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Cosmology and cosmographic analyses with new distance indicators

Author:

Giada Bargiacchi

Supervisors:

Prof. Salvatore Capozziello

Prof. Guido Risaliti

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Coordinator: Prof. Salvatore Capozziello.*



*Se riuscissi a trovare quello che non c'è, allora hai trovato qualcosa di solo tuo. E se
qualcun altro vede quello che vedi tu, allora hai trovato qualcuno che ti vive.
"Ma le stelle quante sono", Giulia Carcasi*

Abstract

Quasars (Quasi Stellar Objects, QSOs) are particularly luminous Active Galactic Nuclei (AGNs), that are extremely persistent powerful sources, characterised by the radiation coming from the accretion onto a supermassive black hole at the centre of a galaxy. Their luminosity can exceed that of the total stellar emission of the galaxy in which they are located. They are observed in all cosmic epochs, from the local Universe up to when it was less than a billion years old, and they are very numerous. These properties make them very interesting objects for cosmology. Similarly, Gamma-Ray Bursts (GRBs) have recently attracted the attention of the cosmological community as incredibly powerful and luminous transient sources that reach up to redshift $z \sim 9$. As high-redshift sources, QSOs and GRBs are potentially the next rung of the cosmic distance ladder beyond type Ia supernovae (SNe Ia), which have been observed up to $z \sim 2$. Indeed, they allow us to investigate the evolution of the Universe in a region that is pivotal to distinguish among predictions of different cosmological models by comparison with observational data. Actually, the recently discovered unsolved cosmological discrepancies, such as the Hubble constant tension, have called into question the reliability of the standard cosmological model and triggered a search for the model that can effectively solve these tensions and reproduce all the observations at different scales. In this scenario, QSOs and GRBs stand as very promising and powerful tools. However, despite our current knowledge, there is still much to be understood about these objects. A deeper insight into them not only broadens our understanding but also enables us to harness them more effectively in cosmological studies.

In this thesis, we analyse the methods and the physical correlations used to turn QSOs and GRBs into cosmological probes and we describe their impact on tests of the standard cosmological model and some of its extensions and alternatives. Additionally, we explore cosmography as a model-independent approach to investigate different cosmological models. In Chapter 1, we introduce the current state of cosmology, together with the shortcomings of the most commonly adopted concordance model and we describe SNe Ia, baryonic acoustic oscillations (BAO), and the cosmic microwave background (CMB) radiation as historically well-established cosmological probes.

In Chapter 2, we define QSOs as astrophysical sources and we focus on their application as standard candles that is based on a relation between the X-ray and Ultraviolet (UV) luminosities of QSOs. In this context, we summarize the most recent results in the literature, proving the reliability of this relation for cosmological purposes. We also present

new results on the choice of the X-ray and UV indicators in the luminosity relation and the origin and the actual value of the intrinsic dispersion of this relation. In particular, although we still do not fully understand the physical mechanism behind this empirical luminosity relation, we strengthen its reliability by proving that it is very tight, with an intrinsic dispersion very close to zero.

In Chapter 3, we describe how the removal of biased objects from the QSO sample allows us to derive a cosmological sample of QSOs that provides more accurate distance measurements. We also analyse the properties of two specific sub-samples, namely one at $z \sim 3$ with high-quality measurements, which confirms the robustness of QSOs as cosmological tools, and another one that turns out to be X-ray weak, for which we investigate the origin of this feature.

In Chapter 4, we introduce GRBs, their features, the physical correlation that turn them into cosmological probes, and the construction of a sample suitable for cosmological studies. In addition, we present new machine-learning based techniques that can be applied to tighten the correlation and increase the sample size. Leveraging these approaches, we show that GRBs can be effectively employed as standalone cosmological probes in the near future.

Chapter 5 focuses on the application of the Efron & Petrosian statistical method to QSOs and GRBs to correct their luminosities for selection biases and redshift evolution, effects that can distort or even induce correlations among physical quantities at high redshift. In this regard, we prove that the relations used for QSOs and GRBs are both trustworthy as intrinsic of their physics and not caused by the above effects. Furthermore, we present a novel method, originally developed by us, to overcome the circularity problem embedded in the Efron & Petrosian method, thus making this treatment independent of the choice of a particular a priori cosmological model.

In Chapter 6, we discuss the search for a “gold” QSO sample, characterized by a reduced intrinsic dispersion and a sufficient number of sources so that it can constrain the matter density parameter in the standard cosmological model as a standalone probe with a precision comparable to that of SNe Ia.

In Chapter 7, we report our cosmological results on several cosmological models obtained with different combinations of QSOs, SNe Ia, GRBs, BAO, and CMB. All these studies provide an interesting common trend towards a statistically significant deviation from the standard cosmological model in favour of a modification of the dark energy behaviour.

A novel approach to dealing with these cosmological analyses by modifying the cosmological likelihood is presented in Chapter 8. Indeed, we prove that the commonly adopted Gaussian cosmological likelihood is not the most appropriate for the case of QSOs, SNe Ia, and BAO since their normalized residuals do not verify the Gaussian assumption. Therefore, if we replace the Gaussian likelihood with the newly found best-fit likelihood for each probe, we significantly reduce the uncertainties on the inferred cosmological parameters. This is a relevant result in the light of the efforts to constrain cosmological parameters with improved precision to investigate the nature and the entity of the current tensions.

In Chapter 9, we introduce the cosmographic approach as a model-independent method to

test cosmological models and we propose a logarithmic cosmographic expansion which overcomes the limitations of the standard approaches also demonstrating its robustness. By applying this cosmographic approach, we obtain a strong tension between the prediction of the standard cosmological model and our data from QSOs, GRBs, SNe Ia, and BAO, thus confirming our previous results. Finally, in Chapter 10, we summarize our work, we draw conclusions, and we outline possible future perspectives.

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1 Introduction

This thesis work is set in the context of the recent and current debates and tensions in cosmology. Therefore, we begin our dissertation by reviewing the current state of cosmology, the concordance cosmological model and some of its extensions, and the cosmological probes that we have employed in our work. However, we devote two separate chapters (Chapters 2 and 4) to quasars (Quasi Stellar Objects, QSOs) and Gamma-Ray Bursts (GRBs), since our original study focuses mainly on their novel use in cosmology and thus they deserve a more complete discussion and treatment.

1.1 Cosmological framework

Currently, the spatially flat Λ Cold Dark Matter (Λ CDM) model [1] is the cosmological model commonly adopted to describe the Universe. It relies on a flat geometry with a cold dark matter (CDM) component and a dark energy corresponding to a cosmological constant Λ [2], and thus it is characterized by an equation of state $w(z) = P_\Lambda/\rho_\Lambda = -1$, with z the redshift, P_Λ and ρ_Λ the pressure and energy density of dark energy, respectively. Indeed, this scenario is grounded on several observational probes: the cosmic microwave background [CMB; e.g. 3], the baryonic acoustic oscillations [BAO; e.g. 4], and the current accelerated expansion of the Universe, discovered from type Ia Supernovae [SNe Ia; e.g. 5, 6]. Nevertheless, the reliability of this model is still being questioned due to well-known longstanding, but also more recent, theoretical and observational issues: the fine-tuning between the current values of the density parameters of matter (Ω_M) and dark energy (Ω_Λ), the cosmological constant problem [7], the origin and properties of dark energy, and the Hubble constant (H_0) tension [see 8, for a review].

The H_0 tension is the most recently discovered shortcoming and it corresponds to a discrepancy between the value of H_0 measured locally from SNe Ia and Cepheids ($H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$, [9]) and that derived from the Planck data of the CMB assuming a flat Λ CDM model ($H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$, [3]). This is a deviation of $\sim 4\sigma$, which increases to up to 6σ depending on the samples considered [10, 11, 12]. Additionally, the investigation of this problem with other cosmological probes has shown a more complex pattern. For example, cosmic chronometers prefer a H_0 value close to that of the CMB [13], while time-delay and strong lensing of QSOs favor that of SNe Ia [14], and other probes, such as QSOs [15], the Tip of the Red-

Giant Branch (TRGB; [16]), and GRBs [17, 18, 19, 20, 21] reveal an H_0 value halfway between the two. This challenging scenario has recently boosted the efforts to solve the H_0 tension, which still remains one of the most studied and puzzling open questions for the astrophysical and cosmological community. One of the main limitations preventing a better investigation of this discrepancy is the limited number of cosmological probes at redshifts intermediate between those of SNe Ia (with a maximum $z = 2.26$, [22]) and the CMB ($z = 1100$). To fill this gap, thus shedding light on the H_0 tension, in this work we have applied GRBs and QSOs as high-redshift cosmological tools.

In this framework, several approaches have been attempted both to investigate the reliability of the flat Λ CDM model and to address its shortcomings. These efforts range from cosmographic analyses [23, 24, 25, 26, 27, 28, 29, 30, 31] to additional relativistic particles, modified theories of gravity, and possible evolution of H_0 [e.g. 32, 33, 34, 35, 36, 37, 38, 19, 39, 40]. In addition, the assumption of a null curvature of the Universe has also been questioned [41, 42, 43, 44, 45, 46, 47, 48, 49], together with a possible evolution of the dark energy [50, 51, 45, 49, 52].

In full generality, the background evolution of the Universe is described by the following expression for $E(z) = H(z)/H_0$, with $H(z)$ the Hubble parameter:

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda \exp \left(3 \int_0^z dz' \frac{1+w(z')}{1+z'} \right) \right]^{\frac{1}{2}}, \quad (1.1)$$

where Ω_r and Ω_k are the density parameters of the radiation and curvature components, respectively. Accordingly, the model-dependent luminosity distance (D_L) is obtained from

$$D_L(z) = \begin{cases} \frac{c}{H_0} (1+z) \frac{\sinh \left[\sqrt{\Omega_k} \int_0^z \frac{dz'}{E(z')} \right]}{\sqrt{\Omega_k}} & \Omega_k > 0, \\ \frac{c}{H_0} (1+z) \int_0^z \frac{dz'}{E(z')} & \Omega_k = 0, \\ \frac{c}{H_0} (1+z) \frac{\sin \left[\sqrt{-\Omega_k} \int_0^z \frac{dz'}{E(z')} \right]}{\sqrt{-\Omega_k}} & \Omega_k < 0, \end{cases} \quad (1.2)$$

where c is the speed of light. Using this formula, we can infer the distance modulus, μ , from D_L as¹ $\mu = 5 \log D_L(z) + 25$, where D_L is assumed in units of Megaparsec (Mpc). These equations can be made explicit for any cosmological model investigated. In particular, with the assumption of the flat Λ CDM model, we obtain

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda \right]^{\frac{1}{2}} \quad (1.3)$$

where the only free cosmological parameter is Ω_M , since $\Omega_\Lambda = 1 - \Omega_M - \Omega_r$ under the flatness condition. Instead, when considering the non-flat curvature extension of the standard Λ CDM model, also Ω_Λ is a free parameter, with the present curvature density parameter $\Omega_k = 1 - \Omega_M - \Omega_r - \Omega_\Lambda$. In this case, we consider the following background

¹Throughout the text, for the sake of simplicity, we use the notation $\log$ to refer to $\log_{10}$.

evolution

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda \right]^{\frac{1}{2}} \quad (1.4)$$

also including a constraint to rule out the region of the parameter space $(\Omega_M, \Omega_\Lambda)$ that does not admit an initial singularity, the so-called “no Big Bang region”. To this end, following [53], we impose

$$\Omega_\Lambda < 4 \Omega_M \coss^3 \left[\frac{1}{3} \arccoss \left(\frac{1 - \Omega_M}{\Omega_M} \right) \right] \quad (1.5)$$

where $\coss = \cos$ if $\Omega_M > 1/2$ and $\coss = \cosh$ if $\Omega_M < 1/2$.

Apart from considering a non-zero curvature, the simplest and most natural extension of the Λ CDM model is the one in which w is still constant in time but it can assume values different from $w = -1$ and allowed by the constraint $w < -1/3$, which is required by the second Friedmann’s equation to explain the present accelerated expansion of the Universe [5, 6] as a dominant effect of dark energy. This is the w CDM model, in which we have

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_k (1+z)^2 + \Omega_r (1+z)^4 + \Omega_\Lambda (1+z)^{3(1+w)} \right]^{\frac{1}{2}} \quad (1.6)$$

where Ω_M and w are the free parameters under the flatness assumption, together with the additional free parameter of Ω_Λ in the non-flat case. The regime $w > -1$ is referred to as “quintessence”, while the one with $w < -1$ as “phantom”. The phantom dark energy scenario predicts a final “Big Rip” for the Universe in which all the matter is ripped apart by the accelerated expansion. This dark energy classification also applies to the models discussed below, in which w evolves with redshift. The following is not intended to be a complete description of the plethora of cosmological models that predict an evolution of the dark energy with time that have been proposed so far, but we focus only on those that we have examined in this work.

The most widely used parameterisation for an equation of state of the dark energy that evolves with time is the Chevallier-Polarski-Linder (CPL) model [54, 55], in which $w(z) = w_0 + w_a z/(1+z)$. This is a smooth function in the redshift range $[0, +\infty)$ that starts from $w = w_0 + w_a$ at $z = +\infty$ and reaches w_0 at $z = 0$, but it diverges in the future as $w(z) = \pm\infty$ for $z \rightarrow (-1)^+$. Substituting in Eq.(1.1), we get

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda (1+z)^{3(1+w_0+w_a)} \exp \left(\frac{-3w_a z}{1+z} \right) \right]^{\frac{1}{2}} \quad (1.7)$$

where Ω_M , w_0 , and w_a are the cosmological free parameters. Another parameterisation with a different dependence on the scale, is the Jassal-Bagla-Padmanabhan (JBP) model [56] where $w(z) = w_0 + w_a z/(1+z)^2$ and $w(z)$ has the same value (i.e. w_0) at present epoch ($z = 0$) and remote past ($z = +\infty$) with a rapid variation at low- z . This model

presents the same divergence in the future as the CPL described above. From Eq.(1.1) we derive

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda (1+z)^{3(1+w_0)} \exp\left(\frac{3w_a z^2}{2(1+z)^2}\right) \right]^{\frac{1}{2}}. \quad (1.8)$$

As for CPL model, Ω_M , w_0 , and w_a are the free parameters. In our analyses we have also considered an “exponential” parameterisation, already introduced in [57], which assumes $w(z) = \frac{w_0}{1+z} e^{\frac{z}{1+z}}$. By construction, $w(z=0) = w_0$ and $w(z) = 0$ for both $z = +\infty$ and $z \rightarrow (-1)^+$. The corresponding $E(z)$ is

$$E(z) = \left\{ \Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda (1+z)^3 \exp\left[3w_0 \left(e^{\frac{z}{1+z}} - 1\right)\right] \right\}^{\frac{1}{2}}. \quad (1.9)$$

In this case, only Ω_M and w_0 are cosmological free parameters. Finally, we also introduce a model presented by [58] and named “Barboza-Alcaniz” (BA) model, which is defined by the equation of the state $w(z) = w_0 + w_a \frac{z(1+z)}{1+z^2}$. In the asymptotic limits of this model, $w(z) = w_0 + w_a$ for $z = +\infty$ and $w(z) = w_0$ in the future. This is a completely smooth function, without any singularity, in which Ω_M , w_0 , and w_a are the free parameters. In this case, from Eq.(1.1) we get

$$E(z) = \left[\Omega_M (1+z)^3 + \Omega_r (1+z)^4 + \Omega_\Lambda (1+z)^{3(1+w_0)} (1+z^2)^{\frac{3}{2}w_a} \right]^{\frac{1}{2}}. \quad (1.10)$$

In addition to these models based on an evolving dark energy, in our work we have also investigated a model that circumvents the problem of the cosmological constant by considering an unified dark sector, which behaves like a decomposed generalized Chaplygin gas (GCG) model, where the Chaplygin gas is a substance that satisfies an equation of state given by

$$p_{GCG} = -\frac{A}{\rho_{GCG}^\alpha}. \quad (1.11)$$

In this equation A is a positive constant, p_{GCG} and ρ_{GCG} are respectively the pressure and the energy density of the decomposed GCG, and α is the interaction parameter between the dark components [59, 60, 61, 62]. The GCG interpolates between a CDM, dominating at early times, and a dark energy component, dominating at late times. The growth rate is suppressed for $\alpha < 0$ due to the homogeneous creation of dark particles from the expanding vacuum, and enhanced when $\alpha > 0$, which implies a clustering of a dynamical dark energy. In this context, the background evolution can be written as

$$\frac{H(z)}{H_0} = \sqrt{\left[(1 - \Omega_M) + \Omega_M (1+z)^{3(1+\alpha)}\right]^{\frac{1}{(1+\alpha)}} + \Omega_r (1+z)^4} \quad (1.12)$$

where $(1 - \Omega_M)$ refers to the dark energy density in the flat geometry assumption. In such a model, the Λ CDM model is recovered for $\alpha = 0$. For a detailed description of this model we refer to [63, 64, 59, 65, 60, 66, 67, 68, 69, 70, 71, 72, 73, 74, 51, 52, 75, 76, 77] and references within. Hereafter, we denote this model as “iDE”, referring to the presence of an interaction in the dark sector.

1.2 Cosmological probes

In the following, we introduce SNe Ia, CMB, and BAO and their use in cosmology as they are among the most reliable cosmological probes discovered so far and we have employed in our work. However, as anticipated, we describe the cosmological development of QSOs and GRBs in two separated chapters (Chapters 2 and 4, respectively) since we performed a relevant original work on these sources and we dedicated our main efforts to their development in the field of cosmology.

1.2.1 Type Ia supernovae as standard candles

SNe Ia are formed by the following process: when a white dwarf in a binary system is increased in mass by its companion or merges with another white dwarf and its mass exceeds the value corresponding to the so-called “Chandrasekhar limit”, which is about $1.44 M_{\odot}$, it can no longer maintain its thermodynamic equilibrium through electron degeneracy pressure and begins to collapse. This leads to the ignition of carbon-burning reactions, triggering a chain reaction (runaway process) because the electron degeneracy pressure is independent of temperature and therefore cannot prevent the temperature increase by expansion. This leads to the explosion of the supernova. The spectrum of supernovae produced by this mechanism is characterised by the absence of hydrogen lines and the presence of strong SiII lines; these properties define the class of “type Ia” supernovae. The fact that the physical mechanism is the same for all SNe Ia, regardless of the redshift and the specific evolution of the physical system, makes these sources standard candles as we explain below.

The SNe Ia light curve, that is the evolution of the absolute magnitude as a function of time, always has the same shape with an initial peak followed by a rapid decline, but the peak brightness value can vary with a spread of about 0.4 magnitudes in the V-band.

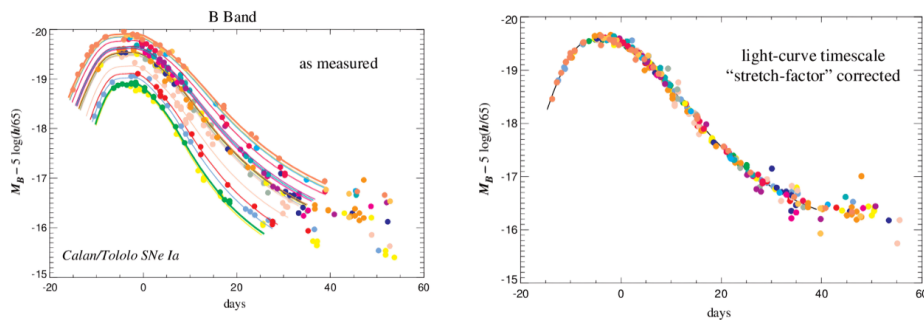


Figure 1.1: Light curves of a SNe Ia sample. The transition from the left to the right panel corresponds to the application of the correction taken from the correlation between peak brightness and rate of decline. As a result of this correction, the light curves are perfectly superimposed.

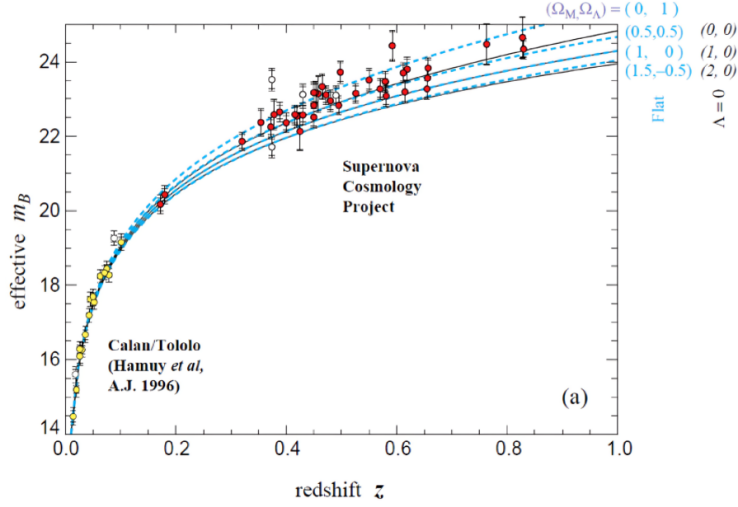


Figure 1.2: Hubble diagram for the sample of 60 SNe Ia from [6]: 42 at high redshift from the Supernova Cosmology Project and 18 at low redshift from the Calan/Tololo Supernova Survey. The solid and dashed curves are the theoretical predictions of the models described in the legend.

Consequently, these sources are not intrinsically standard candles but can be turned into standard candles. Indeed, an empirical relation has been discovered between the peak brightness and the rate of decline of brightness with time [78, 79], which can be used to calibrate light curves. By applying this correction, known as the “Phillips relation”, the light curves of different supernovae overlap perfectly, as shown in Figure 1.1. A further improvement in the measurement of light curves was achieved after the discovery of the need for corrections for extinction and redshift effects which could lead to a misinterpretation of the data. In summary, the application of these corrections has made it possible to reduce the dispersion on the peak in brightness to about 0.1 magnitude in the optical band, thus making SNe Ia excellent “standardised” candles; indeed the absolute magnitude M (equivalent to the intrinsic brightness) obtained in this way can be related to the observed apparent magnitude m by $m - M + 5 = 5 \log D_L$ to derive the luminosity distance of the source.

To obtain the absolute distance, supernovae must be calibrated; this can be done using Cepheid stars, a type of variable stars that can be used as standard candles, in the redshift range where both Cepheids and supernovae are observed ($z \sim 0.05$, corresponding to ~ 20 Mpc). Thus, the intrinsic luminosity and redshift of SNe Ia can be known independently of each other and exploited for the cosmological analysis in which we are interested by means of the “Hubble diagram”, that is the relation between the distance modulus μ (or equivalently luminosity distance D_L) and the redshift. In this diagram, it is indeed possible to compare the trend of the experimental data with that predicted by the different cosmological models as their free parameters vary, as shown in Figure 1.2. It is crucial

to note from this figure that theoretical predictions are indistinguishable in the limit of low redshifts and only deviate from each other with increasing redshift; this implies the need to extend the range of z over which one has data in the Hubble diagram as much as possible by means of high redshift sources to effectively discriminate between different models. To date, SNe Ia have been observed up to a maximum redshift of $z \sim 2$ [80, 81], however very few objects fall at $z > 1$; these are therefore sources that provide information about the Universe at low redshift (i.e. recent epochs) and were indeed at the origin of the discovery of the current accelerated expansion of the Universe.

1.2.2 Baryonic Acoustic Oscillations

BAO are fluctuations in the density of visible baryonic matter caused by acoustic waves in the primordial Universe. The primordial plasma has regions of overdensity, where the density is slightly higher than the uniform density of the Universe, and which therefore correspond to potential potholes. These regions are made up of dark matter, baryons, and photons and evolve under the opposition between the gravitational pull of matter and the pressure of the radiation-matter interaction. As the region gravitationally attracts matter, the radiation-matter interaction increases in intensity until the pressure becomes dominant, pushing matter out. This in turn reduces the intensity of the interaction causing a new dominance of the gravitational force. This juxtaposition then produces oscillations in the primordial Universe, spherical acoustic waves, in which the dark matter component, which interacts only gravitationally, remains at the centre of the overdensity

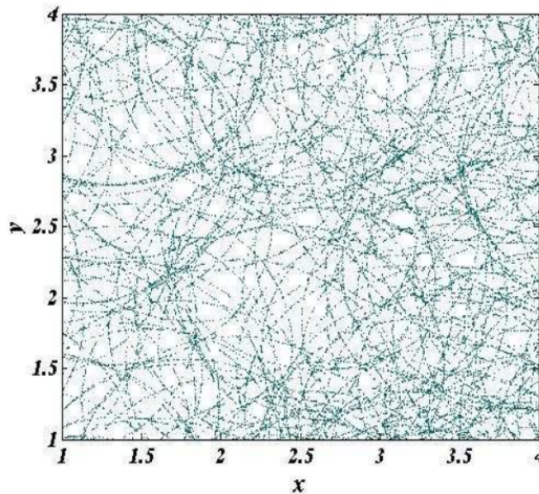


Figure 1.3: Simplified representation of the superposition of several rings of matter generated by overdensities in the early Universe. In a realistic scenario, the rings would be much more numerous, but this would make it more difficult to identify the sound horizon, which is instead easily visible in this figure.

while the other two components, coupled by their relative interaction, expand outwards.

However, since the decoupling of radiation and matter (see Section 1.2.3), the photons freely diffuse and what remains is a shell of baryonic matter under the effect of the only gravitational force. The lack of pressure prevents the baryons from propagating further outwards and the comoving distance travelled by the sound wave between the original overdensity and this outer ring is called the sound horizon, denoted $r_s(z)$. The overall configuration of acoustic oscillations then remains frozen in the radiation emitted at the moment of recombination, the CMB radiation (see Section 1.2.3), and the sound horizon at this epoch defines the length scale on the “last scattering surface”. In the end, after the decoupling, we are left with an overdense region of dark matter at the centre of the original anisotropy and an overdense region of baryonic matter in the outer ring. These regions gravitationally attract matter, thus making them privileged sites for the formation of galaxies. However, from the time of recombination to the present day, the Universe has expanded and with it also the configuration of the acoustic waves. As a consequence, similar to SNe Ia which provide standard candles for cosmological observations, BAO provide a standard ruler, an absolute length scale, resulting from a physical process that is completely independent of redshift, just as in the case of a supernova explosion. The size of this length scale is precisely the sound horizon.

Since the early Universe was formed by multiple overdensities, it is natural to expect that the distribution of galaxies would show a pattern similar to that formed by the superposition of many waves, such as the one in Figure 1.3, where the length scale of the sound horizon is clearly visible. We here stress that this figure is purely an ideal and simplified representation, not corresponding to a real observation, which is here only shown for visual clarity to make the sound horizon clearly visible. Nonetheless, one would expect to observe a non-uniform distribution of galaxies where the probability of finding at redshift z a galaxy at a distance from another equal to the sound horizon $r_s(z)$ is greater than that of finding it at any other distance. In this regard, the study of the distribution of galaxies is based on the calculation of the two-point correlation function $\epsilon(r)$, where r is the comoving distance, which describes the probability of finding a galaxy at a given distance from another in excess of a uniform distribution. The BAO signal should therefore appear as a peak in the function $\epsilon(r)$ at a distance r equal to the sound horizon. From an observational point of view, the sought-after BAO signal is a very weak signal on a large distance scale, which is why it is necessary to map a very large volume of the Universe through surveys to perform a statistical analysis on a large number of galaxies and obtain an accurate distance measurement. This signal has indeed been detected by several surveys in perfect agreement with theoretical predictions and, to date, BAO measurements are available at different redshifts up to $z \sim 2.4$ [82, 83]. Figure 1.4 shows an example of BAO signal detection of $\epsilon(r)$ (in the figure the comoving distance is indicated with s instead of r). The peak corresponds to a comoving distance of the order of $100 h^{-1} \text{ Mpc}$ ², corresponding to $r \sim 150 \text{ Mpc}$, which is therefore a measure of the sound horizon at the effective observational redshift of the survey in question ($z = 0.35$).

In conclusion, BAO allow us to compare the sound horizon at the recombination,

²Throughout this work thesis the symbol h is used to denote the quantity $H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

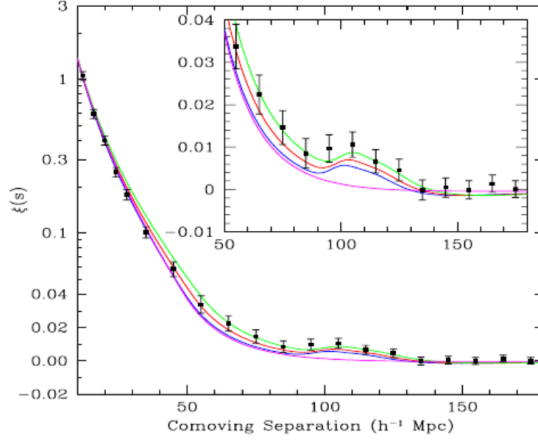


Figure 1.4: Relationship between two-point correlation function and comoving distance (in units of h^{-1} Mpc) for BAO signal detection reported by [84]. The peak at $100 h^{-1}$ Mpc is the searched signal and the coloured curves mark the theoretical predictions from different cosmological models.

measured with extreme precision from the CMB radiation as we will see shortly in Section 1.2.3, with those at very different epochs, obtained from the distribution of large-scale galaxies at a certain redshift z , thus providing a description of the expansion of the Universe under the effect of dark energy between different epochs. As each cosmological model predicts a specific evolution of the Universe and hence a precise correspondence between these distance measurements, BAO allow us to test cosmological models. Indeed, it is possible to compare experimental data with model predictions not only through the function $\epsilon(r)$, as can be seen in Figure 1.4, but also directly through the sound horizon $r_s(z)$ and quantities derived from it as function of cosmological parameters.

In this regard, we here describe the physical quantities of BAO actually reported by the surveys and used for the cosmological analyses presented in this work.

- $D_V(z)$ is a distance measure expressed in units of Mpc and obtained from the following combination of quantities:

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{cz}{H(z)} \right]^{1/3} \quad (1.13)$$

where the angular distance $D_A(z)$ can be derived from the luminosity distance as $D_A(z) = \frac{D_L(z)}{(1+z)^2}$;

- $r_s(z)$ is the comoving sound horizon, which is the maximum comoving distance that the sound waves may have travelled from the Big Bang to the redshift z , and

it is defined as:

$$r_s(z) = \int_z^\infty \frac{c_s(z')}{H(z')} dz' \quad (1.14)$$

where $c_s(z)$ is the sound speed computed as $c_s(z) = \frac{c}{\sqrt{3[1 + \frac{3\Omega_b}{4\Omega_\gamma} \frac{1}{1+z}]}}$ with $\Omega_b h^2 = 0.0224$ and the current photon energy density parameter $\Omega_\gamma = 2.469 \cdot 10^{-5}$;

- z_d is the redshift of the “baryon drag epoch”, the moment when the baryons are released from the ‘drag’ by the photons and decouple from them. This quantity can be computed as

$$z_d = \frac{1291(\Omega_M h^2)^{0.251}}{1 + 0.659(\Omega_M h^2)^{0.828}} [1 + b_1(\Omega_b h^2)^{b_2}] \quad (1.15)$$

where $b_1 = 0.313(\Omega_M h^2)^{-0.419} [1 + 0.607(\Omega_M h^2)^{0.674}]$ and $b_2 = 0.238(\Omega_M h^2)^{0.223}$. Usually the value of r_s is reported measured at $z = z_d$. We here note that in a standard cosmological model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ the value of z_d is ~ 1020 , hence, the integral of Eq. (1.14) is calculated at high redshift, from 1020 to infinity. Therefore, to compute $r_s(z_d)$ we need to account in $H(z)$ also for the presence of the radiation term Ω_r , that becomes non negligible at high redshifts due to the factor $(1+z)^4$ in Eq. (1.1);

- $D_M(z)$ is a distance measure that can be expressed as a combination of the above described quantities as $D_M(z) = D_A(z)(1+z) = D_L(z)/(1+z)$.

Based on these definitions, the combinations of quantities often reported from the surveys and then applied in cosmology are the following ones: $r_s(z_d)/D_V(z)$, $D_V(z) \frac{r_{s,fid}}{r_s(z_d)}$, $D_M(z) \frac{r_{s,fid}}{r_s(z_d)}$, and $H(z) \frac{r_s(z_d)}{r_{s,fid}}$ where $r_{s,fid}$ is the value of r_s in the assumed fiducial cosmology.

1.2.3 The Cosmic Microwave Background

The CMB radiation is the radiation emitted at the so-called “recombination epoch”. In the primordial Universe, radiation and matter are coupled to form a single cosmological fluid by Thomson scattering between photons and free electrons. This prevents both the scattering of radiation, since the mean free path of the photons is too short, and the formation of neutral hydrogen atoms, as the photons are sufficiently numerous and energetic to prevent the formation of the bond between electrons and protons. However, as a result of the expansion and subsequent cooling of the Universe, a condition is reached at $z \sim 1089$ where the rate of recombination in hydrogen atoms exceeds the rate of ionisation; this is known as the decoupling epoch (between radiation and matter), or also as the recombination epoch, and its redshift corresponds to the surface of last scattering. The photons undergo last scattering and then diffuse freely towards us. The radiation from the surface of last scattering is exactly the CMB radiation.

Since its discovery, the CMB has been observed and studied by numerous space missions and ground-based experiments that have provided high-precision measurements.

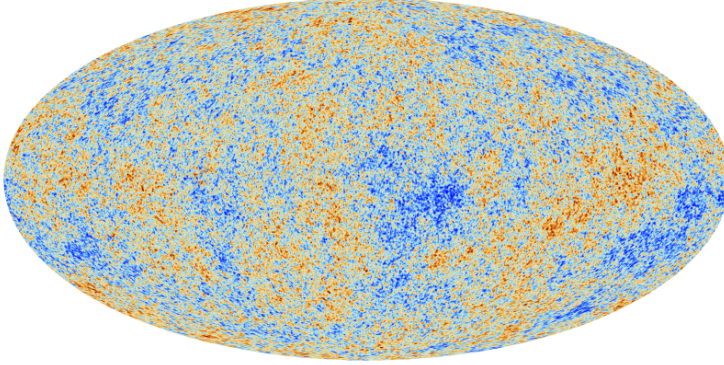


Figure 1.5: Full-sky map of CMB radiation temperature fluctuations obtained in 2013 by the European Space Agency's (ESA) Planck satellite.

Indeed, as an image of the Universe at the time of decoupling, it plays an invaluable role in the study of cosmic evolution, which is complementary with respect to low redshift sources, such as SNe Ia. In the course of this brief description, however, we will not go into the details of its many properties, but only those that are necessary to understand the cosmological analyses we have carried out.

The CMB is remarkably, but not perfectly, isotropic, as shown in Figure 1.5. In fact, a perfect isotropy could not explain the formation of the structures that we observe today, such as galaxies and clusters. Instead, anisotropies are present due to the acoustic waves of the primordial Universe. Indeed, we have already described how the pattern of acoustic waves remains frozen on the surface of last scattering, defining the length scale of the sound horizon $r_s(z_*)$, where z_* is the redshift of the recombination. The acoustic oscillations thus give the CMB its characteristic peak structure from which a great deal of information can be derived, as we will see below.

The mathematical tool used to study temperature anisotropies³ in the CMB is the development in spherical harmonics $Y_{lm}(\theta, \phi)$ of temperature variations $\Delta T(\theta, \phi)$

$$\Delta T(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi) \quad l = 0 \dots +\infty; \quad -l \leq m \leq l \quad (1.16)$$

where θ and ϕ are the angular coordinates, $l = 180^\circ/\theta$ is the multipole order, and a_{lm} is the multipole coefficient relative to a given value of l and one of the $2l + 1$ values of m . The power spectrum showing the intensity of the harmonics as the multipole order (or equivalently the angular scale) varies is described in terms of $C_l = \langle |a_{lm}|^2 \rangle$ by means of the expression shown on the y-axis in Figure 1.6. The peaks produced by the acoustic oscillations of the primordial plasma can be clearly seen in the figure. The position of the first peak allows us to constrain the curvature of the Universe, and the relative

³Fluctuations in temperature track fluctuations in density, which means that they correspond to regions of higher and lower density produced by acoustic waves, and it is these, rather than the density fluctuations, that are directly measured.

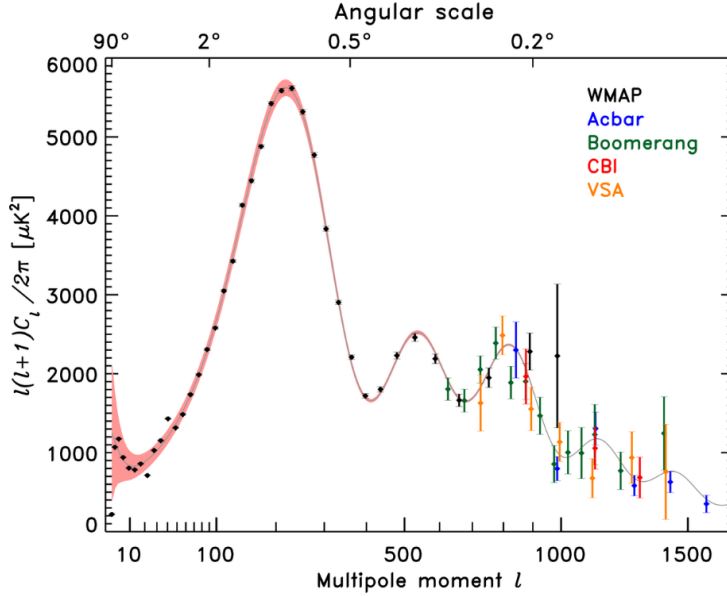


Figure 1.6: Power spectrum of CMB temperature anisotropies in terms of angular scale (upper x-axis) or moments of multiple l (lower x-axis). The points represent experimental data from different measurements according to the legend. The properties of the spectrum depend on cosmological parameters such as Ω_M , Ω_b , and Ω_k and the fit of the data can be used as a test of cosmological models.

properties of the even and odd peaks, such as height and damping rate, depend sensitively and clearly on the total matter and baryon densities. Furthermore, the position of the first acoustic peak provides an accurate measure of the sound horizon at recombination $r_s(z_*)$. Examples of how the variation of cosmological parameters leads to a significant change in the shape and geometrical properties of the spectrum are given in Figure 1.7 for some specific cases described in the caption.

In conclusion, the CMB, as an image of the Universe at high redshift, does not provide direct information about dark energy, which is dominant only in the recent Universe, but it has great power in constraining cosmological parameters such as Ω_M , Ω_b , and Ω_k and allows a comparison between experimental data and predictions of different cosmological models directly through the power spectrum since different models predict a different distribution of structures in the Universe and therefore a different pattern of fluctuations in the CMB. Clearly, the CMB provides complementary information compared to low redshift cosmological probes, such as SNe Ia and BAO, and therefore the correspondence between these independent measurements is crucial for understanding the evolution of the Universe on the time scale between $z \sim 0$ and $z \sim 1100$. For the most recent measurements on the CMB obtained by the Planck satellite and the resulting cosmological implications we refer to [3].

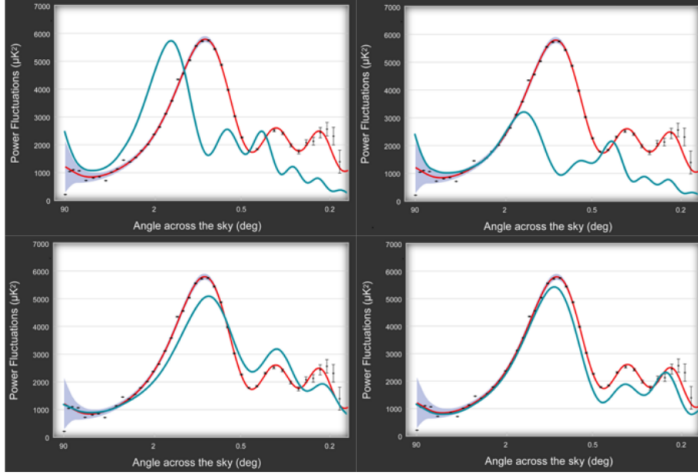


Figure 1.7: Power spectra simulations of CMB temperature anisotropies generated via the web interface of <https://map.gsfc.nasa.gov>. The physical quantity shown on the ordinate is the same as in Figure 1.6. In each panel, the points mark the experimental data with their errors and the red curve the flat Λ CDM model with $\Omega_M \sim 0.3$, $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and $\Omega_b \sim 0.04$. The blue curves are instead obtained under the assumption respectively of a non-flat Λ CDM model with $\Omega_M \sim 0.3$ and $\Omega_\Lambda \sim 1$ corresponding to a closed Universe with $\Omega_k \sim -0.3$ (upper left panel), non-flat Λ CDM with $\Omega_M \sim 0.8$ and $\Omega_\Lambda \sim 0.7$ corresponding to a closed Universe with $\Omega_k \sim -0.5$ (top right panel), flat Λ CDM with $\Omega_M \sim 0.3$ and Ω_b reduced to 0.02 (bottom left panel) and flat Λ CDM with $\Omega_M \sim 0.3$ and $H_0 = 84 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (bottom right panel). It is evident how the variation of these cosmological parameters changes the position, intensity, and relative width of the peaks in the spectrum.

2 Quasars and the X-ray to Ultraviolet relation

This chapter is devoted in its first part to a general introduction to the physics and properties of QSOs and later to a detailed analysis of the empirical relation between X-ray and Ultraviolet (UV) luminosities of QSOs. These are indeed the basis for the theoretical, observational, and cosmological studies carried out so far in the QSO field, and also for future analyses and investigations. Sections 2.2.1 and 2.2.2 are based on our results presented in [85], while Sections 2.2.3 and 2.2.4 report our results from [86] and [87] and [88], respectively.

2.1 Definition, properties and physical mechanism

QSOs belong to the category of Active Galactic Nuclei (AGN), a term used to describe the central region of an “active galaxy”, that is a galaxy in which the emission of large amounts of energy cannot be attributed to the components of a standard galaxy, such as stars, dust and interstellar gas. Despite the peculiarities that are common to all AGN, it is possible to distinguish different types of AGN using a classification based mainly on several observational properties in different spectral bands. According to this classification, QSOs are AGN with such a high bolometric luminosity ($L_{bol} > 10^{45} \text{erg s}^{-1}$) that they obscure the emission of the host galaxy to such an extent that, at the time of their discovery in the late 1950s, they were initially thought to be stellar sources, albeit very peculiar ones (hence the term “quasi-stellar”). QSOs are further distinguished by the intensity of their radio emission: in the case of strong radio emission and the corresponding presence of radio emission lobes connected to the nucleus by collimated relativistic jets of matter, QSOs are defined as “radio loud”, whereas in the case of weak radio emission they are defined as “radio quiet”.

Some of the most important properties that characterise AGN in general, and thus QSOs, are the following:

- Emission across the electromagnetic spectrum, from radio to gamma-ray wavelengths. This distinguishes AGN from normal galaxies, which are dominated by stellar evolution processes and therefore emit almost exclusively in the UV, visible and infrared (IR) bands;

- The presence of broad bands in the emission spectrum. The width of these lines is due to the Doppler broadening phenomenon in the presence of speeds of the emitting regions of the order of thousands of km s^{-1} . This aspect is also very peculiar, since the emission lines of a normal galaxy come from regions with much lower typical velocities on the order of hundreds of km s^{-1} ;
- Diversification of the components contributing to the spectral energy distribution (SED). Indeed, the characteristic shape of the SED is practically constant between 10^{13} and 10^{16} Hz and also constant in the X-ray band, albeit with a brightness reduced by a factor of ~ 30 . However, it exhibits a clear peak in the IR band, generally attributed to thermal emission from dust, and in the UV band, the so-called “Big Blue Bump”, on which we will focus later.

These features of a generic SED of AGN can be visualised in Figure 2.1, where all the photometric measurements across the electromagnetic spectrum are combined to show the peculiarities in each band. In particular, the curve displays the above-described bumps and the X-ray emission together with the distinction between radio loud and radio quiet sources at low frequencies.

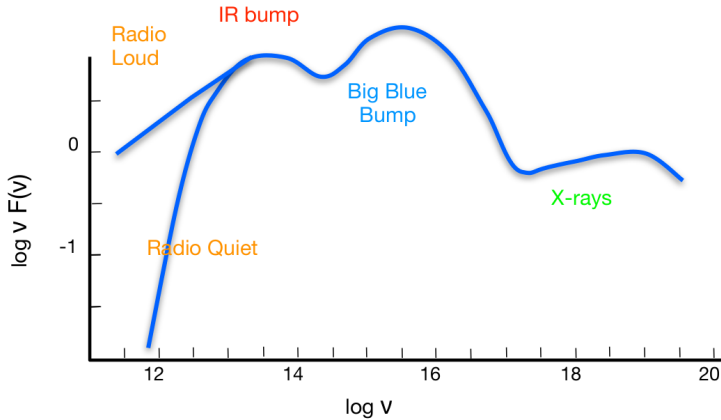


Figure 2.1: Typical AGN SED with main features highlighted. The quantity $\nu F(\nu)$ is reported as a function of the frequency ν (in units of Hz). The blue curve bifurcates at low frequencies to distinguish the cases of radio loud and radio quiet sources.

Regarding the UV spectrum of QSOs, we here also mention the “Baldwin effect”, which consists of the anti-correlation between the brightness of the UV continuum and the equivalent width of the CIV emission line. In principle, this effect can be used for cosmological purposes, as it allows the relative luminosities of QSOs to be calibrated, making them standard candles for measuring cosmological distances. However, this method has not yet yielded convincing and robust results due to the large scatter of this physical relation. Nevertheless, very recent studies are providing new insights into this effect and its cosmological application, which deserve to be further investigated. Another

peculiarity of QSOs, relevant to this study, is the fact that the X-ray and UV spectral properties show no evolution with either redshift or luminosity (see Section 2.2.2).

As mentioned above, the energy at play in AGN cannot be explained by stellar evolutionary phenomena, but must be due to processes of a different nature. So far, the mechanism commonly thought to be responsible for producing such high energies from the extremely compact core region of AGN has been the accretion of matter onto a central supermassive black hole. In this process, the gas surrounding the black hole orbits it in circular co-planar orbits due to the conservation of angular momentum. This generates a disc of matter in which the particles do not permanently occupy an orbit, but instead lose gravitational energy and, due to viscous interactions with other particles, move into orbits with smaller and smaller radii. Hence, the falling material increases the central mass. During this process, some of the lost gravitational energy is converted into radiation, and the efficiency of this physical mass-energy conversion process is so high, between 5% and 40%, depending on the spin of the black hole, that it explains the energies and luminosities of AGN.

