figure 5.3 (a)-(b) show the position trends of the pseudo star $y^*(t) = [\vartheta^*(t), \varphi^*(t)]$, the pre-processed trajectories $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]$ and the telescope positions $y_p(t) = [\vartheta(t), \varphi(t)]$, hence confirming the achievement of the control goal (3.6a)-(3.6b) with small bounded errors (see Figure 5.5 (a)) This is also due to the efficiency of the speed layer control in ensuring the tracking of the telescopes speed measurements $y(t) = [\dot{\vartheta}(t), \dot{\varphi}(t)]$ towards the optimal speed time-varying reference $s(t) = [s_\vartheta(t), s_\varphi(t)]$, being $s_\vartheta(t)$ and $s_\varphi(t)$ the speed reference command for the altitude and azimuth dynamics, respectively (see Figure

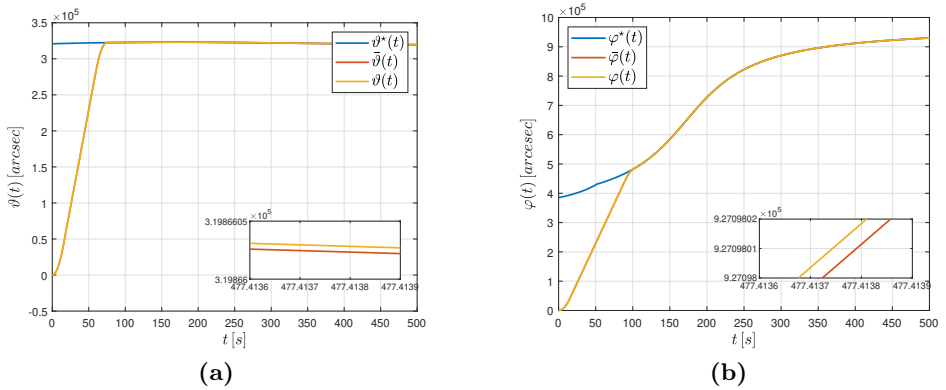


Figure 5.3. Performance analysis of the Optimal Tracking Controller for the TNG case study. Time history of: (a) altitude position trajectories of the star $\vartheta^*(t)$, altitude pre-processed position trajectory $\bar{\vartheta}(t)$ and telescope position $\vartheta(t)$; (b) azimuth position trajectory of the star $\varphi^*(t)$, azimuth pre-processed position trajectory $\bar{\varphi}(t)$ and telescope position $\varphi(t)$.

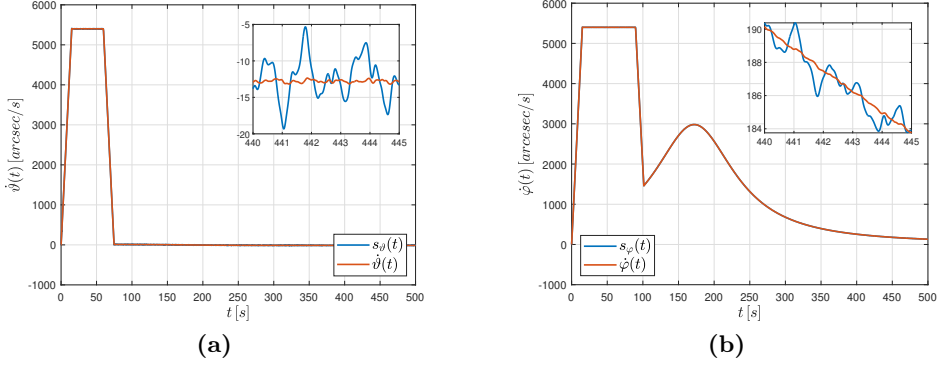


Figure 5.4. Performance analysis of the Optimal Tracking Controller for the TNG case study. Time history of: (a) altitude speed trajectory $s_\theta(t)$ and telescope speed $\dot{\vartheta}(t)$; (b) azimuth speed trajectory $s_\varphi(t)$ and telescope speed $\dot{\varphi}(t)$.

5.4 (a)-(b)) with small bounded errors (see Figure 5.5 (b)). Moreover, for sake of completeness, Figure 5.6 (a) reports the time-history of the computed optimal torques $u(t) = [\tau_e(t), \tau_a(t)]$ while Figure 5.6 (b) reports the estimation errors of the Kalman filter embedded into the positional layer, respectively.

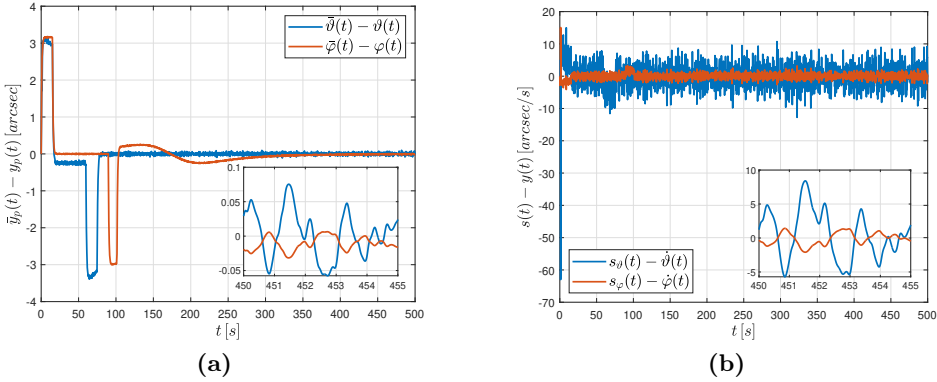


Figure 5.5. Performance analysis of the Optimal Tracking Controller for the TNG case study. Time history of: (a) tracking position errors $y^*(t) - y_p(t)$; (b) tracking speed errors $s(t) - y(t)$.

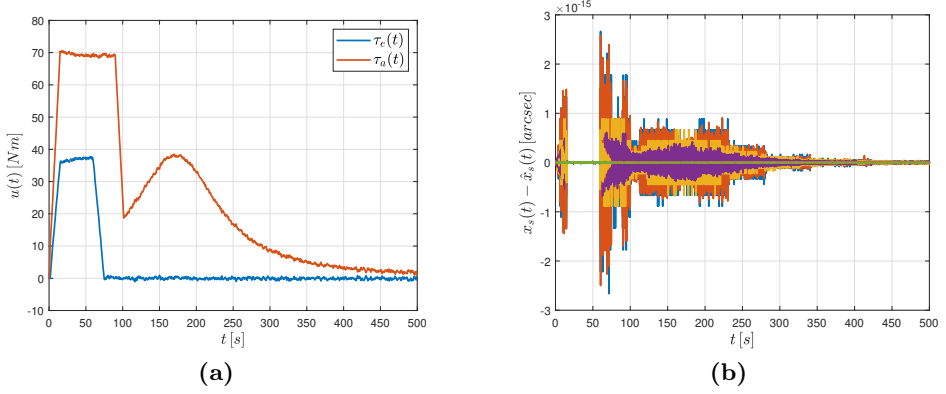


Figure 5.6. Performance analysis of the Optimal Tracking Controller for the TNG case study. Time history of: (c) optimal control torques $u(t)$; (b) estimation state errors $x_s(t) - \hat{x}_s(t)$ via the Kalman filter.