This theoretical model of the mechanism that drives AGN can properly explain and reproduce several typical features of AGN. Indeed, the characteristic wavelength/frequency of the peak of the emission spectrum of a generic accretion disc depends on some specific properties of the central body, such as mass, accretion rate and radius of the last stable orbit; these properties determine the temperature of the disc and hence the emission frequency. If the central body is a supermassive black hole, as in the case of AGN, the emission from the accretion disc is concentrated in the optical UV, producing the excess in this spectral region, the Big Blue Bump. The X-ray emission of AGN is instead attributed to the inverse Compton process, which is produced by an external plasma, called “corona”, of relativistic electrons on the UV photons emitted by the disc. However, the physical nature of this process is still under investigation, since the electron corona, and hence the observed X-ray emission, appears to be stable, although the energy loss due to the X-ray emission should cool the electron plasma in a very short time, causing it to fall onto the central region. An efficient energy transport mechanism from the disc to the corona must be therefore in place to ensure the stability of the corona.

The existence of this connection between the two regions is confirmed by the existence of an empirical relationship between UV and X-ray luminosities (see Section 2.2 for details). In this regard, [89] proposed a slightly modified relation that involves not only the UV and X-ray luminosities but also the emission line full-width half maximum in optical-UV and they interpreted this new relation through a theoretical ad-hoc toy model of accretion disc corona. More specifically, this model is based on previous works in the literature [90, 91] and it attributes the X-ray emission to processes of reconnection and magnetic loops above the accretion disc. However, the actual theoretical interpretation is still debated and needs to be further investigated to provide a robust physical ground for the empirical X-UV relation. In the next section we focus on this relation, which provides the physical basis for employing QSOs to study the cosmic evolution and cosmological models.

2.2 X-ray to Ultraviolet relation

2.2.1 The luminosity relation

The existence of a non-linear relationship between X-ray and UV luminosity in QSOs has been empirically discovered and confirmed by several surveys in these two bands of the electromagnetic spectrum [92, 93, 94]. This relation is usually expressed in a linear form as a function of the logarithms of the luminosities as ¹

$$\log(L_X) = \gamma \log(L_{UV}) + \beta \quad (2.1)$$

where L_X and L_{UV} are the monochromatic luminosities at 2 keV (X-ray band) and 2500 Å (UV band) respectively, γ is the slope and β is the intercept of the relation, and there is no dependence on redshift, a crucial point that we will discuss later. For cosmological analyses, the luminosities are derived from the measurements of the corresponding monochromatic fluxes via the formula $L_i = F_i \cdot 4\pi D_L^2 K$ ($i = UV, X$) and assuming a specific cosmological model to derive D_L , usually a standard flat Λ CDM cosmological model with $\Omega_{M,0} = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. K is the K -correction accounting for the cosmic expansion [95] and for QSOs it is assumed to be $K = 1$ [85]. Monochromatic X and UV fluxes are generally not directly reported in the survey catalogues, but they have to be extrapolated from the available data. They can be derived from a full analysis of the spectra where these are available. When only photometric data are available (or when the samples are too large for full spectroscopic analyses), the following procedure is used (see [28, 85] for a more detailed description):

- The UV flux at 2500 Å is estimated using a procedure that takes into account all photometric information in the different spectral bands from IR to UV. In particular, the monochromatic fluxes at different frequencies are calculated from the corresponding magnitude and corrected for galactic extinction; from the points thus obtained, we interpolate the SED of each source and from this we obtain the flux (or the luminosity) at 2500 Å, either directly or by extrapolation, depending on whether the data cover this wavelength or not. Each of these values is then associated with the corresponding error. In this way it is also possible to calculate the slope of the SED in certain frequency ranges, which is useful in the sample selection process to define extinction indicators in the UV band (see Section 3.1);
- The X-ray flux at 2 keV is estimated from the flux measurements in the “soft” band (0.5÷2 keV) and in the “hard” band (2÷12 keV) and assuming in each of these bands a power-law for the flux as a function of the energy of the form $f(E) = f(E_p)(E/E_p)^\Gamma$. Fitting this function with the free parameters $f(E_p)$ and Γ gives the “pivot” value E_p , which is different for the two frequency intervals, for which the covariance between the two free parameters is zero. It is then possible to compute the fluxes corresponding to these two values of E_p and, assuming a single power-law for the entire X-ray spectrum, to derive the overall slope of the

¹Throughout the text the notation $\log$ is used in place of $\log_{10}$.

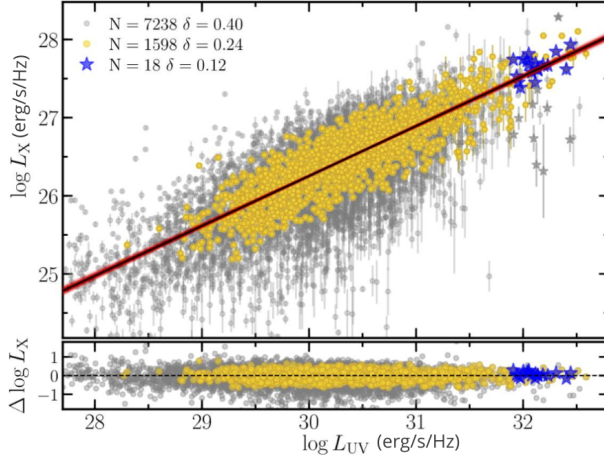


Figure 2.2: The logarithmic X-UV relation presented in [28]. The gray points mark the initial sample of ~ 7300 QSOs, the yellow points are the ~ 1600 sources retained after the selection of a clean sample (see Section 3.1 for further details), and the blue stars are the 18 QSOs at $z \sim 3$ with high-quality observations described in Section 3.2. The red solid line presents the best-fit regression. The lower panel shows the residuals with respect to the best-fit regression line.

law (Γ_X) and hence the value of the flux at 2 keV. This procedure finally gives the monochromatic flux and luminosity and the slope Γ_X , each with its own error.

There are numerous studies in the literature that have tested the X-UV relation on different samples of QSOs using the major X-ray observatories over a wide redshift range and wide ranges of UV luminosity, spanning over 5 orders of magnitudes. All these studies reported the same slope value of ~ 0.6 [96, 97, 98, 99, 100]. However, the dispersion δ of the data in the plane ($\log L_{UV}, \log L_X$) is so large in these studies, with $\delta \sim 0.35/0.40$ dex in logarithmic units, that it casts doubt on the actual validity of the relationship, given that the physical mechanism that is at its origin is not yet fully understood.

In recent years, however, we have shown through various studies that this observed dispersion is not intrinsic to the relation, but is mainly driven by external factors. Indeed, this is mainly caused by the inclusion in the sample of QSOs with low quality observations or X-ray and UV flux measurements, which are implicitly modified compared to the real ones, due to some anomalous properties such as: intense radio emission, presence of broad absorption lines in the spectrum, very low X-ray luminosity, systematic effects and/or contamination in the X/UV band (Eddington bias, absorption by dust, galaxy, gas...). Indeed, [99], [28], and [101] proved that a careful selection of the sample, aimed at removing these anomalous and deviating sources, makes the X-UV relation more evident and significantly reduces the dispersion, confirming its largely observational origin. It is therefore essential to apply very strict selection criteria in order to define a

sample of QSOs with both X-ray and UV measurements that are completely reliable and free from systematic effects, by which it is possible to test the validity of the luminosity relation, to investigate the origin of the physical connection between the accretion disc and the corona, and to perform cosmological analyses. The most recent selection criteria and the selected QSO sub-sample used for cosmological purposes are described in detail in Section 3.1. In this regard, Figure 2.2 shows the linear X-UV relation in the logarithmic space for different QSO samples: the original data set used by [28] in gray, the cleaned sub-sample obtained from the selection procedure described in Section 3.1 in yellow, and the 18 sources at $z \sim 3$ with high-quality observations presented in Section 3.2 with blue stars. The legend also reports the intrinsic dispersion associated with each sample, which decreases as a consequence of the selection of a cleaned sample, as detailed in Chapter 3. The red line in this figure marks the best-fit linear relation and the lower panel shows the residuals with respect to this regression line.

2.2.2 The flux relation and the non-evolution with the redshift

As can be seen from Eq. (2.1), the luminosity relation depends on the cosmological model assumed to calculate luminosities from fluxes, and therefore cannot be used to estimate the value of γ in a model-independent way, nor to prove the redshift independence of the relation. However, this can be done by rewriting the relation in terms of fluxes rather than luminosities. Indeed, by imposing $L_i = F_i \cdot 4 \pi D_L^2$ ($i = \text{UV, X}$), we obtain

$$\log F_X = \gamma \log F_{UV} + \beta + (\gamma - 1) \log 4\pi + 2(\gamma - 1) \log D_L(z). \quad (2.2)$$

When this expression is analysed in sufficiently small redshift intervals, the term of $D_L(z)$ gives a practically constant contribution. More precisely, if the redshift interval is chosen such that the range of values assumed by the luminosity distance is smaller than the intrinsic dispersion of the relation, then $D_L(z)$ can be considered constant and the relation takes the linear form $\log F_X = \gamma \log F_{UV} + \hat{\beta}(z)$ where $\hat{\beta}(z) = \beta + (\gamma - 1) \log 4\pi + 2(\gamma - 1) \log D_L(z)$. Under this assumption, it is therefore possible to test the relation using, instead of luminosities, measured fluxes, which are completely independent of the model. In this way we achieve two goals: we test the fundamental physical assumption in our work of a non-evolution of the X-UV relation with redshift, and we derive an average value for the slope γ .

We repeated the analysis by varying the number of bins and their width in redshift, finding that the final value of γ is insensitive to the specific choice of the redshift bins, provided that the redshift interval is $\Delta \log z < 0.1$, the condition that guarantees the negligibility of the luminosity distance. In particular, the slope obtained with the most recent QSO sample (Section 3.1) is 0.586 ± 0.061 , in agreement with previous works in the literature. The trend of $\gamma(z)$ is shown here in the top panel of Figure 2.3 as reported in [85], where the sample is divided into narrow redshift bins, with a variable step $\Delta z = 0.06 - 0.45$ within the redshift range 0.45–4 to have sufficient statistics and to satisfy the above condition to use fluxes as proxies for luminosities. We can notice that the slope does not show any dependence on the redshift and settles around 0.6 (red horizontal line in the top panel).

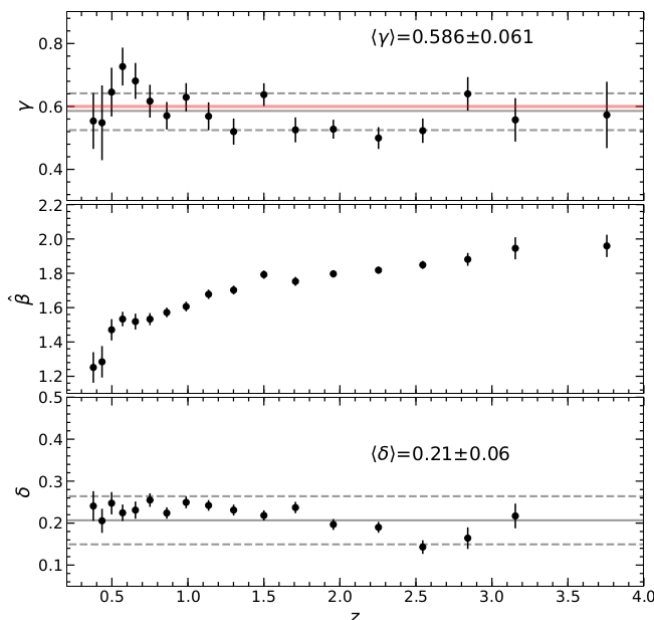


Figure 2.3: Redshift evolution of the slope γ , the intercept $\hat{\beta}$, and the dispersion δ of the linear X-UV flux relation. To perform the regression fit, X-ray and UV fluxes have been normalised to 10^{-28} and $10^{-30} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$, respectively. The data points in each panel are averages in narrow redshift bins ($\Delta z \sim 0.06 - 0.45$). The error bars are the 1σ uncertainty on the mean in each bin. The grey solid and dashed horizontal lines are the means and 1σ uncertainties, respectively, on the slope and the dispersion. The red line marks $\gamma = 0.6$.

However, to test the complete independence of the overall X-UV relation from the redshift it is also necessary to examine the evolution of β with z . Unfortunately, it is not possible to determine what its redshift evolution is from the trend of $\hat{\beta}(z)$. Indeed, by definition, $\hat{\beta}$ depends both from β and $D_L(z)$. Hence, if β was constant with redshift, and thus $\hat{\beta}$ depended only on the luminosity distance, we would obtain $\hat{\beta} \propto -\log D_L(z)$ and consequently a trend for $\hat{\beta}$ that resembles the Hubble diagram. Even though the function $\hat{\beta}$ shown in the middle panel of Figure 2.3 effectively reflects this expected trend, this is not an incontrovertible evidence that β does not evolve with z since the Hubble diagram is only known exactly in the low redshift range covered by SNe Ia and not up to the higher redshifts of QSOs. Nevertheless, we can argue that any dependence of β on z is highly unlikely in the light of several considerations:

- since we have just shown that γ does not depend on the redshift, the only dependence in the relation would be in the normalization factor β ;
- since the spectral properties of QSOs do not vary with the redshift (Section 2.1),

there is no reason to suppose that the X-UV relation does depend on z ;

- the parameter $\log(L_X/L_{UV})$ [92] has been observed to be constant across a wide redshift range (see for instance Figure 4 in [102]);
- if we assume that β is constant in z and we impose the cross-calibration with SNe Ia, the Hubble diagram of SNe Ia and that of QSOs completely overlap in the common redshift region. This guarantees that the X-UV relation is independent of z , at least up to the maximum redshift of SNe Ia, which is $z \sim 2.3$.

In the light of these considerations, we can affirm that the X-UV relation could evolve with the redshift only in the case that this evolution occurs only for $z > 2.3$, which seems to be physically very unlikely. Finally, we have supported the independence of the X-UV relation from the redshift, an essential condition for its application for cosmological studies. Indeed, Eq. (2.2) allows us to infer QSO distances as a function of the measured fluxes and the free parameters of the slope and normalization as

$$\log D_L(z) = \frac{1}{2(\gamma - 1)} \left[\log F_X - \gamma \log F_{UV} - \beta - (\gamma - 1) \log(4\pi) \right]. \quad (2.3)$$

In addition, the lower panel of Figure 2.3 also shows the values of δ obtained in each redshift bin considered, which provide an average value of $\delta = 0.21 \pm 0.06$ for the QSO sample described in [85] (see Section 3.1 for details), thus highlighting the reduction in the dispersion obtained by the selection of the sample.

Concerning the non-evolution of the UV-optical spectral properties of QSOs, we have recently addressed this possible issue by analysing the spectral properties of our cosmological QSO sample. Specifically, we produced composite spectra in different bins of redshift and accretion parameters (black hole mass, bolometric luminosity) to investigate any possible evolution of the spectral properties of the continuum of the composites with these parameters. We found a remarkable similarity amongst the various stacked spectra. Apart from the well-known evolution of the emission lines with luminosity (i.e. the Baldwin effect, see Section 2.1) and black hole mass (i.e. the virial relation), the overall shape of the continuum, produced by the accretion disc, does not show any statistically significant trend with black hole mass, bolometric luminosity, or redshift. Thus, the similarity of the QSO spectra across the parameter space rules out a significant evolution of the average continuum properties with any of the parameters studied, confirming the reliability of our sample for cosmological applications.

In addition, we have further investigated the robustness and reliability of the X-UV relation from different perspectives: i) by analysing different choices of the indicators for the X-ray and UV fluxes other than the luminosities at 2500 Å and 1 keV, respectively, ii) by examining the physical contribution to the dispersion of the relation, and iii) by testing whether the relation still holds when selection biases and redshift evolution of luminosities are taken into account. The results of these recent studies are detailed in Sections 2.2.3, 2.2.4, and 5.3, respectively.

Once proved the reliability of the X-UV relation for cosmological analyses, we can build the Hubble diagram of QSOs. Indeed, Eq. (2.3) depends only on the measured

fluxes, the slope γ , and the intercept β . Hence, we can leave γ and β as free parameters when fitting a cosmological model. As a matter of fact, γ is a physical parameter linked to the QSO physics at play, while β is a scale factor that can be determined only through an external calibrator, such as SNe Ia. Indeed, if we jointly fit QSOs and SNe Ia, we impose a cross-calibration between the two samples that identifies the zero-point of the Hubble diagram and thus the parameter β . An example of the Hubble diagram of QSOs is shown in Figure 2.4. The inner box of this figure clearly shows the match between SNe Ia and QSOs in the common redshift range.

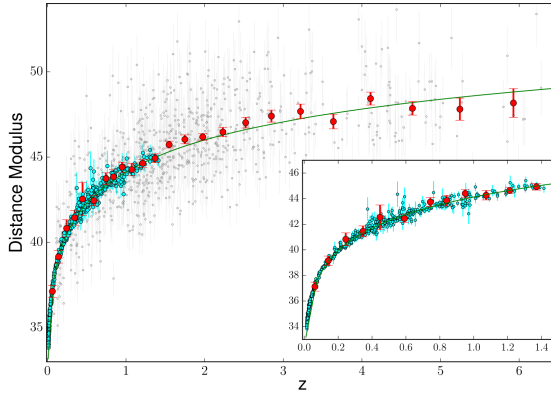


Figure 2.4: Hubble diagram for the QSO sample (small gray points) and SNe Ia (cyan points) from the Union 2.1 sample as taken from [103]. The large red points are QSO averages in small redshift bins. The inner box shows a zoom of the $z = 0 - 1.5$ range to better visualize the match between SNe Ia and QSOs in the common redshift range. The continuous line is obtained from a joint fit of the two samples assuming a standard Λ CDM cosmological model.

Finally, the X-UV relation, also called the Risaliti-Lusso (RL) relation in its cosmological application, establishes QSOs as invaluable cosmological probes. Indeed, although QSOs provide a lower precision for the determination of cosmological parameters, compared to other probes such as SNe Ia, due to the higher dispersion of their relation, they cover a redshift region almost unexplored so far, that, reaching up to $z \sim 7.5$, is pivotal to discern among predictions from different cosmological model and investigate the evolution of the Universe and the current cosmological tensions.

2.2.3 The choice of the X-ray and UV indicators

Concerning the investigation of the X-ray and UV indicators, the choice of the X-ray and UV indicators has so far been somewhat arbitrary and has typically been based on photometric data. Indeed, the 2500 Å monochromatic flux has been adopted since the first studies, but its choice was mainly based on observational considerations. In fact, the rest-frame 2500 Å at $z \sim 1$ corresponds to the observed V-band, which was available for a significant number of sources. Additionally, this rest-frame UV wavelength is less affected by host-galaxy contamination (dominant at low luminosity for $\lambda > 4000$ Å) and intergalactic absorption ($\lambda < 1450$ Å) than other regions of the AGN SED, thus representing the ideal proxy for the intrinsic disc emission. Notwithstanding this, the photometric 2500 Å flux may be contaminated by strong Fe II line emission, which may introduce systematic uncertainties in the photometrically derived F_{UV} values [104]. We also note that, regardless of the details of the physical mechanism driving the X-UV relation, the characteristic UV flux wavelength should be the one most closely related to the global emission from accretion disc, and may not be exactly the rest-frame 2500 Å (see Section 6 of [28]). As we are still far from understanding the nature of the X-UV relation, our photometric fluxes are seen as a more conservative representation of the broadband disc emission, capturing the “true” dependence between the disc and the X-ray corona in AGN.

In this regard, we have investigated the X-ray and UV proxies that provide the smallest dispersion of the relation in order to obtain more precise distance estimates, to better understand the physics behind the X-UV relation, and to confirm the reliability of the X-UV relation as a distance indicator. In this context, we performed a complete UV spectroscopic analysis of a sample of 1764 QSOs from the [85] sample with optical spectra in the UV band and X-ray serendipitous observations. We thus derived monochromatic luminosities at five different wavelengths as well as the emission-line properties. In the X-rays, we analysed the spectra of all the objects in the sample at redshift $z > 1.9$, while at lower redshifts we relied on photometric measurements. As done in previous studies, we analysed the relation in small redshift bins, using fluxes instead of luminosities (see Section 2.2.2).

As a result, we showed that the dispersion δ is slightly higher when using spectroscopically derived monochromatic fluxes as UV fluxes. If the true physical quantity behind the X-UV relation was the monochromatic luminosity at 2500 Å (or any of the wavelengths that we tested) we would have expected better results in terms of the dispersion. Since this does not occur, this probably means that the spectroscopic and the photometric luminosities are simply two different proxies for a different quantity, and are therefore similarly effective when used in the X-UV relation. We further investigated which energies are the best to be used, assuming that both the UV and the X-ray continua of QSOs can be parameterised as power-laws and studying the dependence of the X-UV relation on the respective spectral slopes. Thus, we looked for the best possible proxies for the overall UV (disc) and X-ray (corona) emission. In the X-rays, we found that there is a preference for 1 keV as the characteristic energy, as it is the one less sensitive to the true spectral slope. Indeed, when using the 1 keV fluxes instead of the 2 keV ones, we found a lower dispersion. Unfortunately, given the redshift range of our sample, the

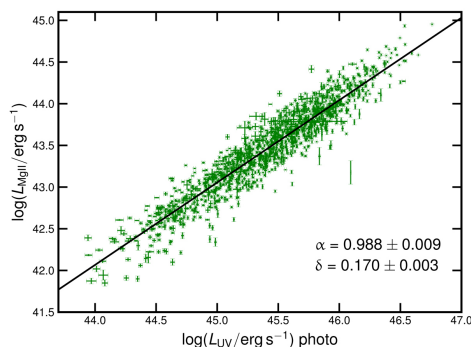


Figure 2.5: Relation between the luminosity of the Mg II line and the luminosity at 2500 Å derived from photometry.

1 keV flux is on average measured with higher uncertainties than the 2 keV ones. This partly undermines the advantage of having found the characteristic energy because the resulting overall dispersion is only slightly lower. Nevertheless, the fact that the total dispersion decreases even if we are using a “worse” proxy in terms of uncertainties means that physically the relation is indeed tighter when we use the 1 keV flux.

In the UV band, we found no conclusive evidence of a specific characteristic wavelength. Consequently, we argue that the best choice is the 2500 Å flux simply because it is the one that results in a slightly lower dispersion. This may be because this wavelength falls in the observed spectra for the wider redshift range in our sample, implying a smaller number of objects for which the fluxes are determined by extrapolation and therefore with larger uncertainties. In addition, when using the integrated Mg II line flux as UV flux, we obtain a higher (steeper) slope value and a lower dispersion ($\delta = 0.16$ dex) compared to all other UV continuum indicator derived from the spectroscopic analysis. This implies that the Mg II emission line flux works as a better proxy when compared to the fluxes in the 1350–5500 Å range. Unfortunately, the Mg II emission line is only available for the objects in our sample up to $z \sim 2.5$. Furthermore, as shown in Figure 2.5, the relation between the luminosity of the Mg II line and the photometric luminosity at 2500 Å shows a slope statistically consistent with unity, which explains the similar behaviour of these two quantities when used as UV proxies in the X-UV relation. Finally, the Hubble diagram obtained from spectroscopic UV data is fully consistent with that obtained from photometric data. This is a validation of the previous results based on “photometric” Hubble diagrams.

In the end, the monochromatic fluxes at 1 keV and 2500 Å are, respectively, the best X-ray and UV continuum indicators among those that are typically available. However, the commonly used monochromatic fluxes at 2 keV and 2500 Å estimated from photometric data provide an almost equally tight X-UV relation, and can be used to derive QSO distances. The Hubble diagram obtained using spectroscopic indicators is fully consistent with those presented in previous papers, based on photometric data.

2.2.4 The origin of the dispersion of the relation

As we have already anticipated the observed dispersion of the RL relation is not mainly intrinsic but observational and this has been strongly confirmed by our recent analyses. Indeed, we have studied the spectroscopic properties of high-redshift QSOs in the X-ray and UV bands. More specifically, we have performed a one-by-one analysis of a sample of 130 QSOs at $z > 2.5$ with high-quality X-ray and UV spectroscopic observations and we found that not only the X-ray to UV correlation still holds at these redshifts, but its intrinsic dispersion is as low as 0.12 dex, compared to the 0.20–0.22 dex of the total sample. Remarkably, for a sample of QSOs at $z \sim 3$ with particularly high-quality observations (see Section 3.2 for further details), the dispersion further drops to 0.09 dex, a value entirely accountable for by intrinsic variability and source geometry effects. This result confirms the observational nature of the observed dispersion. In addition, we have also shown that the composite spectra of these QSOs, both in the X-rays and in the UV, do not show any difference with respect to the average spectra of QSOs at lower redshifts. Thus, the absence of any spectral difference between high- z and low- z QSOs and the tightness of the X-UV relation suggest that there are no evolutionary effects in the relation, as already proved in Section 2.2.2.

As far as the dispersion of the RL relation is concerned, we can notice that it remains significant (i.e. $\delta \sim 0.20$ dex) for the whole QSO sample, especially when compared with the Phillips relation for SNe Ia. In the absence of a comprehensive physical model for this relation, it remains unclear how much of the remaining dispersion is tied to the physical mechanism behind the relation itself and how much can be attributed to other factors, not addressed by the sample selection and by the choice of X-ray and UV indicators. Potential contributing factors include (i) the scatter introduced by the use of X-ray photometric results rather than the spectroscopic ones, (ii) the intrinsic variability of QSOs, and (iii) the inclination of the accretion disc relative to our line of sight. In this regard, we have recently thoroughly examined these three factors and quantified their individual contributions to the observed dispersion.

In particular, concerning the X-ray photometry, we have already detailed in Section 2.2.3, that the use of photometric estimates of the 2 keV monochromatic luminosities instead of spectroscopic ones does not introduce any systematic offset. However, it could still introduce an additional dispersion. Hence, we have tested this effect for a sample of 193 objects at redshift $z > 1.9$ and we found that photometric estimates do not introduce any additional scatter compared to spectroscopic ones. Although the observed scatter is larger with photometric measurements, this is fully accounted for by the larger uncertainties in the photometric data compared to spectroscopic ones. This allows us to use photometric data when spectroscopic ones are not available, with the confidence that we are not introducing additional dispersion.

In relation to the role of variability, the QSO emission is known to be variable in both the UV and the X-ray bands, which inevitably leads to an increase in the observed dispersion. Given the shorter timescales, the X-ray variability is the one that predominantly affects our results. To test the contribution of the variability, we have selected the 289 objects in the sample of [85] that have multiple X-ray observations. We found that the average scatter between different estimates of the 2 keV monochromatic luminosity,

once the measurement uncertainties are taken into account, is $\delta_{var} \sim 0.08$ dex, which we can therefore consider as the variability contribution. We also found, consistently with other results in the literature, that the variability decreases with increasing luminosity, with the high-luminosity sub-sample having a contribution of only $\delta_{var} = 0.02$ dex.

As for the third factor, the inclination of the accretion disc is believed to affect the observed UV luminosity, but not the X-ray luminosity. Therefore, the different QSO inclinations introduce a scatter in the X-UV relation. Unfortunately, we do not have observational methods to measure the inclination and correct the UV luminosities accordingly. Therefore, we have relied on mock-sample estimates to derive the inclination contribution to the observed dispersion. We found a contribution of the inclination to the dispersion of $\delta_{inc} \sim 0.06$ dex, once we account for the presence of an obscuring torus at large inclinations.

We conclude that the characteristic dispersion of the X-UV relation is extremely small, $\delta_{char} \leq 0.06$. This result supports the hypothesis that the physical mechanism governing the X-UV relation is remarkably consistent over a wide range of redshifts and luminosities. Furthermore, this outcome lends additional credibility to the use of QSOs as standard candles in cosmological studies. Moreover, the variability results confirm the above mentioned results: sub-samples with higher luminosities tend to exhibit the lowest observed dispersion values.

Although we can account for the dispersion of the X-UV relation down to ≤ 0.06 dex, the observed value for the current sample of QSOs is still much higher. Obtaining such a small dispersion observationally would give significant additional cosmological power to QSOs. On the one hand, we could try to reduce the variability contribution by obtaining multiple X-ray observations within weeks/months (which is roughly the variability timescale for the UV band), and averaging the X-ray fluxes. Unfortunately, given that our most recent sample (and any foreseeable cosmological sample) contains thousands of objects, this is not feasible in the short term. The use of simultaneous X-ray and UV observations has not proved to be helpful, because of the lag between the changes in one band and the resulting changes in the other [105, 99]. High-luminosity objects show a smaller variability amplitude than low-luminosity ones. Therefore, at the moment, the only possible way to reduce the variability contribution to the overall dispersion is to look at high-luminosity sub-samples, such as the one at $z \sim 3$ previously introduced. Regarding the inclination contribution to the dispersion, we have no way of removing it until a reliable means of estimating the inclination angle for each individual object is found. It could be argued that the brightest objects at any redshift are predominantly seen (almost) face-on, since a high inclination angle would imply an intrinsic luminosity far beyond the Eddington limit. However, there is no definitive method to determine the specific luminosity threshold at which an object can be classified as face-on. In terms of X-ray observations, a possible reduction in the dispersion could come from having more pointed observations instead of serendipitous ones, but further analysis is needed in this respect.

Overall, the best way in the short-term to obtain small dispersion values in the X-UV relation and to increase the cosmological power of QSOs is to obtain pointed observations of the most luminous QSOs at the highest possible redshift. This can

enhance our understanding of the high-redshift Hubble diagram and of the associated cosmological tensions. In the more distant future, the combination of the X-ray data from Athena (or other proposed X-ray facilities) with the optical/UV data that are starting to be collected by Euclid [106] and the Large Synoptic Survey Telescope (LSST; [107]), will provide additional statistics, helping us to characterize the X-UV relation both physically and for its cosmological implementation.

3 The quasar cosmological samples

In this chapter, we detail in Section 3.1 the selection criteria employed to carefully define a QSO sample that can be reliably applied for cosmological purposes and we then focus in Section 3.2 on a specific sample composed of sources at $z \sim 3$ and their peculiarities. These sections are based on our publication of [85] and our series of papers of [108], [109] and [110], respectively.

3.1 Selection of the quasar sample

As anticipated, a careful selection of the QSO sample is needed to reduce the dispersion of the RL relation and thus perform cosmological analyses with sufficient precision. In this section, we briefly describe the data sets and the selection criteria employed to build the most updated QSO sample that we have released for cosmological purposes [85]. These sources have been carefully selected for cosmological studies and we refer to [103], [99], [28], [101] and [85] for a detailed description.

The initial QSO sample we have considered for our analysis was assembled by combining seven different samples from both the literature and the public archives. The first group includes the samples at $z \sim 3.0 - 3.3$ by [108], $4 < z < 7$ by [101], $z > 6$ by [102], and the XMM–XXL North quasar sample published by [111]. We have then complemented this collection by including QSOs from a cross-match of optical (i.e. the Sloan Digital Sky Survey, hereafter SDSS) and X-ray public catalogues (i.e. XMM–Newton and Chandra), which we refer to as SDSS–4XMM and SDSS–Chandra samples hereafter. Finally, we have also included a local sub-set of AGN with UV data (i.e. the International Ultraviolet Explorer) and X-ray archival information. The same order in which these samples were introduced above has been adopted as an order of priority to take into account of any possible overlap. The X-ray fluxes obtained from pointed observations and medium-deep surveys (i.e. XMM–XXL) have been considered first, as they are generally more reliable. Finally, the main parent sample consists of $\sim 19\,000$ objects spanning the redshift range $0.009 < z < 7.54$.

This QSO sample is properly selected to remove as many observational biases as possible. First, we have removed the sources with a signal-to-noise ratio that does not guaran-

tee a well-sampled photometry (i.e. signal-to-noise ratio $S/N < 1$). After this first screening, all QSOs with a SED that shows UV reddening and near-IR host-galaxy contamination are discarded, leaving only sources with extinction $E(B-V) \leq 0.1$. This corresponds to selecting only those sources that satisfy $\sqrt{(\Gamma_{1,UV} - 0.82)^2 + (\Gamma_{2,UV} - 0.40)^2} \leq 1.1$, where $\Gamma_{1,UV}$ and $\Gamma_{2,UV}$ are the slopes of a $\log(\nu)$ - $\log(\nu L_\nu)$ power-law in the rest frame 0.3-1 μm and 1450-3000 $\text{\AA}$ ranges, respectively. The values $\Gamma_{1,UV} = 0.82$ and $\Gamma_{2,UV} = 0.4$ refer to a SED with zero extinction (see [112]). Recently, we have further investigated the contaminants of dust extinction and host-galaxy contribution. Specifically, by comparing the composite spectra of our low- z sources ($z \leq 0.8$) with SDSS objects affected by heavy host-galaxy contamination, we have shown that the composite spectra of our QSO sample are consistent with a negligible amount of host-galaxy emission, that is less than 10%. In addition, the contribution of the intrinsic reddening is also negligible, with a colour excess $E(B-V) \leq 0.01$. We have therefore examined whether unaccounted dust extinction could explain the tension between our data and the concordance ΛCDM model (see Chapters 7 and 9). As a result, the average colour excess required to solve this discrepancy should increase with redshift up to non-physically high values, such as $E(B-V) \sim 0.1$ at $z > 3$, which would imply that the intrinsic emission of QSOs is much bluer and more luminous than ever reported in observed spectra. Lastly, dust reddening cannot account for the observed tension between the Hubble diagram of QSOs and the ΛCDM model. Finally, the spectroscopic analysis that we have carried out corroborates the photometric selection criteria adopted to define a sample of QSOs with typical properties. This, in turn, allows us to keep employing the more efficient photometric cuts rather than a more time-consuming individual spectroscopic analysis.

Concerning the selection criteria in the X-ray band, X-ray observations in which photon indices (Γ_X) are peculiar or indicative of absorption are excluded by requiring $\Gamma_X + \Delta\Gamma_X \geq 1.7$ and $\Gamma_X \leq 2.8$ when $z < 4$ and $\Gamma_X \geq 1.7$ when $z \geq 4$, where $\Delta\Gamma_X$ is the uncertainty on the photon index. Finally, the remaining observations are corrected for the Eddington bias. Indeed, the cleaned sample consists only of sources fulfilling $\log F_{X,exp} - \log F_{min} \geq \mathcal{F}$, where $\mathcal{F}$ is a threshold value and $F_{X,exp}$ is the X-ray flux computed from the observed UV flux, assuming the RL relation with fixed parameters within the flat ΛCDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. F_{lim} is the flux limit of the specific observation estimated from the catalogue. More precisely, as detailed in [99], the minimum detectable flux is computed for each object from the total exposure time of the charge-coupled device (CCD) where the source is detected using the functions plotted in Figure 3 by [113]. The value of $\mathcal{F}$ required in this filter is $\mathcal{F} = 0.9$ for the SDSS Data Release 14–4XMM Newton [114, 115] and XXL [116] sub-samples and $\mathcal{F} = 0.5$ for the SDSS-*Chandra* [117]. All the surviving multiple X-ray observations are ultimately averaged to reduce the effects of the X-ray variability.

The sample that we use is the result of all these selection filters and it consists of 2421 sources covering the redshift range up to $z = 7.54$ [118]. Figure 3.1 shows both the original and the selected QSO samples. The former is represented by gray points, while the latter is marked with different colours according to the corresponding sub-sample, as described in the legend. Remarkably, this careful selection of a cosmological QSO sample significantly reduces the intrinsic dispersion of the RL relation from $\delta \sim 0.40$ to

$\delta \sim 0.20$ (see bottom panel of Figure 2.3) hence enhancing the cosmological power of QSOs in constraining cosmological parameters.

Finally, we here note that we can use the full QSO sample or add a filter of $z > 0.7$ for the sources with a photometric determination of the UV flux. Indeed, the possible uncertainties in the extrapolation from the optical of the QSO UV continuum at low redshift, where the host-galaxy contamination can be relevant, make the 2500 Å monochromatic luminosities less reliable at $z < 0.7$. In particular, this effect is likely to be more pronounced at lower fluxes (or luminosities), where the data quality is also lower. If the continuum slope at UV wavelengths becomes steeper than in the optical, the true 2500 Å flux would be underestimated and this could explain the higher average values of the slope of the X-UV relation that we observe at $z < 0.7$ (see upper panel of Figure 2.3). This point deserves further investigation, so a possible conservative choice is to cut the optical sample for cosmological applications at $z > 0.7$, with the exception of the local sources introduced earlier, whose 2500 Å flux is determined from the UV spectra without extrapolation.

As our method has proven successful in producing large samples of homogeneous sources, we can extend and refine its application to the upcoming data from new facilities. As an example, the extended ROentgen Survey with an Imaging Telescope Array (eROSITA, [119]) has begun to populate the crucial $z \sim 1$ region in the X-ray domain, allowing a more robust cross-calibration between QSOs and SNe Ia. In the mid- to long-term future, the cross-match between Euclid [106] and LSST [107] observations in the optical–UV and Athena (and/or other proposed facilities) in the X-rays will significantly enlarge the sample, helping to fill in the region at intermediate–high redshift. New large and reliable samples will be key to testing the cosmological models in synergy with

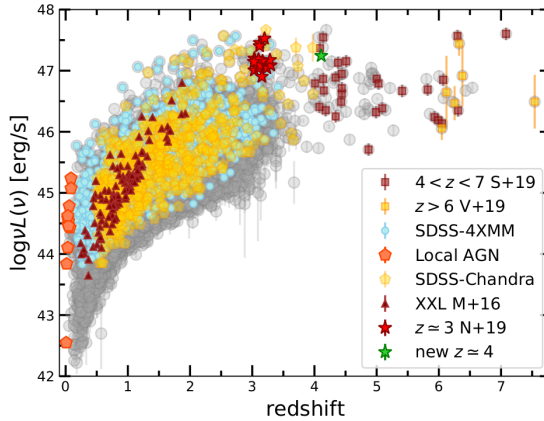


Figure 3.1: Distribution of luminosities at rest-frame 2500 Å as a function of redshift for the main (gray points) and the selected (cleaned) samples. Brown and yellow squares: high- z sample [101, 102], cyan points: SDSS-4XMM, brown triangles: XMM-XXL [111], orange pentagons: the local AGN sample, red stars: $z \sim 3$ QSO sample [108], green star: new $z \sim 4$ QSOs, gold pentagons: SDSS-Chandra.

other well-established probes, with the aim of accurately constraining the cosmological parameters.

3.2 The sub-sample at $z \sim 3$ and the X-ray weak quasars

In this section we focus on the sub-sample of QSOs at $z \sim 3$ already introduced in Sections 2.2.4 and 3.1 and marked with red stars in Figure 3.1. Indeed, our main QSO sample includes 29 bright ($L_{bol} > 10^{47} \text{ erg s}^{-1}$) sources at $z \sim 3$ with X-ray observations obtained from an extensive campaign performed with XMM–Newton (cycle 16, proposal ID:080395, PI: Risaliti). This campaign targeted 30 QSOs in the $z = 3.0 - 3.3$ redshift range for a total exposure of 1.13 Ms. This sample, selected in the optical from the SDSS Data Release 7 to be representative of the most luminous QSOs with a blue continuum emission, typical of radiatively efficient accretion discs, boasts by construction a remarkable degree of homogeneity in terms of optical and UV properties. The X-ray data have been extensively analysed in a recent series of our works.

In particular, we have presented a deep X-ray investigation of this sample that reduces to 25 QSOs since four sources proved to be too faint for a detailed spectral analysis, one of which is formally undetected, and one more object was later found to be radio-loud. The continuum photon index distribution of this final sample is centred at $\Gamma_X \sim 1.85$, in

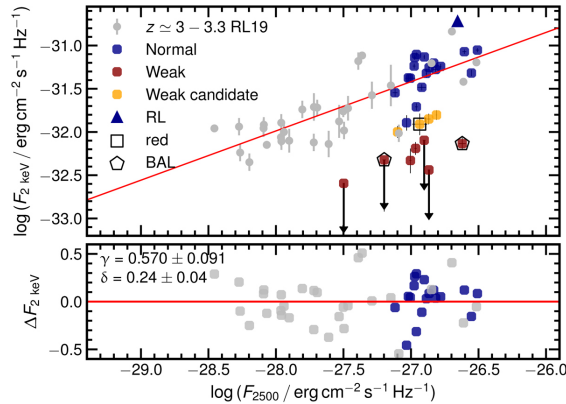


Figure 3.2: Relation between the rest-frame monochromatic X-ray (2 keV) and UV (2500 Å) fluxes for QSOs in the redshift range $z = 3.0 - 3.3$. Blue, brown, and orange symbols represent X-ray normal, weak, and weak candidates, respectively, while the triangle, pentagon, and square symbols mark the radio-bright (J0900+42), BALs (J1148+23, J0945+23), and reddest (J1459+00) QSOs in the sample. Gray points represent the other sources in the same redshift range from the cleaned QSO sample of [28] (30 objects). The red line is the regression best fit for the $z = 3.0 - 3.3$ sources of [28] only. Slope and dispersion values are reported with the residuals from the best fit in the bottom panel (from which X-ray weak, in brown, and weak candidates, in orange, are omitted for clarity).

excellent agreement with those in place at lower redshift, luminosity, and black hole mass, confirming the universal nature of the X-ray emission mechanism in QSOs. Nevertheless, when compared against the X-UV correlation, these QSOs show an unexpectedly diverse behaviour, splitting into two distinct sub-sets. About two thirds of the sources are clustered around the correlation with a minimal scatter of 0.1 dex, while the remaining one third appears to be X-ray underluminous by factors of > 3 –10. This behaviour is shown in Figure 3.2 where blue, brown, and orange symbols represent X-ray normal, weak, and weak candidates, respectively, while the triangle, pentagon, and square symbols mark the radio-bright, broad absorption line (BAL), and reddest QSOs in the sample. Gray points represent the other sources in the same redshift range from the cleaned QSO sample of [28] (30 objects). Such a large fraction ($\sim 25\%$) of X-ray weakness has never been reported in radio-quiet, non-BAL QSO samples. Several factors could contribute to enhancing the X-ray weakness fraction among our $z \sim 3$ blue QSOs, including variability, mild X-ray obscuration, contamination from weak-line QSOs, and missed BALs. However, the X-ray weak objects also have, on average, flatter spectra, with no clear evidence of absorption. Hence, we suggest that, at least in some of our X-ray weak QSOs, the corona might experience a radiatively inefficient phase due to the presence of a powerful accretion disc wind, which substantially reduces the accretion rate through the inner disc and therefore also the availability of seed photons for Compton up-scattering. Finally, the difference in the X-ray spectral slope values between the two groups of QSOs is a viable method to select the sources with the “right” X-ray properties for the analysis of the relation as well as for its cosmological application. As a result of that, only sources with $\Gamma_X > 1.7$ have been included in our final sample (see Section 3.1). In addition, we would like to emphasize here that, as anticipated, the intrinsic dispersion of ~ 0.1 dex shown by the bright sample is significantly lower than the dispersion obtained in all our studies, demonstrating that most of the observed dispersion is due to observational issues rather than to intrinsic effects.

We have then further investigated the origin of the deviations from the X-UV relation by exploring possible correlations between the UV and X-ray properties of these sources. We have fitted the UV spectra from the SDSS of QSOs in our sample and analysed their C IV properties (e.g. equivalent width, line peak velocity) as a function of the X-ray photon index and 2–10 keV flux. As a result, we found that the X-ray weak QSOs deviate from the LX - L C IV main relation, occupying the low X-ray/C IV luminosity part of the plane, thus displaying an excess of C IV line emission with respect to normal sources of similar X-ray luminosities. Consequently, in agreement with our previous analysis, we argued that the X-ray weakness could be interpreted in a starved X-ray corona picture associated with an ongoing disc-wind phase. Indeed, if the wind is ejected close to the black hole, the extreme-UV radiation reaching the corona is depleted, depriving the corona of seeds photons and generating an X-ray weak QSO. Nonetheless, at the highest UV luminosities ($> 10^{47} \text{ erg s}^{-1}$), there will still be an ample reservoir of ionising photons explaining the excess C IV emission observed. This work demonstrates that a systematic study of X-ray spectra, optical to X-ray SEDs, and optical/UV emission line properties in the context of accretion disc models is essential to better understand the observable effects of disc winds.

Based on these results, we have presented new observations with the Large Binocular Telescope (LBT) finding that X-ray weak QSOs display higher Fe II/Mg II ratios compared to typical QSOs. In agreement with the previous two analyses, this study ascribes this peculiarity to wind-related microturbulence and shocks in the outflows, which could enhance the Fe II production in X-ray weak sources.

4 Gamma-Ray Bursts and their application in cosmology

Like QSOs, also GRBs have been recently applied as cosmological tools at high redshifts. In this chapter, we first introduce GRBs as astrophysical sources in Section 4.1, then in Section 4.2 we describe the methodology used to turn them into cosmological probes, and in particular the 3D “Dainotti” relation, and in Section 4.3 we define the GRB samples used in our analyses. Finally, in Sections 4.4 and 4.5 we focus on two machine learning techniques, namely the light curve reconstruction and the redshift prediction, that allow us to tighten the GRB correlations and increase their sample size, and we show how these approaches can be applied to enhance the cosmological power of GRBs in the near future, based on our results from [120].

4.1 Definition, properties and physical mechanism

GRBs are incredibly powerful and luminous sources discovered by the Vela satellites [121] more than 50 years ago and are now observed up to very high redshifts, reaching $z = 8.2$ [122] and $z = 9.4$ [123]. As for QSOs, this marks the promising role of GRBs as a possible new step in the cosmic distance ladder beyond SNe Ia, observed up to $z = 2.26$ [22]. However, in order to turn GRBs into cosmological tools, several issues need to be addressed. Indeed, a proper determination of the GRB classes, the nature of their progenitors, and the energetic mechanisms that drive the GRB emission are still under debate.

In this context, a classification of GRBs based on their measured light curves is crucial for discerning among different possible origins. From a phenomenological point of view, the light curves are usually described by a short prompt high-energy emission followed by an afterglow, that is an emission of longer duration observed in the X-ray, optical, and radio wavelengths [124, 125, 126, 127, 128, 129, 130, 131, 132, 133]. Usually, the prompt emission is detected in high-energy bands, from X-rays up to ≥ 100 MeV γ -rays, but sometimes it has also been observed in the optical band [134, 135].

Based on the duration of the prompt emission, GRBs have historically been divided into two main classes: Short GRBs (SGRBs) and Long GRBs (LGRBs). The former class is characterized by $T_{90} \leq 2$ s, while the latter class by $T_{90} \geq 2$ s, where T_{90} is the

time in which a GRB produces between 5% and 95% of the total number of photons emitted in the prompt [136, 137, 138, 139]. However, these categories have been further divided into several sub-classes since the observations from the Neil Gehrels Swift Observatory (hereafter Swift) satellite [140] have shown the existence of the afterglow phase after the prompt emission and provided evidence in 42% [141, 142] of GRBs for the presence of a plateau, a flat part of the light curve lasting from 10^2 to 10^5 s, in the afterglow phase followed by a power-law decay [143, 144, 145, 146, 147]. The sub-classes identified are the following: ultra-long GRBs (ULGRBs) characterized by $T_{90} > 1000$ s [148, 149, 150, 151], X-ray flashes (XRFs; [152, 153, 154]), with a fluence in the X-ray band (2 – 30 keV) higher than the fluence in the γ -ray band (30 – 400 keV), X-ray rich (XRR), with a ratio between the X-ray fluence and the γ -ray fluence intermediate between standard GRBs and XRFs, SGRBs with extended emission (SEEs; [155, 156, 157],) which exhibit features of both SGRBs [158] and LGRBs, intrinsically short (IS) GRBs with $T_{90}/(1+z) < 2$ s, and GRBs associated with SNe Ib/c (SNe-GRB). Overall, the observed differences among these sub-classes are supposed to arise from different progenitors and/or environments. Indeed, [159] have recently proposed a different classification: Type I GRBs, produced by the collision of two compact objects, and Type II GRBs, caused by the collapse of a massive star, the so-called “collapsar model” [160, 161, 162, 163]. While LGRBs, ULGRBs, XRFs, XRR, and SNe-GRB belong to the latter class, SGRBs, SEEs, and IS GRBs belong the former class. However, some exceptions prevent this classification from properly categorizing all the observed GRBs.

Clearly, the problem of classification is closely related to the problem of identifying the GRB progenitors and the physical processes that drive the GRB emission. In this regard, as anticipated, the origin of GRBs is commonly attributed to two different scenarios. One is the explosion of a massive star [164, 165, 162, 166, 167, 168] followed by a core-collapse SNe [163, 169] and the other is the merging of two compact objects in a binary system [170, 171, 172, 173, 174]. Indeed, the model traditionally used to explain the GRB physics is the “fireball” model [175, 176, 177, 178, 179, 180, 181], in which the central engine (i.e. the core collapsed massive star or the merger in the binary system) produces a relativistic jet that interacts with the external medium. However, this model started to manifest some problems in reproducing the observed light curve after the identification of the plateau by Swift [145, 182, 183]. As a consequence, the search for GRB correlations among physical parameters proves to be very relevant since these relations can be used as model discriminators. Currently, the most physically plausible description is the one in which the central engine powering the GRB is a black hole, a neutron star (NS), or a newly born fast-spinning highly magnetized NS, a magnetar [184, 185, 186, 187, 188, 189, 190].

4.2 Physical correlations and the 3D “Dainotti” relation

Pushed by these open issues, several groups have striven to reveal correlations between prompt, plateau, or both GRB features, in an attempt to turn GRBs into cosmological tools. These efforts are also motivated by the fact that, as already stressed, the flat Λ CDM model is currently being questioned due to well-known longstanding and more recent theoretical and observational problems. In this regard, since the GRB isotropic energies range over eight orders of magnitude and thus GRBs are not intrinsically standard candles [191, 192, 179, 193, 194], it is essential to establish correlations between GRB physical properties that are intrinsic to the physics of GRBs and are grounded on a theoretical model. Such a correlation can then be applied to a well-defined sub-sample of GRBs based on specific and common physical properties to constrain cosmological parameters.

We here focus only on the GRB correlation that we have employed in our analyses and we refer to [195] and [196] for a comprehensive review of the GRB prompt correlations, to [197] for a complete overview of the GRB correlations used in cosmology that correlate afterglow quantities at different wavelengths, and to [198] for a self-consistent compendium that we have recently compiled on the application of GRB correlations in cosmology and their impact on the current knowledge of the Universe. In our work, we have used the most recent, complete, and physically grounded GRB correlation to date, that is the 3D Dainotti relation, also known as “3D fundamental plane”, which has been discovered at different wavelengths. These relations in different wavelength bands have been derived from the previously found two-dimensional relations (2D Dainotti relations) in X-rays [199], optical [200], and radio [201], by adding a third physical quantity.

Regarding the X-ray wavelength, the Dainotti 3D X-ray relation has been proposed in [202] and [203]. This is a correlation between the rest-frame end-time of the X-ray plateau emission, $T_{X,a}^*$, the X-ray luminosity at the end of the plateau emission, $L_{X,a}$, and the additional third quantity of L_{peak} , the peak luminosity of the prompt emission. This relation usually reads as

$$\log L_{X,a} = a \cdot \log T_{X,a}^* + b \cdot \log L_{peak} + C_0, \quad (4.1)$$

where a , b , and C_0 are the slopes and the normalization parameters, respectively, and the luminosities are computed from the corresponding measured fluxes. The coefficients a and b are negative and positive, respectively. The implementation of the 3D correlation yields a reduced scatter compared to the two-dimensional case and defines a tight plane in the $(\log T_{X,a}^*, \log L_{X,a}, \log L_{peak})$ space, which is called the “GRB fundamental plane”, here shown in Figure 4.1.

The extension of the 3D X-ray fundamental plane in optical wavelengths has also been studied [205, 206]. Specifically, as for the three-dimensional fundamental plane in X-rays, also the 3D optical relation has been obtained by adding to the two-dimensional relation the peak luminosity in the optical wavelength and it reads as

$$\log L_{O,a} = a_{Opt} \cdot \log T_{O,a}^* + b_{Opt} \cdot \log L_{peak} + C_{Opt}, \quad (4.2)$$

where a_{Opt} and b_{Opt} are negative and positive, respectively, as in Eq. (4.1). Figure 4.2 shows the 3D representation of this relation, once corrected for selection biases and

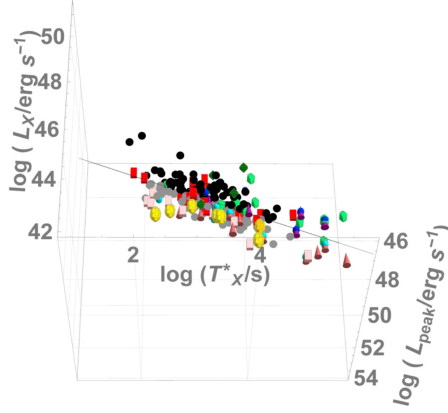


Figure 4.1: Three-dimensional X-ray Dainotti relation for the sample of 222 GRBs with the best-fit plane from [204]. Different symbols mark SNe-LGRBs (purple cones), XRFs (blue spheres), SGRBs (red cuboids), LGRBs (black circles), ULGRBs (green dodecahedrons), Kilonovae-SGRBs (yellow truncated icosahedrons), and GRBs with internal plateau (dark green diamonds). Darker colors indicate sources above the plane, while lighter colors the ones below the plane.

redshift evolution (see Chapter 5), together with the best-fit plane for the optical sample of 58 sources described in [206]. We here notice that the values on the z -axis (i.e. L_{peak} -axis) decrease from bottom to top both in Figure 4.1 and Figure 4.2. Furthermore, [207] updated the 3D GRB fundamental plane relation with a sample of GRBs showing plateaus in γ -rays.

From a physical point of view, the 3D X-ray fundamental plane relation is based on the 2D X-ray relation, which has been interpreted as due to accretion onto a black hole [208, 209, 210], magnetars [211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 187, 223, 224, 225, 226, 227], and within the framework of theories that take into account changes in the microphysical parameters and off-axis jets [228, 229, 230, 231, 232, 233]. Starting from the first one, the accretion model predicts that the GRB emission is generated when the collapsing GRB progenitor is pressed by the outer material of the accretion disc. Instead, the magnetar model is based on a GRB central engine that is a fast-rotating strongly-magnetized pulsar. The magnetar may be formed both in the core-collapse of massive stars and in the merger of NS or white dwarf binaries, or in the accretion-induced collapse of a white dwarf, being thus a plausible progenitor for LGRBs and SGRBs, respectively. In this framework, [215] analytically proved with 159 Swift GRBs that the magnetar model can naturally explain the 2D X-ray anti-correlation if the energy is injected from the central compact object in the forward shock, the shock pushed in the external medium. Indeed, in this scenario the luminosity and the end-time can be expressed as

$$\log L_{X,a} \sim \log(B_p^2 P_0^{-4} R^6) \quad (4.3)$$

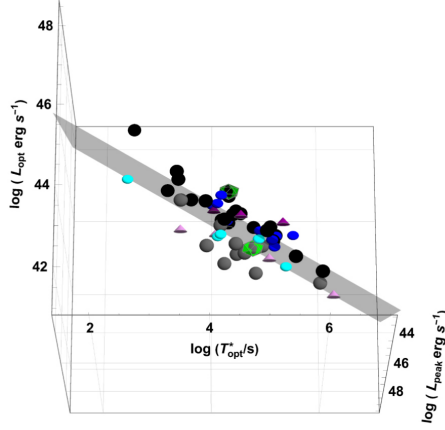


Figure 4.2: Three-dimensional optical Dainotti relation for the sample of 58 GRBs with the best-fit plane of [206]. Both the luminosities and time are corrected for selection effects and redshift evolution as detailed in Chapter 5. Different sub-classes are identified as follows: LGRBs with black circles, SGRBs with red cuboids, GRB-SNe Ic with purple cones, XRFs and XRRs with blue spheres, and ULGRBs with green icosahedrons.

and

$$\log T_{X,a}^* = \log(2.05 I B_p^{-2} P_0^2 R^{-6}), \quad (4.4)$$

where the quantities at play are: $L_{X,a}$ and $T_{X,a}^*$ in units of $10^{49} \text{ erg s}^{-1}$ and 10^3 s , respectively, B_p is the magnetic field strength at the poles in units of 10^{15} G , P_0 is the initial period of the NS in milliseconds, R is its radius in units of 10^6 cm , and I is the moment of inertia in units of 10^{45} g cm^2 . Combining these equations, we obtain $\log L_{X,a} \sim \log(10^{52} I^{-1} P_0^{-2}) - \log T_{X,a}^*$ that yields the anti-correlation $\log L_{X,a} \sim -\log T_{X,a}^*$. Furthermore, [215] attributed the scatter of the relation to the spread of the initial spin periods of the magnetar and also predicted a slope compatible with that intrinsic to the relation.

As anticipated, also other models have been proposed to explain the Dainotti 2D relation. In this framework, [228] studied the “supercritical pile” GRB model [see also 234]. Instead, [229] explained the plateau as a consequence of the synchrotron radiation in a thick shell condition. In addition, a model based on the photospheric emission from stratified jets has been proposed by [231], as well as models related to the microphysical parameters required by the energy-injection model [232] and to jets viewed off-axis [233].

In the last two decades, several efforts have been made to establish correlations for cosmological purposes, to understand their physical meaning and to ensure their reliability. In this regard, we here point out that [20] have shown that, after selection biases and redshift evolution have been taken into account (see Chapter 5), the 3D Dainotti X-ray relation has a reduced intrinsic scatter of 0.18 ± 0.07 . Hence, the 3D GRB fundamental plane represents one of the tightest correlations proposed so far for the standardization of GRBs, independently of the correction for selection biases and

redshift evolution. On the contrary, other correlations are significantly affected by these effects. More precisely, if the correction for selection effects and redshift evolution alters a correlation and it is not negligible, such a relation is not intrinsic and therefore not reliable. Furthermore, the trustworthiness of a correlation must rely on its physical interpretation, as in the case of the magnetar model for the Dainotti 3D correlation. Overall, the 3D X-ray fundamental plane relation represents so far the most complete and reliable GRB correlation developed for cosmological purposes. For this reason, it has been deeply investigated [202, 203, 223, 204, 235, 207, 236, 237] and extensively used in cosmological studies [238, 239, 240, 120, 20, 241, 242, 243, 244, 31, 245, 198].

4.3 The GRB cosmological platinum sample

As already emphasized, similar to the application of SNe Ia in cosmology, for which only a properly defined sub-sample of SNe light curves is considered [80], also in the case of GRBs we need to select a well-established cosmological sample based on specific and common physical features. In this regard, the most recent and refined selection performed to tighten the fundamental plane in X-rays and to identify a GRB sample suitable as a cosmological tool with the 3D X-ray Dainotti relation has led to the “Platinum sample”, first introduced in [204]. This is a refinement of the previous “gold” sample [202, 203] and is inspired by other sub-samples presented in the literature [e.g. 246, 146]. We summarize here the main criteria employed to define this sample [see e.g. 204, 20].

The selection starts with 372 GRBs with X-ray plateau afterglows observed from Swift between 2005 January and 2019 August that have a known redshift, spectroscopic or photometric, catalogued in the Swift+Burst Alert Telescope (BAT)+ X-Ray Telescope (XRT) repository [141]. Among these GRBs, only those with a reliable plateau and a spectroscopic redshift, and that can be fitted with the [145] model are retained, leading to a reduced sample size of 222 sources. As a second step, in order to identify a common mechanism, only LGRBs are considered. After that, five further criteria are required: 1) a direct determination of the end-time of the plateau from the data and not from a fit of the light curve, which means that the end-time must not fall in observational gaps, 2) at least five points at the beginning of the plateau emission, 3) a duration of the plateau phase of at least 500 s, 4) a plateau inclination of $< 41^\circ$, and 5) the absence of flares and bumps in the whole plateau [246]. In the fourth condition the angle of the plateau is obtained from trigonometry as $\Delta_F/\delta_T = F_i - F_{X,a}/T_{X,a} - T_i$, where the index i refers to the time of the onset of the plateau emission. The fulfillment of these requirements reduces the sample size of the platinum sample to 50 GRBs, covering a redshift range between $z = 0.055$ and $z = 5$. We note here that when selecting a sub-sample from an initial data set, it is crucial to verify that the final sample is still representative of the original one, otherwise we introduce biases or significant changes from a physical point of view. In this regard, [120] have verified this condition for the platinum sample.

The platinum sample obeys the 3D fundamental plane relation yielding $a = -0.86 \pm 0.13$, $b = 0.56 \pm 0.12$, $C_0 = 21.8 \pm 6.3$, and a dispersion $\sigma_{int} = 0.34 \pm 0.04$ [204]. A more recent analysis of the platinum sample taking into account selection effects and redshift evolution (see Chapter 5) performed by [20] led to the smallest possible intrinsic scatter

for this relation so far, which is $\sigma_{int} = 0.18 \pm 0.07$. The fact that the sample selection just described significantly reduces the dispersion, as in the case of QSOs (see Section 3.1), is a strong indication that the imposed criteria effectively remove “biased” objects that deviate from the relation due to observational problems and that may be powered by a physical mechanism other than that driving this relation.

4.4 Machine learning techniques for GRBs: light curve reconstruction to tighten the relations and redshift prediction to increase the sample sizes

In the previous sections, we have extensively discussed that only relations that are tight enough can actually be used reliably and effectively to standardize GRBs as cosmological tools. Therefore, one could investigate which are the main drivers for achieving this goal: one is to reduce the uncertainties in the GRB parameters by observations from satellites, such as the Space Variable Objects Monitor (SVOM, [247]) and its follow-up mission, the Transient High Energy Sources and Early Universe Surveyor (THESEUS, [248]), which, if approved, will be launched in 2032. Fortunately, we do not need to wait for a decade as we can actually use machine learning (ML) techniques to reconstruct the light curves of GRBs, thus achieving better precision on the parameters, and to infer the redshift of GRBs for which the redshift is unknown, thus increasing the sample size.

Indeed, for cosmological and theoretical studies, we necessarily need GRB correlations for which the physical variables involved are measured with high precision. This requires high-quality light curves with good coverage. Unfortunately, such light curves are often not available and we face the problem of the presence of gaps in the light curves that prevent a precise determination of the features. However, the development of ML techniques has paved the way to overcome this issue through the light curve reconstruction (LCR) approach. While LCR has already been applied to solve the problem of missing data in several astronomical domains [see e.g. 249, 250, 251, for the time delay in gravitational lensing, Cepheid light curves, and planetary eclipse mapping, respectively], its application to the GRB light curves proved to be more complicated due to the GRB heterogeneous properties. For this reason, the GRB LCR has only recently been developed. In this scenario, [252] have achieved relevant results by employing a stochastic methodology. Indeed, with the reconstructed light curves, they have obtained a significant reduction in the uncertainties of the time at the end of the plateau, the corresponding flux, and the temporal decay index after the plateau that ranges between 33% and 44%.

As anticipated, another factor that limits the employment of GRBs in cosmology is the paucity of GRBs with known redshift, which is only 11% [253]. To overcome this issue, ML algorithms can be employed to predict redshifts [254], as already tested for AGN [255, 256, 257] with reliable predictive results. Indeed, such an approach was recently proposed for GRBs in [258] for X-ray afterglows, in [259] in optical wavelengths, and in [260] for the prompt emission. In particular, [258] have used an ensemble-supervised ML model to predict the redshift of 154 GRBs, thus increasing by 94% the number of

LGRBs with plateaus and known redshift. The robustness of this method is confirmed by the fact that the measured and predicted redshift are strongly correlated, with a Pearson coefficient of 0.93, and a square root of the average squared error of 0.46 with the observed redshifts. Similarly, [259] have addressed the problem of unknown GRB redshift by applying a supervised ML model using observed optical afterglows. This work employed a methodology similar to the previous one, but employing optical variables, and they again obtained a significant correlation, with $r = 0.93$, between the observed and predicted redshifts. Following the same logic, a similar approach has been applied to the prediction of GRB redshifts by [260], but with prompt data only. More specifically, they found that Deep Neural Network with random forest models, which rely on non-linear correlations among parameters, can reliably infer unknown redshifts recovering the observed distribution of GRBs with measured redshift.

4.5 Future role of GRBs as standalone cosmological probes

Based on the above-described cutting-edge ML techniques of LCR and redshift prediction and on additional simulations, we have explored the future role of GRBs as standalone cosmological probes. In the following, we describe our methodology and results in both X-rays and optical. Starting with the X-ray analysis, we used the platinum GRB sample to simulate GRBs and determine how many GRBs are required as standalone probes to constrain Ω_M with closed contours. To this end, the physical quantities $T_{X,a}$, $L_{X,a}$, and L_{peak} of the Dainotti 3D X-ray relation and their corresponding errors are simulated by assuming Gaussian distributions resembling those observed. The minimum number of GRBs required to infer Ω_M with closed contours is found to be ~ 150 and the precision achieved for Ω_M is 0.473, still lower than that reached by SNe Ia. Therefore, we further extended this analysis by investigating the number of GRBs needed to achieve the same precision on Ω_M as the three most recent SNe Ia samples, namely those of [261], [262], and [80]. More precisely, [261] reached an uncertainty on Ω_M of 0.10 with 472 sources, [262] a precision of 0.042 with 740 SNe Ia, and [80] an error of 0.022 with 1048 SNe Ia.

For this computation, we studied two different situations: the one with the original errors on the physical quantities and the one with halved errors, in order to simulate a sample with better-quality measurements. The scenario with divided errors is actually viable as the error bars can be significantly reduced by the LCR (see Section 4.4). Considering the cases with undivided errors, 789, 2653, and more than 3000 are the numbers of GRBs needed to reach the precision of [261], [262], and [80], respectively. Instead, halving the errors, reduces these numbers to 357, 1452, and 2724, respectively. The aforementioned analysis is also repeated by starting not from the whole GRB sample, but only from the 10 GRBs with the lowest intrinsic dispersion (i.e. the sources closest to the fundamental plane) of the 3D Dainotti relation (this method is called “a priori” trim), that still define a plane. This study aims to further reduce the uncertainty in the inferred Ω_M . For the same reason, another trimming technique (called “a posteriori” trim) is also adopted, which optimizes the number of GRBs used as the basis for the simulations,

GRB Sample		Conley (2011)				Betoule (2014)			
		n = 1	n = 2	n = 1 (47.5% LCR)	n = 2 (47.5% LCR)	n = 1	n = 2	n = 1 (47.5% LCR)	n = 2 (47.5% LCR)
PLAT	# GRBs (Plateau)	789	357	374	169	2653	1452	1260	689
	# GRBs (Total)	16789	7596	7975	3608	56455	30898	26816	14676
	Year Achieved	2060	2044	2045	2037	2129	2085	2078	2056
PLAT + ML	# GRBs (Plateau)	8394	3798	3987	1804	28227	15449	13408	7338
	# GRBs (Total)	2045	2037	2038	2026	2080	2058	2054	2043
	Year Achieved	2045	2037	2038	2026	2080	2058	2054	2043
OPT	# GRBs (Plateau)	271	142	128	67	1031	284	489	134
	# GRBs (Total)	4582	2401	2176	1140	17435	4802	8281	2281
	Year Achieved	2046	2038	2038	2026	2093	2047	2060	2038
OPT + ML	# GRBs (Plateau)	2291	1200	1088	570	8717	2401	4140	1140
	# GRBs (Total)	2038	2027	2025	Now	2061	2038	2045	2026
	Year Achieved	2038	2027	2025	Now	2061	2038	2045	2026
PLATrim (10)	# GRBs (Plateau)	847	399	402	189	2705	1788	1284	849
	# GRBs (Total)	18024	8490	8561	4033	57562	38048	27342	18073
	Year Achieved	2062	2045	2046	2038	2131	2097	2078	2062
PLATrim (10) + ML	# GRBs (Plateau)	9012	4245	4280	2016	28781	19024	13671	9036
	# GRBs (Total)	2046	2038	2038	2027	2081	2064	2054	2046
	Year Achieved	2046	2038	2038	2027	2081	2064	2054	2046
OPTrim (10)	# GRBs (Plateau)	330	112	156	53	829	393	393	186
	# GRBs (Total)	5580	1894	2650	899	14019	6646	6659	3156
	Year Achieved	2050	2037	2039	2022	2081	2054	2054	2041
OPTrim (10) + ML	# GRBs (Plateau)	2790	947	1325	449	7009	3323	3329	1578
	# GRBs (Total)	2040	2023	2029	Now	2055	2042	2042	2032
	Year Achieved	2040	2023	2029	Now	2055	2042	2042	2032
PLATrim (20)	# GRBs (Plateau)	646	354	306	168	2699	1466	1282	696
	# GRBs (Total)	13746	7533	6529	3578	57434	31196	27281	14818
	Year Achieved	2055	2044	2045	2037	2131	2085	2078	2057
PLATrim (20) + ML	# GRBs (Plateau)	6873	3766	3264	1789	28717	15598	13640	7409
	# GRBs (Total)	2043	2037	2036	2025	2081	2058	2054	2044
	Year Achieved	2043	2037	2036	2025	2081	2058	2054	2044
OPTrim (25)	# GRBs (Plateau)	244	36	115	17	685	350	325	166
	# GRBs (Total)	4126	608	1959	289	11584	5918	5502	2811
	Year Achieved	2045	2018	2037	Now	2072	2051	2050	2040
OPTrim (25) + ML	# GRBs (Plateau)	2063	304	979	144	5792	2959	2751	1405
	# GRBs (Total)	2037	Now	2024	Now	2051	2040	2040	2030
	Year Achieved	2037	Now	2024	Now	2051	2040	2040	2030

Table 4.1: The first column shows the different samples used in our analysis. The three rows associated with each sample provide the number of GRBs needed that have plateau phases, the total number of GRBs needed once the chosen sample proportionality out of the total samples is considered, and lastly, the year in which that number of GRBs is achieved with observed data. These rows are repeated for samples with the errors undivided ($n=1$), the errors halved ($n=2$), the errors undivided considering LCR, and finally, the errors halved including LCR. The overarching columns three and four give these estimates considering the precision reached by [261] and [262], respectively. A sample “+ ML” implies that machine learning techniques are employed to double the initial sample size by redshift inference.

4.5. Future role of GRBs as standalone cosmological probes

GRB Sample		Scolnic (2018)			
		n = 1	n = 2	n = 1 (47.5% LCR)	n = 2 (47.5% LCR)
OPT	# GRBs (Plateau)	2718	1086	1291	515
	# GRBs (Total)	45964	18365	21833	8723
	Year Achieved	2197	2097	2140	2061
OPT + ML	# GRBs (Total)	22982	9182	10916	4361
	Year Achieved	2113	2063	2069	2046
OPTtrim (10)	# GRBs (Plateau)	2870	1513	1363	718
	# GRBs (Total)	48534	23054	12153	19840
	Year Achieved	2207	2123	2114	2074
OPTtrim (10) + ML	# GRBs (Total)	24267	12793	11527	6076
	Year Achieved	2118	2076	2072	2052
OPTtrim (25)	# GRBs (Plateau)	2104	822	999	390
	# GRBs (Total)	35580	13900	16900	6602
	Year Achieved	2159	2080	2091	2054
OPTtrim (25) + ML	# GRBs (Total)	17790	6950	8450	3301
	Year Achieved	2094	2055	2060	2042

Table 4.2: The first column shows all of the different optical GRB samples used. The three rows associated with each sample show the number of GRBs needed that have 1) plateau phases, 2) the total number of GRBs needed, and 3) the year in which that number of GRBs is achieved with observed data. These rows are repeated for samples with the errors undivided, the errors halved, the errors undivided considering LCR, and finally, the errors halved including LCR. These estimates are given considering the precision reached by [80]. A sample “+ ML” implies that machine learning techniques are employed to double the initial sample size by redshift inference.

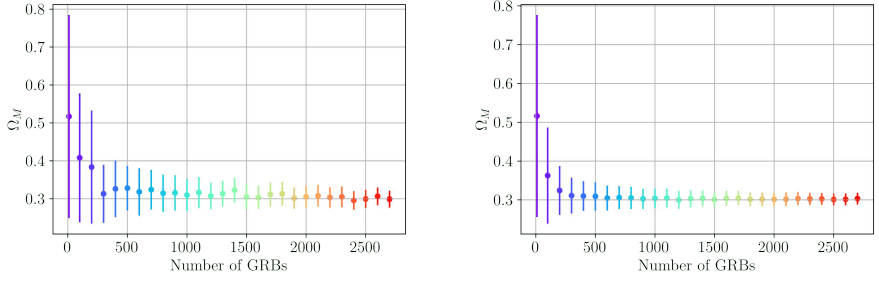
which is chosen after checking the corresponding cosmological results. This method provides an optimized number of 20 GRBs. By employing these trimming approaches and still considering the two cases with and without halved errors, the SNe Ia thresholds of [261], [262], and [80] are achieved with a much smaller number of GRBs compared to using of the full GRB sample. More precisely, these sensitivity limits are reached, respectively, with 847 (399), 2705 (1788), and 2839 (2649) GRBs, considering the a priori trim without (and with) halved errors, and with 646 (354), 2699 (1466), and more than 3000 (2719) GRBs, when the a posteriori trim without (and with) divided errors is applied.

In addition to this computation, we also calculated how many years it will take to obtain the required number of GRB observations based on both current and future surveys. This study focused on the campaigns of SVOM and THESEUS. Taking into account the specifics and the observational capabilities of these missions and the different cases with and without halved errors and the possible use of ML techniques to infer the unknown redshift and LCR (thus doubling the sample size), we computed the number of years we have to wait until GRBs alone can constrain Ω_M with the same precision as SNe Ia (see Tables 4.1 and 4.2). Finally, the best prospect is obtained when trimming a posteriori the GRB sample and considering halved errors, the application of ML to infer the redshift, and the use of LCR. Under these assumptions, the sensitivity of [261] is

already reached in 2025, while that of [262] is achieved in 2044.

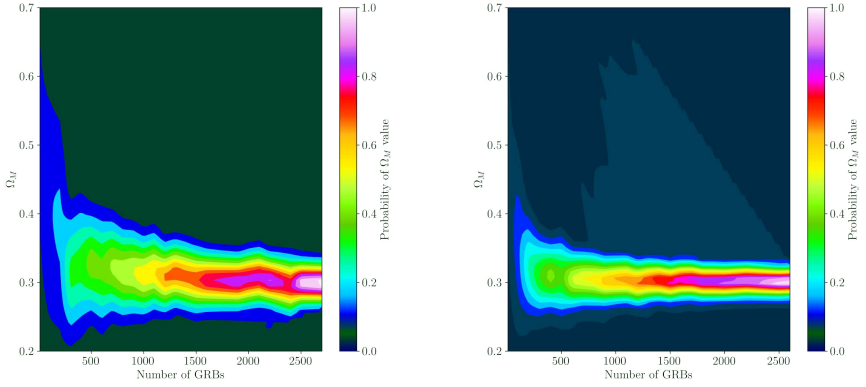
More encouraging results are actually obtained for the optical sample. Indeed, we performed the same study also for the optical GRB sample. The initial total optical sample consists of 45 GRBs, but, as for the X-ray case, we additionally defined a sub-sample of 10 sources with the lowest dispersion through the a priori trim and a sub-sample of 25 GRBs through the a posteriori trim. The methodology applied for the optical analysis is the same as that described above for the X-ray samples and the results are as follows. Simulating data with an assumed Gaussian distribution of the physical quantities, we obtained that we need 271, 1031, and 2718 GRBs to achieve the uncertainties on Ω_M of [261], [262], and [80], respectively, in the case of undivided errors. Instead, with the errors halved, these numbers are reduced to only 142, 284, and 1086. The results obtained with the full optical sample are shown in Figure 4.3 for both undivided (left column) and halved errors (right column). With respect to the a priori trimmed sample, it is possible to achieve the accuracy of [261], [262], and [80] with 330 (112), 829 (393), and 2870 (1513) GRBs, respectively, for the cases without (and with) halved errors. These numbers decrease to 244 (36), 685 (350), and 2104 (822), when the a posteriori trim is used without (and with) divided errors. Regarding how many years are needed to actually have such a GRB sample size available to achieve the SNe Ia precision, we predicted that the precision of [261] would have been reached already in 2022, in the case of the 25 trimmed sources with halved errors and ML techniques, while that of [262] will be achieved in 2026, for the best case of the full optical sample with the employment of halved errors, ML, and LCR. Finally, the current limit of [80] could be reached in 2042 if ML and LCR are applied. All these results are summarized in Tables 4.1 and 4.2.

Overall, this marks the importance of GRBs in the cosmological scenario of the next decades. It is also important to consider that additional analyses and measures can be effectively employed to improve the use of GRB cosmology and this definitely requires a certain sub-set of GRBs that obey a particular model, such as the magnetar or the accretion onto a black hole.



(a) OPTICAL | Simulation Results for the Full OPT Base with Undivided Errors

(b) OPTICAL | Simulation Results for the Full OPT Base with Halved Errors



(c) OPTICAL | Probability Map for Undivided Errors

(d) OPTICAL | Probability Map for Halved Errors

Figure 4.3: Results from the simulations with the optical GRB sample and the 3D fundamental plane relation. Upper panels: cosmological constraints on Ω_M obtained with different numbers of simulated GRBs in the optical sample with undivided (left panel) and halved errors (right panel). Lower panels: the probability density colour map for Ω_M as a function of the number of GRBs by considering real (left panel) and halved errors (right panel).

5 Selection biases and redshift evolution

This chapter is devoted to the problems of selection biases and redshift evolution that arise at high redshifts and thus could affect the cosmological relations used for QSOs and GRBs. In Section 5.1 we introduce the Efron & Petrosian statistical method to correct for these effects and we present a technique to overcome the circularity problem embedded in this method, then in Section 5.2 we describe the application of these approaches in the QSO domain. Still regarding QSOs, in Section 5.3 we validate the RL relation using the results of the Efron & Petrosian method and show that this cosmological relation is intrinsic to the QSO physics and is not induced or biased by the effects of selection and redshift evolution. Finally, in Section 5.4 we report the results of the Efron & Petrosian method for GRBs. The results presented in this chapter are based on our works of [263] and [15].

5.1 The Efron & Petrosian method

High-redshift sources, such as QSOs and GRBs, play an invaluable role in the study of the evolution of the Universe allowing us to explore early epochs, almost uncovered by other probes. Nevertheless, at high redshifts we need to take into account for possible selection biases and evolutionary effects, which in principle can distort or even induce a correlation between physical quantities of a source, leading to an incorrect determination of the cosmological parameters [264]. To correct for these effects, we can use the Efron & Petrosian (EP) statistical method [265].

In this approach, the physical quantities of interest, luminosities for both QSOs and GRBs and also time only in the case of GRBs, evolve with redshift as $L' = \frac{L}{(1+z)^k}$ and $T' = \frac{T}{(1+z)^k}$, where L and T are the observed quantities, L' and T' are the corresponding corrected quantities, and k is the evolutionary parameter. As proved in [266], [267], and in our recent work , the results of this method are not affected by the choice of the power-law function, which could also be replaced by more complex functions of the redshift (see Section 5.2). To determine the value of k that removes the dependence on

the redshift, the Kendall's τ statistic is applied, where the coefficient τ is defined as

$$\tau = \frac{\sum_i (\mathcal{R}_i - \mathcal{E}_i)}{\sqrt{\sum_i \mathcal{V}_i}}. \quad (5.1)$$

Here, the index i refers to all the sources that at redshift z_i have a luminosity greater than the lowest luminosity ($L_{min,i}$) that can be observed at that redshift. This minimum luminosity is computed by choosing a limiting flux. The assumed value must guarantee that the sample retained is at least 90% of the total one and that it resembles the overall distribution as closely as possible, a requirement that can be checked by applying the Kolmogorov-Smirnov test [264, 268, 269, 201, 263, 120]. The rank $\mathcal{R}_i$ in Eq. (5.1) corresponds to the number of data points in the associated set of the i -source, where the associated set is defined by all j -points that satisfy $z_j \leq z_i$ and $L_{z_j} \geq L_{min,i}$. $\mathcal{E}_i = \frac{1}{2}(i + 1)$ and $\mathcal{V}_i = \frac{1}{12}(i^2 + 1)$ are the expectation value and variance, respectively, in the case of the absence of correlation. Therefore, the redshift dependence is removed when $\tau = 0$, and this condition provides us the value of k that eliminates the correlation. When $|\tau| > n$ the hypothesis of no correlation is rejected at $n\sigma$ level, so we obtain the 1σ uncertainty on the k value by requiring $|\tau| \leq 1$. The uncovered dependence on the redshift is then used to derive the de-evolved L' for the total sample. This procedure can be straightforward applied also to the time variable for GRBs by replacing the luminosity with the time.

The de-evolved quantities computed with the obtained k values are thus used in cosmological fits when accounting for a “fixed” evolution. The notation “fixed” refers to the fact that k is determined under the assumption of a specific cosmological model, which is required to compute the luminosities L from the measured fluxes. Usually, the fiducial model is a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. As a consequence, this method suffers from the “circularity problem”: the a-priori assumption of a cosmological model affects the constraints on the cosmological parameters that are inferred by fitting the luminosities computed under this assumption. To overcome this issue, [20] (for GRBs), and our works (for QSOs) have investigated the behaviour of k as a function of cosmology. Specifically, in these works, k is not evaluated for fixed cosmological parameters, but over a set of several values of the cosmological parameters (i.e. Ω_M , H_0 , as well as other parameters for extensions of the flat Λ CDM model), leading to the functions $k(\Omega_M)$ and $k(H_0)$. These analyses show that k does not depend on H_0 , while it does depend on Ω_M . Therefore, the function $k(\Omega_M)$ can be applied in the cosmological fits to vary k together with the cosmological parameters, without the need to fix a cosmology. This method completely solves the circularity problem and we call it “varying” evolution, while we use the notation “fixed evolution” when the cosmological parameters employed to determine the redshift evolution are fixed a-priori.

5.2 Results of the Efron & Petrosian method and the overcome of the circularity problem for quasars

Regarding the correction for selection biases and redshift evolution in the QSO realm, we have computed the values of the evolutionary parameters k for L_{UV} and L_X . In this case we have replaced the simple power-law evolution $(1+z)^k$ with the more complex function $g(z) = (Z^k z_{crit}^k) / (Z^k + z_{crit}^k)$ presented in [266], where $Z = 1+z$, which allows for a more rapid evolution up to redshift z_{crit} than the less rapid one at higher redshifts. This is a good fit for a QSO data set based on the SDSS with many QSOs at $z > 3$ [270, 271, 272]. In particular, we have used this form with a fiducial critical redshift $z_{crit} = 3.7$, which is the most suitable value given the high redshift distribution of QSOs determined in [270], but we have checked our results against the specific functional form allowing also for a simple power-law.

Specifically, assuming a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, we obtained $k_{L_{UV}} = 4.36 \pm 0.08$ and $k_{L_X} = 3.36 \pm 0.07$ using the flux limit of $4.5 \cdot 10^{-29} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$ for the UV and $6 \cdot 10^{-33} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$ for the X-rays, which respectively guarantee retained samples of 2362 (97.6% of the total 2421 sources) and 2379 (98.3% of the total 2421 sources) QSOs. We also verified through the Kolmogorov-Smirnov test that the full and the cut samples in both X-rays and UV come from the same parent population. Indeed, the probability of the null-hypothesis that the two samples are drawn by the same distribution cannot be rejected with a p-value $p = 0.47$ for the UV and $p = 0.85$ for X-rays. The luminosity limiting values for L_{UV} and L_X corresponding to the mentioned values of F_{lim} are shown with a black continuous line in the left and right panels of Figure 5.1, respectively, over the whole set of data points represented by red circles. Instead, the trend of $\tau(k)$ is shown in the left and right panels of Figure 5.2 for the UV and X-rays, respectively. As already explained, $\tau = 0$ and $|\tau| \leq 1$ provide us with the best-fit value and the associated 1σ error for the evolutionary

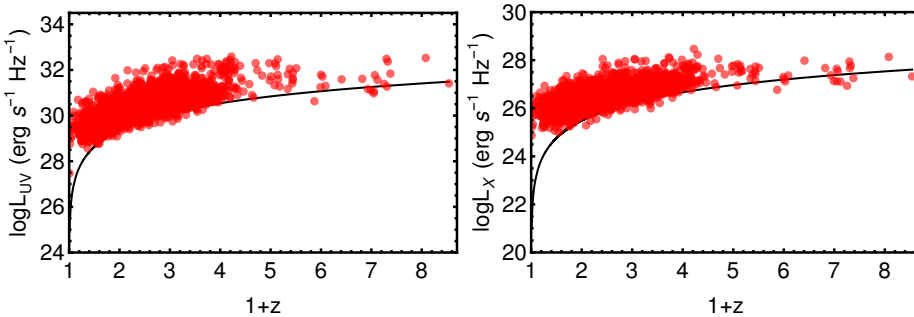


Figure 5.1: Redshift evolution of L_{UV} (left panel) and L_X (right panel) in units of $\text{erg s}^{-1} \text{ Hz}^{-1}$ for the whole QSO sample (red circles). The black line in both panels shows the limiting luminosity chosen according to the prescription described in Section 5.1.

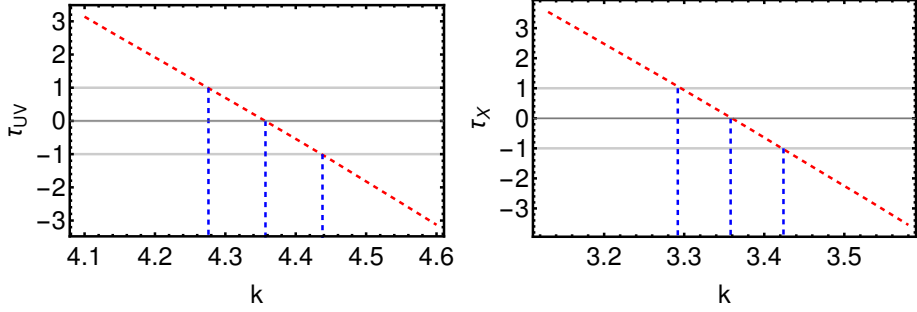


Figure 5.2: $\tau(k)$ function (dashed red line) for both the UV (left panel) and X-ray (right panel) analyses. The point $\tau = 0$ (central gray line) gives us the k parameter for the redshift evolution of L_{UV} and L_X (central dashed blue line), while $|\tau| \leq 1$ (external gray lines) the 1σ uncertainty on it (external dashed blue lines).

parameter k .

Furthermore, we performed an additional test to legitimise our choice for F_{lim} and prove that the evolutionary coefficients depend only weakly on this choice. In both X-ray and UV bands, we computed the evolutionary coefficient k for different limiting values F_{lim} . Specifically, we started with a value of F_{lim} that preserves the original sample and then analyzed a range of values for F_{lim} . If we span within 0.5 orders of magnitude in the UV starting from $F_{lim,UV} = 1.7 \times 10^{-29} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$ and in X-rays starting from $F_{lim,X-rays} = 1.5 \times 10^{-33} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$, we obtain a compatibility within 1σ . Even if we span over one order of magnitude starting from the same F_{lim} values in both UV and X-rays, the evolutionary coefficient remains compatible within 2σ . This analysis proves that the results for the evolutionary coefficients do not depend on the specific choice of F_{lim} for a wide range of their values.

Inserting our values of k in $g(z)$, we then computed the new de-evolved luminosities and the associated uncertainties for the whole original QSO sample. The comparison between these quantities and the original ones is shown in Figure 5.3 in the $(\log L_{UV}, \log L_X)$ plane. Compared to the original quantities, the computed luminosities span a smaller region of the plane and show a slightly larger dispersion ($\delta \sim 0.22$ versus $\delta \sim 0.21$). This fact is expected because the $g(z)$ function, once the best-fit values for k are used, yields a larger correction (i.e. lower de-evolved values) for higher luminosities. In addition, we have accounted for the error in the determination of k by propagating the errors in the $g(z)$ function. This naturally increases the associated uncertainties on the luminosities. Therefore, the correction for $g(z)$ affects the spread of the luminosities and hence the dispersion of the correlation, which is consequently larger.

About the above-mentioned circularity problem intrinsic to the EP method, we have also examined the evolution of $k_{L_{UV}}$ and k_{L_X} as a function of Ω_M , while, as anticipated, there is no dependence on H_0 . These trends are here presented in Figure 5.4, where the 1 , 2 , and 3σ ranges are shown in red, orange, and green, respectively, and the black line marks the value of k for $\Omega_M = 0.3$. This plot shows that the k values increase as

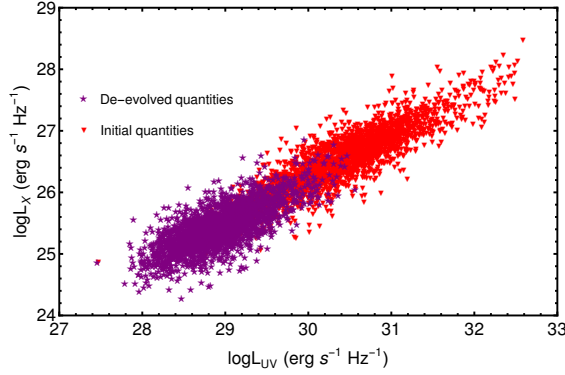


Figure 5.3: Comparison between original (red) and de-evolved (purple) quantities in the $(\log L_{UV}, \log L_X)$ plane.

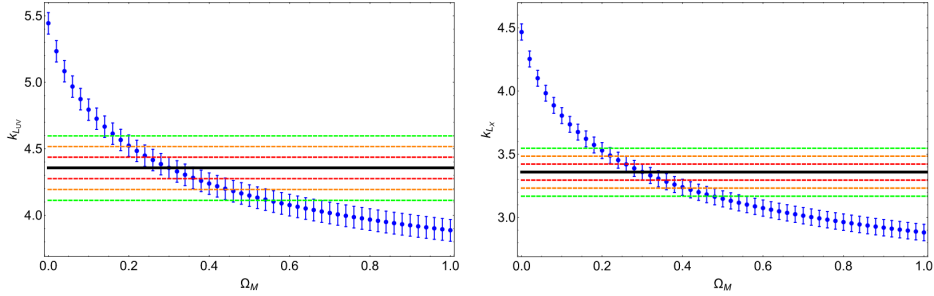


Figure 5.4: The dependence of the evolutionary parameter k for the L_{UV} (left panel) and L_X (right panel) luminosities on Ω_M . The 1, 2, and 3 σ ranges of the values of k are shown in red, orange, and green, respectively. The black line marks the value of k for $\Omega_M = 0.3$ indicated in the text.

Ω_M decreases. Besides this evolution, $k_{L_{UV}}$ and k_{L_X} are compatible within 1 σ with the corresponding values obtained by assuming $\Omega_M = 0.3$ for values of Ω_M between 0.20 and 0.45, for the case of $k_{L_{UV}}$, and between 0.22 and 0.41, for the case of k_{L_X} . Hence, if we consider a range of values of Ω_M based on the latest constraints from SNe Ia [80, 273], the evolution of $k_{L_{UV}}$ and k_{L_X} becomes negligible.

To further apply this model-independent varying evolution procedure to cosmological models more complex than the standard one, we need to derive the dependence of the parameter k in three dimensions, as a function not only of Ω_M but also of couples of free parameters, which are included in the cosmological model under study. To this end, we examined the three-dimensional evolution of $k_{L_{UV}}$ and k_{L_X} in the cosmological parameter spaces of (Ω_M, w) and (Ω_M, Ω_k) for a flat w CDM and a non-flat Λ CDM model, respectively, thus deriving the functions $k_{L_{UV}}(\Omega_M, w)$, $k_{L_X}(\Omega_M, w)$, $k_{L_{UV}}(\Omega_M, \Omega_k)$, and $k_{L_X}(\Omega_M, \Omega_k)$. This implementation allows us to apply the varying

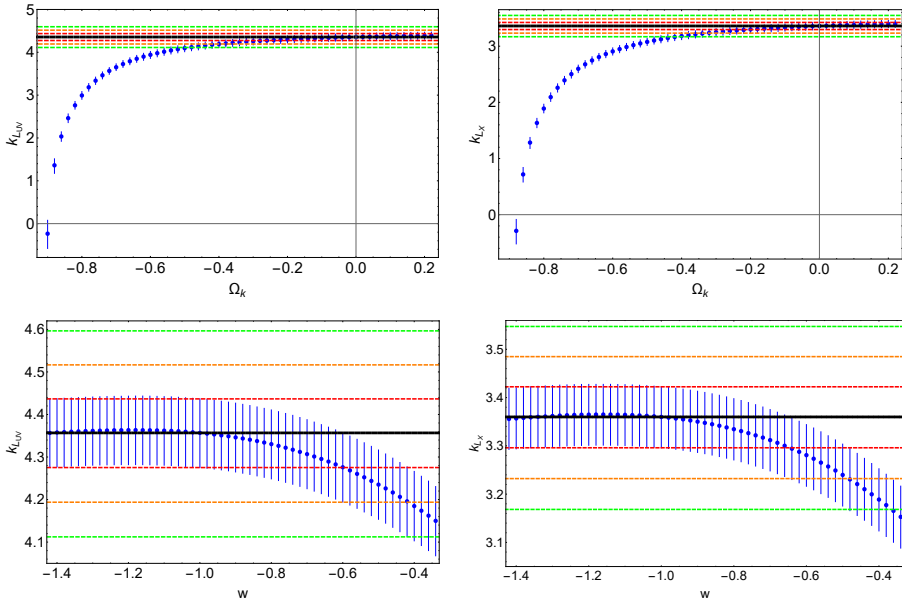


Figure 5.5: Behaviour of the k parameter in UV (left panels) and X-rays (right panels) as a function of Ω_k for a non-flat Λ CDM model with $\Omega_M = 0.3$ (upper panels) and as a function of w for a flat w CDM model with $\Omega_M = 0.3$. We mark with a thick black line the values of k_{LX} and k_{LUV} computed for $\Omega_M = 0.3$, $\Omega_k = 0$, and $w = -1$ together with 1, 2, and 3 σ error bars marked with red, orange, and green dashed lines, respectively.

evolution technique even when both Ω_M and w , in a flat w CDM model, and Ω_M and Ω_k , in a non-flat Λ CDM, are simultaneously free to vary in the cosmological fit. This method can be extended to any cosmological model of interest once the functions linking k to the cosmological parameters have been derived. In terms of the bi-dimensional functions $k_{LUV}(w)$, $k_{LX}(w)$, $k_{LUV}(\Omega_k)$, and $k_{LX}(\Omega_k)$, we show in Figure 5.5 that the k values increase for Ω_k that increases and for w that tends towards less negative values. However, this evolutionary trend is completely negligible for the dependence on w since both k_{LUV} and k_{LX} are compatible within 3 σ from the value obtained from a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ over the whole interval of w values between -1.4 and -0.33. Similarly, the dependence of k on Ω_k is not significant when considering reasonable values of Ω_k . Indeed, k_{LUV} and k_{LX} deviate at more than 3 σ from the values corresponding to $\Omega_k = 0$ only for $\Omega_k \leq -0.7$, thus only for non-physical values of Ω_k .