5.1.2 Comparison Analysis

To better highlight the advantages and the benefits of the proposed optimal control architecture in solving the tracking telescope control problem $\mathcal{C}$, in this section, we compare its closed-loop performance with the one achievable via PID-based control architecture [10]. As simulation scenario, consider the astronomical computation set-up such that the telescope has to reach 10 different altitude configurations, i.e.:

$$\vartheta^*(0) \in (31^\circ, 41^\circ, 51^\circ, 60^\circ, 66^\circ, 71^\circ, 78^\circ, 82^\circ, 87^\circ, 89^\circ).$$

The comparison analysis w.r.t. the PID-based control architecture is performed by taking into account the RMSE KPI at the steady state, namely, when the star is engaged. Due to the physically constrained pre-processed positions computed via the trajectory generator, each target star is engaged approximately in 100 [s]. By taking into account a simulation time of $T_s = 500$ [s], the RMSE is computed for both the altitude and azimuth

Table 5.3. Steady state Root-Mean-Square-Error Comparison

Controller:		LQG-PI		PID	
KPI:		ϑ_e [arcsec]	φ_e [arcsec]	ϑ_e [arcsec]	φ_e [arcsec]
$\vartheta^*(0)$:	31°	0.0087	0.0087	0.0273	0.0273
	41°	0.0089	0.0089	0.0275	0.0275
	51°	0.0086	0.0086	0.0274	0.0274
	60°	0.0084	0.0084	0.0262	0.0262
	66°	0.0091	0.0091	0.0294	0.0294
	71°	0.0083	0.0083	0.0281	0.0281
	78°	0.0084	0.0084	0.0266	0.0266
	82°	0.0082	0.0082	0.0278	0.0278
	87°	0.0084	0.0084	0.0278	0.0278
	89°	0.0191	0.0191	0.0313	0.0313

dynamics as:

$$\vartheta_e = \frac{1}{T_s - t_0} \int_{t_0}^{T_s} \|\vartheta^*(\rho) - \vartheta(\rho)\|_2 d\rho, \quad (5.1)$$

$$\varphi_e = \frac{1}{T_s - t_0} \int_{t_0}^{T_s} \|\varphi^*(\rho) - \varphi(\rho)\|_2 d\rho, \quad (5.2)$$

where the $t_0 = 200$ [s] is the initial computation time of the RMSE, assumed to be greater than the engagement instant.

Results, reported in Table 5.3, disclose the improved performance achieved via the proposed solution. Indeed, the following results, for both altitude RMSE and azimuth RMSE, are obtained w.r.t. the PID-based controller: -68.13% for $\vartheta^*(0) = 31^\circ$; -68.63% for $\vartheta^*(0) = 41^\circ$; -68.61% for $\vartheta^*(0) = 51^\circ$; -67.94% for $\vartheta^*(0) = 60^\circ$; -69.04% for $\vartheta^*(0) = 66^\circ$; -70.46% for $\vartheta^*(0) = 71^\circ$; -68.42% for $\vartheta^*(0) = 78^\circ$; -70.50% for $\vartheta^*(0) = 82^\circ$; -69.78% for $\vartheta^*(0) = 87^\circ$; -38.97% for $\vartheta^*(0) = 89^\circ$.

In conclusion, the comparison results show the benefits and advantages of adopting the proposed optimal tracking control architecture, as well as highlighting the high-tracking performance achievable despite the presence of external input disturbances and measurement noises compared to the ones achievable via the PID approach.

5.2 MPC-based Tracking Control Architecture: Performance Analysis

In this section, we disclose the effectiveness of the proposed MPC-based in letting the TNG telescope track the scientific target as well as its robustness w.r.t. input external disturbances and measurement white noises. With the aim of implementing the MPC-based control algorithm in Problem $\mathcal{F}$, the derived TNG state space representation, described in section 5.0.1, is exploited as prediction model. By leveraging the *Yalmip* tool-box along with *Mosek* optimizer, the MPC closed-loop algorithm is developed according to section ??, with a proper choice of the degree of freedom Q ,

Table 5.4. MPC-based Control Architecture Set-Up

(a) MPC Weighting Matrices and Kalman Filter Observer Gain			
$Q = \begin{bmatrix} 2000 & 0 & 0 & 0 \\ 0 & 1500 & 0 & 0 \\ 0 & 0 & 150 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix} \quad F = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 50 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} 10^{-4} & 0 \\ 0 & 10^{-6} \end{bmatrix}$			
$L_e = \begin{bmatrix} -2.710 \times 10^{-14} \\ 7.6969 \times 10^{-13} \\ -6.775 \times 10^{-13} \\ -1.284 \times 10^{-12} \\ 4.7708 \times 10^{-11} \\ -1.217 \times 10^{-10} \end{bmatrix} \quad L_a = \begin{bmatrix} -2.846 \times 10^{-15} \\ -1.388 \times 10^{-16} \\ -1.254 \times 10^{-15} \\ 4.3253 \times 10^{-15} \\ -1.860 \times 10^{-15} \\ -3.040 \times 10^{-5} \end{bmatrix}$			
(b) MPC and Tracking Generator features			
MPC:	$\Delta t = 0.002 [s]$	$T_p = 1 [s]$	$T_c = 0.002 [s]$
	$u_{min} = -500 [Nm] \quad u_{max} = 500 [Nm]$		
Trajectory generator:	$\Delta t = 0.002 [s]$	$v_{max} = 1.5 [deg/s]$	$a_{max} = 0.1 [deg/s^2]$

R and F reported in Table 5.4(a).

Conversely, the Kalman Filter gain L_∞ is computed according to (4.16) with matrices L_e and L_a reported in Table 5.4(a). The physical constraints involved in the parameter set-up of the trajectory generator are set as in [6] and they are reported as in Table 5.4(b).

Based on the TNG model identified in section 5.0.1, by characterising the Von-Karman disturbance model along with the measurement white noise as in Table 5.4(d), the virtual testing is performed by exploiting an advanced own-made *Matlab&Simulink* simulation platform. This latter exploits a trapezoidal stiffness algorithm solver (ode23t) as integration method with a sample time of $\Delta t = 0.002$ [s].

The virtual testing procedure involves the following star tracking scenarios: *i*) the tracking of the scientific star *TYC 1731-916-1*; *ii*) the tracking of a pseudo star which forces the TNG operating near the singularity condition $\vartheta \simeq 89$ [deg]. Finally, a comparison analysis, w.r.t. a Linear-Quadratic-Gaussian-Proportional-Integrator (LQG-PI) [9] and a Proportional-Integrator-Derivative (PID) [10] based tracking controllers, and a robustness analysis, w.r.t. the model mismatch between the prediction model in (4.20b) and plant (3.1), highlights the advantages and benefits of the proposed MPC-based control architecture.