These results on the evolution of k with the cosmological parameters are also confirmed by the study of the three-dimensional functions, here shown in Figure 5.6. From this figure, we can notice that both k_{LUV} and k_{LX} follow very similar trends. On the one hand, the k values are compatible with the value $k_{LUV} = 4.36 \pm 0.08$ and $k_{LX} = 3.36 \pm 0.07$, obtained by assuming a flat Λ CDM model with $\Omega_M = 0.3$, when

physical regions of the parameter spaces of (Ω_M, w) and (Ω_M, Ω_k) are explored. On the other hand, strong evolution and modifications of the k parameters are observed in exotic regions associated with extreme, non-physical, values of Ω_M , w , and Ω_k , such as for very small Ω_M and large negative values of w . As visible from the plots, we discard a part of the (Ω_M, Ω_k) space. This is due to the fact that the rapid decrease of the k values near this region affects the precision of the interpolation method that we use to compute the function $k(\Omega_M, \Omega_k)$. Nevertheless, we also point out that this discarded region does not impact our analysis since our cosmological results do not fall into this cut-off non-physical region.

In our approach, we also computed the error in the k value as a function of cosmology. Within reasonable ranges, we always observe very small variations of the error with cosmology. As visible in the upper row of Figure 5.5, the error in k for $\Omega_k < -0.8$ is about twice the size of those for $\Omega_k = 0$, but for values of $\Omega_k > -0.55$ the errors are less than 8% bigger than those for $\Omega_k = 0$ for both $k_{L_X}(\Omega_k)$ and $k_{L_{UV}}(\Omega_k)$. Concerning the error $\Delta_k(w)$, we do not observe variations in the errors larger than 4% over the whole range considered for both $k_{L_X}(w)$ and $k_{L_{UV}}(w)$ (see the bottom row of Figure 5.5). For $\Delta_k(\Omega_M, \Omega_k)$, we always observe variations of less than 7% for each computed point both for $k_{L_X}(\Omega_M, \Omega_k)$ and $k_{L_{UV}}(\Omega_M, \Omega_k)$. The same result is obtained for the errors $\Delta_k(\Omega_M, w)$, for both $k_{L_X}(\Omega_M, w)$ and $k_{L_{UV}}(\Omega_M, w)$. The code and a comprehensive technical description of this method can be accessed from the Wolfram Mathematica Notebook Archive [274], which we have made publicly available.

5.3 Validation of the Risaliti-Lusso relation

Once we had overcome the effects of selection biases and redshift evolution, we have tested whether the RL relation still holds between the de-evolved luminosities we have computed. In this regard, we obtained the fitting parameters through a Bayesian technique, the D’Agostini method [275], to check whether the new de-evolved parameters of the normalization, β' , and the slope, γ' , are consistent within 1σ with the original uncorrected parameters, β and γ . We additionally used the Python package *emcee* [276], which is a pure-Python implementation of Goodman & Weare’s affine invariant Monte Carlo Markov Chain ensemble sampler, to further verify our results. These methods have the advantage of accounting for both the error bars on x - and y -axes and also for the presence of the intrinsic dispersion δ . These approaches, or similar ones, are indeed the most appropriate ones since in our case we expect an intrinsic scatter of the RL relation and the error bars on both variables are not negligible. These two fitting techniques give completely consistent results within 1σ . Assuming a linear model of the form $\log L'_X = \gamma' \log L'_{UV} + \beta'$ with intrinsic dispersion δ , the resulting best-fit values for the free parameters and their associated 1σ uncertainties from the D’Agostini fit method are $\gamma' = 0.582 \pm 0.011$, $\beta' = 8.6 \pm 0.3$, and $\delta = 0.223 \pm 0.003$. Remarkably, the evolutionary parameters derived through the EP method with both the simple power-law and the more complex function (see Sections 5.1 and 5.2) lead to values of γ' , β' , and δ parameters that are consistent within 1σ . We have therefore shown that the relation is reliable with respect to the specific choice of $g(z)$ in the EP method.

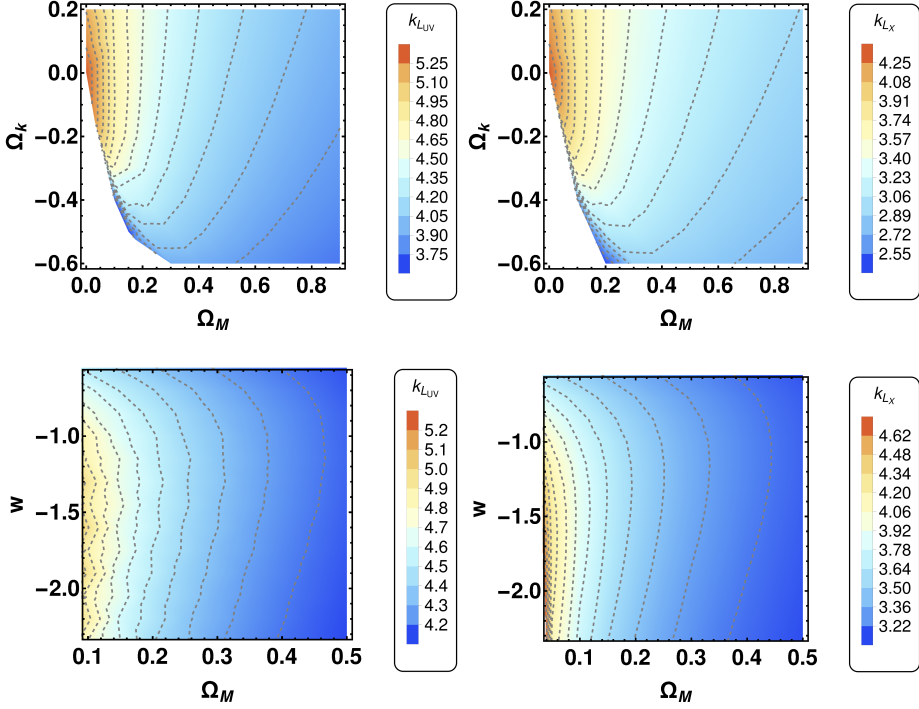


Figure 5.6: Colour-coded behaviour of the k parameter in the parameter spaces (Ω_M, Ω_k) (upper panels) and (Ω_M, w) (lower panels) for both UV (left panels) and X-rays (right panels).

The best-fit values of γ' and δ completely agree with those we obtained with a different and independent approach, where the de-evolved observables are not taken into account. Indeed, as previously detailed in Section 2.2.2, we fitted the $F_X - F_{UV}$ relation in redshift bins so narrow that the spread in luminosity distance is negligible compared to the intrinsic dispersion of the relation and, thus, the fluxes can be considered as proxies for the luminosities. The slope and dispersion determined with this procedure are respectively $\gamma = 0.586 \pm 0.061$ and $\delta = 0.21 \pm 0.06$ (see Figure 2.3). In our previous analysis, the fitted intercept was not β itself, but a combination of β , γ , and the luminosity distance, so we cannot make a direct comparison. However, the value of β' obtained after the correction for the evolution is in agreement with that expected from the literature (i.e. $\beta \sim 8.5$) and the most recent works on QSOs as standardizable candles [see e.g. 28, 85, 100]. Given our results, we have demonstrated that the correlation is intrinsic to the physics of QSOs and not an artifact of selection biases and/or redshift evolution and that it can be used to turn QSOs into reliable cosmological probes.

5.4 Results of the Efron & Petrosian method and the overcome of the circularity problem for Gamma-Ray Bursts

The EP method has also been applied to the GRB field in [20] and in our work. Specifically, we have computed the evolutionary parameters with respect to the 3D fundamental plane for a sample of 222 GRBs. Assuming as limiting fluxes $1.54 \cdot 10^{-8} \text{ erg s}^{-1} \text{ cm}^{-2}$ for the peak phase and $1.5 \cdot 10^{-12} \text{ erg s}^{-1} \text{ cm}^{-2}$ for the plateau phase and the limiting time of 405 s, we obtained $k_{L_{peak}} = 2.24 \pm 0.30$, $k_{T_{X,a}}^* = -1.25^{+0.28}_{-0.27}$, and $k_{L_{X,a}} = 2.42^{+0.41}_{-0.74}$. We also derived the corresponding values for the Platinum sample of 50 GRBs, as shown in Figure 5.7, which are $k_{L_{peak}} = 1.37^{+0.83}_{-0.93}$, $k_{T_{X,a}} = -0.68^{+0.54}_{-0.82}$, and $k_{L_{X,a}} = 0.44^{+1.37}_{-1.76}$ by assuming a limiting flux for the plateau of $4 \cdot 10^{-12} \text{ erg s}^{-1} \text{ cm}^{-2}$, a limiting peak flux of $3 \cdot 10^{-8} \text{ erg s}^{-1} \text{ cm}^{-2}$, and a limiting time of 750 s. These derived values are compatible within 1σ with those obtained for the 222 GRBs, but their associated uncertainties are larger due to the reduced sample size. In addition, we also computed the evolutionary parameters in relation to the 3D optical correlation for 45 GRBs. In this case, we assumed as limiting fluxes $1.40 \cdot 10^{-12} \text{ erg s}^{-1} \text{ cm}^{-2}$ for L_{peak} and $1.5 \cdot 10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2}$ for $L_{O,a}$, and a limiting time of 226 s. The obtained parameters are $k_{L_{peak}} = 3.10 \pm 1.60$, $k_{L_{O,a}} = 3.96 \pm 0.43$, and $k_{T_{O,a}} = -2.11 \pm 0.49$.

As in the case of QSOs (Section 5.2), to overcome the circularity problem of the EP method, [20] investigated for the first time in the GRB realm the behaviour of the k evolutionary parameters of the GRB quantities in the 3D fundamental plane as a function of cosmology. They showed that $k_{L_{peak}}$ and $k_{L_{X,a}}$ do not depend on H_0 , but they do evolve with the cosmological parameters of Ω_M , w , and Ω_k . These evolving trends are displayed in Figure 5.8 for the sample of 222 GRBs. The upper, middle, and lower panels show both $k_{L_{X,a}}$ (left-hand side of each panel) and $k_{L_{peak}}$ (right-hand side of each panel) as a function of Ω_M , w , and Ω_k , respectively. The black error bars on the points are the 1σ uncertainties. The values of $k_{L_{X,a}}$ and $k_{L_{peak}}$ corresponding to the case of a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ are marked with the continuous horizontal purple line and their corresponding 1, 2, and 3 σ uncertainties are displayed with the dashed purple, magenta, and pink lines, respectively. Looking at these plots, we can observe that both $k_{L_{X,a}}$ and $k_{L_{peak}}$ manifest a trend with Ω_M , with k values increasing as Ω_M decreases. However, taking into account the 1σ error bars, this evolution appears to be minimal. Indeed, the values of $k_{L_{X,a}}$ are always within 1σ from the value obtained in the fiducial cosmology of a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Similarly, the values of $k_{L_{peak}}$ are always within 1σ from the value obtained in a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for values of $\Omega_M \geq 0.1$. Regarding the dependence of $k_{L_{X,a}}$ and $k_{L_{peak}}$ on w and Ω_k , the evolutionary trend is completely negligible since the values of the two k parameters are within 1σ from the values computed in the fiducial cosmology over the whole range investigated for w and Ω_k .

In [20] the evolution of k values with the cosmological parameters observed in Figure 5.8 has been interpolated to derive the functions $k_{L_{X,a}}(\Omega_M)$, $k_{L_{X,a}}(w)$, $k_{L_{X,a}}(\Omega_k)$ (first

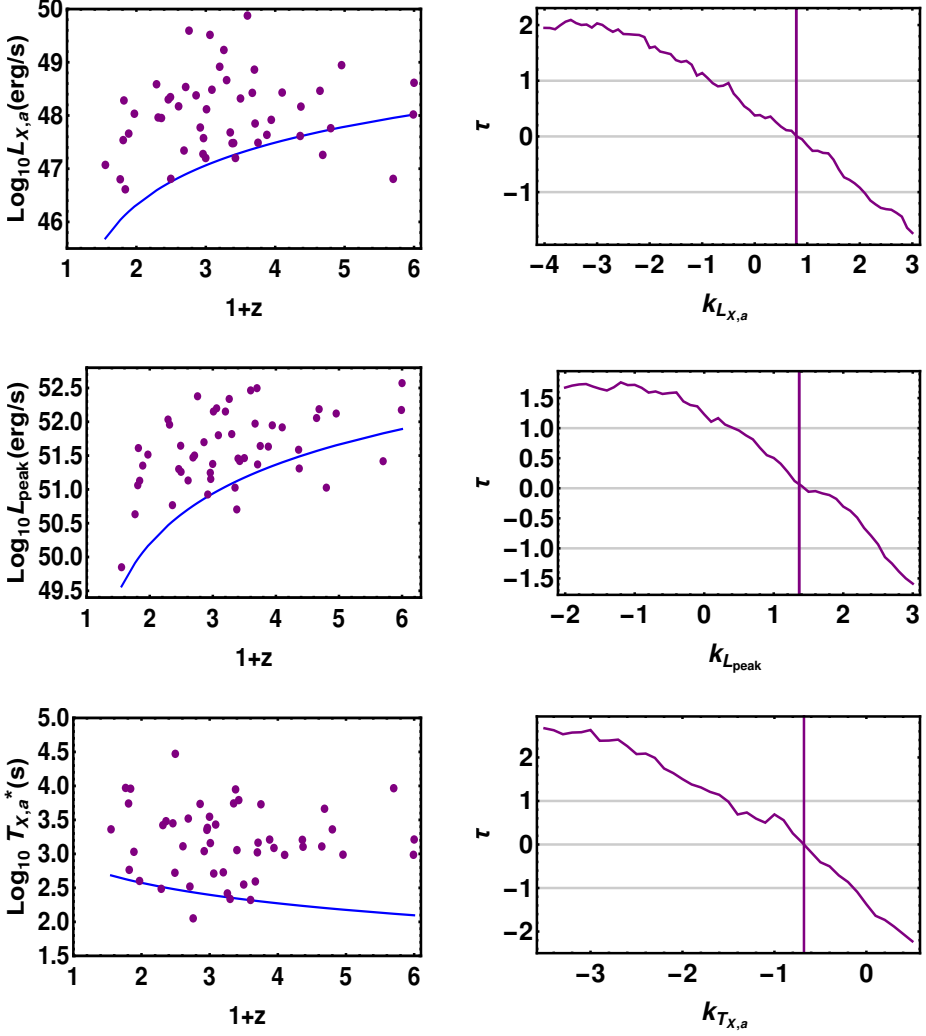


Figure 5.7: The application of the EP method to the GRB platinum sample for the parameters involved in the 3D fundamental plane relation. The left panels show the distribution of the parameters in $(1+z)$ with the limiting lines in blue, while the right panels show the determination of the τ and k parameters. The vertical purple solid lines mark the value of k corresponding to $\tau = 0$, for which the redshift evolution is removed, and the gray horizontal lines provide $\tau = 0$ and its 1σ uncertainty.

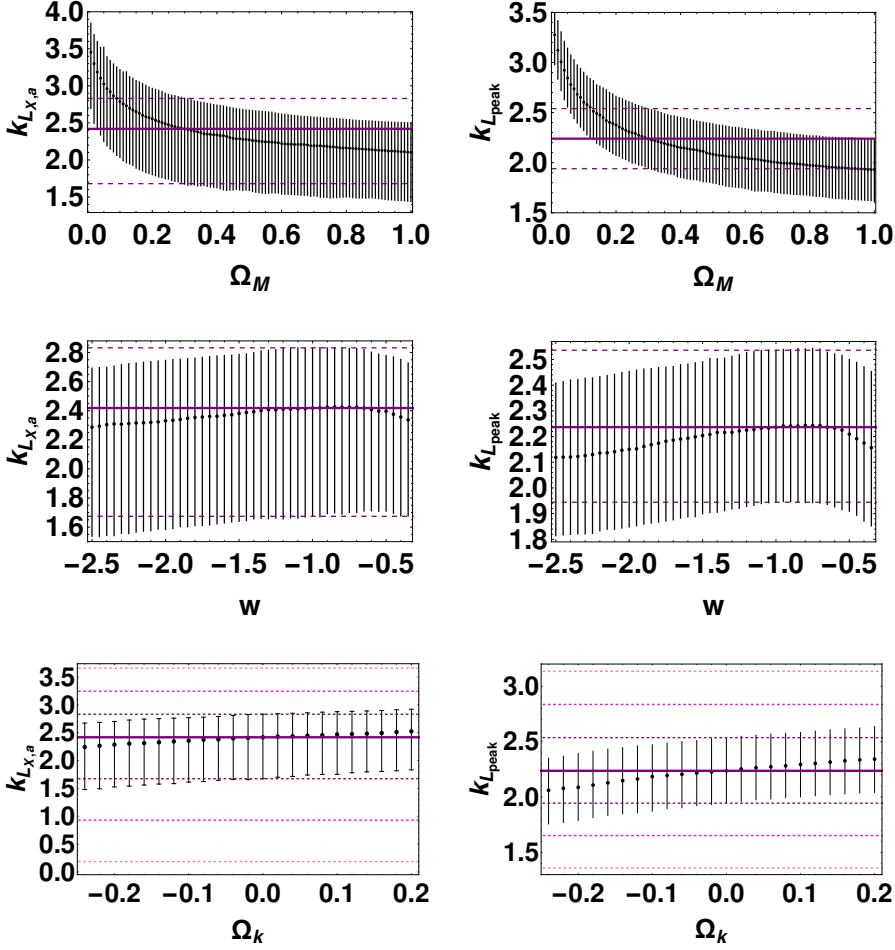


Figure 5.8: Upper panels: $k_{L,X,a}$ (left panel) and $k_{L,peak}$ (right panel) as a function of Ω_M . Middle panels: same as upper panel but as a function of the w parameter. Lower panels: same as above but as a function of Ω_k . In all panels the error bars are computed as the 1σ uncertainty on the points, the continuous purple line marks the values of $k_{L,X,a}$ and $k_{L,peak}$ that remove the redshift evolution in the EP method when assuming a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and the dashed purple, magenta, and pink lines show, respectively, the 1 , 2 , and 3σ uncertainties on this value. These plots are produced for the sample of 222 GRBs.

column of Figure 5.8 starting from the upper panel), $k_{L_{peak}}(\Omega_M)$, $k_{L_{peak}}(w)$, and $k_{L_{peak}}(\Omega_k)$ (second column of the same figure starting from the upper panel) which can then be used to apply the varying evolution method to fit the flat Λ CDM and w CDM models and the non-flat Λ CDM model. We have then further improved this procedure extending the study of the trend between the evolutionary parameters k and the cosmological parameters from a two-dimensional to a three-dimensional analysis, similar to the case of QSOs (Section 5.2).

We here also notice that the parameters of the 3D Dainotti relation not corrected for selection biases and redshift evolution and those obtained by applying the correction are consistent within 0.3σ . Therefore, as for the RL relation, this relation is intrinsic to the GRB physics and is not induced or biased by the effects described above, which establishes this relation as a reliable cosmological tool. Furthermore, the correction for selection biases and redshift evolution strongly reduces the intrinsic dispersion of the relation by a factor of 37%, from 0.34 to 0.18 [20], thus significantly tightening this correlation.

6 The hunt for a quasar sub-sample to be used as a standalone cosmological probe

As noticed so far, the dispersion of our entire cosmological QSO sample, although significantly reduced by a careful selection and removal of biased objects (see Section 3.1) and largely accounted for by factors not tied to the mechanism behind the X-UV relation (see Section 2.2.4), it is still considerable, with a value of $\delta \sim 0.2$ dex. This limits the use of QSOs as standalone cosmological probes, requiring the combination with other probes to improve the precision of the derived cosmological parameters. In this scenario, we here describe some recent efforts to obtain a QSO sub-sample that can be used as a standalone cosmological probe with enhanced power in constraining cosmological parameters. These analyses relies on different methodologies that select a QSO sub-sample consisting only of sources that are closer to the best-fit RL relation, and thus show a reduced dispersion. As a consequence, the smaller dispersion of the identified sub-set, if balanced with a sufficient number of remaining sources, allows us to constrain cosmological parameters with reduced uncertainties. In the following, we detail the two techniques we have employed and the results we have presented in [277] and [278], in Sections 6.1 and 6.2 respectively.

6.1 A quasar sub-sample from the σ -clipping selection procedure

The first method employed to trim the QSO sample is the σ -clipping technique. However, before applying this procedure, we have searched for possible outliers by checking the ratio between the luminosities and their associated error. More precisely, we have computed the maximum value of $\Delta \log L'_{UV} / \log L'_{UV}$ and $\Delta \log L'_X / \log L'_X$, where L' denotes the luminosities corrected for selection biases and redshift evolution with a fixed correction, as detailed in Chapter 5. Indeed, points with large uncertainties on luminosities would not be removed by the σ -clipping and would have a higher weight in the cosmological fits compared to points with smaller uncertainties. This check revealed that our total sample

of 2421 sources does not contain any such outliers. Irrespective of the Ω_M value assumed for the fixed correction, the maximum values of $\Delta \log L'_{UV}/\log L'_{UV}$ and $\Delta \log L'_X/\log L'_X$ are 0.022 and 0.013, respectively, with only one source with $\Delta \log L'_{UV}/\log L'_{UV} > 0.02$ and four sources with $\Delta \log L'_{UV}/\log L'_{UV} > 0.01$. As these values are lower than $\sim 1-2\%$, we do not remove any QSO from the initial sample. We therefore apply the σ -clipping selection to all the 2421 sources in our sample.

The σ -clipping allows us to reduce the intrinsic scatter of the RL relation by removing the sources with a vertical distance from the best-fit relation greater than a chosen threshold value. It is used when dealing with relations presenting an intrinsic dispersion in order to remove possible outliers in the sample and ensure a better determination of the free parameters of the relation. This procedure has already been successfully applied to QSOs to constrain cosmological parameters [85, 29, 279]. Here we detail this method for our case. First, we fit the RL relation with the Bayesian D'Agostini method [275] using the entire QSO sample and assuming a flat Λ CDM model with $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and three different choices for Ω_M , namely $\Omega_M = 0.3$, $\Omega_M = 0.1$, and $\Omega_M = 1$, to compute the X-ray and UV luminosities. From this fit, we obtain the best-fit values of γ , β , and δ and their associated 1σ uncertainties. Then, for each QSO at z_i , we evaluate the quantity

$$\Sigma = \frac{|\log L'_{X,i} - (\gamma \log L'_{UV,i} + \beta)|}{\sqrt{\Delta^2 \log L'_{X,i} + \gamma^2 \Delta^2 \log L'_{UV,i} + \delta^2}} \quad (6.1)$$

where we use the best-fit values previously obtained for γ , β , and δ . This quantity is exactly the one that is minimized in the fitting algorithm to determine the parameters of the RL relation, and is therefore the most appropriate to estimate the discrepancy between the measured X-ray luminosity and the one predicted by the RL relation. Once we have computed Σ for each source, we select only QSOs with $\Sigma \leq \sigma_{\text{clipping}}$, where σ_{clipping} is a chosen threshold, and we repeat the fit of the RL relation with this reduced sample. Since the fit to this new sample yields different best-fit values of γ , β , and δ from those of the previous fit, and thus different Σ values for each source, there will be QSOs in the sample considered at this step that no longer satisfy the $\Sigma \leq \sigma_{\text{clipping}}$ condition. Hence, we iterate this procedure until all sources in the selected sample satisfy this condition. After this σ -clipping process, we obtain a final QSO sample, with the corresponding best-fit values and 1σ uncertainties of γ , β , and δ .

We have chosen several σ_{clipping} values up to 2 to investigate how the assumed σ_{clipping} , the best-fit value of δ , and the number of surviving sources are related. We have also applied the same method to select the QSO samples using the observed fluxes instead of the luminosities. In this case, we fitted a linear relation between $\log F_X$ and $\log F_{UV}$ for which γ_F , β_F , and δ_F are the slope, intercept, and intrinsic dispersion, respectively. As we mainly aim at constraining Ω_M with high precision, we are not only interested in achieving the smallest dispersion of the QSO sub-sample, but also in relying on a statistically sufficient number of sources. Therefore, we need to find a compromise between these two antagonistic factors. Indeed, increasing the data set leads to a larger intrinsic dispersion and vice versa. As a result, when we consider the case of luminosities in which we assume a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, the

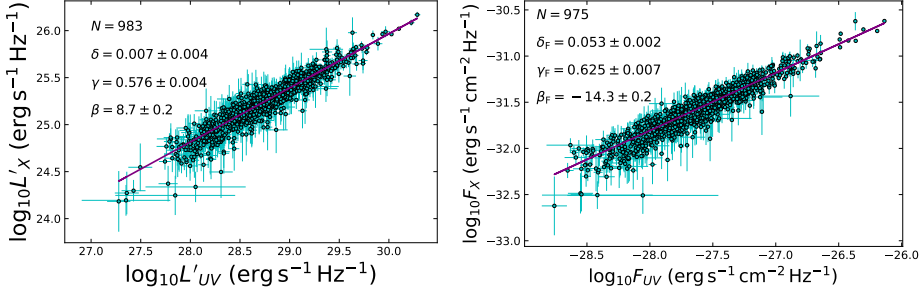


Figure 6.1: Left panel: The final sample of 983 QSOs obtained with the σ -clipping on the $L_X - L_{UV}$ relation assuming a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The resulting best-fit values of the parameters are: the slope $\gamma = 0.576 \pm 0.004$, the normalization $\beta = 8.7 \pm 0.2$, and the dispersion $\delta = 0.007 \pm 0.004$. Right panel: The final sample of 975 QSOs generated with the σ -clipping on the $F_X - F_{UV}$ relation with the best-fit parameters being $\gamma_F = 0.625 \pm 0.007$, $\beta_F = -14.3 \pm 0.2$, and $\delta_F = 0.053 \pm 0.002$. In both panels blue points are the sources with error bars representing the statistical 1σ uncertainties and the best-fit linear relation is drawn with a purple line.

optimal number of sources is 983, obtained by requiring a threshold of $\sigma_{\text{clipping}} = 1.788$. This sample presents a scatter $\delta = 0.007$ and is shown in the left panel of Figure 6.1. Instead, in the case of fluxes, a $\sigma_{\text{clipping}} = 1.78$ identifies an optimal sample of 975 sources with $\delta_F = 0.053$, that is represented in the right panel of Figure 6.1.

For each of these final QSO samples produced by the σ -clipping we have then fitted a flat Λ CDM model. By construction, when fitting the sample obtained from the luminosities we employed a fixed evolution treatment to infer the parameters, while, when considering the outcome of the procedure with fluxes, we applied the more general varying evolution to avoid any circularity problem (see Chapter 5 for more details). In these fits, H_0 is fixed at $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ while Ω_M is treated as a free parameter with a broad uniform prior between 0 and 1 and also γ and β are also free to vary. We therefore obtained the best-fit values of Ω_M , γ , β , and δ with their associated 1σ uncertainties. Considering the sample of 983 QSOs derived from the luminosity approach, we obtained $\Omega_M = 0.268 \pm 0.022$ which carries the same uncertainty on Ω_M as found with the Pantheon sample (1048 SNe Ia) assuming the same cosmology [80], as shown by the horizontal blue line in the left panel of Figure 6.2. Furthermore, we recovered the Ω_M we have assumed to build this sample (i.e. $\Omega_M = 0.3$) within 0.68σ . To check this result against the cosmological settings, we also tested the assumptions of $\Omega_M = 1$ (a Universe filled by matter with $\Lambda = 0$), and $\Omega_M = 0.10$ (very close to the De Sitter Universe). We obtained that the best-fit values of Ω_M are consistent within less than 3σ with the a-priori assumption. In fact, when considering the sample of 980 QSOs derived with the $\sigma_{\text{clipping}} = 1.788$ assuming $\Omega_M = 0.10$, we found $\Omega_M = 0.083 \pm 0.009$, while when using the sample of 968 QSOs with the $\sigma_{\text{clipping}} = 1.785$ assuming $\Omega_M = 1$, we constrained $\Omega_M = 0.910 \pm 0.055$. Instead, when fitting the sample of 975 QSOs

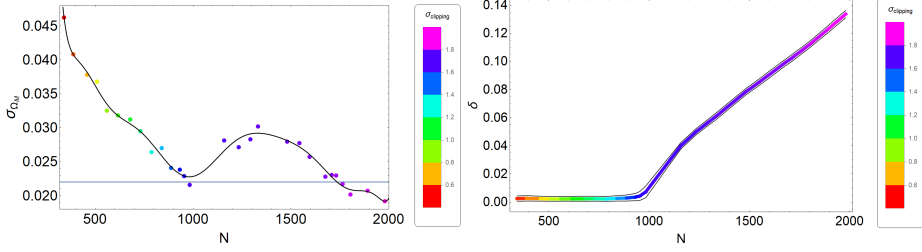


Figure 6.2: Left panel: the uncertainty on Ω_M as a function of the number of sources N and of the threshold for the σ -clipping showed with the colour bar on the right. The solid black line shows the best-fit of the decreasing trend of the uncertainty on Ω_M vs. the number of sources, while the horizontal blue line denotes the uncertainty on Ω_M achieved by the Pantheon SNe Ia sample. This is obtained for our sample derived from the luminosity approach under the assumption of a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Right panel: the intrinsic dispersion of the RL relation δ as a function of the number of sources N and of the threshold for the σ -clipping indicated on the right as a colour bar. Black lines mark the 1σ uncertainty on the δ values.

derived from the fluxes, we obtained $\Omega_M = 0.107 \pm 0.047$. In addition, to ensure that our results are not biased by the low- z QSOs ($z < 0.7$), which could be affected by host galaxy contamination and lower data quality, as described in Section 3.1, we have removed these sources from our sample obtaining $\Omega_M = 0.125 \pm 0.040$ and thus a completely consistent result.

To assess to what extent the number of sources impacts the values and the uncertainty on Ω_M , we show in the left panel of Figure 6.2 the values of σ_{Ω_M} as a function of the number of sources N and of the threshold for the σ -clipping shown as a colour bar. In particular, the smaller the σ -clipping value, the smaller the sample size obtained. Considering sources from 300 to 1000 and from 1300 to 2000, the uncertainty on Ω_M decreases as the threshold value increases and, when the σ -clipping is very small, then the uncertainties on Ω_M become larger (see for example the red point at $\sigma_{\Omega_M} = 0.045$ in the left panel of Figure 6.2). It is clear from this figure that the σ -clipping of 1.8 applied to our final sample of 983 sources leads a precision on Ω_M that reaches that of the SNe Ia Pantheon sample (blue horizontal line in the left panel). Since the σ -clipping and the number of sources also determine the intrinsic dispersion of the RL relation, we show in the right panel of Figure 6.2 the intrinsic scatter as a function of these two factors. We can see that the dispersion of the RL relation increases monotonically with increasing sample sizes starting from around 1000 sources, while to achieve a smaller dispersion the trend is rather flat from 300 to 1000 sources. This increasing trend of the dispersion of the RL relation as a function of the number of sources is reflected by the increasing values of the uncertainties on Ω_M in the range between 1000 and 1300. However, the highly non-linear process of obtaining the cosmological parameters, in this case the value of σ_{Ω_M} , and the truncation caused by the σ -clipping induces in the initial sample does not allow for a straightforward comparison between the left and right panels

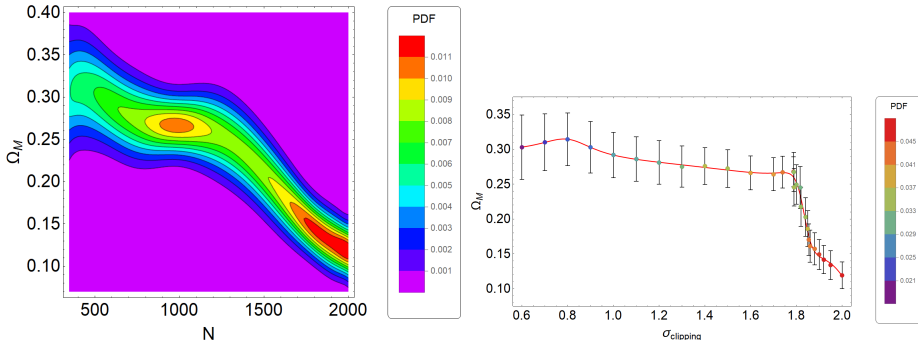


Figure 6.3: Left panel: The values of Ω_M and their associated uncertainties vs. the number of QSOs. The colour bar on the right shows the normalized PDF, indicating for each sample size the most probable value of Ω_M , thus the smallest uncertainty on Ω_M . This plot is derived considering the sample obtained from luminosities. Right panel: Values of Ω_M with the corresponding 1σ uncertainties as a function of the σ -clipping threshold and the PDF shown with the colour bar on the right side. The red line is the best-fit of the Ω_M points.

of Figure 6.2. In summary, the number of 975 sources for the sample derived from fluxes (and the corresponding number of sources for the final samples obtained with the other approaches) is the optimal compromise to obtain the smallest uncertainty on Ω_M taking into consideration the dispersion.

Continuing with the evaluation of how many number of sources are needed to obtain the most probable value of Ω_M , we show the colour map in the left panel of Figure 6.3 where the probability density function (PDF) of Ω_M is plotted as a function of the number of sources N . This plot is obtained for the case of the sample obtained from luminosities. We can see from this figure that 983 sources lead to the optimal sample, as it provides closed cosmological contours with the highest corresponding probability. We here notice that another red region, corresponding to a high probability, appears for $N \sim 2000$ and Ω_M close to 0.1. However, compared to the contours obtained with 983 sources, in this case, Ω_M is not constrained since the contours are not closed due to the large intrinsic dispersion associated with such a large sample. Hence, the approach based on the luminosities favours a value of Ω_M close to 0.3. To complement this information, we also plot in the right panel of Figure 6.3 the Ω_M values as a function of the σ -clipping and of the probability density for Ω_M shown in the colour bar.

Our results highlight that our circularity-free analysis with the fluxes favours a value of Ω_M (i.e. 0.107 ± 0.047) compatible with the De-Sitter Universe at 2.3σ level and not compatible with the current value of $\Omega_M = 0.338 \pm 0.018$ found by [273] at the level of 4.59σ . Furthermore, although the uncertainty of 0.047 is 2.14 times larger than that obtained by the Pantheon sample of SNe Ia [80], that is 0.022, we can still obtain uncertainties comparable to the 740 SNe Ia presented by [262] where $\sigma_{\Omega_M} = 0.042$. Finally, for the first time with QSOs alone, we have obtained a value of Ω_M with a higher

precision than previous analyses, while applying a circularity-free methodology when using the fluxes. In addition, the analysis we performed by using the luminosities and assuming a given cosmological model is intended to demonstrate the great potentiality of QSOs to be used as standardizable candles, even at present, when an appropriate sample size and reduced uncertainties are considered.

6.2 A quasar sub-sample with Huber regressor

We have performed a similar search for a sub-sample of QSOs which acts as a standalone probe to constrain Ω_M with improved precision by applying a different selection procedure. This selection follows the steps here listed:

- We have first divided the initial QSO sample of 2421 sources into redshift bins to fit a linear relation between the logarithms of the fluxes in each redshift bin. This procedure is the one already detailed in Section 2.2.2 and thus the binning must be chosen so as to verify the specific condition that, in each bin, the contribution of $\log D_L$ is negligible compared to the dispersion of the relation. However, we also need to fine-tune our choice of bins to ensure that there are enough sources (we require a minimum of 10), at least in the majority of the bins, to reliably perform the fit in each of them. The specific choice of 10 sources is arbitrary. Indeed, we could apply a different threshold, that would allow sufficient statistics to perform the fit. In fact, we have performed our analysis also changing the minimum number to 4, 5, 6, and 10, without any change in our results. We here also notice that in order to reduce the scatter of the relation in each bin, the condition of narrow redshift bins is very relevant, since the smaller the bins, the smaller the difference in $\log D_L$. Therefore, we can reduce the intrinsic dispersion up to the limit imposed by $\log D_L$.

Following these prescriptions, we have defined our optimal division into redshift bins in terms of $\log(1/(1 + \Delta z))$ with a bin width of $\Delta z = 0.042$. We have adopted the division as $\log(1/(1 + z))$ as it retains the same division in volume. In this way, we can keep the bin constant, and we do not need to derive arbitrary bins. With this division in redshift, we obtained 32 bins with at least 10 sources. Furthermore, we have investigated also different choices for the division into bins and their impact on the cosmological results, as we will discuss later.

We emphasize here that, as described in Section 2.2.2, binning the data into redshift intervals is necessary to use fluxes instead of luminosities. This is a crucial point because it enables us to perform a circularity-free analysis. Indeed, fluxes are measured quantities which, unlike luminosities, do not require any cosmological assumptions. As a consequence, the use of fluxes in the selection of the sample guarantees that our cosmological results are not induced by any a priori cosmological assumption.

- After binning, we have fitted the linear relation between $\log F_X$ and $\log F_{UV}$ in each bin consisting of at least 10 sources. This fit was performed using the Kelly

method [280], which accounts for the uncertainties in both quantities as well as for the intrinsic dispersion of the correlation. More specifically, this is a Bayesian method accounting for measurement errors in linear regression of astronomical data. It allows for possibly correlated measurement errors and intrinsic scatter in the regression relationship and it is based on deriving a likelihood function for the measured data, primarily in the case when the intrinsic distribution of the independent variables can be approximated using a mixture of Gaussian functions. The Kelly method can be also generalized to incorporate multiple independent variables, nondetections, and selection effects.

To check that the condition on D_L is satisfied, the best-fit value of δ obtained from the fit is compared with the maximum difference in $\log D_L$ for the sources in the investigated redshift bin. This difference is computed by assuming a flat Λ CDM model, however the assumption of a specific cosmological model for this computation does not affect the results, since we are considering a difference between two luminosity distances. Then, we have retained unmodified sources in the redshift bins that do not provide enough statistics (i.e. less than 10 QSOs) to perform a reliable fit. In the following, we denote these sources with the notation “untouched”. In this regard, we have distinguished two cases: one in which we do not include these sources in our analysis and another one in which we have added them to the final selected sample. The strategy here is to balance and compromise among the smallest bin so that $\Delta \log D_L < \delta$, but still sufficiently large so that the number of sources is at least 10 or more.

- At this stage, as the presence of outliers can affect the performance and accuracy of a least-squared-loss error-based regression, we have employed the consolidated statistical technique of the Huber algorithm [281, 282, 283]. Indeed, the Huber regressor is a method for estimating the parameters of a model, in this case the $F_X - F_{UV}$ relation, detecting the outliers and weighting them less in the evaluation of the best-fit parameters of the fitted model. Thus, compared to traditional fitting procedures, such as the D’Agostini [275] or the Kelly [280] methods, the Huber regression identifies outliers and recognizes the actual best-fit based on the inliers of the relation. This algorithm has the advantage of not being heavily influenced by the outliers, while not completely ignoring them. For these reasons, the Huber technique is often used for robust regression problems. In summary, this allows us to estimate the actual slope and intercept of the relation, not altered by the outliers, and simultaneously identify the sources that are outliers in the model. Hence, we discarded these sources from the QSO sample in each redshift bin. We here clarify that we do not use the notation of “outliers” in a strict statistical sense, but rather to refer to those QSOs that show more discrepancy from the RL relation line. The sensitivity of the Huber regressor in identifying the so-called outliers can be handled through the parameter ϵ embedded in the regressor method, which will be treated later in this section.

After this selection, we have also checked the following two criteria in each bin:

- i) the null hypothesis that the populations of both UV and X-ray fluxes are drawn

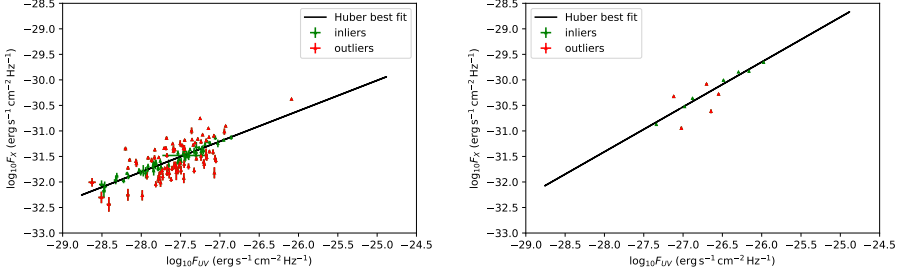


Figure 6.4: Results of the Huber regression in redshift bins. Left panel: Outliers (in red) and inliers (in green) sources for the second most populated redshift bin together with the Huber best-fit shown with the black line. Right panel: Same as the left panel but showing the second least populated redshift bin.

from the initial ones should not be rejected with a p-value $> 5\%$ according to the Anderson two-sample test and ii) the distribution of the residuals around the best-fit line must be Gaussian according to the Anderson-Darling normality test with an acceptance significance level of 5% (see e.g. [284] for the Gaussianity discussion). To showcase the Huber regressor's advantage in each bin and how effectively it removes the outliers in Figure 6.4 we show the selected sample in green and the sources identified as outliers in red. The two bins examined in the left and right panels of this figure are the second most populated one and the second least populated one, respectively.

- As anticipated, we have finally defined two final samples: one with only the sources retained by the steps detailed above and another obtained by combining the sources retained in each bin with the unmodified sources of the bins without sufficient statistics. In this way, we generated the final selected QSO samples consisting of 1065 and 1132 sources, respectively.

Concerning the application of the Huber regressor, we can also take advantage of the above-introduced parameter ϵ , that is a parameter of this method that can be set arbitrarily in the range $[1, \infty)$. This parameter controls the number of sources to be classified as outliers: the smaller ϵ , the more robust the Huber regression is in defining the outliers. We have indeed performed our selection by trying different values of ϵ and we have obtained compatible results in all the attempts. Thus, we have identified the value of $\epsilon = 1.2$ as the one that leads to the cosmological result with the smallest uncertainty on Ω_M .

With the above-described division of the sample into bins we have proved that there is no particular trend or significant behaviour of the slope as a function of the redshift, since it fluctuates around an average value of $\gamma = 0.60$. As a further check, we have fitted the luminosity relation by using both the two final selected QSO samples to check the best-fit value of the slope. In both cases, the value of γ is compatible with 0.6 within 1σ . These results on the slope are in full agreement with previous works in the literature

and with our analyses on the validation of the X-UV relation presented in Section 2.2.2 and Figure 2.3.

As anticipated, to further investigate the impact of our choice for the division in redshift bins (i.e., $\log(1/(1 + \Delta z))$ with $\Delta z = 0.042$), we have also selected the QSO sample by using three other choices for the division in bins (see e.g. [285] for a discussion on the binning). We here briefly summarize these methods and their results compared to the previous ones:

- The first alternative binning choice consists of dividing the sources into bins whose width depends on the redshift. Specifically, we have divided the original sample according to the following prescription: $\Delta \log z = 0.03$ when $z \leq 1$, $\Delta \log z = 0.04$ when $1 < z \leq 2$, $\Delta \log z = 0.05$ when $2 < z \leq 4$, and $\Delta \log z = 0.06$ when $z > 4$. The choice of increasing the width of the bins at higher redshifts is justified by the trend of the luminosity distance $D_L(z)$, which is much steeper at low z compared to higher z , where the function $D_L(z)$ flattens; thus, we need to impose narrower $\Delta \log z$ at low redshifts to verify the condition on $\Delta \log D_L$. In addition, the specific values of $\Delta \log z$ in each redshift range have been chosen to guarantee the fulfillment of all the required criteria detailed above. Applying this division, we generated 29 bins with at least 10 sources. Again, the value $\epsilon = 1.2$ for the Huber regressor proved to be the one that leads to the cosmological results with smaller uncertainties on Ω_M . Furthermore, the results obtained by applying different values of ϵ are fully compatible with each other. Finally, using this procedure, we identified two final samples: one of 1084 QSOs, which does not contain the sources in the bins where some of the conditions are not fulfilled, and one of 1125 QSOs, which also includes these sources. While this analysis is a good alternative to the previous method, it does not allow us to reach the same precision on Ω_M as with our main method. These results are reported in Table 6.1.
- To avoid the arbitrary choice of the division into bins that is inherent in the two procedures above, we have also developed an approach to optimize the binning. More specifically, we start from the source at the highest redshift in the initial sample or the source at the lowest redshift (we have investigated both cases). We define the edge of the bins so that the maximum number of sources is included in each bin and still the same criteria of the previous procedures are fulfilled. After the first bin has been identified, the same procedure is repeated to create another bin adjacent to it in redshift and this algorithm continues until the size of the initial sample is reached. Following this recursive approach, this algorithm automatically creates bins to divide the QSO sample without any arbitrary choice. When we reach the point where almost the whole sample is divided into bins, we might face the problem that the number of data points that still do not belong to any bin would be less than our threshold of 10. Thus, it is impossible for these points to generate a reliable bin. We store these sources in a separate set called “untouched”. Alternatively, this could also be the case in which the number of untouched sources is greater than 10, but the condition between the $\Delta \log(D_L(z))$ and the intrinsic scatter cannot be fulfilled for the set of these sources. In this case, we treat these sources just like the untouched sources in bins without sufficient statistics.

As in the previous approaches, we have tested different values for the ϵ parameter of the Huber regressor. As the results proved to be fully compatible, we have chosen the values $\epsilon = 1.5$ and $\epsilon = 1.6$ for the methods starting from the highest redshift and the lowest redshift, respectively, since these choices guarantee the cosmological results with the smallest uncertainty on Ω_M . Using this approach, we obtained the following final QSO samples (see Table 6.1): 1843 and 1858 sources, respectively, excluding and including the sources not belonging to any bin, for the method starting from the highest redshift, and 1965 and 1980 QSOs, respectively, for the corresponding cases when starting from the lowest redshift. Similarly to the previous method, this procedure does not allow us to achieve the same precision as the binning division in $\log(1/(1 + \Delta z))$, as shown in Table 6.1.

- Finally, we have developed an additional method that allows us to completely free our analysis from the possible issues of the binning approach. Indeed, in this procedure, we do not actually assign each QSO to a specific bin, but we consider each of the initial 2421 QSO sources as the center of an interval. This interval is symmetric in redshift, so it consists of the five sources next to the central one at lower redshifts and the five sources adjacent to the central one at higher redshifts. In this way, each interval consists of a total of 11 sources. Then, we performed the same steps detailed for our main binning choice (i.e. $\log(1/(1 + \Delta z))$) within each of these intervals. In particular, we define a source as an inlier only if the Huber regressor selects it as an inlier in all the intervals to which it belongs. Again, we have checked that choosing different values of the parameter ϵ for the Huber regression does not impact our analysis, leading to compatible results. Therefore, we have chosen $\epsilon = 1.3$ as the value that leads to the cosmological result with the smallest uncertainty on Ω_M . Finally, we obtained the following samples, summarized in Table 6.1: 811 and 825 sources, respectively, when we exclude and include the sources belonging to intervals that do not fulfill our criteria. Once again, this method does not achieve the same precision on Ω_M as the one described at the beginning.

In summary, we have applied four different and independent methods to divide the initial total QSO sample into redshift bins and trim it to define the final samples used for the cosmological analysis. The employment of multiple approaches allows us to draw a comprehensive scenario and gain full insight into the process of the QSO selection. Indeed, we leverage the advantages of all the methods described above, which also allow us to free our analyses and results from the arbitrariness of a fixed chosen binning. Furthermore, the simultaneous use of all these procedures enables us to check the validity of our results and the robustness of our analysis.

In the following, we focus on the cosmological analysis performed with the final samples obtained from the first method with the binning in $\log(1/(1 + \Delta z))$ with $\Delta z = 0.042$. This is indeed the approach that, among all the methods investigated, leads to the cosmological result with the smallest uncertainty on Ω_M . However, we have checked that our cosmological results are not affected by the binning choice. We have employed the final QSO samples to fit with the Kelly method a flat Λ CDM model, in which we fix

Method	Sample	N	$\Omega_M \pm \Delta\Omega_M$
Bins in $\log(1/(1 + \Delta z))$	Without sources	1065	0.231 ± 0.122
	With sources	1132	0.256 ± 0.089
Bins wider in z	Without sources	1084	0.317 ± 0.162
	With sources	1125	0.229 ± 0.061
Bins optimized in width from highest z	Without sources	1843	0.362 ± 0.162
	With sources	1858	0.285 ± 0.110
Bins optimized in width from lowest z	Without sources	1965	0.482 ± 0.222
	With sources	1980	0.349 ± 0.149
Bins centered on each QSO	Without sources	811	$0.490 \pm 0.226 *$
	With sources	825	0.315 ± 0.124

Table 6.1: Comparison of the results obtained from the different selection approaches. The first column details the method applied to divide the initial QSO sample in bins, the second column specifies whether the sample includes or not the sources in bins that do not fulfill the criteria required by our trimming analysis. The third and fourth columns report, respectively, the number of sources (N) in the considered sample after the removal of the outliers and the estimated Ω_M with its 1σ uncertainty. The symbol “*” marks the cases in which Ω_M is not constrained.

$H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and we consider Ω_M as a free parameter with a broad uniform prior between 0 and 1. We have also left γ , β , and δ free to vary, we have imposed on them the uniform priors $0 < \gamma \leq 1$, $0 < \beta < 20$, and $0 < \delta < 1$, and we have applied the best-fit cosmological likelihoods: a Gaussian likelihood for the sample of 1065 sources and a logistic likelihood (whose PDF is provided by Eq. (8.3)) for the sample of 1132 QSOs. These choices are justified in light of the results presented in Chapter 8. Indeed, we have checked the normality assumption for our final QSO samples obtaining that it is verified by the sample of 1065 QSOs, but not by the sample of 1132 sources. In fact, for this sample the best-fit likelihood is a logistic one, as for the initial 2421 QSOs (see Section 8.2.2). In the fitting procedure, we have also accounted for the effects of the evolution in redshift of the QSO luminosities through the varying evolution approach (see Chapter 5).

Concerning the sample of 1132 QSOs, the flux dispersion is $\delta_F = 0.22$, which is 24% less than the dispersion of the original sample ($\delta_F = 0.29$), and the luminosity dispersion is $\delta = 0.07$ vs. $\delta = 0.09$ for the total sample. It still covers the whole redshift range from $z = 0.009$ to $z = 7.54$, and it is smaller in size compared to the initial sample (53% of sources are discarded). From the cosmological fit we obtained $\Omega_M = 0.256 \pm 0.089$, with a 58% improvement in precision compared to 0.210, derived from the full QSO sample. This precision is also slightly improved compared to that achieved by the SNe Ia sample of [261], which is 0.10, although in the case of SNe Ia Ω_M is not the only free parameter of the fit. However, it is also possible to achieve a larger reduction of the intrinsic dispersion, compared to that of the 1132 QSOs, if we discard the untouched sources. Following this approach, we have defined a QSO sample of 1065 sources with reduced redshift coverage between $z = 0.2$ and $z = 3.4$ and an intrinsic dispersion of the

flux relation $\delta_F = 0.18$, which is 18% smaller than the dispersion of the 1132 sources and 38% less than the dispersion of the original sample. Compared to the sample of 1132 sources, the sample of 1065 QSOs, leads to $\Omega_M = 0.231 \pm 0.122$.

A remarkable difference between the two samples of 1132 and 1065 QSOs is that in the smaller sample the highest redshift is $z = 3.435$, while the larger sample covers the whole redshift range and contains 45 more QSOs, which are in the range of $3.435 \leq z \leq 7.5413$ and are absent in the 1065 QSO sample. These additional high- z QSOs are distributed by redshift as follows: 9 QSOs within $3.435 < z \leq 4$, 19 within $4 < z \leq 5$, 6 within $5 < z \leq 6$, and 11 within $6 < z \leq 7.5$. This difference in the redshift distribution reflects in the intrinsic dispersion of the two samples. Indeed, the smaller sample is obtained only by considering inliers determined through the Huber regression, while the larger sample includes sources at low ($z < 0.2$) and high ($z > 3.4$) redshift, for which, due to the insufficient statistics of the corresponding redshift bins, it was not possible to perform the fit and thus remove the outliers. The addition of these sources, which cannot be identified either as inliers or outliers, increases the dispersion. Nevertheless, the 1065 QSOs lead to a larger uncertainty on Ω_M , 0.12, compared to the uncertainty of 0.09 achieved by the sample of 1132 sources. The reason is that the latter sample covers the whole redshift range of the initial QSO sample, including also the low-redshift interval that allows the Hubble diagram to be anchored at the zero-point near $z = 0$, thus better constraining the value of the matter density today (at $z = 0$). The difference in the redshift coverage of the two QSO samples is also reflected in the different best-fit cosmological likelihoods: the larger sample is better fitted with a logistic likelihood, as the original QSO sample, since it resembles the initial 2421 sources in the redshift coverage, while the smaller sample fulfils the Gaussianity assumption as it ranges over a reduced redshift interval. Based on these results, we claim that with more QSOs at low and high redshifts ($z < 0.2$ and $z > 3.4$), which would allow us to remove outliers in these redshift ranges, we could achieve a reduced intrinsic dispersion and smaller uncertainty on Ω_M , which highlights the importance of future QSO surveys and observations.

In conclusion, we have shown that QSOs with the RL relation can be promoted to reliable standalone probes for the measurement of cosmological parameters, such as Ω_M , provided that a suitable sub-sample is defined. However, a lot of work is still required in this direction. Indeed, we need to uncover the common spectral properties and physical features that characterize such a sub-set, compared to the whole QSO sample. Only this information, that requires a huge research effort, could allow us to collect all the QSO observations that exhibit the uncovered proper features and build such a “gold” cosmological QSO sample.

7 Cosmology with quasars and other probes

QSOs have recently attracted the attention of the cosmological community as very promising cosmological tools since they provide information at high redshifts. Indeed, in the previous chapters, we have described the RL relation, that allows us to turn QSOs into cosmological probes, and validated this relation from different perspectives: the non-evolution of the parameters involved (Section 2.2.2), the physical factors responsible for the observed dispersion which reduce the intrinsic dispersion to < 0.06 dex (Section 2.2.4), the homogeneity of the cosmological QSO sample (Section 3.1), the tightness of the relation for the sub-sample at $z \sim 3$ (Section 3.2), and the robustness of the relation against selection biases and redshift evolution of luminosities (Section 5.3). Therefore, QSOs now stand as invaluable cosmological sources that probe the Universe up to $z \sim 7.5$, at much higher redshift than $z \sim 2.3$ reached by the most distant SNe Ia, thus shedding light on a previously unexplored redshift interval. For this reason, QSOs are currently the most promising cosmological probes to investigate the evolution of the Universe, the nature and origin of its constituents, and the current puzzling tensions between the predictions and the observational data in search of a cosmological model that can effectively explain and reproduce the observations at different scales.

In this thesis work, we have delved into this challenging scenario by performing several cosmological analyses both with QSOs alone and with QSOs combined with other cosmological probes. In this chapter we describe these studies and their results focusing on the analyses performed with a “traditional” cosmological approach, that is the inference of cosmological parameters through the commonly adopted Gaussian likelihood. Instead, in Chapter 8, we introduce the use of non-Gaussian likelihoods when they prove to be more appropriate for the specific cosmological probes under investigation. This chapter is structured as follows: Section 7.1 outlines the cosmological analyses of the concordance model and its extensions that we have carried out with QSOs as standalone probes and with QSOs combined or calibrated with SNe Ia, while in Section 7.2 we also discuss the combination with BAO. Finally, in Section 7.3 we describe our original implementation of the QSO analyses in Monte Carlo Markov Chain algorithms, such as CosmoMC, Montepython, and Cobaya, which include Boltzmann solver codes. In this context, we also present the CMB and Dark Energy Survey (DES) data both alone and in combination with QSOs and we provide insights on a possible interaction in the

dark sector. Sections 7.1, 7.2, and 7.3 are respectively based on our publications of [286] and [15], [279], and [287].

7.1 Cosmological constraints from quasars alone and quasars combined with SNe Ia

The non-negligible dispersion of the RL relation limits the power of QSOs as standalone probes to constrain the cosmological parameters compared to other well-established probes such as SNe Ia. However, we can still employ QSOs alone in cosmological analyses under certain specific assumptions that reduce the dimensionality of the parameter's space and thus allow QSOs to infer the free parameter with sufficient precision. As an example, we have used QSOs alone to constrain Ω_M in a flat Λ CDM model with H_0 fixed to $70 \text{ km s}^{-1}, \text{ Mpc}^{-1}$. In this analysis, we have also examined the impact of low- z and high- z QSOs on the derived Ω_M . Indeed, we have considered four sub-samples of QSOs each reaching $z = 0.7$, $z = 0.8$, $z = 0.9$, and $z = 1$, consisting of 398, 543, 678, and 826 sources, respectively. Then, we have fitted Ω_M with each of these sub-samples. The values obtained for Ω_M are as follows: $0.411^{+0.342}_{-0.259}$ for $z < 0.7$, $0.511^{+0.305}_{-0.275}$ for $z < 0.8$, $0.601^{+0.248}_{-0.250}$ for $z < 0.9$, and $0.717^{+0.184}_{-0.231}$ for $z < 1$. Hence, QSOs alone confirm $\Omega_M \sim 0.3$, as expected from SNe Ia, in the low redshift regime up to $z \sim 0.7$ but they favour an Einstein-de Sitter Universe with $\Omega_M \sim 1$ at higher redshifts (i.e. $z \sim 1$).

We have further extended this analysis by using the full QSO sample, without the cut at $z = 0.7$ explained in Section 3.1, to fit a flat and a non-flat Λ CDM model and a flat w CDM model. In particular, we have considered three cases: QSOs alone, QSOs calibrated to Pantheon SNe Ia, and QSO not calibrated but combined with Pantheon SNe Ia. Furthermore, we have distinguished the three cases for the correction of redshift evolution and selection biases described in Chapter 5: without correction, with fixed evolution, and with varying evolution. In addition, we have considered different combinations of the cosmological free parameters when SNe Ia are also included, as shown in Table 7.1, while only the case of a flat Λ CDM model with Ω_M free parameter has been studied for QSOs alone, as reported in Table 7.2. We here outline our main results.

Assuming a flat Λ CDM model, the case of QSOs alone calibrated with SNe Ia and without any correction for evolution favours high values of Ω_M , even if compatible in 3σ with $\Omega_M = 0.3$, when H_0 is fixed, and high values of H_0 when Ω_M is fixed, whereas when both parameters are free, we do not obtain convergence on the Ω_M parameter which tends to assume very high values even outside our uniform prior bound. Including a fixed correction for the evolution instead, Ω_M always tends to lower values, for both fixed and free H_0 ($\Omega_M = 0.251 \pm 0.054$ and $\Omega_M = 0.167 \pm 0.062$, respectively), while H_0 ranges from 67.55 to $72.09 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in 1σ when Ω_M is fixed, corresponding to intermediate values between that measured from SNe Ia ($H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$) and that obtained from the Planck data on the CMB under the assumption of the same cosmological model ($H_0 = 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$). The case with both parameters free does not converge for any of the two parameters yielding too low Ω_M values ($\Omega_M = 0.167 \pm 0.062$), compatible within 3σ with a De Sitter Universe where $\Omega_M = 0$,

and too high H_0 values ($H_0 = 76.40 \pm 3.33 \text{ km s}^{-1} \text{ Mpc}^{-1}$). Taking into account the variation of the correction for evolution together with the cosmological parameter Ω_M , Ω_M becomes consistent with $\Omega_M = 0.3$, when H_0 is fixed, also due to large uncertainties, but Ω_M does not converge when H_0 is also free to vary, approaching to the upper limit of our uniform prior. In this case, H_0 is compatible in 1σ with the value from the CMB.

Still in a flat Λ CDM model, when combining non-calibrated QSOs with SNe Ia we do not deal with convergence issues in any of the cases. Specifically, without accounting for the correction for the evolution, we find Ω_M close to 0.3 (within 1σ and 2σ when H_0 is fixed and free, respectively) and H_0 close to 70 (within 1σ and 2σ when Ω_M is fixed and free, respectively). When accounting for a fixed correction for the evolution, we still have Ω_M close to 0.3 even if within 2σ and 3σ when H_0 is fixed and free, respectively, and H_0 close to 70, within 1σ and 3σ when Ω_M is fixed and free, respectively. Letting the evolution vary together with the cosmological parameter (i.e. $k(\Omega_M)$), we obtain Ω_M and H_0 always compatible within 1σ with 0.3 and 70, respectively. Indeed, we here remind that k does not depend on H_0 so we do not need to study the case where only H_0 is free.

Only in the case of a flat Λ CDM model, we also consider the data set composed of QSOs alone not calibrated with SNe Ia. These results are presented in Table 7.2, where H_0 is fixed at $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. If we do not consider the correction for the evolution, Ω_M is not constrained and reaches the upper limit $\Omega_M = 1$ of the range of the uniform prior. Instead, considering a fixed correction for the evolution, we obtain closed

Only QSOs calibrated with SNe Ia				Combination of SNe Ia with QSOs without calibration			
Results without correction for evolution							
Ω_M	H_0	w	Ω_k	Ω_M	H_0	w	Ω_k
0.443 ± 0.054	70	-1	0	0.305 ± 0.008	70	-1	0
0.3	73.76 ± 2.18	-1	0	0.3	69.97 ± 0.14	-1	0
0.663 ± 0.108	62.77 ± 2.52	-1	0	0.338 ± 0.022	69.44 ± 0.32	-1	0
0.3	70	-0.696 ± 0.132	0	0.3	70	-1.003 ± 0.019	0
0.3	70	-1	-0.531 ± 0.275	0.3	70	-1	-0.01 ± 0.018
0.522 ± 0.052	70	-1.966 ± 0.395	0	0.462 ± 0.034	70	-1.552 ± 0.166	0
0.517 ± 0.06	70	-1	-0.740 ± 0.106	0.656 ± 0.048	70	-1	-0.762 ± 0.101
Results with fixed correction for evolution							
Ω_M	H_0	w	Ω_k	Ω_M	H_0	w	Ω_k
0.251 ± 0.040	70	-1	0	0.291 ± 0.008	70	-1	0
0.3	69.82 ± 2.27	-1	0	0.3	69.99 ± 0.14	-1	0
0.167 ± 0.062	76.40 ± 3.33	-1	0	0.251 ± 0.02	70.72 ± 0.34	-1	0
0.3	70	-1.113 ± 0.182	0	0.3	70	-1.006 ± 0.018	0
0.3	70	-1	0.01 ± 0.122	0.3	70	-1	0 ± 0.018
0.151 ± 0.099	70	-0.765 ± 0.267	0	0.059 ± 0.038	70	-0.662 ± 0.041	0
0.24 ± 0.05	70	-1	0.076 ± 0.104	0.118 ± 0.032	70	-1	0.377 ± 0.069
Results with correction for evolution as function of cosmology							
Ω_M	H_0	w	Ω_k	Ω_M	H_0	w	Ω_k
0.382 ± 0.178	70	-1	0	0.299 ± 0.008	70	-1	0
0.543 ± 0.251	67.50 ± 2.80	-1	0	0.3 ± 0.022	69.99 ± 0.35	-1	0
0.3	70	-1.074 ± 0.205	0	0.3	70	-1.005 ± 0.019	0
0.3	70	-1	-0.041 ± 0.102	0.3	70	-1	-0.006 ± 0.018
0.546 ± 0.172	70	-1.591 ± 0.500	0	0.358 ± 0.054	70	-1.168 ± 0.156	0
0.591 ± 0.217	70	-1	-0.235 ± 0.191	0.415 ± 0.063	70	-1	-0.250 ± 0.134

Table 7.1: Cosmological results obtained for all the cases investigated when SNe Ia are included in the analyses. The errors reported in the table correspond to the 1σ uncertainties. Bold values mark the cosmological parameters that are fixed in the fit.

Results obtained with QSOs alone without calibration	Ω_M
Results without correction for evolution	0.934 ± 0.059
Results with fixed correction for evolution	0.067 ± 0.017
Results with correction for evolution as a function of Ω_M	0.500 ± 0.210

Table 7.2: Cosmological results obtained with only the QSO sample without calibration. The errors reported in the table correspond to the 1σ uncertainties. In this computation we have fixed the parameters as: $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_k = 0$, and $w = -1$.

contours on this parameter and $\Omega_M = 0.067 \pm 0.017$, so a very small value although not compatible with $\Omega_M = 0$ within 3σ . Unsurprisingly, the case with the correction $k = k(\Omega_M)$ allows us to overcome the convergence issue and the non-physical behaviour of Ω_M close to 0 leading to $\Omega_M = 0.500 \pm 0.201$, which is compatible within 1σ with $\Omega_M = 0.3$. The results obtained without the correction and with the varying evolution confirm the trend anticipated above, with QSOs alone over the whole redshift range up to $z \sim 7.5$ favouring a Ω_M greater than 0.3 and close to 1.

When considering $\Omega_k \neq 0$ in a non-flat Λ CDM model, QSOs alone calibrated with SNe Ia point toward negative values of Ω_k , if we do not correct for the luminosity evolution in redshift. More precisely, $\Omega_k = -0.531 \pm 0.275$ when Ω_M is fixed, while when both are free parameters of the fit Ω_M tends to 0.52 but Ω_k does not converge due to very low values in our range of uniform priors. Very interestingly, when we correct for a fixed evolution, the convergence is achieved for both Ω_k and Ω_M free to vary, however with large uncertainties, and Ω_M shifts towards 0.3 and Ω_k towards 0. Instead, when Ω_k is the only free parameter of the fit it is not constrained, as it is when the correction that varies with Ω_k is also taken into account. This can be ascribed to the fact that QSOs are more sensitive to the value of Ω_M , which is better determined by probes at intermediate redshifts, rather than the value of Ω_k , which is instead constrained at very high redshifts as from the CMB; thus when also Ω_M is free QSOs can constrain Ω_M and, as a consequence of their degeneracy, also Ω_k . Varying both parameters and taking into account a correction for evolution that varies together with these parameters, does not constrain Ω_M , which tends to take values close to $\Omega_M = 1$.

In the case of the non-flat Λ CDM model and non-calibrated QSOs combined with SNe Ia, Ω_k is always compatible with $\Omega_k = 0$ within 1σ when Ω_M is fixed to 0.3 (as expected for a flat Universe), both with and without the correction for the evolution, while, when also Ω_M is free, it tends to negative values (i.e. $\Omega_k = -0.762 \pm 0.101$) if we do not correct for the evolution, and to positive values (i.e. $\Omega_k = 0.377 \pm 0.069$) if we consider a fixed correction. In these two cases, Ω_M is shifted to high and low values, respectively. When accounting for the correction with $k(\Omega_M, \Omega_k)$, we observe an intermediate behaviour: $\Omega_M = 0.415 \pm 0.063$, within 2σ from $\Omega_M = 0.3$, and $\Omega_k = -0.250 \pm 0.134$, within 3σ from a zero curvature. Very similar behaviour can be observed in the cases with calibrated QSOs. We see here that, after applying the correction for the evolution as a function of cosmology, the results are much closer to the flat Λ CDM model parameters obtained with SNe Ia alone than those without this

correction. In order to drive clear conclusions on the flatness of the Universe more work still needs to be done in the future with a larger sample of QSOs and the addition of more high-redshift probes.

In the framework of a flat w CDM model, if we consider QSOs alone calibrated with SNe Ia and we do not take into account any correction for the evolution, we obtain very different results regarding the dark energy scenario, depending on whether or not we fix the parameter Ω_M . In particular, if $\Omega_M = 0.3$ we constrain $w \sim -0.7$, corresponding to the quintessence behaviour (even if compatible within 3σ with $w = -1$), whereas if Ω_M is a free parameter of the fit, we enter the phantom dark energy region with $w \sim -2$ and $\Omega_M \sim 0.5$, although in this case w does not converge achieving very negative values in our uniform prior. On the contrary, we observe the opposite trend when accounting for a fixed correction for the evolution. In this case, indeed, the phantom regime is obtained with Ω_M fixed (but at only 1σ from a quintessence case with $w = -0.931$) and the quintessence regime (at 1σ from $w = -1$) with Ω_M as a free parameter, although in the latter case Ω_M does not converge hitting the barrier at $\Omega_M = 0$. When considering $k = k(w)$ with Ω_M fixed, w is compatible with both scenarios in 1σ . Instead, if Ω_M is free to vary, both parameters are unconstrained.

When fitting jointly non-calibrated QSOs and SNe Ia, we obtain that w is always consistent within 1σ with $w = -1$ when Ω_M is fixed to 0.3 (as expected in a flat Λ CDM model), regardless the treatment of the correction for the evolution. Instead, when varying also Ω_M , we obtain $w \sim -1.6$ and $\Omega_M \sim 0.46$ without the correction, $w \sim -0.7$ and $\Omega_M \sim 0.06$ with a fixed correction, although in this last case Ω_M does not converge, and $w \sim -1.2$ and $\Omega_M \sim 0.36$ with a correction that varies with the cosmological parameters. This last case is compatible with $\Omega_M = 0.3$ and $w = -1$ within 2σ .

We also would like to emphasise that, as shown in Figure 7.1, if we consider non-calibrated QSOs combined with SNe Ia in both cases where H_0 and Ω_M change together and only H_0 changes, then we can safely state that the resulting H_0 values are all compatible with each other and they lie midway between the value of H_0 determined by the Planck collaboration and that from SNe Ia (see yellow, dark blue, pink, brown, and purple points in the figure). Instead, for calibrated QSOs, if we consider only H_0 varying without the correction for the evolution (blue point), we have compatibility with SNe Ia, while when we fix the evolution (green point), the H_0 value again lies in the middle between the values obtained from the CMB and SNe Ia. From this analysis, we still cannot conclude that the evolution is the driving factor that pushes H_0 closer to the Planck values. Indeed, when both parameters are allowed to vary freely, the results with and without the evolution lies on the opposite side of the Planck (without evolution) and SNe Ia values for H_0 (for fixed evolution).

In conclusion, among all the data sets and methodologies investigated, the case with non-calibrated QSOs combined with SNe Ia and with the correction for the redshift evolution varying together with the cosmological parameters is the one that turned out to be the most reliable and complete. Indeed, including the SNe Ia sample and considering this correction for the redshift evolution removes the convergence issues and leads to tighter constraints on the fitted parameters in all the cosmological models studied, also allowing us to overcome the circularity problem. In this case, our results are always

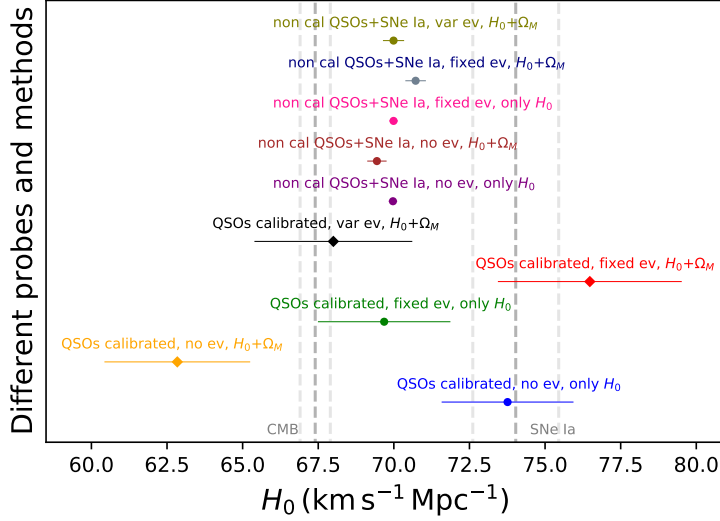


Figure 7.1: Comparison between the values of H_0 obtained in all our analyses. Beside each point, we report both the corresponding data set and methodology used for the computation. The two vertical dashed gray lines with the corresponding dashed light gray lines are the reference points of CMB and SNe Ia measurements with their 1σ uncertainties. The points marked with “ $\diamond$ ” correspond to the cases in which the analyses do not converge for one or more parameters.

consistent with a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$, without any hint of a dark energy with $w \neq -1$ nor a non-flat Universe. More precisely, we always obtain compatibility with the concordance model within 1σ , except for two cases: the flat w CDM and the non-flat Λ CDM models with both cosmological parameters free to vary, which show a 2σ discrepancy from $\Omega_M = 0.3$ and $w = -1$ and from $\Omega_M = 0.3$ and $\Omega_k = 0$, respectively. In addition, we have shown that QSOs alone can be used as standalone probes without any cut in redshift nor calibration. Indeed, for the first time for such a sample, we have obtained closed contours in 2σ for the computation of Ω_M . Ultimately, this analysis could represent a leap forward in shedding light on the H_0 tension and investigating whether the current tension could be attributed to a constant H_0 value that stands between that of SNe Ia and CMB.

7.2 The combination of quasars, SNe Ia and BAO in view of alternative cosmological models

As an extension to the previous analyses of the standard cosmological model and some of its alternatives, we have also included BAO in our data set of QSOs and SNe Ia. In particular, we have analyzed the w CDM, CPL, JBP, BA, and an exponential models (see Section 1.1) which are based on specific physical and theoretical assumptions, such as i) the dark energy component must be dominant only at very low redshifts and negligible in early epochs, due to the radiation-dominated and matter-dominated regimes that have preceded the recent dark energy domination along the Universe's evolution; this implies a well-bounded behaviour of the dark energy; ii) $w(z)$ must be negative to give rise to negative pressure and repulsive force able to counter the gravitational attractive force; in particular, according to the second Friedmann's equation, we need $w(z) < -1/3$ at low z to explain the present accelerated expansion of the Universe [5, 6] as a dark energy domination effect. Specific constraints on the future behaviour of the dark energy ($z \rightarrow (-1)^+$) are not required because, in general, cosmological models can show convergence or singularity (e.g. CPL parameterisation) in this limit.

Our results are summarised in Table 7.3, where we report the mean values of the cosmological free parameters with the associated 1σ error. We also show in the figures of this section the behaviour of Ω_Λ and Ω_M in the non-flat curvature assumption for the Λ CDM and w CDM models, as well as the dependence of the parameters of the equation of state both on Ω_M and on the other dark energy parameters (see Figures 7.2, 7.3, 7.4, 7.5, 7.6, 7.7, and 7.8, respectively). The confidence levels are always the 1, 2, and 3 σ intervals, corresponding to the 68.3, 95, and 99.7% confidence level, respectively. We point out that the H_0 value is fixed in all the models of our analysis. This leads to some differences (e.g. the degeneracy direction of the confidence contours shown in Figure 7.2 for the BAO data in the non-flat Λ CDM model) in our results with other similar analyses already presented in the literature [e.g. 4, 288]. Furthermore, our fitting procedure is based on a simpler treatment of the cosmological model than the CosmoMC [289] and Montepython [290] algorithms (which are among the most widely used codes). Therefore, care should be taken when comparing our results with previous works in the literature that make use of the cosmological packages listed above, especially when employed with different data sets and calibrations of the probes. The implementation of the presented analysis in CosmoMC and Montepython is presented in the following section.

Our analysis shows that the constraints on the cosmological parameters obtained from the SNe+QSOs+BAO sample are not always a simple average of those derived from SNe+QSOs and BAO separately (see Table 7.3). In other words, the 2D contours of the combined sample are not necessarily the geometric intersection of the individual projections. This effect is caused by the relative orientation of the contours in the multi-dimensional space defined by the free model parameters. The overall consistency for the SNe+QSOs and BAO data sets thus requires the individual constraints to be compatible in the full multi-dimensional space, since the mere agreement of pairs of cosmological parameters could be misleading [see 288]. The combination of these two data sets is

7.2. The combination of quasars, SNe Ia and BAO in view of alternative cosmological models

Model	Data set	Ω_M	Ω_Λ	w_0	w_a
Flat Λ CDM	SNe+QSOs	$0.295^{+0.013}_{-0.012}$			
	BAO	$0.373^{+0.056}_{-0.048}$			
	SNe+QSOs+BAO	0.300 ± 0.012			
Non-flat Λ CDM	SNe+QSOs	0.504 ± 0.029	$1.107^{+0.051}_{-0.052}$		
	BAO	$0.376^{+0.057}_{-0.049}$	$0.638^{+0.071}_{-0.079}$		
	SNe+QSOs+BAO *	$0.364^{+0.022}_{-0.021}$	0.829 ± 0.035		
Flat w CDM	SNe+QSOs	$0.403^{+0.022}_{-0.024}$		$-1.494^{+0.132}_{-0.143}$	
	BAO	$0.381^{+0.057}_{-0.050}$		$-1.049^{+0.098}_{-0.116}$	
	SNe+QSOs+BAO	$0.369^{+0.022}_{-0.023}$		$-1.283^{+0.094}_{-0.108}$	
Non-flat w CDM	SNe+QSOs	$0.280^{+0.041}_{-0.037}$	$1.662^{+0.041}_{-0.048}$	$-0.667^{+0.024}_{-0.027}$	
	BAO	$0.301^{+0.080}_{-0.072}$	$0.463^{+0.072}_{-0.058}$	$-2.850^{+1.459}_{-1.441}$	
	SNe+QSOs+BAO *	$0.224^{+0.018}_{-0.017}$	$1.667^{+0.040}_{-0.047}$	$-0.626^{+0.012}_{-0.013}$	
CPL	SNe+QSOs	$0.447^{+0.023}_{-0.027}$		$-1.267^{+0.196}_{-0.191}$	$-3.771^{+2.113}_{-2.496}$
	BAO	$0.420^{+0.073}_{-0.070}$		$-0.821^{+0.469}_{-0.349}$	$-1.269^{+1.835}_{-2.608}$
	SNe+QSOs+BAO	$0.354^{+0.032}_{-0.030}$		$-1.323^{+0.103}_{-0.112}$	$0.745^{+0.483}_{-0.974}$
JBP	SNe+QSOs	$0.441^{+0.025}_{-0.028}$		$-1.250^{+0.223}_{-0.209}$	$-4.282^{+2.680}_{-3.283}$
	BAO	$0.384^{+0.103}_{-0.098}$		$-1.091^{+0.923}_{-0.727}$	$0.235^{+4.922}_{-6.612}$
	SNe+QSOs+BAO	$0.354^{+0.032}_{-0.030}$		-1.371 ± 0.141	$1.127^{+1.293}_{-1.547}$
Exponential	SNe+QSOs	$0.395^{+0.023}_{-0.026}$		$-1.481^{+0.141}_{-0.147}$	
	BAO	$0.371^{+0.058}_{-0.051}$		$-1.067^{+0.102}_{-0.119}$	
	SNe+QSOs+BAO	$0.359^{+0.023}_{-0.024}$		$-1.271^{+0.092}_{-0.107}$	
BA	SNe+QSOs	$0.452^{+0.022}_{-0.025}$		$-1.316^{+0.172}_{-0.168}$	$-2.654^{+1.329}_{-1.626}$
	BAO	$0.410^{+0.086}_{-0.081}$		$-0.930^{+0.464}_{-0.333}$	$-0.423^{+1.064}_{-1.671}$
	SNe+QSOs+BAO	$0.307^{+0.044}_{-0.055}$		$-1.303^{+0.115}_{-0.106}$	$1.010^{+0.152}_{-0.466}$

Table 7.3: Best-fit values and 1σ uncertainties for the cosmological free parameters in each model and data set. The symbol * identifies the data sets for which the analysis is not statistically justified due to the tension between the probes (even if the results are here reported for sake of clarity).

then likely possible and meaningful only if the derived constraints are consistent, which means that the separate confidence levels overlap (within approximately 2σ) both in the marginalised posterior distributions of each of the shared free parameters and in the 2D projections of each pair of them. By comparing both the 1D and 2D information, we find that only the non-flat models show a significant discrepancy between the two probes. Below we summarise the main results of our analysis.

Starting from the flat concordance model, the only cosmological free parameter is Ω_M . The best-fit values obtained separately with SNe+QSOs and BAO are consistent within 2σ (see Table 7.3) and the combination of the two data sets gives $\Omega_M = 0.300 \pm 0.012$. This result completely agrees with the latest cosmological evidence [e.g. 291, 3, 80]. When the flatness condition is removed, the SNe+QSOs and BAO data sets show a strong discrepancy on the bound value of the cosmological parameters (see Table 7.3 and Figure 7.2). Indeed, the red confidence levels from BAO alone in Figure 7.2 are

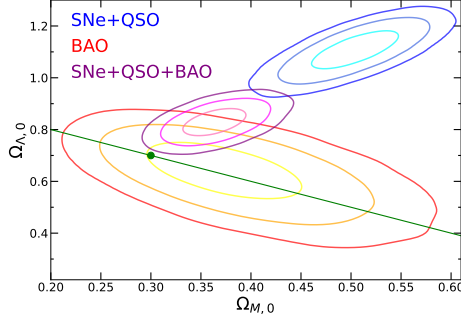


Figure 7.2: Bi-dimensional contour plot (Ω_M, Ω_Λ) with $1-3\sigma$ confidence levels obtained from each data set in the legend for the non-flat Λ CDM model analysis. The green line is the place of points corresponding to a flat model and the green point on it is located at $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$, close to the values expected in a flat Λ CDM model. The confidence levels for the SNe+QSOs+BAO data set are shown here just for sake of clarity since the two probes show strong incompatibility and their combined analysis is not meaningful (see the main text).

completely consistent with the flat constraint $\Omega_\Lambda + \Omega_M + \Omega_r = 1$ represented by the green line, although with a preference for $\Omega_M > 0.3$ and $\Omega_\Lambda < 0.7$, corresponding to a 1σ discrepancy from the flat Λ CDM model with $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$ (green point), the values expected from the latest observational evidence. Instead, the blue contours from SNe+QSOs favour a closed Universe (i.e. $\Omega_k < 0$) with high values for both Ω_M and Ω_Λ , in agreement with the results of [42, 292]. As already stated in [288], in this case, where BAO and SNe+QSOs data do not give compatible results in 2σ , it is not appropriate to consider the joint SNe+QSOs+BAO data set, due to the tension between the two data sets. Nevertheless, just for the sake of clarity, Table 7.3 also reports the mean values from the combined sample and Figure 7.2 shows the corresponding constraints in the bi-dimensional free parameter space (purple contours).

The results for the flat w CDM model are reported in Table 7.3 and the behaviour of Ω_M with w is shown in Figure 7.3. The SNe+QSOs data set (in blue) shows a tension greater than 3σ with the green point located at $(0.3, -1)$, that corresponds to a cosmological constant and $\Omega_M = 0.3$, preferring a higher Ω_M and a more negative w . The best-fit value $w \sim -1.5$ corresponds indeed to a phantom dark energy scenario. The BAO data set (in red) instead gives a w distribution consistent with $w = -1$, but with Ω_M always greater than $\Omega_M = 0.3$. The combination of the two data sets, justified by their consistency within 2σ , retains the same anti-correlation between Ω_M and w as that of SNe+QSOs, with a resulting 3σ discrepancy from the flat Λ CDM model with $\Omega_M = 0.3$, which favours a larger value of Ω_M and $w < -1$.

In the case where both a constant equation of state of dark energy not fixed at $w = -1$ and a non-flat curvature of the Universe are considered, we obtain that the SNe+QSOs and BAO data cannot be combined due to their strong inconsistency. Indeed, Table 7.3 and Figure 7.4 show that the two probes give compatible values of Ω_M , but completely

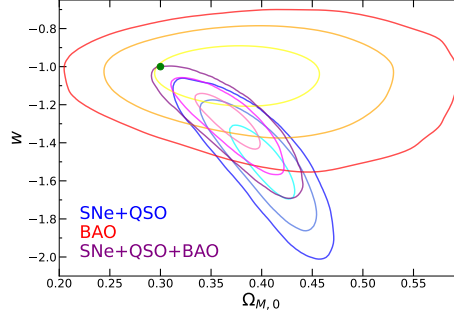


Figure 7.3: Bi-dimensional contour plot (Ω_M, w) with 1-3 σ confidence levels obtained from each data set in the legend for the flat w CDM model analysis. The green point is located at $\Omega_M = 0.3$ and $w = -1$, corresponding to the cosmological constant case.

discrepant values of Ω_Λ and 2D contour plots (Ω_M, Ω_Λ) at more than 3 σ tension. As BAO are not sensitive to the equation of state parameter of the dark energy, and thus such a data set cannot constrain w , we do not show the behaviour of w with the other parameters. Anyway, we are mostly interested in studying the implications for the spatial curvature of the Universe. In this regard, as for the non-flat Λ CDM model, the BAO alone are consistent with a flat Universe within 2 σ with a preference towards $\Omega_M \sim 0.3$ and $\Omega_\Lambda \sim 0.5$, while SNe+QSOs prefer a negative curvature (i.e. a closed Universe) with $\Omega_M \sim 0.3$ but with a high value of Ω_Λ . Despite the inconsistency, as in the case of the non-flat Λ CDM case, we report in Table 7.3 and Figure 7.4 also the results obtained

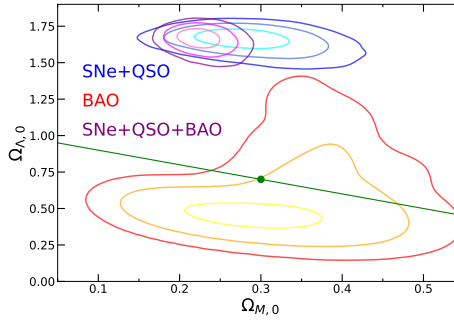


Figure 7.4: Bi-dimensional contour plot (Ω_M, Ω_Λ) with 1-3 σ confidence levels obtained from each data set in the legend for the non-flat w CDM model analysis. The green line is the place of points corresponding to a flat model and the green point on it is located at $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$, close to the values expected in a flat Λ CDM model. The confidence levels for the SNe+QSOs+BAO data set are shown here just for sake of clarity since the two probes show strong incompatibility and their combined analysis is not meaningful, as explained in the main text.

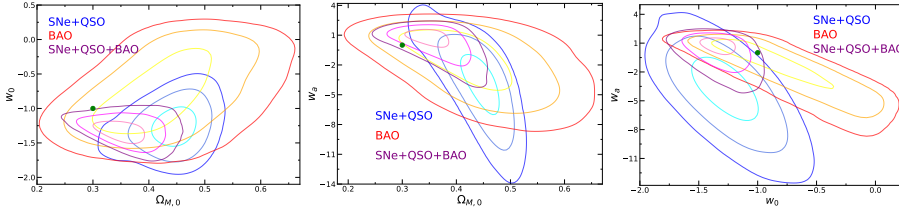


Figure 7.5: Contour plots for all the pairs of cosmological free parameters in the flat CPL model. The green point corresponds to a flat Λ CDM model with $\Omega_M = 0.3$. The colours-data sets association is reported in the legend.

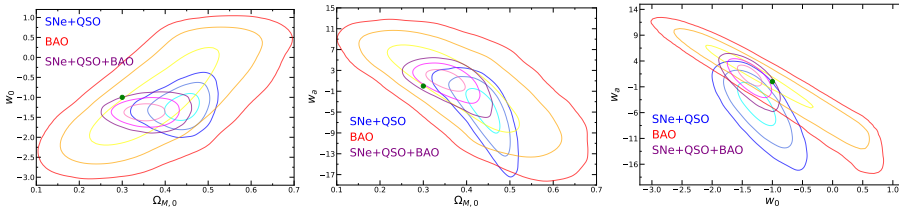


Figure 7.6: Bi-dimensional contour plots with $1-3\sigma$ confidence levels obtained from each data set in the legend for the flat JBP model. The green point corresponds to a flat Λ CDM model with $\Omega_M = 0.3$.

from the joint sample.

Concerning the flat CPL model, Figure 7.5 shows the 2D contour plots for all the cosmological free parameters in each of the data sets considered. Indeed, the constraints from SNe+QSOs (blue contours) and BAO alone (red contours) are always completely consistent (also due to the large uncertainties) and we can combine them. We can note that both the data sets favour $\Omega_M > 0.4$, while we obtain $w_0 < -1$ and $w_a \sim -4$ from SNe+QSOs and w_0 consistent with -1 and $w_a \sim -1.3$ from BAO, at more than 3σ and at about 1σ from the flat Λ CDM model with $\Omega_M = 0.3$ for each pair of free parameters, respectively. The joint fit (in purple) yields tighter constraints with Ω_M shifted towards $\Omega_M = 0.3$, $w_0 < -1$ completely consistent with SNe+QSOs and w_a slightly positive, with an overall statistical significance of $2-3\sigma$ relative to the reference flat Λ CDM prediction. The change in sign of w_a from negative to positive values (or at least values consistent with $w_a = 0$) is mainly due to the inclusion of the BAO and it agrees with the analysis shown in [293, 3]. Compared to the results of the w CDM model, the best-fit values of Ω_M and w are fully consistent in each data set, while their correlation is different due to the presence of the additional w_a parameter of the CPL model.

Figure 7.6 shows the results of the flat JBP model for each pair of the cosmological free parameters. The SNe+QSOs data set (blue contours) again shows a strong discrepancy with the flat Λ CDM model with $\Omega_M = 0.3$, favouring $\Omega_M > 0.3$, $w_0 < -1$, and $w_a \sim -4$. The BAO alone (red contours) instead are consistent at 1σ level with the reference model.

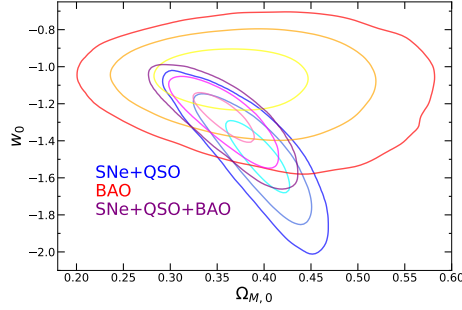


Figure 7.7: Bi-dimensional contour plot (Ω_M, w_0) with $1-3\sigma$ confidence levels obtained from each data set in the legend for the flat exponential model.

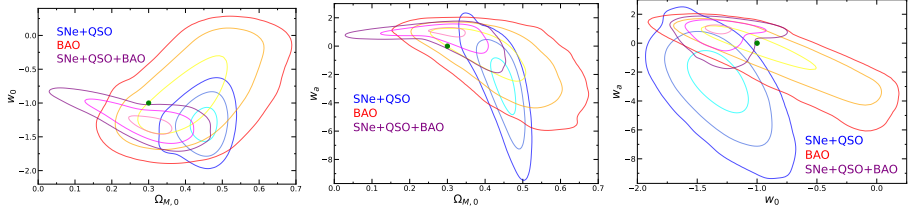


Figure 7.8: Bi-dimensional contour plots with $1-3\sigma$ confidence levels obtained from each data set in the legend for the flat BA model. The green point corresponds to a flat Λ CDM model with $\Omega_M = 0.3$.

Despite this difference, the constraints from the two samples are compatible, also due to the large uncertainties (mainly in the BAO); this allows us to fit them together. The combined fit (purple contours) provides a $2-3\sigma$ discrepancy from $\Omega_M = 0.3$, $w_0 = -1$, and $w_a = 0$, towards a more negative w_0 and larger values of Ω_M and w_a . The shift in sign of w_a and the correlations (or anti-correlations) between each pair of parameters are the same already seen in the CPL model. The best-fit values of all the free parameters in each data set agree with those obtained in the previous models.

In the exponential model, we deal with only two cosmological free parameters: Ω_M and w_0 . Figure 7.7 shows the results of the fit with the same colours-data sets association used above. Unlike the other models studied, in this case, we cannot make a direct comparison with the flat Λ CDM model in the bi-dimensional contour plot of Figure 7.7. Indeed, there is no value of w_0 that turns Eq. (1.9) into Eq. (1.3). This is the reason why we do not show the green point in the contour plot, as in the other figures. Nevertheless, Ω_M and w_0 always have the same physical meaning; this means that we can compare their best-fit values with those expected in a flat Λ CDM model. The analysis of this model shows that the SNe+QSOs sample (blue contours) favours $\Omega_M \sim 0.4$ and $w_0 \sim -1.5$, corresponding to a phantom dark energy scenario, while the BAO sample (red contours) tends towards a w_0 consistent with $w_0 = -1$, although with a value of Ω_M always close to $\Omega_M = 0.4$. The best-fit from the joint sample (purple contours) keeps the same Ω_M ,

while w_0 is pushed to values more negative than $w_0 = -1$. Correspondingly, the best-fit values of Ω_M and w_0 are at 2-3 σ from $\Omega_M = 0.3$ and $w_0 = -1$. These results are in full agreement with those of the flat w CDM model, for both the best-fit values and the correlation among the parameters in each data set. The best-fit values of Ω_M and w_0 are also consistent with those of the CPL model.