Remark 8 Note that, the resolution of the position encoders equipped on the TNG is 139,06 [arcsec]. However, it is important to remark two crucial aspects: *i*) the real data related to TNG dynamics, used to identify the TNG dynamical model, are obtained via the tachometer measurements whose resolution is higher than the encoder. Indeed, the conversion factor between the encoder and the related tachometer is 1 encoder count is $1.1e^{-4}$ [arcsec] for the related tachometer; *ii*) the own-realized simulation platform does not take into account the emulation of the encoder since all the measured physical quantities are derived from the tachometers. This latter point is an essential aspect for better understanding why the results in Table 5.5 are of one order of magnitude less than the one ensured by the encoders themselves.

Tracking of *TYC 1731-916-1*

Considering the TNG observatory, located at latitude $28^\circ 45' 14'' N$ and longitude $17^\circ 53' 17'' W$, the scientific star target trajectories of *TYC 1731-916-1* are obtained via the astronomical computation with Right Ascension (RA) of $00 : 29 : 50$ [hh : mm : ss], Declination (DEC) of $23 : 45 :$

09 [dd : mm : ss] in the observation date 22 : 03 : 2024 [dd : mm : yy] at time 13 : 40 : 12 [hh : mm : ss]. Note that the real scientific star altitude trajectory is within the operative range of the identified TNG model, i.e. $\theta^*(t) \in (80, 90) [deg]$. Moreover the trajectory generator in Section ??, computes the admissible telescope trajectories $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]$ to reach the star in less then 10 [s] from the rest position, i.e. initial state condition, $\vartheta(0) = 90 [deg]$ and $\varphi(0) = \kappa 360 [deg] \forall \kappa \in \mathbb{N}$.

Results are depicted in Figure 5.7 - 5.8 - 5.9 - 5.10 and disclose the high-tracking performance ensured by the proposed optimal control architecture despite the presence of the external disturbance $\delta(t)$ and the noise measurements ω . More specifically, Figure 5.7(a)-(b) show the position trends of the scientific target $y^*(t) = [\vartheta^*(t), \varphi^*(t)]$, the pre-processed trajectories $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]$ and the telescope positions $[\vartheta(t), \varphi(t)]$, hence confirming the achievement of the control goal (3.6a) with small bounded errors (see Figure 5.10(a)). This high-tracking performance is also disclosed in Figure 5.8(a)-(b). Herein, it is highlighted the capability of the proposed solution in ensuring the tracking of the speed measurements $y(t) = [\vartheta(t), \varphi(t)]$ towards the target star velocities, i.e. $\dot{y}^*(t) = [\dot{\vartheta}^*(t), \dot{\varphi}^*(t)]$, and the pre-processed trajectories, i.e. $\bar{y}_s(t) = [\dot{\bar{\vartheta}}(t), \dot{\bar{\varphi}}(t)]$, hence confirm-

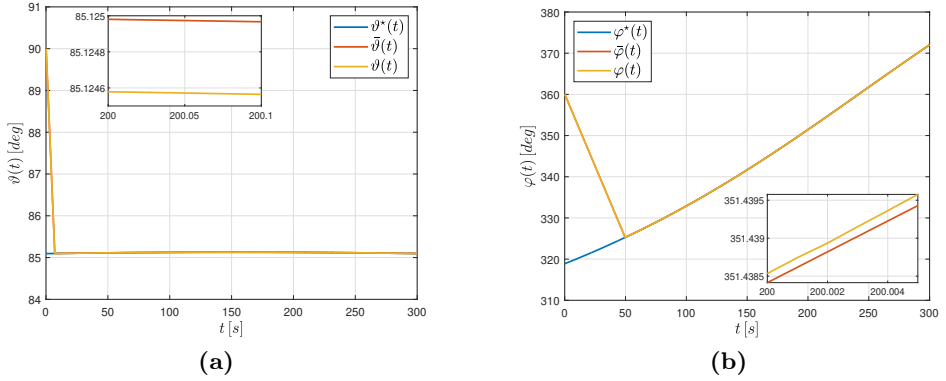


Figure 5.7. Performance analysis for the TNG case study. Tracking of *TYC 1731-916-1*. Time history of: (a) altitude position of the star $\vartheta^*(t)$, trajectory generator $\bar{\vartheta}(t)$ and telescope $\vartheta(t)$; (b) the azimuth position of the star $\varphi^*(t)$, trajectory generator $\bar{\varphi}(t)$ and telescope $\varphi(t)$.

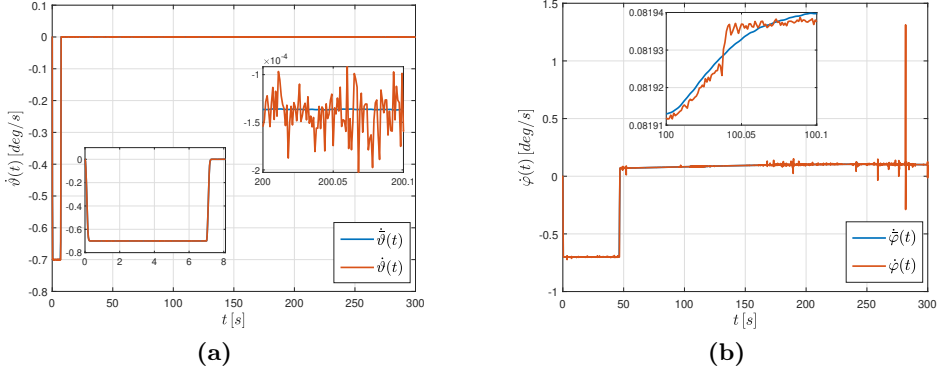


Figure 5.8. Performance analysis for the TNG case study. Tracking of *TYC 1731-916-1*. Time history of: (a) altitude speed of the star $\dot{\vartheta}^*(t)$, trajectory generator $\dot{\vartheta}(t)$ and telescope $\dot{\vartheta}(t)$; (b) azimuth speed trajectory $\dot{\varphi}(t)$ and telescope speed $\dot{\varphi}(t)$.

ing the achievement of the control goal (3.6b) with small bounded errors (see Figure 5.10(b)). Moreover, for sake of completeness, Figure 5.9(a) reports the time-history of the computed optimal torques $u(t) = [\tau_e(t), \tau_a(t)]$ while Figure 5.9(b) show the effectiveness of the Kalman filter in ensuring