Regarding the flat BA model, Figure 7.8 shows the bi-dimensional contour plots obtained from all data sets. Compared to a flat Λ CDM model with $\Omega_M = 0.3$, the best-fits from SNe+QSOs and BAO samples show respectively a tension greater than 3 σ and a consistency at 1 σ level. Nevertheless, their constraints are consistent. Their combination changes the sign of the best-fit value of w_a , as in the previous models, and shifts Ω_M towards values consistent with $\Omega_M = 0.3$ and w_0 towards values more negative than $w_0 = -1$, with an overall tension of about 3 σ from the reference flat Λ CDM prediction. In all the data sets the best-fit values are in agreement with those from the other models and the correlation (or anti-correlation) between the parameters is the same as that obtained from the CPL and JBP models.

In summary, under the assumption of a spatially flat Λ CDM model, Ω_M is fully consistent with $\Omega_M = 0.3$ in all the data sets, as expected from recent cosmological observations [e.g. 291, 3, 80]. However, all the other models show a deviation from this prediction, with a statistical significance of the order of 2-3 σ for the combined sample of SNe+QSOs+BAO. We point out that the cosmological parameters obtained from the separate samples of SNe+QSOs and BAO are consistent (within 2σ) in all the models so that they can be combined, except for the non-flat cases studied. In these models, indeed, BAO confirm the flatness condition while SNe+QSOs show evidence of a closed Universe (Figures 7.2 and 7.4). The models with a dark energy density evolving in time show a conclusive common trend with respect to the prediction of the Λ CDM model (Figures 7.5, 7.6, 7.7, and 7.8). Indeed, the combination SNe+QSOs+BAO always favours $\Omega_M > 0.3$, $w_0 < -1$, and w_a larger but consistent with $w_a = 0$. This phantom dark energy behaviour is mainly driven by the contribution of SNe+QSOs, while the BAO data set is in general statistically in agreement with the prediction of the flat Λ CDM model (also due to larger uncertainties) and is responsible for the shift towards $\Omega_M = 0.3$ and $w_a = 0$. In conclusion, while the BAO measurements are in agreement with the standard model, the combined analyses with SNe+QSOs+BAO rule out the flat Λ CDM model with $\Omega_M = 0.3$, green point in the figures, at 2-3 σ , with evidence for a phantom dark energy scenario in all the models where the dark energy density is allowed to vary with time.

7.3 The comparison with the CMB and the interaction in the dark sector

As anticipated, we have also originally implemented the use of QSOs as cosmological probes in the commonly adopted algorithms of CosmoMC, Montepython, and Cobaya, which allow us to further include new probes, such as the CMB and DES data, into our analyses. Therefore, we have fitted several cosmological models by employing separately and, when justified by the relative consistency (see Section 7.2), jointly QSOs, Pantheon SNe Ia, BAO, DES, and CMB. All the above algorithms are Monte Carlo Markov Chain codes using Boltzmann solver codes such as “Cosmic Linear Anisotropy Solving System” [CLASS; 294] for the theory predictions.

More specifically, we have analyzed the CMB measurements from the Planck data [295], using “TT,TE,EE+lowE” data by combination of the temperature power spectra (TT) and the cross-correlation TE and EE polarization spectra over the multipole range $\ell \in [30, 2508]$, the low- ℓ temperature Commander likelihood, and the low- ℓ SimAll EE likelihood [see 295, for details]. Additionally, in our analysis we have considered the lensing reconstruction power spectrum from the latest Planck satellite data release [295, 296]. For the Pantheon SNe Ia, we have included both statistical and systematic uncertainties, also marginalising over the nuisance parameter M since the theoretically modelled apparent magnitude is calculated as $m_{\text{th}} = \mu_{\text{th}} + M = 5 \log D_L + M$, where μ_{th} is the theoretical distance modulus. M depends on H_0 and the absolute magnitude M and for the Pantheon sample it is estimated to be $M = -19.263 \pm 0.049$ [297]. Hereafter, we indicate this data set with “Pth”. For the DES data, we have used the first-year release (DES-1Y) results that combine configuration-space two-point statistics from three different cosmological probes: cosmic shear, galaxy–galaxy lensing, and galaxy clustering, using data from the first year of DES observations. We have used the Cobaya implementation of the DES likelihood with covariance matrix, power spectra measurements, and nuisance parameters in agreement with [298], [299], and [300]. We refer to this data set as “DES”.

In the following, we also detail the use of QSOs, which we have originally implemented for the Monte Carlo Markov Chain algorithms. First, we fit the combined sample of Pth and QSOs with the cosmographic orthogonal fifth-order logarithmic polynomial described later in Chapter 9. This procedure is completely independent of a cosmological model since it uses a cosmographic approach. In addition, it marginalises over the free parameters γ , β , and δ . Thus, by fixing γ and β to the values obtained with this cosmographic method, we avoid any dependence on an assumed cosmological model which could bias our analysis and affect our results. The best-fit values obtained from the cosmographic analysis are: $\gamma = 0.591 \pm 0.011$, $\beta = -31.475 \pm 0.008$, and $\delta = 0.209 \pm 0.004$. At this point, we can insert these values into Eq. (2.3) to compute D_L for the QSO sample, for which we consider the cut at $z = 0.7$ (see Section 3.1). Then, we can compute the corresponding distance moduli and their associated uncertainties $\Delta(\mu) = \sqrt{\sigma_\mu^2 + \delta_\mu^2}$, where σ_μ is the statistical error propagated from D_L to the distance modulus and δ_μ is the intrinsic dispersion of the X-UV distance modulus-redshift relation and is related to

δ by $\delta_\mu = 5\delta/(2|\gamma - 1|)$, with γ and δ fixed at their best-fit values. Following these steps, we end up with the distance moduli and their associated uncertainties for each QSO source, since we do not have to account for any systematic error; this is the QSO physical quantity that we use as input in the Monte Carlo Markov Chain algorithms. Although the parameters of the X-UV relation are fixed in this procedure, we have to include and marginalise over a calibration nuisance parameter “ k ”, just as required by the Pantheon code for SNe Ia. It is worth noting that our results are not affected by fixing the values of γ and β . Indeed, in principle we could also marginalize over these parameters or use the value of γ obtained by fitting QSOs in narrow redshift bins, as detailed in Section 2.2.2. Nevertheless, if we marginalize over γ and β , the presence of a tension between the observational data and the investigated cosmological model could be alleviated or completely removed by changing the best-fit values evaluated for these parameters. More precisely, it could happen that the cosmological model is not in agreement with the probes, but it seems to reproduce the data well, and thus that there is no discrepancy, since the tension is shifted to the values of γ and β , which are altered compared to their intrinsic values. Indeed, γ and β are not arbitrary, but are intrinsic to the X-UV relation and are therefore determined by the physics of this relation. To avoid this problem, we have fixed γ and β through the above-described cosmology-independent technique.

The goodness of our novel procedure has been checked by comparing the CosmoMC/Montepython/Cobaya analysis results with those obtained with the Python package emcee [276]. The weak differences detected between the two different analyses are due to the fact that the latter package is not implemented with Boltzmann solver code, as the first ones. For example, it does not include cosmological free parameters such as $\Omega_b h^2$ and $\Omega_{\text{cdm}} h^2$, where Ω_b and Ω_{cdm} are the density parameters of the baryon and cold dark matter components, respectively. However, the results are completely statistically consistent. With the data sets just described, we start our investigation from the standard flat and non-flat cosmological model and then we extend it by considering the flat and non-flat w CDM models, the flat CPL, and the “iDE” models, introduced in Section 1.1.

In our analysis, we first aim to test the physical consistency among the individual probes, which is a crucial point to validate a joint analysis, as detailed in the previous section. Therefore, we have performed a parameter estimation study with each data set presented assuming a specific cosmological model and then repeated this procedure for each model considered in our work. To this purpose, we need to select a single set of initial conditions and parameter priors that can be used for all analyses, which means to ensure an exploration of the parametric space as similar as possible for all models and for all data sets, and thus a more robust comparison of the results. This can prove to be a challenging choice that needs to be carefully evaluated, as the results can be affected by the initial conditions and choices made. On the one hand, each data set analysed provides information on a specific epoch, and thus it is sensitive to a specific set of parameters while leaving the others largely unconstrained. On the other hand, our goal is to identify a set of parameters that can be left free for all the considered data sets, thus ensuring the use of consistent assumptions for each model as far as possible. We will later discuss the impact of the initial assumptions on the results of the analysis.

As a matter of fact, in our analyses involving only large-scale data, such as SNe Ia,

QSOs, BAO, and DES, we choose to work with three free parameters, which are Ω_M , Ω_b , and H_0 using wide and flat priors in the following ranges:

$$\Omega_M = [0.01 : 0.90] \quad \Omega_b = [0.01 : 0.20] \quad H_0 = [20 : 200]$$

where H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$. Also, we assume the big bang nucleosynthesis (BBN) consistency, interpolating the primordial helium abundance value provided by table from PARthENoPE fits¹ and we impose a Gaussian prior on the baryon density parameter motivated by the observations of the D/H abundance [301], which is $\Omega_b h^2 = 0.0222 \pm 0.0005$. At the same time, we fix the remaining cosmological parameters, namely the spectral tilt, n_s , the optical depth, τ , the primordial scalar amplitude, A_s , and the curvature density, Ω_k , to the following values, as constrained by [3]:

$$n_s = 0.965 \quad \tau = 0.055 \quad A_s = 2.21 \times 10^{-9} \quad \Omega_k = 0.$$

Indeed, the large scale data from SNe Ia, QSOs, BAO, and DES are only sensitive to the components of the background evolution, the $H(z)$ equation, with no information about the primordial or re-ionization parameters. In addition, we consider a free curvature density in the non-flat standard cosmological model and the w CDM one, in order to test the compatibility of the models under this condition as well. Instead, when considering the CMB data, all the vanilla cosmological parameters, namely the set of $\{\Omega_b, \Omega_{cdm}, n_s, A_s, \tau, \theta\}$, where θ is the ratio between the sound horizon and the angular diameter distance at the decoupling, are free to vary over a wide flat range. Therefore, we consider models with more free parameters with respect to models analysed with large-scale data, since we assume that fixing them to defined values, although in agreement with the Planck data, would have introduced an unjustifiable bias.

Before presenting our results, it is important to point out that, as anticipated, a correct consistency test cannot be performed by investigating only 2D contours separately, but must take into account all the parameter space simultaneously. Actually, 2D graphs are only projections of spaces onto surfaces, that is, slices of analysis, and such projections can sometimes be misleading. Therefore, we will only show a few contour planes to illustrate our main results, but our analysis has been extended to a complete study of the entire parametric space of the model under consideration [279, 302]. In this regard, we show in the plots and tables only the results obtained from data sets that are found to be compatible, which means that they are in agreement within a 3σ statistical level, as pointed out by [302]. Then, we consider *in agreement* measurements compatible up to 2σ , while we report a *difference* when the analysis meets at 3σ . In particular, we show the results assuming a flat spatial curvature in Table 7.4, while the results obtained by considering the curvature as a free parameter are provided in Table 7.5. Below, we detail the results of our analyses.

We start by examining the standard cosmological model. Assuming a flat geometry of the Universe (see Table 7.4), we find a general agreement between the CMB and DES constraints within 1σ , while the values of the H_0 parameter meet at 2σ for CMB

¹The primordial abundances of helium and deuterium are calculated as a function of the baryon density and the extra relativistic number of species. [PARthENoPE website](#).

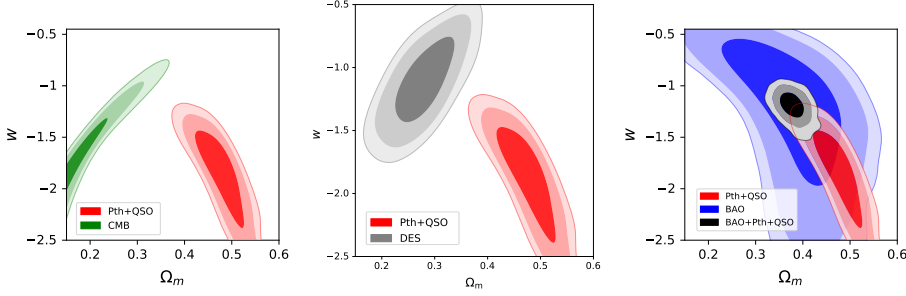


Figure 7.9: Bi-dimensional contour plot (Ω_M , w) at 1, 2, and 3 σ confidence levels obtained from each data set in the legend for the flat w CDM model analysis.

and BAO. Also, in both the Hubble and the matter density values a 2σ agreement is reached between CMB and Pth+QSOs. We note a preference of the late-time data (i.e. Pth+QSOs and BAO), for a higher amount of matter density than that estimated by the CMB, and this effect is in agreement with the results of previous works in the literature [286, 15, 244]. In general, we note that the probes are in agreement in at least within 2σ , although they still show slightly different preferences. This could be read as a problem of the standard model, which should include extra physics, implying more degrees of freedom to increase the compatibility between the data sets. Indeed, the rigidity of this model, due to the small number of free parameters, does not seem to allow it to capture the greater complexity that emerges from the observations. This may hint at the need to extend the simplest cosmological model by considering additional mechanisms and parameters.

The first extension we have considered is a non-flat Universe with the non-flat Λ CDM model (see Table 7.5), where we adopted a prior range for the curvature within $[-0.5, 0.5]$. We note from our results that Pth+QSOs and CMB meet at 3σ on H_0 , while Pth+QSOs are in agreement at 2σ on H_0 and Ω_M with DES. The single probes are compatible on the Ω_k parameter with the exception of Pth+QSOs and CMB, which prove to be inconsistent at more than 3σ . Indeed, Pth+QSOs meet DES within 1σ and BAO within 2σ . Thus, only the combinations of Pth+QSOs+DES and Pth+QSOs+BAO are shown in Table 7.5. Concerning these combined analyses, the joint data set of Pth+QSOs+BAO significantly reduces the uncertainty on H_0 and leads to values of Ω_M and H_0 compatible with those of the single probes and a negative value of Ω_k driven by Pth+QSOs. Furthermore, the Pth+QSOs+DES data set provides a value of Ω_M close to that preferred by DES alone, a H_0 intermediate between the two data sets, and a Ω_k closer to the flatness assumption compared to the negative values favored by Pth+QSOs and DES alone. Noteworthy, Pth+QSOs constrain the matter density and H_0 values in agreement within 1σ with the flat case, while the CMB prefers higher values, recovering the flat Universe in 2σ , as also shows in other works [296, 42, 279].

The simplest extension of the vanilla Λ CDM assumption is to consider the w CDM model. As extensively analysed, the CMB data from the latest Planck release constrain a value of w other than -1 in 1σ [3], favouring non-standard values. Instead, when

the CMB is combined with other data, such as BAO, a value fully compatible with -1 is recovered. Therefore, it is reasonable to leave this parameter free and explore the w CDM model as the first test of any alternative analysis; hence, we consider w as a free parameter of the theory, within the flat prior in the range $[-3,0]$. Below, we report the results of our analysis in two specific situations: considering a flat Universe and relaxing this assumption.

In the case of a flat Universe, the constraints obtained with the Pth+QSOs data set (red contours in Figure 7.9) are in tension at more than 3σ with the CMB (green contours) and DES (gray contours) on the Ω_M parameter, while they are compatible with the constraints obtained from BAO (blue contours) within 1σ on all the free parameters. The combined BAO+Pth+QSOs analysis is placed as the geometric intersection between the analysis of two data sets alone and follows, in terms of the anti-correlation between the parameters, the Pth+QSOs analysis, showing that this sample plays the leading role in constraining the $(w - \Omega_M)$ plane. Our results are compatible with our analysis described in Section 7.2, the small differences due to what has already been described. Table 7.4 also shows the compatibility of the Pth+QSOs data set with BAO. As already discussed, we adopt the strategy of considering not statistically justified to combine probes that independently do not show compatibility within 3σ , as is in the case of the CMB. The latter data set alone shows a weak power on constraining H_0 and w , so leaving the equation of state parameter free leads to a significant deterioration in the parameter constraint. Given the incompatibility in the Ω_M estimate between CMB and Pth+QSOs, as well as DES and Pth+QSOs, it is not statistically and physically correct to explore the combined CMB+Pth+QSOs and DES+Pth+QSOs data set. Such a joint analysis is thus not indicated in Table 7.4, nor shown in Figure 7.9. These results reveal that a one-parameter extension in the dark energy sector, compared to the standard cosmological model, preserves compatibility between the late-time data but proves to be unsuitable to describe both CMB and Pth+QSOs observations, or even a combination of DES and Pth+QSOs.

Relaxing the flatness assumption, the Pth+QSOs data set prefers large total matter density values and high and negative curvature densities (see Table 7.5), with $0.397 < \Omega_M < 0.663$ and $\Omega_k < -0.395$ at 99% confidence level. While Pth+QSOs provide only an upper limit of the curvature density, BAO, DES, and CMB give narrow constraints on the curvature, as reported in Table 7.5. Such constraints are in tension with the Pth+QSOs upper limit at more than 3σ , and therefore the combined analyses are not reliable. Let us deserve a further comment about these analyses. In this case, we see that the Pth+QSOs data set is unable to jointly constrain the matter density and curvature, due to the degeneracy of these two parameters and the low sensitivity of this probe to Ω_k . The analyses force Ω_k to the lower values allowed by the adopted prior, and, at the same time, to high values of Ω_M . Extending the Ω_k prior ranges would shift the values further. Pth+QSOs analyses are therefore *unreliable*, and their compatibility (or non-compatibility) with other probes is merely driven by the chosen priors.

The CPL parameterisation is the simplest model to allow for a dynamic evolution of the dark energy. As this model describes a late physics mechanism, the CMB is not very sensitive to its degrees of freedom, and more decisive in constraining their values

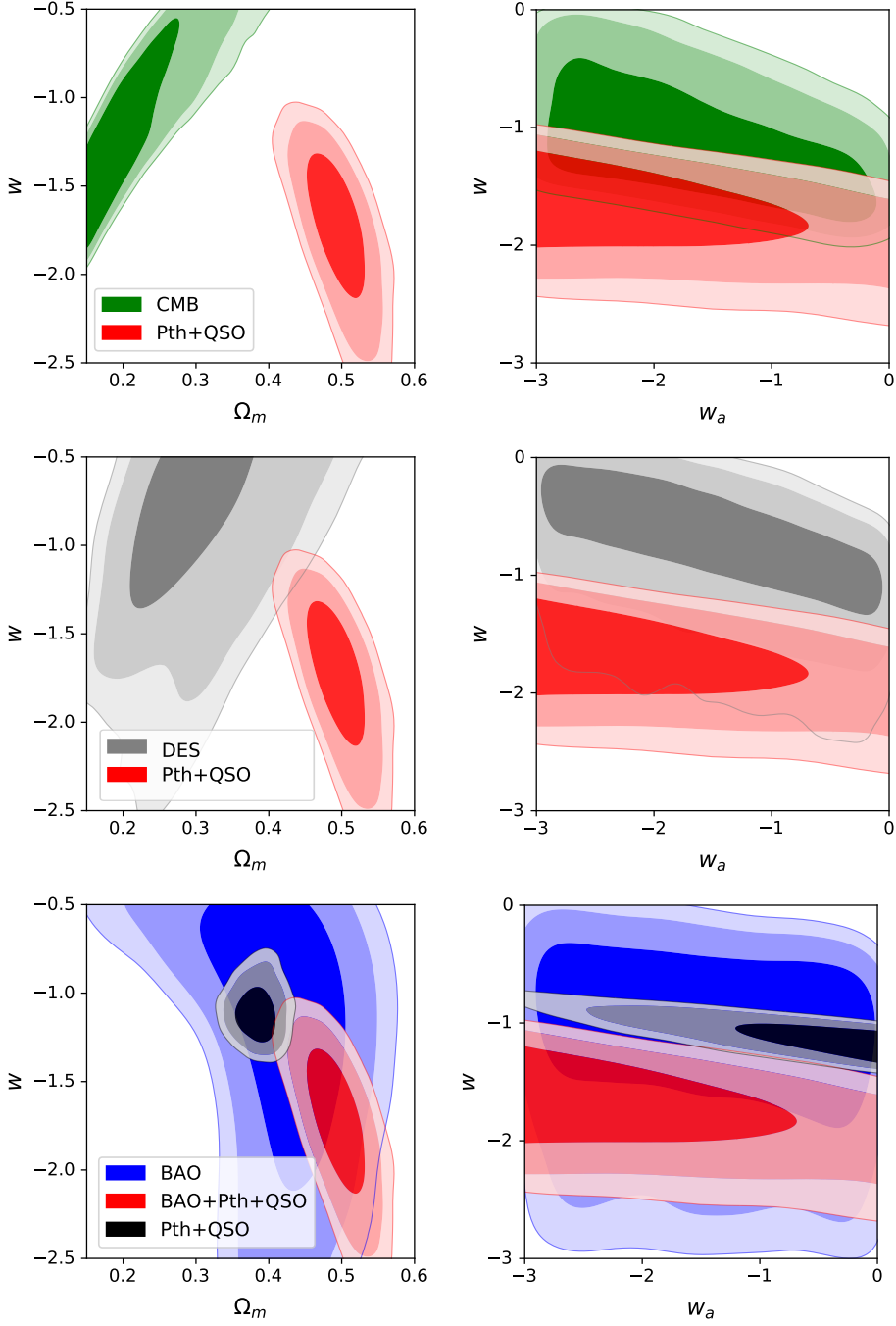


Figure 7.10: Bi-dimensional contour plot at 1, 2, and 3 σ confidence levels obtained from each data set in the legend for the flat CPL model analysis.

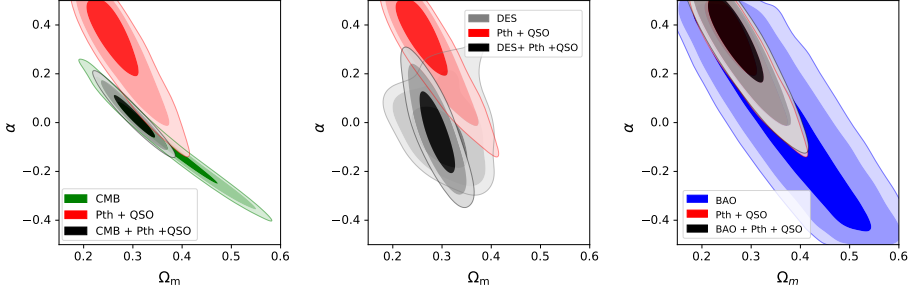


Figure 7.11: Bi-dimensional contour plot at 1, 2, and 3 σ confidence levels obtained from each data set in the legend for the flat iDE model analysis.

are the measurements of BAO, Redshift Space Distortion (RSD), SNe Ia (see the full analysis by [3]) and Fast Radio Bursts [FRBs; 303]. These data constrain a w_a value consistent with zero, although slightly negative values are preferred. The current data of CMB+BAO+SNe Ia constrain $w = -0.957 \pm 0.080$ and $w_a = -0.29^{+0.32}_{-0.26}$ [3], while the Pantheon + SNe Ia sample with SH0ES Cepheids distance provides $w = -1.81^{+1.71}_{-0.60}$ and $w_a = -0.4^{+1.0}_{-1.8}$ [273]. In our study, we find that no data set imposes constraints on w_a . As shown by the plots on the right side of Figure 7.10, w shows no significant correlation with w_a across all the data sets. Since w_a is unconstrained, the CPL model restricts a parametric space very similar to that of the w CDM model, and the addition of a dark energy scale dependence does not lead to a compatibility between CMB and Pth+QSOs (top panel of Figure 7.10) and DES and Pth+QSOs (middle panel of Figure 7.10), leaving instead the BAO consistent with Pth+QSOs (bottom panel of Figure 7.10). In addition, the CMB (and DES) do not constrain the w parameter, imposing only an upper (lower) limit on the dark energy equation of state.

Finally, we consider a GCG model that allows for non-gravitational interactions between the dark components via a dissipation mechanism. Such a mechanism can generate either the creation of dark particles from the expanding vacuum or an increasing effect of dynamical dark energy. The dissipation is characterized by the parameter α , which behaves at the background level like cold matter in the early times and tends to become a cosmological constant in the asymptotic future. A value of α less than zero would indicate the creation of dark matter from dark energy, whilst a positive value would indicate the opposite. Previous works have shown that late-universe data favour negative values of α [304], while the inclusion of the CMB calls for values more compatible with zero [63, 73, 51, 52, 75, 305, 306], and SNe+QSOs favour positive values [307, 308].

In our analyses, we explore this model using a wide, flat prior in the α range of $[-0.5, 0.5]$. Our results show that this model allows reconciling CMB, BAO, and DES with Pth+QSOs, as shown in Figure 7.11 and Table 7.4. Interestingly, we note that the prediction of the standard model of a larger amount of Ω_M using the Pth+QSOs sample than that predicted by the CMB is solved by introducing the GCG dissipation mechanism. Indeed, the high positive values of α implies a dissipation of vacuum energy

Model	Data set	Ω_M	H_0 (km s ⁻¹ Mpc ⁻¹)	w	w_a	α
Flat Λ CDM	Pth+QSOs	0.357 ± 0.025	71.79 ± 1.74	-1	-	-
	CMB	0.316 ± 0.007	67.34 ± 0.54	-1	-	-
	BAO	$0.371^{+0.044}_{-0.053}$	$72.06^{+3.53}_{-2.68}$	-1	-	-
	DES	0.300 ± 0.049	66.76 ± 7.79	-1	-	-
	Pth+QSOs+CMB	0.316 ± 0.008	67.33 ± 0.60	-1	-	-
	Pth+QSOs+BAO	0.358 ± 0.024	71.14 ± 2.30	-1	-	-
Flat w CDM	Pth+QSOs+DES	0.285 ± 0.014	69.44 ± 2.35	-1	-	-
	Pth+QSOs	0.478 ± 0.041	71.70 ± 4.97	-1.87 ± 0.28	-	-
	CMB	$0.130^{+0.018}_{-0.067}$	$111.70^{+28.43}_{-20.70}$	< -1.24	-	-
	BAO	$0.377^{+0.081}_{-0.053}$	$78.31^{+12.50}_{-16.54}$	$-1.26^{+0.62}_{-0.27}$	-	-
	DES	0.281 ± 0.033	68.53 ± 4.73	-1.10 ± 0.21	-	-
	Pth+QSOs+BAO	0.378 ± 0.017	75.33 ± 1.74	-1.19 ± 0.08	-	-
Flat CPL	Pth+QSOs	0.494 ± 0.027	73.37 ± 1.76	-1.74 ± 0.28	unconstrained	-
	CMB	$0.219^{+0.028}_{-0.091}$	82.92 ± 10.71	< -0.28	unconstrained	-
	BAO	$0.414^{+0.067}_{-0.045}$	$82.87^{+13.98}_{-17.12}$	$-1.18^{+0.76}_{-0.38}$	unconstrained	-
	DES	$0.306^{+0.053}_{-0.069}$	65.74 ± 8.57	> -1.48	unconstrained	-
	Pth+QSOs+BAO	0.383 ± 0.018	75.54 ± 1.73	-1.11 ± 0.10	> -1.96	-
	Pth+QSOs+DES	0.282 ± 0.009	70.56 ± 1.61	-0.90 ± 0.05	> -0.54	-
Flat iDE	Pth+QSOs	$0.264^{+0.018}_{-0.025}$	73.35 ± 1.82	-1	-	$0.34^{+0.15}_{-0.06}$
	CMB	$0.364^{+0.027}_{-0.081}$	$65.61^{+2.73}_{-2.15}$	-1	-	-0.08 ± 0.11
	BAO	$0.359^{+0.045}_{-0.060}$	71.25 ± 3.83	-1	-	$0.04^{+0.44}_{-0.20}$
	DES	$0.303^{+0.032}_{-0.038}$	68.77 ± 5.55	-1	-	-0.06 ± 0.09
	Pth+QSOs+CMB	0.302 ± 0.024	67.79 ± 0.91	-1	-	0.03 ± 0.05
	Pth+QSOs+BAO	$0.271^{+0.031}_{-0.040}$	$73.98^{+2.03}_{-1.99}$	-1	-	$0.34^{+0.15}_{-0.06}$
	Pth+QSOs+DES	0.289 ± 0.021	71.17 ± 3.21	-1	-	-0.04 ± 0.10

Table 7.4: Mean values and 1σ uncertainties for the cosmological free parameters in each model with zero curvature and data set considered. When upper limit values are indicated, these refer to the value in 2σ . Furthermore, when the probability distribution is not Gaussian, the value is indicated with the two distinct limits of the 1σ error.

into dark particles, and thus captures the extra amount of CDM predicted by the QSOs, thus reconciling the cosmological parameter estimates. In general, the dissipation values for each analysis are always compatible with zero in 1σ and thus they recover the Λ CDM model, with the exception of Pth+QSOs, which instead recover the standard model in 3σ . It is also worth noting that this model leads to a broadening of the error bars of the cosmological parameters using the CMB data, compared to the Λ CDM model, so that in 3σ it arrives at the maximum value of $H_0 = 70.92 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Combining the data from Pth+QSOs with the other probes makes α compatible with the standard model, with the exception of the Pth+QSOs+BAO data set, which instead shows a clear preference for H_0 values compatible with the direct observations of SNe Ia and a non-zero dissipation mechanism between the dark components. This is because the BAO are not sensitive to the dissipation parameter (see right panel of Figure 7.11) while it is the Pth+QSOs data set that drives the α parameter constraint. The combined analyses shown in Figure 7.11 lie as the geometric intersection of the contour levels of the two data sets analysed independently only for the combination with BAO. As for DES, the combined analysis lies on DES and it acquires the parameter correlation of Pth+QSOs. As for the CMB, the combined analysis lies on the CMB but α is limited to the negative values as shown by Pth+QSOs.

As anticipated, we here also discuss the role of the assumptions in our analysis.

7.3. The comparison with the CMB and the interaction in the dark sector

Model	Data set	Ω_M	H_0 (km s ⁻¹ Mpc ⁻¹)	w	Ω_k
non-flat Λ CDM	Pth+QSOs	0.376 ± 0.024	72.50 ± 2.04	-1	< -0.061
	CMB	0.351 ± 0.024	63.61 ± 2.27	-1	-0.010 ± 0.007
	BAO	0.362 ± 0.060	72.04 ± 7.5	-1	-0.005 ± 0.120
	DES	0.285 ± 0.044	67.42 ± 6.0	-1	-0.034 ± 0.068
	Pth+QSOs+BAO	0.366 ± 0.018	73.63 ± 1.59	-1	< -0.049
	Pth+QSOs+DES	0.280 ± 0.011	70.17 ± 1.65	-1	-0.007 ± 0.033
non-flat w CDM	Pth+QSOs	0.536 ± 0.054	73.87 ± 1.76	-1.11 ± 0.12	< -0.433
	CMB	$0.237^{+0.053}_{-0.178}$	$88.27^{+18.21}_{-35.54}$	< -0.57	$-0.007^{+0.010}_{-0.001}$
	BAO	0.376 ± 0.056	76.41 ± 8.22	-1.31 ± 0.38	$0.048^{+0.104}_{-0.074}$
	DES	0.291 ± 0.047	67.23 ± 6.75	-0.92 ± 0.17	$-0.067^{+0.067}_{-0.101}$

Table 7.5: Mean values and 1σ uncertainties for the cosmological free parameters in each model with curvature and data set considered. When upper limit values are indicated, these refer to the value in 2σ . Furthermore, when the probability distribution is not Gaussian, the value is indicated with the two distinct limits of the 1σ error.

Indeed, particular attention should be deserved to the initial conditions, as the setting of parameter priors and boundary choices significantly influence the results. While it is true that the choice of fixing certain parameters while allowing others to vary is often taken arbitrarily in a given model, attention should be paid to select the parameters of the theory that are sensitive to the data sets analysed, since leaving one of them fixed necessarily means spoiling the results. Analogously, it is also important to choose a sufficiently high prior range so that the parametric space is adequately explored.

As a didactic example, we show the significant changes in the parameter constraints for the flat w CDM model when we vary some of the assumptions used in our analyses ². For our purposes, we study the analysis with the BAO data. We emphasize here that the BAO measurements of $D_V(z)$ depend on the $H(z)$ evolution and $D_M(z)$ depends on the H_0 value (see Section 1.2.2). This means that considering the parameters of the matter densities and the local value of the Hubble constant is essential for an analysis with BAO data using measures of $D_V(z)$ and $D_M(z)$ [309]. In addition, it is important to break the degeneracy between the baryonic and the dark matter using the prior provided by the primordial abundance of the BBN, as well as impose the constraint on the baryonic matter that can reproduce the current observations. We plot the analysis with the characteristics used in our main analysis in blue in Figure 7.12, compared with the results obtained by changing the initial assumptions, marked with different colours. For example, we can notice that reducing the prior of the H_0 parameter to a maximum of $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (green curves), as it is generally employed in cosmological analyses,

²As already explained, we consider as free parameters of the MCMC analysis Ω_m , Ω_b , and H_0 , imposing wide and flat priors as $\Omega_m = [0.01 : 0.60]$, $\Omega_b = [0.01 : 0.20]$, and $H_0 = [20 : 200] \text{ km s}^{-1} \text{ Mpc}^{-1}$. Also, the BBN consistency and a Gaussian prior motivated by the observations of the D/H abundance [301] are considered.

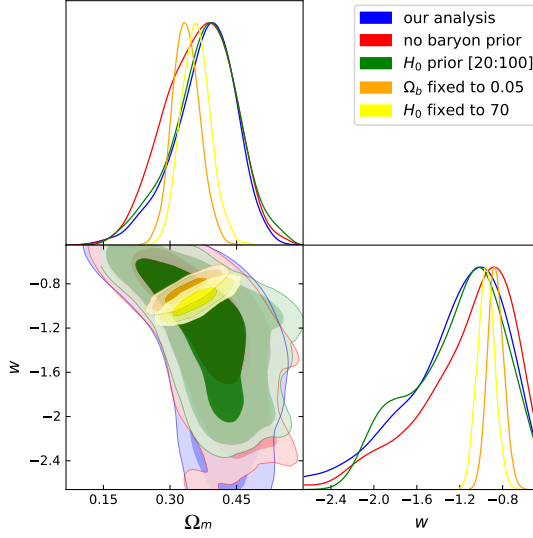


Figure 7.12: Analysis of the flat w CDM model, using BAO data and different assumptions with respect to the ones used in our main analysis, which consist of leaving free parameters Ω_M , Ω_b , and H_0 , and assuming the BBN consistency and the D/H abundance prior on $\Omega_b h^2$ (this analysis is drawn with blue line).

reduces the parametric space exploration of the equation of state parameter w in 2σ contour levels. This can be significant but not decisive, since large deviations from $w = -1$ are generally not taken into account. Conversely, fixing H_0 to a specific value (i.e. $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in our case) tightens the constraints on both w and the total density of matter also changing the correlation among parameters, as shown by the yellow curves, with a significant impact on the results and their physical interpretation. Even neglecting the prior on the baryonic density can slightly change the parameter space (red curves), while fixing the value of the baryonic matter density (orange curves) significantly alters the results with a similar impact as the choice of fixing H_0 .

We can conclude from these tests that reducing the parameter space exploration of the current Hubble constant value or the current baryonic density can lead to misleading results both in terms of the compatibility of the data sets and in the proper comparison of different models, if BAO data are used. Finally, from our analysis, summarized in Tables 7.4 and 7.5 and Figures 7.9, 7.10, and 7.11, we have shown that:

- the Λ CDM model is able to reconcile the Pth+QSOs data with the other probes considered, constraining the free parameters values within 2σ .
- in the non-flat Λ CDM model, BAO and DES probes are still compatible with Pth+QSOs, while the CMB is in tension at more than 3σ . The Pth+QSOs only impose an upper limit on the density curvature parameter. These results indicate a clear failure of this model to simultaneously describe information at all scales.

- extending the dark energy sector with a constant equation of state that is not fixed at $w = -1$ does not make the CMB and DES consistent with Pth+QSOs, and it only gets worse if we extend this model by introducing a non-zero curvature. Indeed, none of the probes are consistent with Pth+QSOs in the non-flat w CDM model, since this data set favours large, negative values of the curvature density.
- in the CPL model, the data do not appear to be sensitive to the parameter governing the variation of the equation of state with the scale, w_a , so the parametric space restricted by this model is essentially the same as that of the w CDM model (where $w_a = 0$), with equal or larger error bars depending on the probe considered.
- the GCG iDE model here considered reconciles the CMB and Pth+QSOs data sets by preferring an interaction that generates dark matter particles from vacuum energy. Furthermore, the DES and BAO probes are in agreement with all the other data sets.

Our results point out that where the simplest models of a dark energy extension fail to describe the Universe evolution at all scales, the more complicated model involving a dark energy-dark matter interaction can succeed. This would therefore show the path to be explored to make several cosmological and astrophysical observations compatible, especially in the light of future QSO data that will be available from the Euclid and Athena collaborations [310, 311].

8 A statistical analysis on cosmological likelihoods and its impact on the determination of cosmological parameters

In the literature, it is common practise to perform cosmological analyses using the well-known Gaussian likelihood to constrain cosmological parameters. However, this approach relies on the assumption that the normalized residuals of the physical quantity under study, which depends on the probe considered, are actually Gaussianly distributed. This requirement is generally taken for granted and is not checked each time different probes and/or data sets are investigated. We have therefore questioned the Gaussianity assumption on the likelihood traditionally used for SNe Ia from both the Pantheon and Pantheon + samples and for QSOs, GRBs, and BAO. We here discuss our methodology in Section 8.1 and its application to SNe Ia (Section 8.2.1) and to QSOs, GRBs, and BAO (Section 8.2.2). Finally, we focus on the impact of our results on the determination of cosmological parameters in Section 8.3. These topics are presented in details in our papers of [312], [243], and [244].

8.1 The methodology

To investigate the Gaussianity assumption, we apply the Anderson-Darling [313, 314] and Shapiro-Wilk [315] normality tests, we compute the skewness and kurtosis of the distributions of the normalized residuals, and we employ the skewness test [316], the kurtosis test [317], and a test that combines both features [318], which are embedded in the `scipy` Python package. The Anderson-Darling and Shapiro-Wilk tests determine whether a data sample is drawn from the Gaussian distribution and they are frequently applied in the literature (e.g. [206] in astrophysics and [319, 320] in statistics). An important property of these tests is their consistency: they can identify any small deviation from normality, if the sample size is large enough [313, 321]. Therefore, the larger the sample size, the more these tests tend to reject the null hypothesis of a normal distribution [322],

even in case of very small deviations from the Gaussianity, such as in the presence of ties caused by limited measurement precision (i.e. the number of decimals). This weakness limits the applicability of the Anderson-Darling and Shapiro-Wilk tests on large samples and need to be accounted for.

Therefore, we include investigations on the skewness and kurtosis. The skewness is the third standardized moment and measures the asymmetry of the distribution as it identifies extreme values in one tail compared to the other. The kurtosis is the fourth standardized moment and measures extreme values in both the tails: a large kurtosis corresponds to more populated tails than those of a Gaussian distribution, small kurtosis to less populated tails than those of a Gaussian distribution. A normal distribution has skewness and kurtosis equal to zero¹. Hence, the skewness and kurtosis of a distribution provide information about the deviation from normality. Based on this, we use the skewness and kurtosis tests, to determine whether the values of the skewness and kurtosis of the investigated distribution are so statistically close to zero that it can be well approximated by a Gaussian distribution. Furthermore, we apply the “skewness+kurtosis” test, which simultaneously detect deviations of skewness and kurtosis from the Gaussian case. These complementary methods are crucial to obtain reliable and robust results when testing the Gaussianity assumption.

8.2 The investigation of the likelihood for different cosmological probes

8.2.1 The case of type Ia supernovae

Focusing on SNe Ia, we investigate the Gaussianity of the Pantheon and Pantheon + samples by examining the χ^2 quantity used to evaluate the cosmological parameters, that is defined as [80]:

$$\chi^2 = \Delta\mu^T C^{-1} \Delta\mu. \quad (8.1)$$

In this formula, $\Delta\mu = \mu_{th} - \mu_{obs}$, where μ_{th} and μ_{obs} are the distance moduli predicted from an assumed cosmological model and those measured, respectively, and C is the uncertainty matrix constructed as $C = D_{stat} + C_{sys}$, where D_{stat} is the diagonal matrix of statistical errors and C_{sys} the systematic covariance matrix, both provided by the SNe Ia releases [80, 273, 81]. As a first step, we investigate whether the argument $\Delta\mu_{norm} = C^{-1/2} \Delta\mu$ [323] of the likelihood (hereafter denoted with $\mathcal{L}$) of the SNe Ia distance moduli in the Pantheon and Pantheon + satisfies the Gaussianity condition within a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We define $\Delta\mu_{norm}$ as before to investigate the normalized residuals of the distance moduli while accounting for both the diagonal and the non-diagonal terms of the C matrix, which include both statistical and systematic uncertainties. Since the computation of $\Delta\mu_{norm}$ requires the assumption of a specific cosmological model, we have chosen to report here the results obtained assuming a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Indeed, this value of

¹We consider the Fisher’s definition of the kurtosis, in which 3 is subtracted from the result to give 0 for a normal distribution.

Ω_M is the one reported in [80] and the value of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the one obtained in several works that combine SNe Ia with other probes [e.g. 324, 325, 8]. However, we have checked the impact of this cosmological assumption on our results by computing the distribution of $\Delta\mu_{norm}$ by imposing other values of Ω_M . More specifically, we have tested the two extreme cases of $\Omega_M = 0.1$ and $\Omega_M = 1$. Under these new assumptions, as expected, the values of skewness and kurtosis of the distribution of $\Delta\mu_{norm}$ change, but our results on the Gaussianity tests, the best-fit likelihoods, as well as the cosmological results, do not depend on the assumption of a specific cosmological model.

Both the Pantheon and Pantheon + SNe Ia samples do not pass neither the Anderson-Darling nor the Shapiro-Wilk test. Therefore, the null hypothesis that $\Delta\mu_{norm}$ comes from a Gaussian distribution should be rejected. The Anderson-Darling test rejects the Gaussianity hypothesis at a significance level $> 15\%$. This rejection is confirmed by the Shapiro-Wilk test as its p-value is $< 5\%$, the minimum value required to accept the null hypothesis. Given the above-mentioned limits of these methods, we compute the skewness and kurtosis of the $\Delta\mu_{norm}$ distributions. The Pantheon sample has skewness $= -0.08$ and kurtosis $= 0.44$ corresponding to an excess of the tail on the left, while the Pantheon + shows a skewness $= -0.50$ and more pronounced tails with kurtosis $= 3.9$ (see Figure 8.1). As also visible from the right panel of Figure 8.1, the skewness of Pantheon + is larger than that of Pantheon. Furthermore, none of the samples passes the skewness, kurtosis, and “skewness+kurtosis” tests. As anticipated, we have also verified that the results of all the normality tests are independent of the assumed cosmology. All of these independent normality tests produce coherent and consistent results, guaranteeing the common outcome and strongly proving that the $\Delta\mu_{norm}$ of SNe Ia is not normally distributed, neither for the Pantheon nor for the Pantheon + sample. As a consequence, the Gaussianity assumption for SNe Ia $\Delta\mu_{norm}$ is not statistically legitimated and it should be checked for each SNe Ia sample used. Therefore, the $\Delta\mu_{norm}$ distributions are not well fitted by the Gaussian PDF, defined as:

$$\text{PDF}_{\text{Gaussian}} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\hat{x}}{\sigma}\right)^2} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\chi^2} \quad (8.2)$$

where $\hat{x}$ is the mean, σ is the standard deviation, $x - \hat{x} = \Delta\mu$, and χ^2 is defined in Eq. (8.1).

Since the tests described above fail, we further extend our analysis. First, we fit the $\Delta\mu_{norm}$ values to find the best-fit distribution with the Mathematica FindDistribution tool, which finds the best distribution among the first 20 best-fitting distributions. We have checked the reliability of these results with python as well. More specifically, we have employed the Mathematica tool by requiring that each distribution investigated is compared with the others by all possible properties evaluated by FindDistribution, including the logarithm of the likelihood value, the Bayesian Information Criterion (BIC), the Akaike Information Criterion (AIC), and the p-values of the Pearson χ^2 and Cramer Von Mises tests. After this comparison, the fitting distributions are determined starting from the best-fit one to those that reproduce the data with the highest accuracy. Indeed, the value of the likelihood and the BIC and AIC criteria provide an estimate of the quality of the statistical model investigated for a given set of data. Additionally,

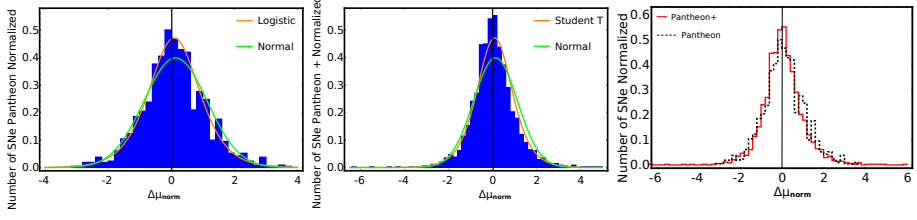


Figure 8.1: Normalized $\Delta\mu_{\text{norm}}$ histograms, defined as $\Delta\mu_{\text{norm}} = C^{-1/2} \Delta\mu$, for the 1048 SNe Ia in the Pantheon sample (left panel) and the 1701 SNe Ia in the Pantheon + sample (middle panel). The green curve is the best-fit Gaussian distribution, while the orange curves are the best-fit logistic (left panel) and Student's t (middle panel) distributions. The right panel shows the superimposition of the Pantheon and Pantheon + distributions as in the legend. In all the panels the vertical black line marks the zero line.

the Pearson χ^2 and the Cramer Von Mises tests include, respectively, the Pearson and Cramer Von Mises goodness-of-fit tests with the null hypothesis that the data are drawn from a population with a specific distribution. For these two tests, if the p-value is small enough (usually $p < 0.05$ by convention), then the null hypothesis is rejected and we conclude that the observed data do not follow this distribution.