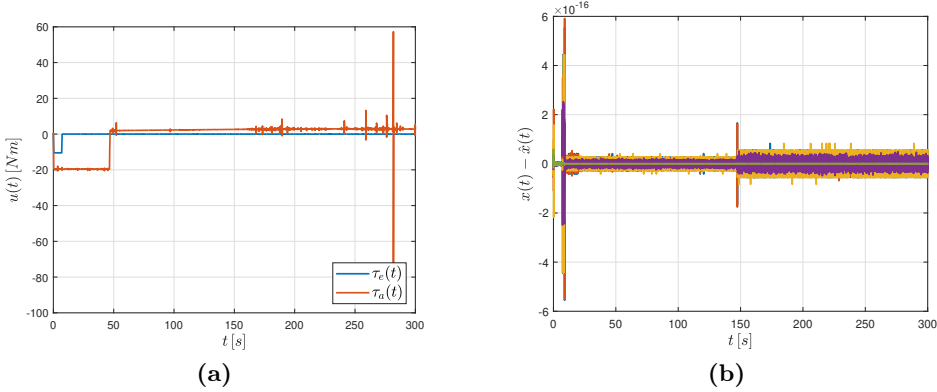


Figure 5.9. Performance analysis for the TNG case study. Tracking of *TYC 1731-916-1*. Time history of: (a) optimal control torques $u(t)$; (f) estimation state errors $x(t) - \hat{x}(t)$ provided by Kalman filter.

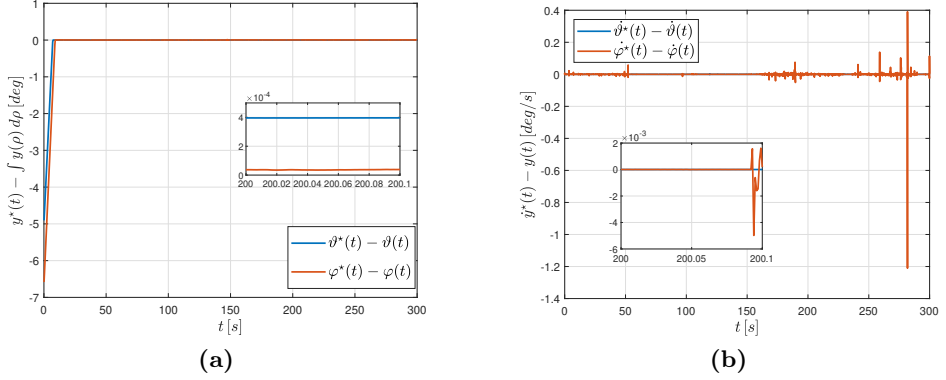


Figure 5.10. Performance analysis for the TNG case study. Tracking of *TYC 1731-916-1*. Time history of: (a) tracking position errors $y^*(t) - y_p(t)$; (b) tracking speed errors $\dot{y}^*(t) - \dot{y}(t)$.

the convergence of the estimation errors towards zero.

Pseudo Star in Singularity Condition

The effectiveness of the MPC-based control architecture is here disclosed by considering the more challenging tracking scenario for a ground-based telescope, namely, the one close to the singularity condition $\vartheta(t) \simeq 90 [deg]$.

Hence, considering the TNG observatory, the pseudo-scientific star target trajectories are emulated via the astronomical computation with Right Ascension (RA) of $21 : 25 : 00 [hh : mm : ss]$, Declination (DEC) of $27 : 60 : 00 [dd : mm : ss]$ in the observation date $9 : 10 : 2023 [dd : mm : yy]$ at time $20 : 53 : 52.72 [hh : mm : ss]$. In doing so, the altitude of the pseudo star is about $89 [deg]$ while the telescope is in rest position, i.e. $\vartheta(0) = 90 [deg]$ and $\varphi(0) = \kappa 360 [deg] \forall \kappa \in \mathbb{N}$.

According to the initial setting for the TNG telescope and the scientific target position $\bar{y}_p(t) = [\bar{\vartheta}(t), \bar{\varphi}(t)]$, the trajectory generator, in Section ??, computes the admissible telescope trajectories to reach the star in less than $50 [s]$. Results are depicted in Figure 5.11 - 5.12 - 5.13 - 5.14 and disclose the high-tracking performance ensured by the proposed optimal control architecture despite the presence of the external disturbance $\delta(t)$

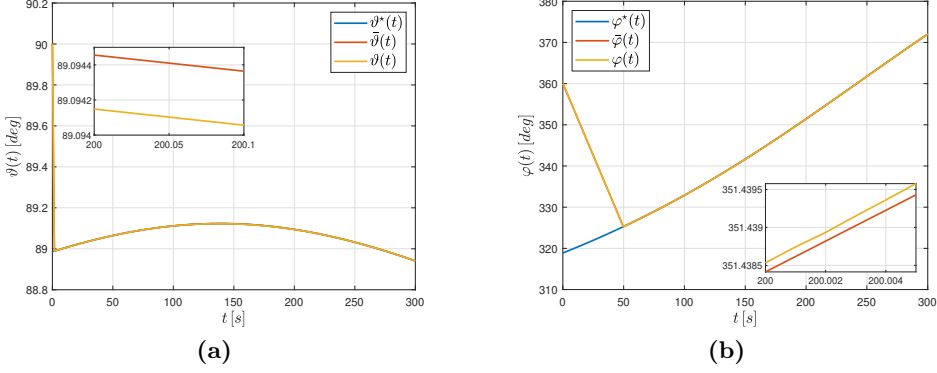


Figure 5.11. Performance analysis for the TNG case study. Tracking of the pseudo star in singularity condition. Time history of: (a) altitude position of the star $\vartheta^*(t)$, trajectory generator $\bar{\vartheta}(t)$ and telescope $\vartheta(t)$; (b) the azimuth position of the star $\varphi^*(t)$, trajectory generator $\bar{\varphi}(t)$ and telescope $\varphi(t)$.

and the noise measurements ω .

More specifically, Figure 5.11(a)-(b) show the position trends of the pseudo star $y^*(t) = [\vartheta^*(t), \varphi^*(t)]$, the pre-processed trajectories $\bar{y}_p(t) =$

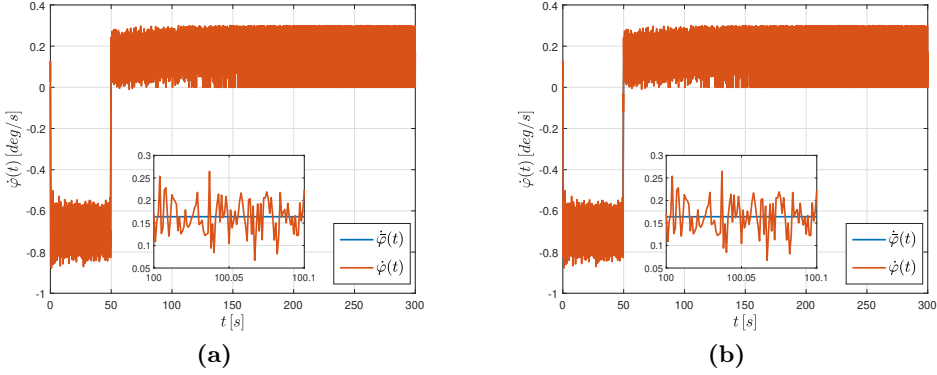


Figure 5.12. Performance analysis for the TNG case study. Tracking of the pseudo star in singularity condition. Time history of: (a) altitude speed of the star $\dot{\vartheta}^*(t)$, trajectory generator $\dot{\bar{\vartheta}}(t)$ and telescope $\dot{\vartheta}(t)$; (b) azimuth speed trajectory $\dot{\bar{\varphi}}(t)$ and telescope speed $\dot{\varphi}(t)$.

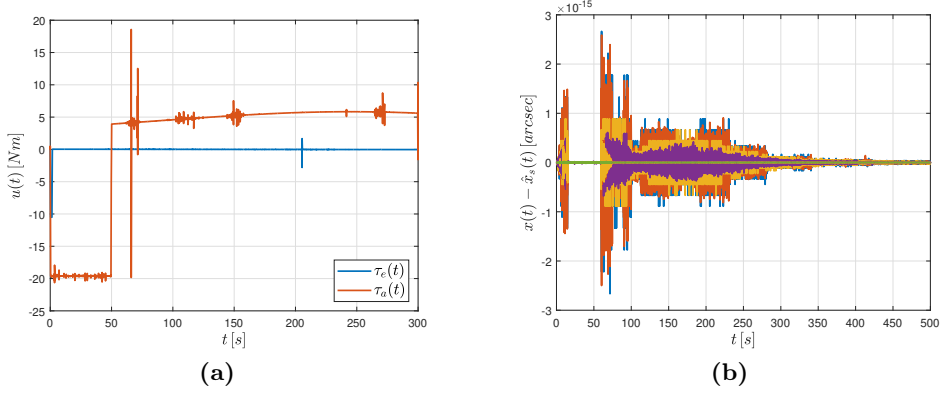


Figure 5.13. Performance analysis for the TNG case study. Tracking of the pseudo star in singularity condition. Time history of: (a) optimal control torques $u(t)$; (b) estimation state errors $x(t) - \hat{x}(t)$ provided by Kalman filter.

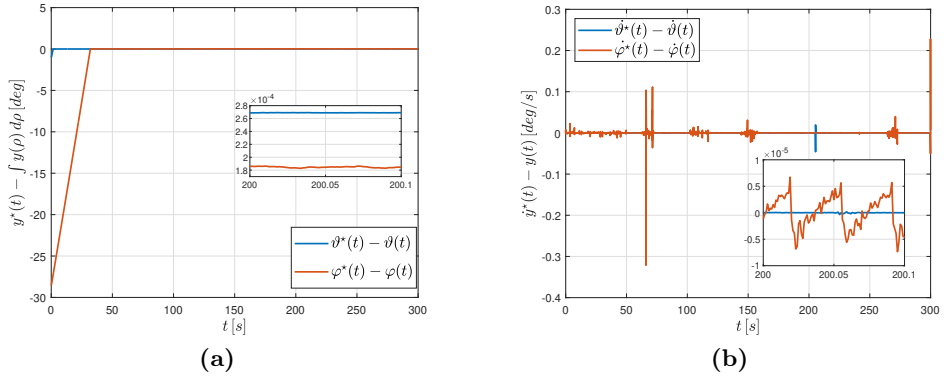


Figure 5.14. Performance analysis for the TNG case study. Tracking of the pseudo star in singularity condition. Time history of: (a) tracking position errors $y^*(t) - y_p(t)$; (b) tracking speed errors $\dot{y}^*(t) - \dot{y}(t)$.

$[\bar{\vartheta}(t), \bar{\varphi}(t)]$ and the telescope positions $[\vartheta(t), \varphi(t)]$, hence confirming the achievement of the control goal (3.6a) with small bounded errors (see Figure 5.11(a)). Moreover, Figure 5.12(a)-(b), reporting the velocities trends of the pseudo star $\dot{y}^*(t) = [\dot{\vartheta}^*(t), \dot{\varphi}^*(t)]$, the pre-processed speed trajec-

tories $\bar{y}_s(t) = [\dot{\vartheta}(t), \dot{\varphi}(t)]$ and the telescope speeds $y(t) = [\dot{\vartheta}(t), \dot{\varphi}(t)]$, show the capability of the MPC-based tracking control in ensuring the speed tracking of the target pseudo star, hence confirming the achievement of the control goal (3.6b) with small bounded error (see Figure 5.14(b)). Moreover, for sake of completeness, Figure 5.13(a) reports the time-history of the computed optimal torques $u(t) = [\tau_e(t), \tau_a(t)]$ while Figure 5.9(b) show the effectiveness of the Kalman filter in ensuring the convergence of the estimation errors towards zero.

5.2.1 Comparison Analysis

To better highlight the advantages and the benefits of the proposed MPC-based control architecture in solving the tracking telescope control problem, in this section, we compare its closed-loop performance with both the one achievable via the LQG-PI-based double layer control architecture recently proposed in [9] and the one obtainable via PID-based control approach in [10]. For more details about the control design, the tuning rules and the values of the control gains for both the controllers in comparison, see [9].

The comparison analysis is performed by considering 4 different pseudo star trajectories which are selected such that $\vartheta^*(0)$ is within the operative range of the TNG identified model, i.e. $\vartheta^*(0) = (78, 82, 87, 89)$ [deg] respectively.

To disclose the improved performance achievable via the proposed approach, the following KPIs are exploited: *i)* the *Tracking Index* (TI), which evaluates the effectiveness of a controller in tracking the reference behaviour imposed by a trajectory generator [77]; *ii)* the *root-mean-square error* (RMSE) which assesses the performance of a controller in tracking the pseudo star at the steady state, i.e. after the engagement of this latter [78];

More specifically, the TI, which weights the speed and position tracking

error, is formalized as:

$$TI|_{\vartheta} = \frac{1}{T_s} \int_0^{T_s} \left| \varpi_p \cdot (\bar{\vartheta}(\rho) - \vartheta(\rho)) + \varpi_s \cdot (\dot{\bar{\vartheta}}(\rho) - \dot{\vartheta}(\rho)) \right| d\rho \quad (5.3)$$

$$TI|_{\varphi} = \frac{1}{T_s} \int_0^{T_s} \left| \varpi_p \cdot (\bar{\varphi}(\rho) - \varphi(\rho)) + \varpi_s \cdot (\dot{\bar{\varphi}}(\rho) - \dot{\varphi}(\rho)) \right| d\rho \quad (5.4)$$

where ϖ_p and ϖ_s are positive weights that represent the sensitivity to the position and the velocity error and are set as $\varpi_p = 1$ and $\varpi_s = 10$.