With this procedure, we fit the $\Delta\mu_{\text{norm}}$ histograms and we obtain as the best fit for the Pantheon sample a logistic distribution, with the following PDF

$$\text{PDF}_{\text{logistic}} = \frac{e^{-\frac{(x-\hat{x})}{s}}}{s \left(1 + e^{-\frac{(x-\hat{x})}{s}}\right)^2} = \frac{e^{-\chi'^2}}{s \left(1 + e^{-\chi'^2}\right)^2} \quad (8.3)$$

where s is the scale and the variance σ^2 is $\sigma^2 = (s^2 \pi^2)/3$. Indeed, in the case of the logistic distribution, the χ^2 is the same as defined in Eq. (8.1) but where the elements of C are multiplied by $3 \sigma^2 / \pi^2$. Hence, χ'^2 is defined as $\chi'^2 = \Delta\mu^T (3 \sigma^2 C / \pi^2)^{-1} \Delta\mu$. The left panel of Figure 8.1 shows the best-fit logistic distribution with $\hat{x} = -0.019$ and $s = 0.55$ (orange curve) and the Gaussian one with $\hat{x} = 0.030$ and $\sigma = 0.99$ (green curve). The same analysis is then repeated for the Pantheon + data. In this case, the distribution that best fits the histogram of $\Delta\mu_{\text{norm}}$ is a Student's t-distribution, for which the generalized PDF is given by

$$\text{PDF}_{\text{student}} = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu \pi} s \Gamma\left(\frac{\nu}{2}\right)} \left[1 + \frac{((x-\hat{x})/s)^2}{\nu}\right]^{-\frac{\nu+1}{2}} = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu \pi} s \Gamma\left(\frac{\nu}{2}\right)} \left[1 + \frac{\chi'^2}{\nu}\right]^{-\frac{\nu+1}{2}} \quad (8.4)$$

where Γ is the gamma function, ν are the degrees of freedom, and $\sigma^2 = (s^2 \nu)/(\nu - 2)$. As explained for the logistic case, this relation between s and σ allows us to substitute C used in χ^2 of Eq. (8.1) for the Student's t. More specifically, $\chi'^2 = \Delta\mu^T [(\nu - 1) \sigma^2 C / \nu]^{-1} \Delta\mu$. In our case, $x = \Delta\mu$ and $\hat{x} = 0$, as before. The best-fit distributions of the Student's t with $\hat{x} = 0.01$, $s = 0.20$, and $\nu = 4.2$ (in orange) and

the Gaussian with $\hat{x} = 0.018$ and $\sigma = 0.30$ (in green) are shown in the middle panel of Figure 8.1.

As visible from Figure 8.1, for the Pantheon and Pantheon + respectively, the logistic and the Student's t distributions better reproduce the $\Delta\mu_{norm}$ in all features (e.g. peak, width, and tails) compared to the Gaussian PDF and these results remain the same if we change the cosmological assumption of a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, as already anticipated. For visual clarity, the right panel of Figure 8.1 shows the comparison between the normalized residuals $\Delta\mu_{norm}$ of Pantheon (in dashed gray) and Pantheon+ (in continuous red). This superimposition clearly proves that the Pantheon+ distribution is more peaked and shows heavier tails compared to Pantheon. This difference is due to the sources included in Pantheon + and not in Pantheon. Furthermore, this discrepancy causes the difference between the two corresponding best-fit distributions, namely the change from the logistic distribution for the Pantheon SNe Ia to the Student's t for the Pantheon +.

8.2.2 The case of quasars, Gamma-Ray Bursts, and Baryonic Acoustic Oscillations

Following the procedure described in the previous section, we extended the investigation of the Gaussianity assumption to QSOs, GRBs, and BAO. For the sake of clarity, we here generically denote with Δ the normalized residuals computed for each probe with its own physical quantity: $\Delta\mu_{\text{SNe Ia}} = \mu_{obs, \text{SNe Ia}} - \mu_{th, \text{SNe Ia}}$ for SNe Ia, $\Delta\mu_{\text{GRBs}} = \mu_{obs, \text{GRBs}} - \mu_{th, \text{GRBs}}$ for GRBs, $\Delta\log L_X = \log L_{X, obs} - \log L_{X, th}$ for QSOs, and $\Delta d_z = d_{z, obs} - d_{z, th}$ for BAO. As anticipated for the case of SNe Ia (Section 8.2.1), the computation of the theoretical quantities requires the assumption of a specific cosmological model, hence we here assume a flat Λ CDM model with $\Omega_M = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Nevertheless, we test our results with respect to the assumptions for Ω_M and H_0 . In addition, the computation of $\mu_{obs, \text{GRBs}}$ and $\log L_{X, obs}$ requires to fix the values of the parameters of the 3D Dainotti and RL relations, respectively. Thus, we use the values obtained from the fit of these relations in the different evolutionary cases. We also notice that, for the study of the Gaussianity assumptions, the two treatments of the evolution (i.e. fixed and varying) are equivalent since the evolutionary parameter k of the fixed correction is also obtained from the assumption of $\Omega_M = 0.3$ and the function $k(\Omega_M)$ is computed for $\Omega_M = 0.3$, as assumed in our Gaussianity tests.

Regarding the Gaussianity tests of Anderson-Darling, Shapiro-Wilk, skewness, kurtosis, and skewness+kurtosis, GRBs pass all these tests, regardless of the treatment of the redshift evolution and the values assumed for Ω_M and H_0 . Indeed, the distribution in the left panel of Figure 8.2 might appear to be approximately non-Gaussian, but this is only due to the binned representation of the few data points. On the contrary, it turns out to be Gaussian under our statistical investigations, independently of the small size of the GRB sample. The computation of the skewness and kurtosis is not affected by the assumptions on the cosmological parameters, but it is slightly impacted by the approach employed for the redshift evolution. More precisely, without considering the evolution, the skewness and kurtosis of the $\Delta\mu_{\text{GRBs}}$ distribution are 0.34 and -0.74, respectively,

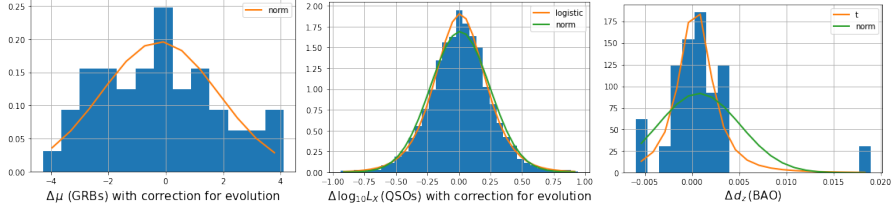


Figure 8.2: Normalized Δ histogram for the 50 GRBs (left panel) corrected for the redshift evolution, the 2421 QSOs corrected for the redshift evolution of luminosities (middle panel), and the 26 BAO (right panel). In the left panel, the orange curve is the best-fit distribution, which is Gaussian. In the middle and right panels, the green curve is the Gaussian distribution with the parameters that best reproduce the data, while the orange curves are the best-fit logistic and Student's t distributions, respectively.

with a central value shifted towards a positive value. Indeed, in this case, the best-fit Gaussian distribution has $\hat{x} = 1.38$ and $\sigma = 2.0$. When the correction for the evolution is included, the distribution becomes more symmetric around 0 with a skewness of 0.25, kurtosis of -0.65, and best-fit Gaussian parameters $\hat{x} = -0.21$ and $\sigma = 2.04$ (see left panel of Figure 8.2).

When examining the QSO residuals, the results of the normality tests do not depend on the fixed values of Ω_M and H_0 , nor on the approach for the redshift evolution. However, in all cases, the null hypothesis of the Gaussianity of $\Delta \log L_X$ is rejected by the Anderson-Darling, Shapiro-Wilk, kurtosis, and “skewness+kurtosis” tests. Only the skewness test is satisfied because the value of the skewness is very close to 0. Indeed, without the correction for the evolution, the skewness is 0.07 (and the kurtosis is 0.84), so the test is fulfilled with $p\text{-value} = 0.14$. Correcting for the evolution, the skewness and kurtosis become 0.0007 and 0.79, respectively, and the $p\text{-value}$ of the skewness test significantly increases up to 0.99. For a direct visualization, we show in the middle panel of Figure 8.2 the histogram obtained including the redshift evolution correction. It is indeed interesting to note from the computed skewness values that the $\Delta \log L_X$ distribution is symmetric around 0 and this symmetry increases when accounting for the redshift evolution. As a matter of fact, the best-fit Gaussian distribution (green curve in the middle panel of Figure 8.2) has $\hat{x} = 0.009$ and $\sigma = 0.24$. Nevertheless, due to the high kurtosis, the combination of the two features, considered in the skewness+kurtosis test, confirms the non-Gaussian distribution of $\Delta \log L_X$. Similarly, BAO do not pass any of the normality tests, regardless of the a-priori values assumed for Ω_M and H_0 to compute $d_{z,th}$. Despite its asymmetry, the distribution is centred on 0. Indeed, the best-fit Gaussian curve (in green in the right panel of Figure 8.2) has $\hat{x} = 0.001$ and $\sigma = 0.004$.

Since both the QSO and BAO residuals proved to follow a non-Gaussian distribution, we employ the methodology described in Section 8.2.1 to determine their proper best-fit distributions. Fitting the QSO $\Delta \log L_X$, we find that, for all the evolutionary cases, the best-fit distribution is a logistic distribution, whose PDF is that in Eq. (8.3). The middle

panel of Figure 8.2 shows the normalized histogram, once accounted for the correction for evolution, superimposed with the best-fit logistic curve (in orange) with $\hat{x} = 0.009$ and $s = 0.13$ and with the best-fit Gaussian curve (in green) with $\hat{x} = 0.009$ and $\sigma = 0.24$. In the case without evolution, the best-fit parameters for the logistic and the normal distributions are $\hat{x} = 0.0006$ and $s = 0.13$ and $\hat{x} = 0.002$ and $\sigma = 0.24$, respectively. For BAO, the best-fit of the Δd_z histogram is the Student's t distribution in Eq. (8.4). The best-fit Student's t curve shown in orange in the right panel of Figure 8.2 has $\hat{x} = 0.0003$, $s = 0.002$, and $\nu = 2.23$, while the best-fit Gaussian curve in green has $\hat{x} = 0.001$ and $\sigma = 0.004$. As for the Pantheon and Pantheon + SNe Ia, for both QSOs and BAO, the logistic and the Student's t distributions, respectively, better reproduce Δ in all the features compared to the Gaussian PDF. To further discuss what a deviation from non-Gaussianity might mean and why the functional form is different for the different probes, we have compared the histograms of the observed and predicted distance moduli of the GRBs and SNe Ia, the observed and predicted angular distance of BAO, and the observed and predicted luminosities for QSOs. By comparing the distributions of measured and modeled quantities for these probes, we notice that the observed distribution presents a larger variance than the theoretical distribution. This variance originated from the larger uncertainties and the intrinsic dispersion of the measurements. The level of discrepancy between the two distributions causes the rejection of the null hypothesis for the statistical tests in the cases of SNe Ia, QSOs, and BAO. This means that their actual observed physical quantities are not exactly reproduced by a Gaussian, but require another kind of distribution, even though the physics behind this result is still under investigation. We here again stress that these investigations on the Gaussianity assumption and the appropriate distributions are crucial to validate the results of the cosmological analyses and must be carried out on every probe used and also on future new samples.

8.3 Implication for cosmology

In this scenario of non-Gaussianity, the appropriate choice of the cosmological likelihood is crucial to obtain statistically reliable results and to better constrain the cosmological parameters, thus reducing their uncertainties. Therefore, we here describe the cosmological implications of our results on the likelihoods of SNe Ia, QSOs, GRBs, and BAO.

8.3.1 The case of SNe Ia alone

Starting from the case of SNe Ia, the newly found distributions are used in $\mathcal{L}$ (denoted with $\mathcal{L}_{new}$) instead of the Gaussian PDF. Thus, we fit the flat Λ CDM model with the Kelly method [280] using $\mathcal{L}_{new}$ and considering these cases: H_0 fixed and Ω_M free, Ω_M fixed and H_0 free, and both parameters free. We impose the wide uniform priors $0 \leq \Omega_M \leq 1$ and $50 \text{ km s}^{-1} \text{ Mpc}^{-1} \leq H_0 \leq 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ to ensure that the parameter's space is explored in all physical regions. We then repeat the fit with the Gaussian likelihood, $\mathcal{L}_{Gauss}$, thus comparing the outcomes. Furthermore, we test how the cosmological constraints vary when we change the assumptions on the fixed parameter in the cases of one free parameter. Specifically, we consider $\Omega_M = 0.30$ and $\Omega_M = 0.34$,

8.3. Implication for cosmology

$\mathcal{L}_{logistic}$			$\mathcal{L}_{Gauss}$		
Pantheon	H_0	Ω_M	H_0	Ω_M	
	70	0.295 ± 0.007	70	0.300 ± 0.009	
	73.04	0.173 ± 0.006	73.04	0.144 ± 0.007	
	70.00 ± 0.13	0.3	70.00 ± 0.13	0.3	
	69.47 ± 0.12	0.34	69.43 ± 0.14	0.34	
	70.21 ± 0.20	0.285 ± 0.012	69.99 ± 0.34	0.300 ± 0.021	
$\mathcal{L}_{Student}$			$\mathcal{L}_{Gauss}$		
Pantheon+	H_0	Ω_M	ν	H_0	Ω_M
	70	0.532 ± 0.010	3.35 ± 0.07	70	0.587 ± 0.011
	73.04	0.345 ± 0.007	3.05 ± 0.06	73.04	0.349 ± 0.010
	73.50 ± 0.11	0.3	3.07 ± 0.06	73.51 ± 0.12	0.3
	73.06 ± 0.11	0.34	3.05 ± 0.06	73.06 ± 0.13	0.34
	72.93 ± 0.16	0.352 ± 0.011	3.06 ± 0.06	72.85 ± 0.24	0.361 ± 0.019

Table 8.1: Best-fit values with 1σ uncertainties for the Pantheon and Pantheon+ SNe Ia samples, for all cosmological cases and $\mathcal{L}$. Fixed cosmological parameters are in bold. H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$.

respectively from [80] and [273], and we investigate $H_0 = 73.04 \text{ km s}^{-1} \text{Mpc}^{-1}$ from [9], and $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$ based on several works [e.g. 325, 8].

Table 8.1 reports the results of these analyses showing the best-fit values with 68% confidence level for each case considered. In addition, the left panel of Figure 8.3 shows in blue the corner plots from the logistic likelihood ($\mathcal{L}_{logistic}$), while the right panel of Figure 8.3 shows those from the Student's t likelihood ($\mathcal{L}_{Student}$), also in blue. For the Student's t, we treat the degrees of freedom ν as a free parameter by imposing a wide uniform prior in the range $0 < \nu < 6$. Indeed, ν does not manifest any correlation with the cosmological free parameters, as can be seen in the right panel of Figure 8.3, hence the estimates of Ω_M and H_0 are independent on the treatment of ν , which is if we fix it or leave it free. Thus, we decided to apply the more general approach of using ν as a free parameter, which also allows us to perform a more accurate analysis, without the need for knowing the value of this parameter a priori. The results of the fit of the flat Λ CDM model with $\mathcal{L}_{Gauss}$ are reported in Table 8.1 and in red in Figure 8.3.

Considering the Pantheon sample, the cosmological parameters obtained with $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{logistic}$ are compatible within 1σ , except for Ω_M when $H_0 = 73.04 \text{ km s}^{-1} \text{Mpc}^{-1}$, that shows a $2 - 3\sigma$ discrepancy. Indeed, this H_0 value is not the one favoured by Pantheon, that fixes $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$. Regarding the uncertainties, while the errors are similar in the cases with only one free parameter, $\mathcal{L}_{logistic}$ shows significantly tighter constraints when both Ω_M and H_0 are free parameters: the errors on Ω_M and H_0 are reduced by 43% (from 0.021 to 0.012) and 41% (from 0.34 to 0.20), respectively. The same conclusions can be drawn for the Pantheon + sample. The only difference is that the 3σ discrepancy in Ω_M emerges when $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$. This can be attributed to the value of $H_0 = 73.04 \text{ km s}^{-1} \text{Mpc}^{-1}$ assumed for Pantheon +. Again, for the Pantheon + SNe Ia, the use of $\mathcal{L}_{Student}$ reduces the errors on Ω_M and H_0 , when both are free parameters, by 42% (from 0.019 to 0.011) and 33% (from 0.24 to 0.16), respectively. For

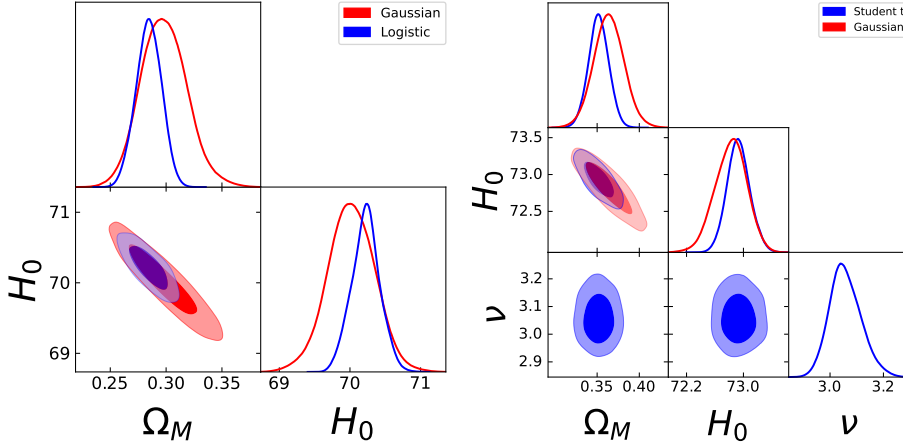


Figure 8.3: Fit of the flat Λ CDM model with Ω_M and H_0 free parameters. Left panel shows the results for Pantheon SNe Ia with both $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{logistic}$ as in the legend. Right panel shows the contours for the Pantheon + sample with both $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{Student}$ as illustrated in the legend.

comparison, [3] reported $\Omega_M = 0.315 \pm 0.007$. Thus, the discrepancy with the values in Table 8.1, when Ω_M and H_0 are free, increases using $\mathcal{L}_{new}$ due to the reduced errors. Quantitatively, $\mathcal{L}_{logistic}$ shows a 2σ discrepancy compared to 1σ of $\mathcal{L}_{Gauss}$, while $\mathcal{L}_{Student}$ increases the discrepancy from 2 to 3σ .

Our results steadily demonstrate that the newly found distributions perform better on the data than the Gaussian distribution. This result has a significant repercussion in the SNe Ia field, the approach implicitly adopted in SNe Ia cosmology, and future SNe Ia collections for which the Gaussianity assumption will have to be carefully inspected. Remarkably, we have demonstrated that the choice of the appropriate distribution for each SNe Ia sample is the key to substantially improving the constraints on the cosmological parameters. In an era in which the cosmological debate is focused on the tensions in the cosmological parameters, the reduction of $\sim 40\%$ in Ω_M and H_0 represents a breakthrough in constraining cosmological parameters more precisely, thus shedding light on the effective extent of these tensions.

$\mathcal{L}_{new}$		$\mathcal{L}_{Gauss}$		
Pantheon	H_0	Ω_M	H_0	Ω_M
	70.21 ± 0.20	0.285 ± 0.012	69.99 ± 0.34	0.300 ± 0.021
GRBs+QSOs+BAO+Pantheon	H_0	Ω_M	H_0	Ω_M
No Evolution	70.01 ± 0.14	0.300 ± 0.007	69.78 ± 0.15	0.315 ± 0.008
Fixed Evolution	70.09 ± 0.14	0.291 ± 0.007	69.88 ± 0.15	0.305 ± 0.007
Varying Evolution	70.03 ± 0.14	0.297 ± 0.007	69.82 ± 0.15	0.311 ± 0.007
Pantheon +	H_0	Ω_M	H_0	Ω_M
	72.93 ± 0.16	0.352 ± 0.011	72.85 ± 0.24	0.361 ± 0.019
GRBs+QSOs+BAO+Pantheon +	H_0	Ω_M	H_0	Ω_M
No Evolution	72.83 ± 0.14	0.361 ± 0.009	72.74 ± 0.15	0.372 ± 0.009
Fixed Evolution	72.96 ± 0.13	0.347 ± 0.009	72.83 ± 0.14	0.359 ± 0.009
Varying Evolution	72.86 ± 0.13	0.356 ± 0.009	72.77 ± 0.14	0.367 ± 0.009

Table 8.2: Best-fit Ω_M and H_0 values with 1σ uncertainty for GRBs+QSOs+BAO together with Pantheon or Pantheon + SNe Ia, as specified in the left column, for all the evolutionary cases and both likelihoods $\mathcal{L}_{new}$ and $\mathcal{L}_{Gauss}$. For an easier comparison, we also report the results from only Pantheon and Pantheon + samples as described in Section 8.3.1. In the corresponding part of the table, $\mathcal{L}_{new}$ is the likelihood with the logistic and the Student's t distribution, respectively for Pantheon and Pantheon +. H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$.

8.3.2 The case of quasars, Gamma-Ray Burst, Baryonic Acoustic Oscillations, and SNe Ia combined

The impact of the likelihoods on the cosmological constraints in the standard cosmological model

In the scenario described above, we fit a flat Λ CDM model with Ω_M and H_0 as free parameters using the combined samples of SNe Ia (from Pantheon) + GRBs + QSOs + BAO and SNe Ia (from Pantheon +) + GRBs + QSOs + BAO, considering both $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$. As stressed in Section 8.3.1, the comparison between the results obtained with $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$ is pivotal to test how the change in the likelihood affects the cosmological results. Indeed, it is particularly interesting to investigate whether and to what extent $\mathcal{L}_{new}$ changes the values of the cosmological parameters and if it performs better, even reducing the uncertainties of the cosmological parameters.

All the results are presented in Table 8.2. For an easier and more direct comparison, we also include in this table the results described in Section 8.3.1 with only Pantheon and Pantheon + SNe Ia samples. In these cases, $\mathcal{L}_{new}$ corresponds to the likelihood with the logistic and the Student's t distribution, respectively for Pantheon and Pantheon +. We here discuss our main findings.

- For both SNe Ia samples and independent of the evolutionary treatment, the best-fit values of Ω_M and H_0 are compatible within 1σ when comparing $\mathcal{L}_{new}$ and $\mathcal{L}_{Gauss}$, even though the use of $\mathcal{L}_{new}$ slightly lowers the Ω_M values. This result is

completely consistent with the previous analysis on SNe Ia, where the values of the cosmological parameters proved not to be affected by the choice of the likelihood function. Concerning the uncertainties on Ω_M and H_0 , they do not show any dependence on the correction for the evolution and the likelihoods used. More precisely, we obtain $\Delta\Omega_M \sim 0.007$ and $\Delta\Omega_M \sim 0.009$ when including Pantheon and Pantheon + samples, respectively, and $\Delta H_0 \sim 0.14$ with both SNe Ia samples.

Indeed, we here consider the combination of SNe Ia, GRBs, QSOs, and BAO, increasing the sample of SNe Ia by 2497 additional sources. The combination of different probes and the increase in the number of data, compared to the use of SNe Ia alone, already reduces the uncertainties up to a value lower than that obtained with the new likelihoods for SNe Ia only. Specifically, using the new proper likelihoods for SNe Ia alone, we reach $\Delta\Omega_M = 0.012$ and $\Delta H_0 = 0.20$ with the Pantheon sample, and $\Delta\Omega_M = 0.011$ and $\Delta H_0 = 0.16$ with the Pantheon + sample. In this analysis, instead, we achieve $\Delta\Omega_M \sim 0.08$ and $\Delta H_0 = 0.15$ just by combining all the probes with the standard $\mathcal{L}_{Gauss}$. Hence, the increase in the number of data (i.e. 3545 with Pantheon and 4198 with Pantheon +) and the combination of different probes are the main reasons for the reduction in the uncertainties of the cosmological parameters, making the choice of the proper likelihood of a secondary impact. However, the search for the proper likelihood still represents a crucial point for validating the cosmological analysis on a statistical basis and for precision cosmology, especially when dealing with new probes, small data sets, different combinations of probes, and new samples.

We here also emphasize that, between the two primary factors, the increased statistics is the one that less contributes to reducing the uncertainties, while the use of independent probes has the strongest impact. In this regard, the addition of BAO has the most relevant effect. Indeed, [20] have already shown that the addition of BAO to Pantheon SNe Ia reduces the uncertainties up to $\Delta\Omega_M = 0.07$ and $\Delta H_0 = 0.14$ (see their Table 5). Nevertheless, increasing the number of cosmological data sets could still play a significant role in future analyses, depending also on the cosmological probes investigated. Quantitatively, the value we obtain for $\Delta\Omega_M$ and ΔH_0 respectively corresponds to a reduction by a factor of 42% and 30% compared to only Pantheon SNe Ia with $\mathcal{L}_{new}$, 67% and 56% with respect to only Pantheon SNe Ia with $\mathcal{L}_{Gauss}$, 18% and 19% related to only Pantheon + SNe Ia with $\mathcal{L}_{new}$, and 53% and 42% in comparison to only Pantheon + SNe Ia with $\mathcal{L}_{Gauss}$ (see Table 8.2).

- Regardless of the approach for the likelihood and the correction for evolution, we can identify a specific trend when comparing the two cases with the inclusion of the Pantheon and Pantheon samples provided in Table 8.2. Indeed, we find that the inclusion of SNe Ia from Pantheon leads to $\Omega_M \sim 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while considering the Pantheon + sample we obtain $\Omega_M \sim 0.36$ and $H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1}$. More precisely, the values we obtain for Ω_M and H_0 show a discrepancy always greater than 3σ between the data sets with Pantheon and Pantheon + SNe Ia, with the largest inconsistency being 10

σ in the H_0 value. As previously stressed, this difference can be ascribed to the a-priori assumption on the absolute magnitude M for the two data sets. Indeed, [80] assume $M = -19.35$ corresponding to $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while [81] use $M = -19.253$, which corresponds to $H_0 = 73.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$, as obtained by [9]. Since Ω_M and H_0 are related, an assumption on H_0 directly affects the constraints on Ω_M .

- Our best-fit results and those presented in the previous section with SNe Ia alone are fully compatible, even though we have added GRBs, QSOs, and BAO. Indeed, GRBs and QSOs alone cannot constrain the cosmological parameters, while BAO provide best-fit values of the cosmological parameters that are compatible in 1σ with those of SNe Ia, but with larger uncertainties [326, 327, 279], due to the small number of data. Thus, in our combination of four cosmological probes, SNe Ia play the leading role in determining Ω_M and H_0 .
- Comparing our results for Ω_M with $\Omega_M = 0.315 \pm 0.007$ from [3], $\Omega_M = 0.298 \pm 0.022$ from [80], and $\Omega_M = 0.338 \pm 0.018$ from [273], we find that our Ω_M values are always compatible within 3σ with each of these values, without any dependence on the SNe Ia sample, likelihood, or correction for the evolution considered. More specifically, when we use the Pantheon sample, the compatibility with $\Omega_M = 0.315 \pm 0.007$ is within 1σ with $\mathcal{L}_{Gauss}$ and within 2σ with $\mathcal{L}_{new}$, while, using the Pantheon + sample, the difference increases to $2\text{--}3 \sigma$ for both likelihoods. Compared to $\Omega_M = 0.298 \pm 0.022$, the consistency is within 1σ for the case with Pantheon SNe Ia and within 2σ with Pantheon +. In relation to $\Omega_M = 0.338 \pm 0.018$, both data sets show a $1\text{--}2 \sigma$ compatibility.
- As this is the first time that QSOs have been used in combination with the most recent samples of GRBs and SNe Ia, and BAO to constrain cosmological parameters, we explore the QSO role in the overall data set by comparing our results with those of [20], where the combination of the same samples of GRBs and BAO as in our analysis and SNe Ia from Pantheon is used to constrain Ω_M and H_0 both without correction for the evolution and with a fixed correction. In this regard, we find compatibility within 1σ for the values of Ω_M and H_0 . Moreover, also the uncertainties on these parameters do not show any difference. This implies that QSOs are still too weak, mainly due to the intrinsic dispersion of the RL relation, to significantly contribute to constraining the cosmological parameters, compared to the dominant power of SNe Ia and BAO. As a matter of fact, the same conclusion can be drawn for GRBs. Indeed, as shown in [20], the inclusion of GRBs in the BAO + Pantheon SNe Ia data set does not impact the cosmological results, neither in the best-fit values nor in the associated uncertainties (see their Table 5). However, [20] showed that the addition of BAO to SNe Ia significantly reduces the uncertainties on these parameters, as already stressed above. Thus, in the full data set used in our study, BAO and SNe Ia play the dominant role in constraining the cosmological parameters. The non-predominant role of QSOs and GRBs is also the reason why different treatments of the evolution in redshift of their physical quantities do not affect the results, leading to best-fit values of

Ω_M and H_0 that are compatible within 1σ , regardless of the likelihood and the SNe Ia sample considered.

Despite their secondary role in this analysis, GRBs and QSOs are still very promising cosmological tools. Indeed, as the methodology to standardize them as cosmological probes has been developed only very recently, they cannot be as powerful as the probes traditionally used. Nonetheless, they manifest an incredible potential to significantly contribute to the cosmological analysis. First, as high- z probes, they allow us to investigate a previously unexplored region in the Universe evolution, which is the one crucially needed to discriminate between the predictions of different cosmological models, which are indistinguishable in the low- z range of SNe Ia. In this regard, as already mentioned, GRBs and QSOs also fill in the redshift gap between SNe Ia and CMB, and thus can potentially help in understanding or solving the H_0 tension. Additionally, their power in constraining cosmological parameters shows significant margins of improvement from various points of view. We here list a few: an increase in the number of sources, new and higher-quality observations, a refinement in the selection of the samples, which would lead to a reduced intrinsic dispersion of the Dainotti and RL relations (and thus to tighter constraints on the cosmological parameters). Lastly, and very importantly, new studies that can further improve the understanding of their physical mechanisms and backgrounds will better validate their application in cosmology.

In conclusion, as stated for the analysis of SNe Ia, this study shows the importance of inspecting the Gaussianity assumption for all cosmological probes and future new samples in order to apply the most appropriate likelihood when constraining the cosmological parameters.

The impact of the likelihoods on the cosmological constraints in alternatives cosmological models

We have further extended the study described in the previous section by investigating cosmological models alternative to the standard one, namely a non-flat Λ CDM model and a flat w CDM model. All our cosmological fits are performed with the Kelly method and we consider all the free parameters of the investigated cosmological model free to vary contemporaneously. As for the concordance model, we perform the fits with all the evolutionary cases and with both the traditional Gaussian likelihood, $\mathcal{L}_{Gauss}$, and the newly found likelihoods, denoted by $\mathcal{L}_{new}$, where SNe Ia, from Pantheon and Pantheon + samples, QSOs, and BAO are treated with their own non-Gaussian best-fit likelihood, as detailed in Sections 8.2.1 and 8.2.2. We present in Table 8.3 the best-fit values and 1σ uncertainties of the cosmological free parameters in all the cases studied.

Here we start the discussion by considering the results obtained by applying the $\mathcal{L}_{Gauss}$ likelihood to the non-flat Λ CDM model. As shown in the upper part of Table 8.3, the values obtained for Ω_k are always compatible within 1σ with $\Omega_k = 0$, regardless of the SNe Ia sample and the treatment of the redshift evolution. Therefore, our results significantly favour a flat Universe, in agreement with [3] and some other recent works [e.g. 4, 288]. However, all our analyses show a common trend towards $\Omega_k < 0$, suggesting

8.3. Implication for cosmology

$\mathcal{L}_{Gauss}$ likelihoods:		Non-flat Λ CDM			flat w CDM		
GRBs+QSOs+BAO+Pantheon	H_0	Ω_M	Ω_k	H_0	Ω_M	w	
No Evolution	69.98 ± 0.32	0.310 ± 0.010	-0.018 ± 0.025	69.90 ± 0.40	0.312 ± 0.010	-1.012 ± 0.038	
Fixed Evolution	70.20 ± 0.33	0.297 ± 0.009	-0.027 ± 0.025	70.45 ± 0.37	0.295 ± 0.009	-1.058 ± 0.035	
Varying Evolution	70.10 ± 0.30	0.304 ± 0.010	-0.024 ± 0.024	70.12 ± 0.38	0.306 ± 0.010	-1.031 ± 0.036	
GRBs+QSOs+BAO+Pantheon +	H_0	Ω_M	Ω_k	H_0	Ω_M	w	
No Evolution	72.94 ± 0.23	0.366 ± 0.011	-0.023 ± 0.021	72.80 ± 0.24	0.371 ± 0.010	-1.011 ± 0.030	
Fixed Evolution	73.02 ± 0.23	0.354 ± 0.010	-0.021 ± 0.021	73.07 ± 0.25	0.354 ± 0.010	-1.035 ± 0.029	
Varying Evolution	72.94 ± 0.24	0.362 ± 0.011	-0.021 ± 0.022	72.91 ± 0.25	0.364 ± 0.010	-1.020 ± 0.030	
$\mathcal{L}_{new}$ likelihoods:		Non-flat Λ CDM			flat w CDM		
GRBs+QSOs+BAO+Pantheon	H_0	Ω_M	Ω_k	H_0	Ω_M	w	
No Evolution	70.34 ± 0.23	0.299 ± 0.008	-0.040 ± 0.022	70.31 ± 0.24	0.300 ± 0.008	-1.043 ± 0.028	
Fixed Evolution	70.37 ± 0.22	0.287 ± 0.007	-0.027 ± 0.017	70.47 ± 0.24	0.289 ± 0.007	-1.046 ± 0.024	
Varying Evolution	70.33 ± 0.23	0.294 ± 0.008	-0.033 ± 0.020	70.37 ± 0.25	0.295 ± 0.008	-1.046 ± 0.027	
GRBs+QSOs+BAO+Pantheon +	H_0	Ω_M	Ω_k	H_0	Ω_M	w	
No Evolution	72.99 ± 0.17	0.361 ± 0.009	-0.026 ± 0.017	72.93 ± 0.18	0.362 ± 0.010	-1.025 ± 0.026	
Fixed Evolution	73.03 ± 0.17	0.347 ± 0.008	-0.011 ± 0.018	73.06 ± 0.19	0.348 ± 0.009	-1.019 ± 0.024	
Varying Evolution	72.99 ± 0.17	0.356 ± 0.009	-0.019 ± 0.017	72.99 ± 0.18	0.357 ± 0.009	-1.023 ± 0.025	

Table 8.3: Best-fit values and 1σ uncertainty of the cosmological free parameters of the non-flat Λ CDM and flat w CDM model for GRBs+QSOs+BAO together with Pantheon or Pantheon + SNe Ia, as specified in the left column, for no evolution, fixed and varying evolutionary cases studied with the $\mathcal{L}_{Gauss}$ likelihood (upper part of the table) and $\mathcal{L}_{new}$ likelihood (lower part of the table). H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$.

a slight preference for a closed Universe, as claimed in [41], [42], [43], and [45]. This hint at $\Omega_k < 0$ is also in agreement with the analysis of a non-flat Universe reported by [80] with only SNe Ia, although they have larger uncertainties compared to those obtained in our work (see their Table 8). As for Ω_M and H_0 , we recover the values assumed for the SNe Ia sample used. Specifically, if we consider the Pantheon SNe Ia, Ω_M and H_0 are compatible with 0.3 and $70 \text{ km s}^{-1} \text{Mpc}^{-1}$, respectively, as imposed in [80], while when using the Pantheon + SNe Ia, we obtain compatibility with $\Omega_M = 0.338 \pm 0.018$ and $H_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{Mpc}^{-1}$, as in [273] and [9], respectively. This dependence of the results on the calibration choice for the SNe Ia samples has been already mentioned in the previous sections.

Moving the discussion to the results obtained by applying the $\mathcal{L}_{new}$ likelihood, shown in the lower part of Table 8.3, we can state that these results confirm those obtained with $\mathcal{L}_{Gauss}$ since the best-fit values of the cosmological parameters are consistent in both cases. Besides this, the use of $\mathcal{L}_{new}$ remarkably reduces the uncertainties on H_0 , Ω_M , and Ω_k , as proved by the computation of the percentage difference provided in Table 8.4. This value (i.e. $\Delta\%$) is derived as $\Delta\% = (\Delta_{\text{comparing}} - \Delta_{\text{reference}}) / \Delta_{\text{reference}}$, where $\Delta_{\text{comparing}}$ is the uncertainty on the parameter considered obtained with the $\mathcal{L}_{new}$ likelihood and $\Delta_{\text{reference}}$ that obtained with $\mathcal{L}_{Gauss}$. Specifically, the uncertainties on Ω_M are reduced up to 27%, those on H_0 up to 30%, and those on Ω_k up to 32%, when considering both Pantheon and Pantheon + samples. All the results described above are valid independently of the treatment of the correction for redshift evolution.

Focusing on the flat w CDM model and starting from the case of the $\mathcal{L}_{Gauss}$ likelihood, the upper part of Table 8.3 provides the following results. All the values of w are consistent with $w = -1$, reaching a maximum discrepancy of 1.7σ in the case with

	Non-flat Λ CDM			flat w CDM		
GRBs+QSOs+BAO+Pantheon	$\Delta_{\%}(H_0)$	$\Delta_{\%}(\Omega_M)$	$\Delta_{\%}(\Omega_k)$	$\Delta_{\%}(H_0)$	$\Delta_{\%}(\Omega_M)$	$\Delta_{\%}(w)$
No Evolution	-0.28	-0.20	-0.16	-0.34	-0.20	-0.26
Fixed Evolution	-0.30	-0.22	-0.32	-0.35	-0.22	-0.31
Varying Evolution	-0.23	-0.20	-0.17	-0.34	-0.20	-0.25
GRBs+QSOs+BAO+Pantheon +	$\Delta_{\%}(H_0)$	$\Delta_{\%}(\Omega_M)$	$\Delta_{\%}(\Omega_k)$	$\Delta_{\%}(H_0)$	$\Delta_{\%}(\Omega_M)$	$\Delta_{\%}(w)$
No Evolution	-0.26	-0.27	-0.19	-0.25	0	-0.13
Fixed Evolution	-0.26	-0.20	-0.14	-0.24	-0.10	-0.17
Varying Evolution	-0.29	-0.18	-0.23	-0.28	-0.10	-0.17

Table 8.4: Percentage difference of the uncertainties on the best-fit values of the cosmological parameters obtained when using $\mathcal{L}_{new}$ instead of $\mathcal{L}_{Gauss}$ likelihood. $\Delta_{\%}$ is computed as $\Delta_{\%} = (\Delta_{\text{comparing}} - \Delta_{\text{reference}})/\Delta_{\text{reference}}$, where $\Delta_{\text{comparing}}$ is the uncertainty on the parameter considered obtained with the $\mathcal{L}_{new}$ likelihood and $\Delta_{\text{reference}}$ that obtained with $\mathcal{L}_{Gauss}$.

Pantheon SNe Ia and fixed correction for the evolution. This confirms the standard flat Λ CDM model, although with a visible hint at values $w < -1$ pointing towards a phantom dark energy scenario. Regarding the values of Ω_M and H_0 , the same considerations drawn for the non-flat Λ CDM model are still valid since the determination of these parameters is driven mainly by the SNe Ia samples and their corresponding calibration. As for the non-flat Λ CDM model, the results just described are not significantly affected by the different cases studied for the correction of the redshift evolution. Our results for Ω_M and w are compatible within less than 1σ with those provided only by SNe Ia in [80] (see their Table 8), where again w points towards $w < -1$, but with large uncertainties (i.e. $w = -1.090 \pm 0.220$). We can now compare these results obtained with $\mathcal{L}_{Gauss}$ with those obtained with $\mathcal{L}_{new}$, which are shown in the lower part of Table 8.3. As for the non-flat Λ CDM model, the best-fit values of the cosmological parameters are still not affected by the change in the likelihood, but we reach a significantly higher precision on the cosmological parameters with $\mathcal{L}_{new}$. Indeed, the uncertainty on H_0 is reduced up to the 35%, that on Ω_M up to 22%, and that on w up to 31%, when considering both SNe Ia samples.

In conclusion, in this epoch of cosmological tensions, it is important to carefully reconsider the definition of the likelihood themselves. Indeed, when fitting cosmological models, and especially alternatives and extensions to the flat Λ CDM model, the use of the best-fit likelihood function for each probe considered is important not only from a statistical point of view, but also to obtain reliable and intrinsic results and reduced uncertainties on cosmological parameters with the currently available data, as also pointed out in the previous sections.

9 Cosmography: a model-independent approach for cosmology

In the framework of the current puzzling cosmological tensions, model-independent approaches have been proposed in place of the commonly adopted cosmological analyses to investigate the correct form of the dark energy without any assumptions about its physical origin or the constituents of the Universe. Among these techniques, *cosmography* [328] is one of the most consolidated. The cosmographic approach is based only on the isotropy and homogeneity of the Universe through the assumption of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. This is a purely geometrical description of the kinematics of the Universe where all the physics is hidden in the scale factor $a(t)$. A cosmographic approach does not depend on any cosmological assumption other than the Cosmological Principle, as it does not require any a priori postulated equation of state of the cosmic fluid, but only an analytical expression for the scale factor $a(t)$ in the FLRW metric. This technique allows us to study the evolution of the Universe in a completely model-independent way.

On the one hand, the cosmographic approach suffers from some well-known shortcomings: convergence issues at $z \geq 1$, an arbitrary truncation of the cosmographic polynomial which may limit its predictive power and lead to misleading results, a high-dimensional space of the free parameters that causes degeneracy and a difficult estimation of the parameters, and a complex comparison between the cosmographic best-fit over the whole redshift range of the data and the prediction of a specific cosmological model, which is only expanded around $z = 0$. On the other hand, cosmography leverages several advantages: a high accuracy in fitting the data if the expansion is flexible enough, the possibility to reduce the degeneracy of the cosmological models, and, more importantly, the capability of testing any cosmological model avoiding cosmological assumptions. We refer to [23], [329], [32], and [330] for an extensive description of the cosmographic technique and its limitations and advantages.

The traditional cosmographic approach consists of a Taylor expansion of $a(t)$ around the current epoch t_0 (i.e. $z = 0$) that, if truncated at the fourth order, depends on the Hubble constant H_0 , the deceleration parameter q_0 , the jerk parameter j_0 , and the snap

parameter s_0 [331]. These parameters describe, respectively, the current expansion rate of the Universe, the current acceleration or deceleration phase, a transition between the acceleration and deceleration regime, and a possible dark energy evolution. Therefore, the best-fit values obtained by fitting the Taylor cosmographic function to the data provide information about the physics of the Universe [332, 333, 25, 334, 335, 336]. Furthermore, if we write the Taylor expansion around $z = 0$ of the luminosity distance in a flat Λ CDM model, a one-to-one relation between q_0 and Ω_M and between s_0 and Ω_M is found, which can be used to infer Ω_M from the cosmographic fit. An example of this approach is the fit of the Hubble diagram of SNe Ia with a second-order Taylor expansion, whose free parameters are H_0 and q_0 [332]. The negative value of q_0 obtained in this analysis has provided the fundamental information of the present acceleration of the expansion of the Universe. Moreover, when q_0 is expressed as a function of Ω_M in a flat Λ CDM model, we obtain $\Omega_M \sim 0.3$ and hence $\Omega_\Lambda \sim 0.7$. A further step in this approach is the inclusion of the third-order term j_0 in the cosmographic expansion, which is expected to be equal to 1 in a flat Λ CDM model. The fit of this third-order polynomial not only provides a more precise determination of q_0 , but is also a test of the model itself: any significant deviation from $j_0 = 1$ would be an indication of a tension between the data and the flat Λ CDM model (see e.g. [25]).

In principle, this method can be used to test any cosmological model. However, in the last years its application has become less straightforward due to the increased redshift range of the observational data. Indeed, it is clear that, by definition of the Taylor expansion, if the investigated data set consists of sources at $z > 1$, it is not correct to employ this cosmographic approach since the Taylor expansion of D_L is only valid in the limit $z \leq 1$. This convergence issue has only recently emerged due to the new observations of SNe Ia at $z > 1$ [80] and the standardization of high-redshift sources, such as GRBs and QSOs, which has significantly extended the Hubble diagram. This has introduced two new problems. First, the number of terms needed to reproduce the observational data with a cosmographic expansion has increased to four or even five [25, 330, 30], which makes the estimation of the significance of each parameter (and the comparison with the expectations of a physical model) complicated due to the degeneracy among the cosmographic parameters. Second, even if the degeneracy issues are solved, it is not guaranteed that the results of a cosmographic fit can be compared with the theoretical prediction of a given physical model which is calculated around $z = 0$ (see e.g. [337]).

In this scenario, cosmographic analyses involving high-redshift cosmological probes are only viable if such problems are solved by expansions other than the Taylor one. In this regard, different cosmographic techniques have been proposed and tested. For example, the use of rational polynomials extends the convergence radius, allowing us to reach higher redshifts. This is the case of the Padé [23, 337, 32, 338], Chebyshev [25, 24, 32, 26], and Bezi er polynomials [339]. Indeed, [23] have shown that while the Taylor expansion diverge at the third, fourth, and fifth orders for $z \geq 2$, the Pad  approximations at some specific orders guarantee an excellent agreement with the standard flat Λ CDM model and the flat w CDM model (see their figures 1 and 2). If, on the one hand, the Pad  polynomials effectively overcome the convergence problems, reduce the error

propagation at $z > 1$, and give the chance to choose the appropriate order, on the other hand, their exact convergence is not known, different series can be degenerate among each other, and they work better for non-smooth functions, which is not the case for cosmic distances. Similar considerations apply to the Chebyshev polynomials, as pointed out in [32].

In this thesis, we propose a novel technique. We try to overcome the weakness of cosmography at high redshifts by replacing the Taylor expansion with a new analytical function. More precisely, in Section 9.1 we expand the luminosity distance in orthogonal polynomials of logarithmic functions and in Section 9.1.1 we investigate the advantages of this cosmographic technique in reproducing and testing cosmological models and we validate our cosmographic model. Having demonstrated the robustness of the method, we also apply it to test the flat Λ CDM model by combining QSOs with SNe Ia in Section 9.2 and joining QSOs with SNe Ia, GRBs, and BAO in Section 9.3. This chapter is based on our publications of [29] and [31].

9.1 Cosmographic logarithmic approach

The cosmographic technique we propose is an extension of that already described in some previous works [28, 27]. It is based on an expansion of the luminosity distance as a power series of the quantity $\log(1+z)$. With respect to our previous works, we have modified the expression of $D_L(z)$ to make the cosmographic coefficients “orthogonal”, a notation we use to mean “without covariance”. This makes our method unique, since in the other cosmographic approaches all the coefficients are correlated as they are combinations of those at lower orders. The main advantage of the orthogonal polynomials is that a change in the truncation order of the series does not change the values of the cosmographic coefficients a_i , since they are uncorrelated. This allows us to test the significance of a possible additional term in the expansion (through its deviation from zero) and to evaluate the tension of the cosmographic fit with a given point in the parameter space without taking into account the correlation amongst the free parameters.