Conversely, for the computation of the RMSE, we take into account the time windows starting from $t_0 = 100$ [s], namely the time required by the pre-processed trajectories in engaging the target star, until the end of the simulation scenario, i.e. the simulation time $T_s = 300$ [s]. Hence, the RMSE, for both position trajectories, is formulated as:

$$RMSE|_{\vartheta} = \frac{1}{T_s - t_0} \int_{t_0}^{T_s} \|\vartheta^*(\rho) - \vartheta(\rho)\|_2 d\rho, \quad (5.5)$$

$$RMSE|_{\varphi} = \frac{1}{T_s - t_0} \int_{t_0}^{T_s} \|\varphi^*(\rho) - \varphi(\rho)\|_2 d\rho, \quad (5.6)$$

Comparison results, reported in Table 5.5, demonstrate the high-performance achieved and the improvement of the MPC-based tracking control w.r.t. both the LQG-PID and the PID-based architectures in terms of TI and RMSE, despite the presence of external disturbance $\delta(t)$. Furthermore, as shown by the RMSE improved performance, the proposed strategy is able to properly reject the noise measurement and hence ensure a precise tracking of the appraised scientific star target.

5.2.2 Robustness analysis

Herein, with the aim of demonstrating the benefits and the drawbacks of the proposed MPC-based control architecture, we present a robustness analysis w.r.t. the model mismatch between the prediction model in (4.20b), exploited for solving the open-loop optimal problem in $\mathcal{F}$, and the TNG plant in (3.1).

In doing so, as in [79], the model mismatch is taken into account by considering the following inertial matrix $\check{A} = A + \iota A$ being A the nominal inertial

Table 5.5. Comparison results: Tracking Index, Root-Mean-Square-Error and improvement [%] w.r.t. LQG-PI [9] and PID [10].

axis :		$\vartheta(t) [arcsec]$			
Configuration $\vartheta^*(0)$:		78°	82°	87°	89°
MPC	TI	$6.24e^{-5}$	$6.24e^{-5}$	$5.99e^{-5}$	$5.99e^{-5}$
	RMSE	$1.02e^{-4}$	$1.13e^{-4}$	$2.92e^{-4}$	$2.92e^{-4}$
LQG-PI	TI	100.88	94.655	91.257	98.632
	Improvement	-99.99%	-99.99%	-98.63%	-98.63%
	RMSE	0.0084	0.0082	0.0084	0.0191
	Improvement	-87.85%	-86.21%	-96.52%	-98.90%
PID	TI	75.438	71.121	66.962	72.657
	Improvement	-99.99%	-99.99%	-99.99%	-99.99%
	RMSE	0.0266	0.0278	0.0278	0.0313
	Improvement	-96.16%	-96.33%	-98.95%	-99.07%
axis :		$\varphi(t) [arcsec]$			
Configuration $\vartheta^*(0)$:		78°	82°	87°	89°
MPC	TI	0.125	0.125	0.1164	0.1164
	RMSE	$2.26e^{-4}$	$2.47e^{-4}$	$2.11e^{-4}$	$2.11e^{-4}$
LQG-PI	TI	12.245	11.762	11.259	9.422
	Improvement	-98.97%	-98.93%	-98.97%	-82.65%
	RMSE	0.0084	0.0082	0.0084	0.0191
	Improvement	-97.30%	-96.98%	-97.49%	-98.90%
PID	TI	47.545	46.874	43.676	34.021
	Improvement	-99.73%	-99.73%	-99.73%	-99.66%
	RMSE	0.0266	0.0278	0.0278	0.0313
	Improvement	-99.15%	-99.11%	-99.24%	-99.33%

matrix reported in Table 5.1 while $\iota \in (0, 1)$ representing the uncertainties. The tracking of TYC 1731-916-1 is selected as simulation scenario and to better highlight the robustness to the model mismatch, no wind disturbance is considered, i.e. $\delta(t) = 0$. Moreover, the simulation time is set to $T_s = 10 [s]$. We consider model uncertainties of 10%, 20% and 30% ,i.e. $\iota = 0.1, 0.2, 0.3$ respectively. The KPI, described in Section 5.2.1, are exploited to assess the benefits/limitations of the proposed MPC-based control architecture.

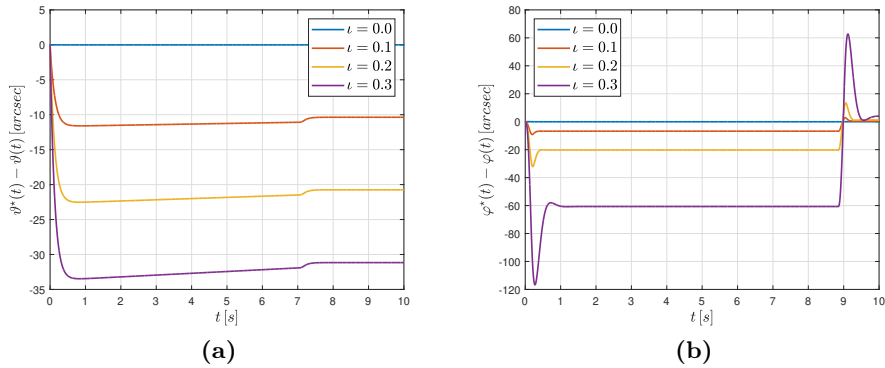


Figure 5.15. Robustness analysis of the proposed MPC-based control architecture w.r.t. 10%, 20% and 30% model mismatch, i.e. $\iota = 0.1, 0.2, 0.3$. Time history of: (a) altitude position error $\bar{\vartheta}(t) - \vartheta(t)$; (b) azimuth position error $\bar{\varphi}(t) - \varphi(t)$.

$\iota \rightarrow [\%]$	TI		RMSE	
	$\vartheta(t)$	$\varphi(t)$	$\vartheta(t)$	$\varphi(t)$
0.0 $\rightarrow$ 0%	$2.23e^{-4}$	$1.29e^{-4}$	$2.42e^{-4}$	$1.04e^{-4}$
0.1 $\rightarrow$ 10%	0.0102	0.0058	0.0058	0.0018
0.2 $\rightarrow$ 20%	0.0204	0.0176	0.0195	0.0054
0.3 $\rightarrow$ 30%	0.0306	0.05318	0.1429	0.0165

Table 5.6. Robustness analysis results: Tracking Index, Root-Mean-Square-Error w.r.t. model mismatch of 10%, 20% and 30%, i.e. $\iota = 0.1, 0.2, 0.3$.

Simulation results are reported in Figure 6, where the error position trend for the altitude axes and azimuth axes are disclosed in Figure 5.15(a) and Figure 5.15(b), respectively. From them, it is possible to observe that, although the proposed architecture performances are slightly downgraded (since the model mismatch between the prediction model increases), the star tracking is still ensured with small bounded errors. This is also confirmed by the performed TI and RMSE indexes reported in Table 5.6.

Chapter 6

The TNG Tracking Control System Upgrade Project

All the above work was carried out as part of the TNG tracking control system upgrade project. In this context, as future work, we aim to test and validate the proposed solutions on the real TNG telescope, especially the MPC-based tracking controller whose simulative results disclose higher tracking performances.

To this end, in this chapter, we first present the upgrading program then the development procedure of the controller solutions is provided.

6.1 Tracking System Upgrade Program

During the last few years, a modernization plan for the Telescopio Nazionale Galileo (TNG) Control System [80, 81] is in process. Within this framework, the control system installed at 1995s is updated in terms of hardware devices and software tools.

Nevertheless, the control algorithm is still the one designed in the 1995s. In doing so, as at the beginning (see Fig. 6.1), both TNG tracking control axes are based on a double loop tracking control approach where the inner loop [82] aims to properly control the speed of the telescope, while, the outer loop [83], based on the reference trajectory computes the appropriate speed command such that the telescope position pursuit the scientific

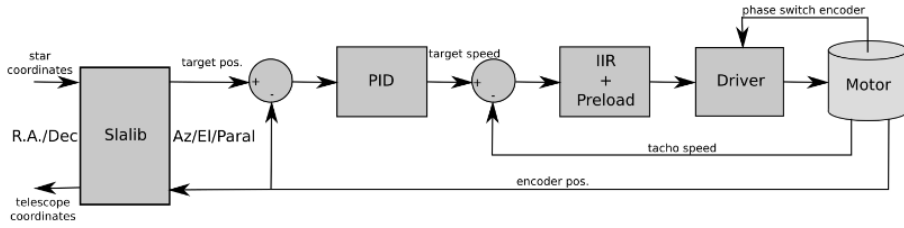


Figure 6.1. Overview of the TNG tracking control algorithm of both axes. From [8].

target. In detail, the speed loop is composed of a PI controller with the addition of a double low-pass filter, both implemented as IIR filters which are running on a MS-DOS PC. Conversely, the position loop is a PID loop with variable gains depending on the position error value, running on a VME crate with PDOS operating system. Everything is programmed in C language.

To this end, since the control loops are deployed on different devices, the upgrading program aims to modernize both hardware and software by changing one loop at a time and installing them on one device (see Fig.??). To this end, the speed loop has been the first one to be upgraded. Indeed, by taking the speed command from the existing VME system, i.e. the position loop, and closing the speed loop on the new hardware. Then, the same procedure has been proposed for the position loop.

Indeed, both axes' tracking control algorithms have been developed on a cRIO-9039 from National Instruments running NI Linux Real-Time OS which leverages a Xilinx Kintex-7 FPGA in order to communicate with the different input/output modules.

Moreover, the speed and position closed-loop algorithms of both axes were translated according to LabVIEW graphical language.

It is worth noting that, thanks to the higher computational power of the novel control CU, both the speed loop and the position loop were implemented on it. Moreover, the novel CU allows to include on the same device all the necessary tasks which are required for ensuring the TNG axes control. However, while on the FPGA are developed the double loop tracking control and the emergency system, on the real-time Linux OS,

the astronomic functions (slalib), the network interface, the log manager as well as the cooling control system are implemented. Then, to monitor the right running of all the system tasks, a watchdog is deployed on the CompactRIO.

At the same time, while the old designed control algorithm has been translated on the new system, the sensing and communication hardware have been upgraded so to improve the closed-loop performance. In this view, the new EIB749 encoders from Heidenhain have replaced the old IK320 boards. These new encoders, since allow the data read only via the Ethernet interface, have forced the system network design to exploit the real-time Linux OS as a bridge since the FPGA is not able to exploit this specific communication protocol.

Concerning the drive train system, the motor drivers have been changed with the one from Aerotech, since the old custom-built drivers were running out of spare parts. The new drivers have some computational power by themselves, they can be programmed to automate certain functions, although we are using them as simple current amplifiers. Finally, once

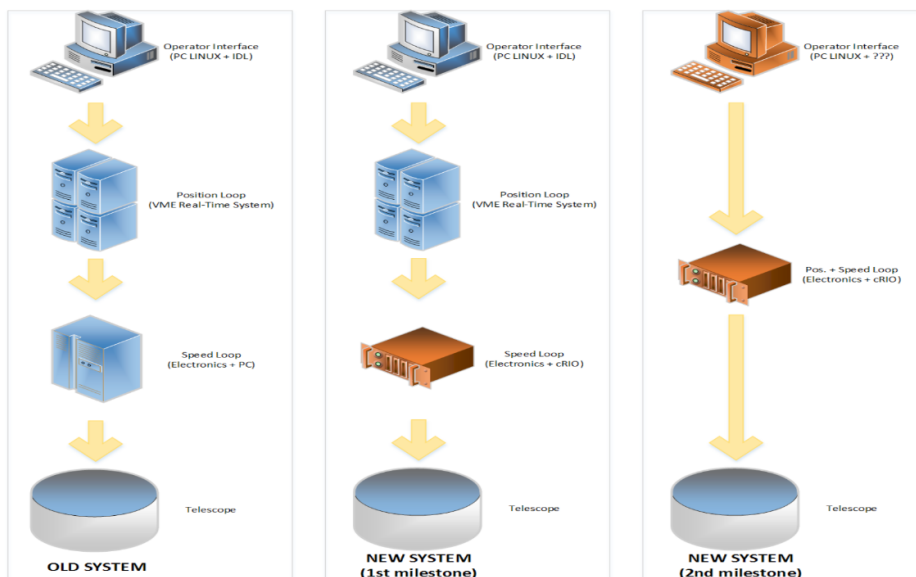


Figure 6.2. TNG tracking control system upgrade project stages. From [8].

the whole hardware and software have been upgraded, the new system has been tested without any retuning procedure (only the motor drivers were tuned to get coupled to our motors). The testing results, reported in [8], have disclosed the right functioning of the novel system despite the small deteriorated performance.

To this end, according to the tuning procedure exploited for the designing of the TNG old system [55], the retuning of the control parameters of the new system has been done and has led to improved tracking performances.

6.2 Toward the Implementation of the Control Solutions for the TNG

Once the TNG's current hardware and software tools are presented, we discuss about the implementation procedure of both control solutions. Specifically, as the TNG upgrade program highlights, today all the routine/functionality, that we want to deploy and test on the real plant, can be provided or in Labview graphical language or in C#/C++.

In this view, it is worth noting that, the native National Instrument program language, i.e. Labview graphical, lacks of functionalities. Moreover, even if the native library provides an optimiser, it does not guarantee the real-time feasibility. To this end, every optimal control problem is implemented in C++ by leveraging the well-known optimizer qpOASES¹ [84] which ensure the real-time feasibility of QP optimal problems of the form:

$$\min_x x^\top H x + g x \quad (6.1a)$$

$$\text{s.t.}$$

$$A x = b \quad (6.1b)$$

$$A x \leq b. \quad (6.1c)$$

Within this framework, while the OTCA's optimal problems are easy to translate into a QP optimal problem in (6.1), the translation of the MPC control algorithm is more challenging. Indeed, this latter is provided by exploiting the predictor-based approach with integral action [85].

¹<https://www.coin-r.org/qpOASES/doc/3.1/manual.pdf>

Moreover, it is worth noting that, since the MPC testing procedure is still ongoing work, this latter in the first stages is implemented by relaxing the local optimal problem omitting the input and the terminal constraints which will be implemented as future works.

In this view, with the aim of implementing the MPC control algorithm via the qpOASES tool, we want to ensure that the telescope position and speeds $y(k) \in \mathbb{R}^m$ tracks the desired references, computed via the trajectory generator, with null steady-state error. Therefore, the telescope model in (3.1) is readapted by assuming the steady-state error as an external unknown disturbance, i.e.:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + v, \\ y(k) &= Cx(k) + w, \end{aligned} \quad (6.2)$$

where $x(k) \in \mathbb{R}^n$ are the telescope states, $u(k) \in \mathbb{R}^r$ the torques inputs and v and w are the unknown disturbance representing the output steady-state errors.

To this end, to properly ensure the null steady-state error, the incremental telescope prediction model is introduced, hence, the MPC control algorithm aims to optimize the future control input increments $\Delta u(\cdot|k)$, being k the sampled time. Therefore, the incremental telescope prediction model is formalized as:

$$\Delta x(k+1) = A \Delta x(k) + B \Delta u(k) \quad (6.3a)$$

$$y(k) = y(k-1) + C \Delta u(k), \quad (6.3b)$$

where $\Delta x(k) = x(k) - x(k-1)$.

Then, by introducing the augmented state $\tilde{x}(k) = [\Delta x(k), y(k)]^\top$ the following equivalent state space model is given:

$$\begin{aligned} \tilde{x}(k+1) &= \underbrace{\begin{bmatrix} A & 0_{n \times m} \\ C & I_m \end{bmatrix}}_{\tilde{A}} \tilde{x}(k) + \underbrace{\begin{bmatrix} B \\ 0_{m \times r} \end{bmatrix}}_{\tilde{B}} \Delta u(k) \\ y(k) &= \underbrace{\begin{bmatrix} C & w \end{bmatrix}}_{\tilde{D}} \tilde{x}(k). \end{aligned} \quad (6.4)$$

The state space model in (6.3), (6.3b) (or equivalently in (6.4)) may be used to define a Prediction Model (PM) of the form:

$$y(\cdot|k+1) = p_L + F_L \Delta u(\cdot|k), \quad (6.5)$$

where L is the discrete prediction horizon while

$$p_L = O_L \tilde{A} \tilde{x}(k), \quad (6.6)$$

$$F_L = [O_L \tilde{B} \quad H_L^d] \quad (6.7)$$

and where O_L is the extended observability matrix of the pair $(\tilde{A}, \tilde{D})$ and $H_L^d \in \mathbb{R}^{mL \times (L-1)r}$ is the Toeplitz matrix of impulse response matrices $\tilde{D} \tilde{A}^{i-1} \tilde{B} \in \mathbb{R}^{m \times r}$ with $i = 0, \dots, L$.

Finally, thanks to the PM, now we can translate the MPC local optimal problem into QP problem.

Therefore, the cost function in (4.21) can be reshaped into the form:

$$J(k) = \Delta u(\cdot|k)^\top H \Delta u(\cdot|k) + 2 f_k^\top \Delta u(\cdot|k) + J_0 \quad (6.8)$$

being

$$H = F_L^\top Q F_L + P \quad (6.9)$$

$$f_k = F_L^\top Q (p_L - r(\cdot|k+1)) \quad (6.10)$$

while J_0 can be neglected since it is not a function of the optimization variable $\Delta u(\cdot|k)$.

Once, the algorithms were implemented in C++, according to the mentioned procedure, the cross-compiling method tailored to carry out the executable control program for the compactRIO target hardware was performed. Then, the resulting executable programs have been downloaded on the control unit and tested. To this end, several issues occurred. Specifically, the C++ mangling properties have limited the progress of testing forcing to move to C#. Moreover, for safety reasons, all the implemented source files have been tested only on the real-time Linux-based OS, this has led to computational limitations for the MPC testing.

In conclusion, due to issues that occurred during the implementation procedure along with the short time available for the testing on the real plant –given that the TNG telescope is one of the most operational in

our hemisphere— the validation of the proposed solutions is still ongoing.

Remark 9 From the first test on the real facility has came out as the identification procedure presented in section 5.0.1 is still far from guaranteeing the correct tuning and functioning of the proposed solutions. to this end by linking to the remark 2.2.4 the right identification procedure able to carry out a telescope dynamical model such that the advanced control strategy can be designed is still an open challenge.

Conclusions

This thesis addresses the tracking control problem for the class of ground-based telescopes in the presence of wind disturbance along with measurement noises.

The aim is to promote and lead the research in focusing on the exploitation of advanced tracking control strategy, hence, filling the gap between the telescope technical literature and the control one. This is also highlighted by the future steps in which the astronomic research wants to move on, i.e. the study of even more depth space. To this end, an in-depth analysis of the exploited tracking approaches along with their limitations and drawbacks is provided.

Then, in order to properly present the proposed solutions, the tracking control problem for ground-based telescopes is stated. In this regard, the two architectural control solutions are designed with the aim of guaranteeing high tracking performance despite the presence of external wind disturbance and measurement noise.

Both solutions embed a trajectory generator to ensure the physically admissible telescope trajectories and leverage the optimal control theory to achieve high tracking performance. To this end, the first solution, by inheriting the traditional tracking control scheme, adopts an LQG-PI approach. Besides, the second solution, by revolutionising the tracking control scheme for ground-based telescopes, leverages a single-loop approach based on the tracking MPC algorithm.

The solutions have been tailored for the TNG telescope located at *“Rogue*

de los Muchacho”, La Palma (Spain). To this end, by leveraging real telescope data, the TNG dynamical behaviour has been identified via the non-iterative subspace approach whose good prediction properties have been disclosed. The effectiveness of the proposed solutions in ensuring the optimal star tracking of the TNG telescope has been disclosed via virtual simulation, carried out via an advanced own-made *Matlab&Simulink* platform, for different star tracking scenarios, also involving the real scientific target *TYC 1731-916-1*.

Results prove the capability of the architectures in achieving the control goals with high tracking performance despite the presence of external disturbances and measurement noise with significant improvements w.r.t. state-of-art control strategies.

Finally, future works include: *i*) the improvement of the identification procedure so to overcome the limitation stated in Remark 7 and rise to the challenge stated in remark 2.2.4, 9; *ii*) extension of the own-realized Matlab-based simulation platform by including dedicated modules emulating the TNG sensors behaviour and limitations. In doing so, the test and the validation of the proposed control architectures on the real TNG telescope can be performed as final future work.

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