We have estimated the fifth-order of the orthogonal logarithmic polynomial expansion as¹

$$D_L(z) = \frac{\ln(10)}{H_0} \left\{ \log(1+z) + a_2 \log^2(1+z) + a_3 \left[k_{32} \log^2(1+z) + \log^3(1+z) \right] + \right. \\ \left. + a_4 \left[k_{42} \log^2(1+z) + k_{43} \log^3(1+z) + \log^4(1+z) \right] + \right. \\ \left. + a_5 \left[k_{52} \log^2(1+z) + k_{53} \log^3(1+z) + k_{54} \log^4(1+z) + \log^5(1+z) \right] \right\}. \quad (9.1)$$

In our analysis, we have tested the need for the different orders in this expansion proving the significance of the fifth-order, while the sixth-order term turned out to be consistent

¹Throughout the text we use the notation $\ln$ to refer to the natural logarithm.

with zero, as will be shown later. This is the reason why we consider a fifth-order expansion throughout this chapter. The terms $\frac{\ln(10)}{H_0}$ and $a_1 = 1$ are needed to reproduce the Hubble's law locally, that is $D_L(z) = \frac{\ln(1+z)}{H_0}$. The coefficients k_{ij} in this formula are not free parameters and they depend on the data set we want to fit and, in particular, on its redshift distribution. They are determined by a step-by-step procedure, as follows, to make all the a_i in Eq. (9.1) uncorrelated.

First, the data set is fitted with a second-order non-orthogonal polynomial $P_2 = \frac{\ln(10)}{H_0} \left[\log(1+z) + a'_2 \log^2(1+z) \right]$ to obtain a'_2 . Then, the same fit is performed with a third-order non-orthogonal polynomial $P_3 = \frac{\ln(10)}{H_0} \left[\log(1+z) + a'_2 \log^2(1+z) + a''_3 \log^3(1+z) \right]$ to derive a''_2 and a''_3 . The comparison between the orthogonal and the non-orthogonal polynomial expressions leads to the constraint $a''_2 = a'_2 + a''_3 k_{32}$. Continuing with the fourth-order non-orthogonal polynomial $P_4 = \frac{\ln(10)}{H_0} \left[\log(1+z) + a''_2 \log^2(1+z) + a'_3 \log^3(1+z) + a'''_4 \log^4(1+z) \right]$ we determine a'''_2 , a'''_3 , and a'''_4 , and with the fifth-order non-orthogonal polynomial $P_5 = \frac{\ln(10)}{H_0} \left[\log(1+z) + a'''_2 \log^2(1+z) + a'''_3 \log^3(1+z) + a'''_4 \log^4(1+z) + a''''_5 \log^5(1+z) \right]$ we obtain a''''_2 , a''''_3 , a''''_4 , and a''''_5 , so we require that

$$a'''_2 = a'_2 + a''_3 k_{32} + a'''_4 k_{42}, \quad (9.2)$$

$$a'''_3 = a'_3 + a'''_4 k_{43}, \quad (9.3)$$

$$a''''_2 = a'_2 + a''_3 k_{32} + a'''_4 k_{42} + a''''_5 k_{52}, \quad (9.4)$$

$$a''''_3 = a'_3 + a'''_4 k_{43} + a''''_5 k_{53}, \quad (9.5)$$

$$a''''_4 = a'_4 + a''''_5 k_{54}. \quad (9.6)$$

The solutions of these six equations are the six unknown coefficients k_{ij} . The same procedure can be applied to determine the values of k_{ij} of the logarithmic orthogonal expansion of any order and, not just the fifth. Since this method uses the fits to the data set, the fixed coefficients k_{ij} depend on the specific sample considered and the procedure should be repeated for each different sample considered. More specifically, the data set used for this analysis consists of the sample of QSOs described in Section 3.1 with the discussed cut at $z > 0.7$.

To better visualize the effect of our orthogonalization procedure, we show in Figure

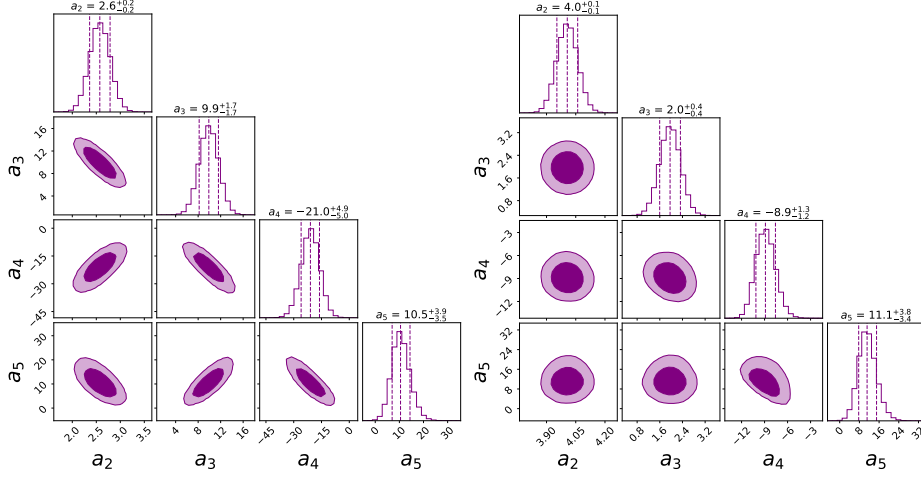


Figure 9.1: Contour plots of the cosmographic free parameters a_i for the non-orthogonal (left panel) and orthogonal (right panel) logarithmic fits to the fifth-order. The purple contours are the confidence levels at 1 and 2 σ . The best-fit values are very different in the two cases except for a_5 , which is consistent within the errors. This is expected from Eq. (9.1) as a_5 is not multiplied by any factor k_{ij} and thus must be equal to the same coefficient in the non-orthogonal formula. The apparent correlation in the (a_3, a_4) and (a_4, a_5) planes in the right panel is due to the prior $D_L > 0$, which allows only specific values of a_i .

9.1 the comparison between the orthogonal and non-orthogonal² cosmographic logarithmic fits to our data set composed of QSOs and Pantheon SNe Ia. In the latter case, as we can see from the slanting contours (at 1 σ and 2 σ) in the parameter spaces in the left panel, the coefficients a_i are correlated. Consequently, changing the order of the polynomial would modify the best-fit values of the parameters. We can see this directly by comparing the non-orthogonal fourth-order fit shown in Figure 9.2 with the fifth-order fit in Figure 9.1. Instead, in the case of the orthogonal polynomial (right panel of Figure 9.1), there is no covariance at all between the parameters. As anticipated, the results of the fits in Figure 9.1 show that the fifth order is significant, with a deviation from zero of about 3 σ , and so it is required to fit our set of data.

²The formula for D_L in the fifth-order non-orthogonal case is simply

$$D_L(z) = \frac{\ln(10)}{H_0} \left[\log(1+z) + a_2 \log^2(1+z) + a_3 \log^3(1+z) + a_4 \log^4(1+z) + a_5 \log^5(1+z) \right]. \quad (9.7)$$

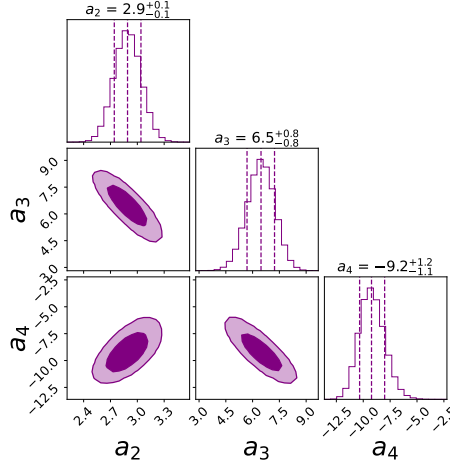


Figure 9.2: Same as Figure 9.1, but for the fourth-order non-orthogonal fit. The best-fit values of a_i are different from those of the fifth-order non-orthogonal fit shown in the left panel of Figure 9.1, which is a consequence of the correlation between the parameters. These values are also inconsistent with those of the fifth-order orthogonal fit, except for a_4 (as already explained for a_5 ; see the caption of Figure 9.1).

9.1.1 Validation of the cosmographic logarithmic approach

The cosmographic function is fitted over a redshift range much larger than the convergence radius of the logarithmic expansion. Therefore, we need to check whether this expansion is still useful for testing physical models. We note that we are only interested in validating the cosmographic approach within the redshift range of the observational data, while we do not discuss the extrapolation of the model to higher redshifts or the associated convergence issues. To compare a cosmographic fit with the predictions of a physical model, we use the relations that hold between the cosmographic and the physical free parameters. However, this technique is strictly valid only in a redshift range up to $z \sim 1$ (due to the expansions centred at $z = 0$). The comparison with a fit on the whole redshift range may not be strictly correct. Nevertheless, it could be possible that the cosmographic expansion represents a good approximation even if the redshift range extends beyond the convergence radius, just because its shape is similar to that of the cosmological model. We have investigated this problem in two ways: first, we compare the cosmographic expansion of the flat Λ CDM model with the model itself for different values of Ω_M , then we compare the predictions of the cosmographic expansion with the fits of mock samples in the $z = 0 - 7.5$ range.

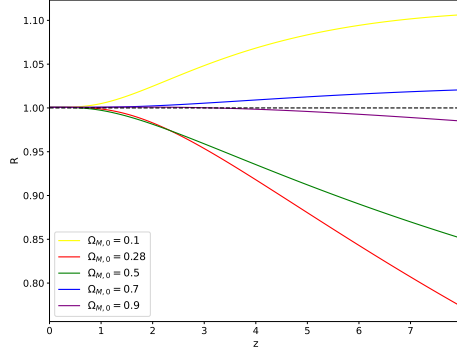


Figure 9.3: Luminosity distance ratio (R) of a fifth-order cosmographic expansion of a flat Λ CDM model to the model itself, for different values of Ω_M as shown in the legend. In each case the expansion very precisely reproduces the cosmological model for $z < 1$ ($R = 1$). The discrepancy from $R = 1$ at $z > 1$ depends on the specific value of Ω_M : the worst case is for $\Omega_M = 0.28$, while the best case is for $\Omega_M = 0.9$.

Cosmographic expansion of physical models

To check whether the cosmographic logarithmic expansion is a good approximation of the Λ CDM model beyond its convergence radius, we have analyzed the ratio of the physical model to its logarithmic expansion, as a function of the redshift. The results are shown in Figure 9.3. The parameter R is indeed the ratio of the luminosity distance estimated from the fifth-order cosmographic approximation of a specific flat Λ CDM model with $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and five different choices of Ω_M ($\Omega_M = 0.1, 0.28, 0.5, 0.7, 0.9$) to the luminosity distance from the corresponding cosmological model. The redshift range considered corresponds to that spanned by our data, up to $z \approx 7.5$. The numerator of R is obtained using the relations between a_i and Ω_M calculated by the expansions at $z = 0$. These relations depend on the truncation order of the expansion and on the sample, due to the presence of the coefficients k_{ij} . The exact expressions of the functions $a_i(\Omega_M)$ in a logarithmic expansion at the fourth order under the assumption of a flat Λ CDM model are as follows:

$$a_4 = \ln^3(10) \left(-\frac{135}{64}\Omega_M^3 + \frac{18}{4}\Omega_M^2 - \frac{47}{16}\Omega_M + \frac{5}{8} \right), \quad (9.8a)$$

$$a_3 = -k_{43}a_4 + \ln^2(10) \left(\frac{9}{8}\Omega_M^2 - 2\Omega_M + \frac{7}{6} \right), \quad (9.8b)$$

$$a_2 = -k_{32}a_3 - k_{42}a_4 + \ln(10) \left(-\frac{3}{4}\Omega_M + \frac{3}{2} \right). \quad (9.8c)$$

The extension to the fifth-order is given by

$$a_5 = \ln^4(10) \left(\frac{567}{128}\Omega_M^4 - \frac{729}{64}\Omega_M^3 + \frac{315}{32}\Omega_M^2 - \frac{25}{8}\Omega_M + \frac{31}{120} \right), \quad (9.9a)$$

$$a_4 = -k_{54}a_5 + \ln^3(10) \left(-\frac{135}{64}\Omega_M^3 + \frac{18}{4}\Omega_M^2 - \frac{47}{16}\Omega_M + \frac{5}{8} \right), \quad (9.9b)$$

$$a_3 = -k_{53}a_5 - k_{43}a_4 + \ln^2(10) \left(\frac{9}{8}\Omega_M^2 - 2\Omega_M + \frac{7}{6} \right), \quad (9.9c)$$

$$a_2 = -k_{32}a_3 - k_{42}a_4 - k_{52}a_5 + \ln(10) \left(-\frac{3}{4}\Omega_M + \frac{3}{2} \right). \quad (9.9d)$$

From this analysis we conclude that the cosmological model is well reproduced up to $z \sim 1$ for all the assumed values of Ω_M . This implies that it is correct to compare the best-fit values of the cosmographic free parameters and the prediction of the cosmological model only for fits at $z \leq 1$. Moreover, the logarithmic expansion always shows a limited deviation from the Λ CDM model within the whole redshift range: the larger deviation from $R = 1$ is ~ 0.2 at the maximum redshift $z \sim 7.5$ for the case with $\Omega_M = 0.28$, which corresponds to a relative difference in the distance modulus of 0.01. This implies that the logarithmic approximation and the reference cosmological models have the same shapes, that is to say the same trend with the redshift.

Comparison with fits of mock samples

A direct way to check the applicability of a cosmographic analysis to a wide redshift range is through mock samples of data. This procedure allows us to simulate data sets with the same redshift distribution, statistical errors, and intrinsic dispersion in distance modulus as the real sample and which follow a specific cosmological model. This sample is very different from the ideal one used in Figure 9.3, where the statistical errors and the dispersion are not taken into account and where we have assumed an almost continuous distribution in redshift. As a consequence, a mock sample can be used to fit the cosmographic model to a real data set that assumes a specific cosmological model, and this fit can be compared with the theoretical prediction obtained by the analytic expansion around $z = 0$. This approach allows us to turn the qualitative comparison shown in Figure 9.3 into a quantitative comparison in the a_i two-dimensional spaces. This is a crucial test to investigate whether the prediction from an expansion at $z = 0$ can be extrapolated up to redshifts $z \sim 8$. We now explain in detail the procedure we have followed, starting with the creation of the mock samples of QSOs and SNe Ia.

Regarding SNe Ia, we calculated the mock distance moduli using the redshifts from the Pantheon sample and $D_L(z)$ from a flat Λ CDM model with $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and two values of Ω_M : $\Omega_M = 0.28$ and $\Omega_M = 0.7$. We also included in the calculation of the distance moduli the contribution of a Gaussian distribution with an intrinsic dispersion around the central value of 0.16 dex. The statistical errors associated with the distance moduli are those of the real sample. The mock samples of QSOs have the same redshifts and UV luminosity as the real data set and the X-ray fluxes are obtained from the X-UV relation assuming a flat Λ CDM model with the same values of H_0 and Ω_M assumed for the mock samples of SNe Ia. As for SNe Ia, we also included an intrinsic dispersion. Specifically, we assumed a Gaussian distribution in the distance modulus around 1.3 dex. Again, the statistical errors on the distance moduli are those from the real sample. The number of sources in these mock samples is increased for both QSOs and SNe Ia

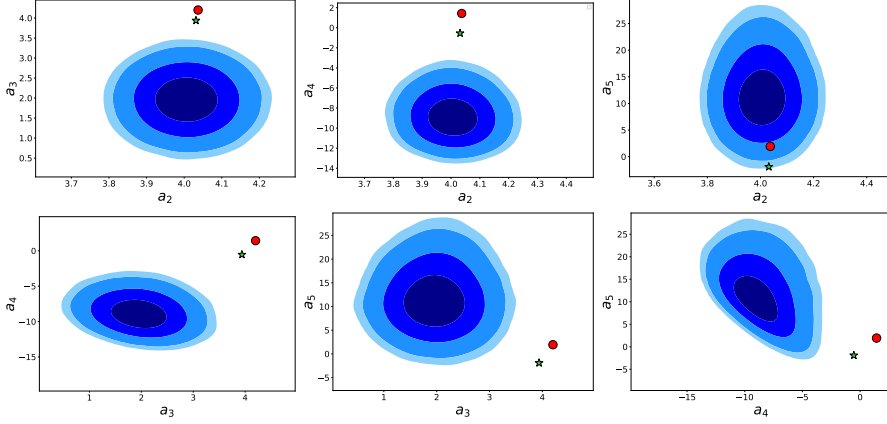
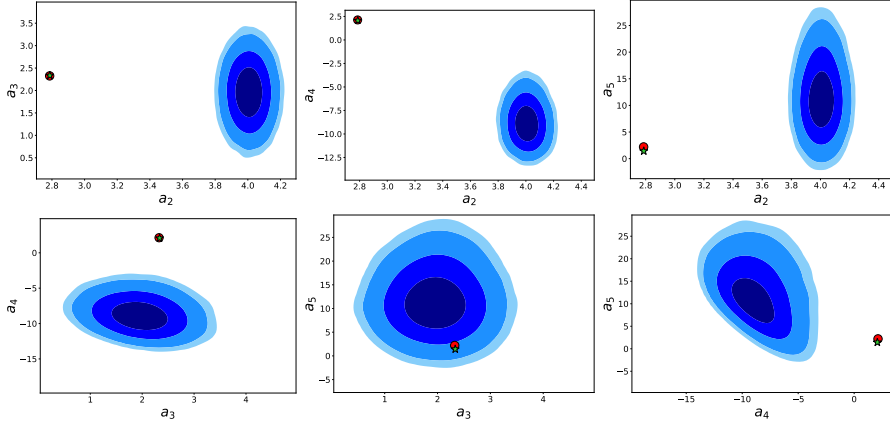


Figure 9.4: Bi-dimensional spaces for the cosmographic free parameters a_i . In each panel, the blue contours are the confidence levels at 1, 2, 3, and 4 σ for the best fit on the real set of data (QSOs+SNe Ia), the red point is the best fit from the mock sample with $\Omega_M = 0.28$ and the green star corresponds to the prediction of the corresponding Λ CDM model from the expansion at $z = 0$.

in order to constrain the free parameters of the fits with higher precision, while keeping the relative statistical weight of QSOs and SNe Ia. For each redshift we generated 100 sources with slightly different distance moduli due to the Gaussian distribution of the intrinsic dispersion. This allowed us to improve the precision of the best fit of the free parameters by a factor of 10 making the uncertainties negligible compared to those of the real data. As a consequence the error bars are too small to be plotted on the red point in Figures 9.4 and 9.5.

Following this method, we built two joint mock samples of QSOs and SNe Ia assuming $\Omega_M = 0.28$ and $\Omega_M = 0.7$, respectively. These choices are justified by the fact that, looking at Figure 9.3, these are the cases where the cosmological model is worst and best (just after $\Omega_M = 0.9$) reproduced by the expansion around $z = 0$, respectively. The conclusions we obtain for these two extreme cases can be generalized to all the other intermediate cases. Moreover, $\Omega_M = 0.28$ is the value actually favoured by several cosmological probes, and thus it is of general interest to test our method on this specific value. As a self-consistency check, we first recover the cosmological parameters through a direct fit of a flat Λ CDM model to the mock data. This test quantifies the degree of randomness introduced in the mocking procedure and confirms that the data are fully consistent with the specific flat Λ CDM model assumed. Then, the mock samples of QSOs and SNe Ia were fitted jointly with a fifth-order orthogonal logarithmic function studying separately the two cases of $\Omega_M = 0.28$ and $\Omega_M = 0.7$.

Figure 9.4 summarizes the results for the case with $\Omega_M = 0.28$. Each panel in this figure shows a bi-dimensional free parameter space of the a_i parameters where the confidence levels from 1 to 4 σ are obtained from the cosmographic analysis on the real data set of QSOs and Pantheon SNe Ia (and are exactly those already shown on


 Figure 9.5: Same as Figure 9.4, but for a reference flat Λ CDM model with $\Omega_M = 0.7$.

the right-hand side of Figure 9.1), the red point is the best fit from the mock sample, while the green star is the analytic prediction from the expansion of the standard model around $z = 0$ (i.e. Eqs. (9.9) for $\Omega_M = 0.28$). Focusing on the comparison between the red point and the green star, we can see that they perfectly match on the parameter a_2 , while the green star underestimates the discrepancy with the observational data of 0.7σ in a_3 and 1.5σ in a_4 and overestimates the same discrepancy of 1σ in a_5 . This means that the prediction in the limit $z < 1$ is in good agreement with the correct results also when evaluated over the whole redshift range. Furthermore, this analysis shows something more interesting on closer inspection. The discrepancy between the data and the Λ CDM model with $\Omega_M = 0.28$ (green star) is at more than 6σ and 7σ for a_3 and a_4 , respectively. Moreover, for these two parameters, the prediction of the expansion at $z = 0$ (green star) affects the results with an underestimation of the discrepancy if compared to the fit on the mock sample (red point). As a consequence, the claim for the existence of a tension coming from the comparison between the data and the analytic prediction would be correct and confirmed at even more sigmas by the use of mock samples. The results for the parameter a_5 , instead, require a more complex interpretation. In this case, the discrepancy between the data and the red point (i.e. the best fit from the mock sample) and the green star (i.e. the prediction from the expansion) is at 2.7σ and 3.8σ , respectively. However, we must bear in mind that a_5 is the less significant parameter, and that it is less constrained because of the large errors. Therefore, if a larger sample of QSOs at high redshifts (where this term is dominant) were available, we expect that a_5 could be better constrained leading to a much more significant discrepancy from the cosmological model, as observed in a_3 and a_4 . If this happens the argument for a_3 and a_4 could also be generalized to this parameter. Therefore, it is evident that future observations of QSOs at high redshifts could play a crucial role in making this analysis easier and clearer.

The complexity of this analysis and the care required to take all aspects into account lead us to demonstrate the great advantages of the logarithmic approach. On the one

hand, the fifth-order orthogonal logarithmic function can accurately reproduce the given physical model, as proved by the fit to the mock data. On the other hand, since we are only interested in comparing the data with cosmological models to test for a possible tension, we can directly compare the fit over the whole redshift range with the prediction from the expansion at $z = 0$ without the need of mock samples, since the approximation introduced by this method is negligible. We have demonstrated this for $\Omega_M = 0.28$, but, as previously pointed out, this is the case where the analytic prediction has the maximum deviation from the model (see Figure 9.3), and hence these conclusions can be generalized to all the other flat Λ CDM models with values of Ω_M between 0.1 and 0.9. Moreover, in all the other cases we expect a better agreement with the theoretical prediction and a solid confirmation of the results described above. As an example of this statement, we show in Figure 9.5 the same analysis for a flat Λ CDM model with $\Omega_M = 0.7$. The main result is that the green star and the red point completely overlap in each of the parameter spaces, leading to a perfect match between the physical model and the prediction at $z = 0$. This is a strong indication that the cosmographic extrapolation can be used at redshifts $z > 1$. This allows us to avoid fitting mock samples for each possible value of Ω_M to reproduce the curve predicted by a flat Λ CDM model in the parameter spaces by replacing this long-time computational procedure with the simple analytic calculation at $z \sim 0$.

In summary, we have studied in detail the features of our cosmographic approach showing that the logarithmic function has excellent properties and can reproduce physical models over the whole redshift range of the data considered. As a consequence, we can use this method to analytically describe the Hubble diagram up to high redshift and compare it with the predictions of cosmological models. The method we have just described is completely general and can be applied to any cosmographic function to test whether its theoretical prediction at $z = 0$ can be compared to the cosmographic fit over the whole redshift range. This provides us an unbiased measure of the usefulness and the power of different cosmographic approaches.

9.2 A cosmographic test of the standard cosmological model with quasars and SNe Ia

Once we have demonstrated the robustness of the cosmographic logarithmic technique, the next step is to use it as a test for cosmological models. We have proved that our cosmographic function provides a reliable reproduction of a flat Λ CDM model when we assume this model as the reference, but we can also compare the prediction of the standard cosmological model with the observational data taking advantage of the degrees of freedom and the flexibility of the logarithmic function. This means fitting the data with both the cosmological and the cosmographic models to study: 1) the deviations of the distance moduli; 2) the different trends of the two best-fit curves in the Hubble diagram; and 3) the discrepancy between the best-fit values of a_i and the values of a_i predicted by the cosmological model. Regarding this last point, we have already shown in the previous section that it is correct to compare the results obtained with the data over

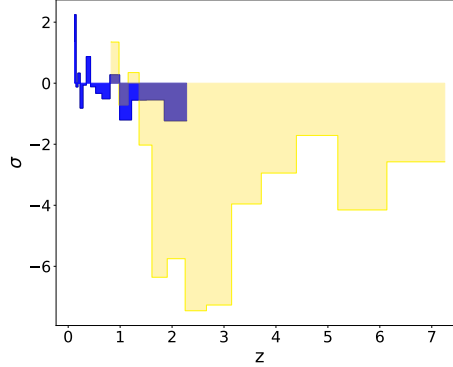


Figure 9.6: Deviations (in unity of σ) of the cosmographic distance moduli of QSOs (yellow bars) and SNe Ia (blue bars) from the best-fit Λ CDM model up to $z = 1.1$ and extrapolated to higher redshifts. The values of σ are redshift-averaged in narrow logarithmic bins. A discrepancy between the data and the model emerges at $z \geq 1$ and it remains significant at higher redshifts. This proves the tension between the Λ CDM model and the observations.

the whole redshift range with the theoretically predicted values of $a_i(\Omega_M)$ obtained by the expansion around $z = 0$. Such an analysis is only possible because we have defined a model that has good convergence properties and that guarantees a purely analytic reconstruction of the data. In principle, this cosmographic model can be used as a test for any cosmological model, but here we focus only on a flat Λ CDM model.

As a first step, we fitted the distance moduli of the combined sample of QSOs with the cut at $z > 0.7$ (see Section 3.1) and Pantheon SNe Ia using the cosmographic logarithmic orthogonal expansion to the fifth order and assuming a flat Λ CDM model. The fit of the flat Λ CDM model is performed not only over the entire redshift range of the data, but also in the restricted interval at $z < 1.1$, where SNe Ia predominate, in order to get a deeper insight into the comparison between this cosmological model and the observational data. In Figure 9.6, we plot the redshift-averaged sigma deviations between the cosmographic distance moduli of QSOs (yellow bars) and SNe Ia (blue bars) and the best-fit Λ CDM model up to $z = 1.1$ and extrapolated to higher redshifts. The discrepancy between the Λ CDM model and the data is clearly due to a deviation from the Hubble diagram of QSOs at $z > 1$. This tension increases with redshift, peaking at $z = 2 - 3$ and remaining statistically significant up to the highest redshifts. This is a step forward from the results presented in [27], where the same discrepancy peaks at $z \sim 3$, the redshift of high-quality pointed observations (see Section 3.2), but it almost vanishes at $z > 3$, where the data quality is lower (see their Figure 5). The authors already suggested that observations of QSOs at $z \sim 4$ were required to verify this trend. In our sample of QSOs new observations at $z > 4$ have been included with a tension that is still high at these redshifts, confirming the discrepancy at high redshifts. As a final comment, it should be noted that QSOs are not the only sources responsible for this discrepancy. In fact, SNe

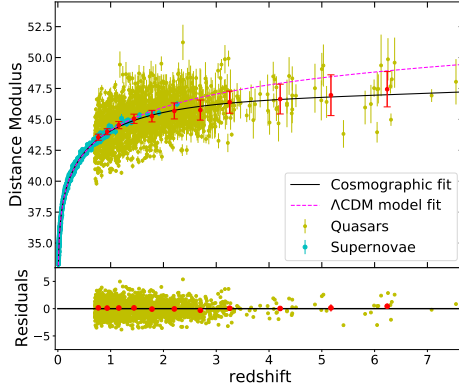


Figure 9.7: Top panel: Hubble diagram of SNe Ia (cyan points) and QSOs (yellow points). The red points are averages, along with their uncertainties, of the distance moduli of QSOs in narrow logarithmic redshift intervals and they are shown for visualization purposes only, without any statistical application. The dashed magenta and continuous black lines show respectively the best-fit curves of the flat Λ CDM and the logarithmic models over the whole redshift range. Bottom panel: residuals with respect to the cosmographic fit for QSOs (yellow points) and averages of the residuals (red points) over the same redshift intervals.

Ia at $z > 1$ (~ 23 SNe Ia) already display the same trend and the same evidence of a deviation. As SNe Ia are the most powerful and reliable standard candles, this is a very strong piece of evidence for the existence of a tension between the Λ CDM model and the observations. These results, already reported by [28] and [27], are now confirmed with greater statistical significance thanks to our new sample of QSOs.

These considerations are reinforced if we analyse the fit of the Λ CDM model over the whole redshift range, and not only in the interval $z < 1.1$. In Figure 9.7 we show the Hubble diagram of QSOs (yellow points) and SNe Ia (cyan points) and the best-fit curves of the logarithmic function (continuous black line) and the flat Λ CDM model (dashed magenta line) over the whole range of z . Compared to the previous analysis on the data at $z < 1.1$, we find that the best-fit values of Ω_M are consistent within the error bars, since we obtained $\Omega_M = 0.281 \pm 0.013$ for $z < 1.1$ and $\Omega_M = 0.296 \pm 0.013$ over the whole range of z , and the best-fit curves on the Hubble diagram are indistinguishable with a maximum absolute difference in the distance modulus of 0.04. More importantly, the best-fit curves of the cosmological and cosmographic models shown in Figure 9.7 overlap completely in the low-redshift region at $z < 1$, whilst the tension becomes significant at $z > 1.5$, where the prediction of the Λ CDM model (dashed magenta line) is much steeper than that of cosmography (continuous black line). In order to quantify the discrepancy between these two curves we can compare the best-fit values of the a_i parameters from the cosmographic fit with those predicted by the Λ CDM model (see Eqs. (9.9)). This comparison is exactly the one between the blue contours and the green star already shown in Figure 9.4 for the concordance model ($\Omega_M = 0.28$) and in Figure 9.5 for the case with

$\Omega_M = 0.7$. Focusing on the concordance model, as it is the most accepted and successful one, we note that there is no tension on a_2 , the dominant term at low redshifts. This is further evidence of the agreement between the Λ CDM model and the data up to $z \sim 1$, as already shown in the analysis above. However, due to the increasing discrepancy in the other parameters, and thus at higher redshifts, the overall tension between the cosmographic fit and the cosmological model is much larger than 4σ . This confirms the results of [28] and [27] with evidence of a significant discrepancy between the standard cosmological model and the data.

9.3 An extended cosmographic test of the standard cosmological model with quasars, SNe Ia, GRBs, and BAO

We have then extended the previous cosmographic analysis by considering both QSOs alone and the combination of QSOs with Pantheon + SNe Ia, GRBs, and BAO. Furthermore, in this study, we have implemented the use of both the Gaussian and the newly found cosmological likelihoods, $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$, respectively, as detailed in Sections 8.2.1 and 8.2.2, and we have included the treatment of the redshift evolution and selection biases (see Chapter 5). However, in this case we cannot apply the varying correction, but only the fixed correction, where we assume a specific cosmological model. Indeed, we are not dealing with cosmological fits in which the cosmological parameters are free to vary and hence we can let the evolutionary parameter k vary with them, but we are investigating a cosmographic model, in which the only free cosmological parameter is H_0 , which does not affect k . Nevertheless, to provide a complete and comprehensive picture of our analysis, we consider four possible treatments of the redshift evolution: without correction and with fixed correction by assuming three different values of Ω_M , namely $\Omega_M = 0.3$, $\Omega_M = 0.1$, and $\Omega_M = 0.9$. The investigated value of 0.3 is justified as that expected from the latest observations [80, 273]. Instead, the values of $\Omega_M = 0.1$ and $\Omega_M = 0.9$ have been chosen as illustrative examples corresponding to the highest and lowest values, respectively, that k can assume when Ω_M is in the physical interval between 0.1 and 0.9. Indeed, the function $k(\Omega_M)$ monotonically decreases with increasing Ω_M , as shown in Figure 5.4 for QSOs and in figure 5 of [20] for GRBs. We emphasize here that the reason for investigating these three cases of the fixed correction is that, since we cannot apply the model-independent technique for the correction for redshift evolution, we test how our results depend on the a-priori assumption of a particular cosmology. This provides an insight into the effects of our cosmological assumptions and allows us to interpret our results and draw conclusions.

We here outline the results obtained by fitting QSOs alone, which are summarized in Tables 9.1 and 9.2.

- The first step in our cosmographic procedure is to determine the order of the orthogonal logarithmic polynomial needed to fit the data properly. As discussed in Section 9.1, the maximum order required depends on the sample under investiga-

	$\mathcal{L}_{Gauss}$				$\mathcal{L}_{new}$			
	a_2	a_3	a_4	H_0	a_2	a_3	a_4	H_0
No Evolution	1.12 ± 0.37	-2.83 ± 1.62	10.53 ± 6.31	74.73 ± 14.45	1.19 ± 0.38	-3.18 ± 1.56	11.38 ± 6.00	74.94 ± 15.50
$\Omega_M = 0.3$	10.23 ± 2.82	9.56 ± 2.19	29.70 ± 18.43	75.24 ± 14.51	13.64 ± 2.89	10.12 ± 2.04	27.30 ± 17.53	75.97 ± 14.45
$\Omega_M = 0.1$	8.50 ± 3.38	19.08 ± 3.27	52.57 ± 27.82	75.61 ± 14.49	13.86 ± 3.52	20.24 ± 3.14	48.56 ± 27.09	74.63 ± 14.35
$\Omega_M = 0.9$	11.57 ± 1.94	3.14 ± 2.39	21.59 ± 18.00	74.51 ± 14.59	12.61 ± 2.00	3.19 ± 2.09	17.53 ± 13.91	74.26 ± 14.22

Table 9.1: Best-fit values of the a_i coefficients together with their 1σ uncertainty from QSOs alone and for all cases investigated. H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$, but it is not constrained by QSOs alone.

	$\mathcal{L}_{Gauss}$			$\mathcal{L}_{new}$		
	a_2	a_3	a_4	a_2	a_3	a_4
Flat Λ CDM, No Evolution	6.45	4.93	1.12	6.48	4.94	1.12
Flat Λ CDM, Fixed Evolution ($\Omega_M = 0.3$)	3.81	4.40	1.12	5.03	4.40	1.12
Flat Λ CDM, Fixed Evolution ($\Omega_M = 0.1$)	2.51	8.21	4.57	5.13	8.19	4.57
Flat Λ CDM, Fixed Evolution ($\Omega_M = 0.9$)	4.46	2.43	1.08	4.51	2.43	1.08

Table 9.2: Values of a_i predicted in a flat Λ CDM model according to Eqs. (9.8) for only QSOs and all cases studied in this work.

tion. Unlike the previous study, which examined QSOs combined with Pantheon SNe Ia, when using only QSOs, we obtain that the fifth order is negligible, since the best-fit value of a_5 is consistent with 0 within much less than 1σ in all the cases studied. Hence, we truncate the formula of Eq. (9.1) to the fourth order imposing $a_5 = 0$. Indeed, as visible from Table 9.1, a_4 is significant since it is always different from $a_4 = 0$ at $> 1\sigma$ level.

- Comparing the results of the different likelihoods in Table 9.1, we can observe that $\mathcal{L}_{new}$ leads to reduced uncertainties on the parameters compared to $\mathcal{L}_{Gauss}$, in agreement with our previous cosmological analyses. More specifically, $\mathcal{L}_{new}$ lowers the errors on the higher-order a_i coefficients, reaching a reduction of $\sim 23\%$. Nevertheless, QSOs alone cannot provide tight constraints on the coefficients a_i , which are indeed associated with large 1σ uncertainties.
- Regarding the only cosmological parameter, H_0 , it is not constrained in any of the cases studied with QSOs alone. Indeed, QSOs alone not calibrated to SNe Ia, which behave as anchors for the low- z regime, cannot constrain H_0 .
- For both the likelihoods considered, a trend is exhibited between the three cases (i.e. $\Omega_M = 0.3$, $\Omega_M = 0.1$, and $\Omega_M = 0.9$) with fixed correction for the evolution and the case without correction: the application of the correction leads to higher uncertainties of the a_i since the evolutionary parameter k introduces an additional uncertainty. Moreover, the cases with correction significantly change the best-fit values of the coefficients compared to those obtained without the correction for the evolution. We can also notice that the case with correction with $\Omega_M = 0.9$ is the one that leads to best-fit values closer to those obtained without evolution.

9.3. An extended cosmographic test of the standard cosmological model with quasars, SNe Ia, GRBs, and BAO

	$\mathcal{L}_{Gauss}$					$\mathcal{L}_{new}$				
	a_2	a_3	a_4	a_5	H_0	a_2	a_3	a_4	a_5	H_0
No Evolution	3.90 ± 0.05	2.40 ± 0.38	-9.40 ± 1.57	9.69 ± 5.83	73.19 ± 0.37	3.56 ± 0.04	2.57 ± 0.38	-7.89 ± 1.31	3.88 ± 5.60	72.77 ± 0.27
$\Omega_M = 0.3$	3.98 ± 0.05	5.37 ± 0.41	5.77 ± 2.13	22.17 ± 10.25	73.54 ± 0.38	3.59 ± 0.04	4.84 ± 0.36	8.63 ± 1.89	7.99 ± 8.70	73.11 ± 0.28
$\Omega_M = 0.1$	4.01 ± 0.05	5.56 ± 0.45	13.21 ± 2.42	35.76 ± 11.93	73.57 ± 0.39	3.63 ± 0.04	5.41 ± 0.38	16.85 ± 2.24	21.22 ± 10.19	73.16 ± 0.28
$\Omega_M = 0.9$	3.96 ± 0.05	4.37 ± 0.40	0.06 ± 1.88	13.59 ± 8.66	73.48 ± 0.37	3.62 ± 0.04	4.18 ± 0.32	3.00 ± 1.63	1.16 ± 6.96	73.06 ± 0.27

Table 9.3: Best-fit values of the a_i coefficients together with their 1σ uncertainty from all probes together and all the cases investigated. H_0 is in units of $\text{km s}^{-1} \text{Mpc}^{-1}$.

	$\mathcal{L}_{Gauss}$				$\mathcal{L}_{new}$			
	a_2	a_3	a_4	a_5	a_2	a_3	a_4	a_5
Flat Λ CDM, No Evolution	3.76	3.67	-0.85	-1.82	3.65	3.68	-0.91	-1.82
Flat Λ CDM, Fixed Evolution ($\Omega_M = 0.3$)	3.76	3.68	-0.55	-1.82	3.68	3.69	-0.52	-1.82
Flat Λ CDM, Fixed Evolution ($\Omega_M = 0.1$)	4.67	7.67	5.40	0.94	4.56	7.41	5.39	0.94
Flat Λ CDM, Fixed Evolution ($\Omega_M = 0.9$)	2.29	2.15	1.66	0.61	2.27	2.17	1.67	0.61

Table 9.4: Values of a_i predicted in a flat Λ CDM model according to Eqs. (9.9) for all the probes together and all cases studied in this work.

This means that QSOs alone prefer values of Ω_M towards 0.9. Indeed, the two configurations without correction and with correction with $\Omega_M = 0.9$, are the ones that bring the lowest uncertainties on a_2 , a_3 , and a_4 .

- Comparing the best-fit values of the a_i coefficients of the same order for $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$ and equal cases for the correction, we notice that all the a_i are consistent within less than 1σ , thus the two likelihoods yield compatible best-fit values, in agreement with our previous results.
- Using Eqs. (9.8), we compute for the corresponding cases the discrepancy between the obtained best-fit values of a_i reported in Table 9.1 and the theoretically predicted values in a flat Λ CDM model (Table 9.2). There is only one case of compatibility within 1σ : a_3 with the correction for redshift evolution with $\Omega_M = 0.9$ for both likelihoods. More precisely, in this case we obtain compatibility in 0.3σ and 0.4σ , for $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$, respectively. In all the other cases studied, a tension with the flat Λ CDM model emerges with a significance level $> 1\sigma$, which reaches up to $\sim 14\sigma$ on a_2 in the case without the correction for redshift evolution for both the likelihoods.

We now discuss instead the results obtained by combining QSOs, GRBs, Pantheon + SNe Ia, and BAOs, analyzing the different cases studied, the differences and similarities with the results from QSOs alone, and comparing with previous works in the literature. These results are reported in Tables 9.3 and 9.4.

- With regard to the maximum order of the orthogonal logarithmic polynomial required to fit all the probes together, we notice from Table 9.3 that in all the cases with $\mathcal{L}_{Gauss}$ a_5 is significant with a deviation from $a_5 = 0$ always greater than 1.5σ . We have further checked that instead the addition of the sixth order leads to a_6 compatible with $a_6 = 0$, and it is thus negligible. Therefore, in these cases we have

used the fifth-order function, as in Eq. (9.1). The deviation of a_5 from 0 is reduced when using $\mathcal{L}_{new}$, reaching the minimum of 0.2σ in the case with the correction for evolution applied by assuming $\Omega_M = 0.9$. This shows that the use of $\mathcal{L}_{new}$, in place of $\mathcal{L}_{Gauss}$, reduces the dimensionality of the cosmographic polynomial. Nevertheless, we still fit a fifth-order cosmographic model when applying $\mathcal{L}_{new}$ to be consistent with the case of $\mathcal{L}_{Gauss}$. As already stated for QSOs alone, we can also notice that $\mathcal{L}_{new}$ reduces the uncertainties on the parameters compared to $\mathcal{L}_{Gauss}$, mainly on the higher-order coefficients, up to a maximum reduction of $\sim 20\%$, and on H_0 , with an uncertainty reduced of $\sim 28\%$. This reduction in the error on H_0 obtained through $\mathcal{L}_{new}$ confirms our previous results.

- Still focusing on H_0 , we observed that, unlike the case of QSOs alone, the combination of the four probes allows us to obtain close contours on H_0 in all the cases studied. The value of H_0 is always around $73 \text{ km s}^{-1} \text{ Mpc}^{-1}$ within 0.5σ , since it is mainly driven by Pantheon + SNe Ia and BAOs [see e.g. 9, 20, and our previous results], and it is compatible within 0.5σ with the value $H_0 = 73.04 \pm 1.04$ reported by [9]. In addition, the use of $\mathcal{L}_{new}$ slightly lowers the value of H_0 if we compare corresponding cases for the treatment of the redshift evolution.
- Regardless of the likelihood, we can identify, similar to the case of QSOs alone, a trend between the three cases with fixed correction for evolution and the case without correction. Indeed, the inclusion of the correction leads to higher uncertainties of the coefficients a_i of the highest orders (mainly a_4 and a_5), due to the fact that also the additional uncertainties of the evolutionary parameters are taken into account. Moreover, since the correction for the redshift evolution affects more the high redshifts, the cases where this correction is applied lead to best-fit values of the highest-order coefficients that are significantly different from those obtained without accounting for the correction. However, this discrepancy is partially mitigated by the large errors in the values of the a_i of the highest orders. From the point of view of this comparison between the cases with and without correction, it is also visible that, among the different applications of the correction for the redshift evolution, the one that assumes $\Omega_M = 0.9$ is the one that is closer to the case without evolution in terms of best-fit values, as outlined in the case of QSOs alone. Again, these two cases are the ones that lead to the lowest uncertainties on a_3 , a_4 , and a_5 .
- Regarding the a_i coefficients, we obtain best-fit values of a_2 that are compatible within 1σ among all the evolutionary cases for each likelihood studied, while a_3 , a_4 , and a_5 show discrepancies among each other at different significance levels when comparing different treatments of the redshift evolution. Indeed, this discrepancy reaches a maximum of 5.7σ and 7σ , for a_4 with $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$, respectively, between the cases without correction and with correction and $\Omega_M = 0.1$. Similarly, if we compare the cases without correction and with correction and $\Omega_M = 0.3$, the maximum discrepancy is obtained for a_4 at a significance level of 4.1σ and 5.3σ , with $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$, respectively. For the cases without correction and with correction and $\Omega_M = 0.9$, a_4 shows a

discrepancy, which is of 3σ and 3.8σ , with $\mathcal{L}_{Gauss}$ and $\mathcal{L}_{new}$, respectively. Comparing instead the best-fit values of the coefficients a_i of the same order for the two different likelihoods and the same evolutionary treatments, we observe that a_3 , a_4 , and a_5 are compatible within 1σ , while a_2 shows a discrepancy $\geq 4\sigma$, also due to the smaller uncertainties on a_2 compared to those of the other coefficients.

- Using Eqs. (9.9), we can also quantify the tension with the prediction of a flat Λ CDM model by comparing the best-fit a_i values in Table 9.3 with those reported in Table 9.4 for the corresponding cases. The only cases in which the obtained values are compatible with the theoretical predictions within 1σ are the following two: a_4 in the case of correction for redshift evolution with $\Omega_M = 0.9$ and $\mathcal{L}_{Gauss}$ (at 0.9σ level), and a_4 and a_5 in the case of correction for redshift evolution with $\Omega_M = 0.9$ and $\mathcal{L}_{new}$ (at 0.8 and 0.1σ level respectively). All the other cases show a tension with the flat Λ CDM model at $\geq 1\sigma$, reaching a maximum discrepancy $> 30\sigma$ on a_2 in the case of correction for redshift evolution with $\Omega_M = 0.9$ for both the likelihoods.
- Based on the previous points, we can now compare our results with previous works in the literature using QSOs in cosmography along with other probes. In this regard, [27] used a less updated sample of 1598 QSOs between $z = 0.04$ and $z = 5.1$, SNe Ia from the Pantheon sample, and 162 GRBs in the range $z = 0.03 - 6.67$ and standardized through the Amati relation [340]. In the work of [27], the cosmographic model is a fourth-order non-orthogonal logarithmic polynomial and only the Gaussian likelihood is employed without any correction for the redshift evolution of QSOs and GRBs. Beside these differences, they still found a strong overall discrepancy ($> 4\sigma$) with a flat Λ CDM with $\Omega_M = 0.3$. Moreover, in our study, thanks to the orthogonalization of the cosmographic polynomial, we have been able to reduce the uncertainties on a_2 , a_3 , and a_4 compared to [27]. Furthermore, we can compare the results presented in this chapter with our analysis reported in Section 9.2, where Pantheon SNe Ia were combined with the same QSO sample but with only sources at $z > 0.7$, without any correction for evolution and with a Gaussian likelihood. Comparing the corresponding cases for $\Omega_M = 0.3$, besides the differences in the data set and the analysis, we find compatibility between the best-fit a_i values within 1σ . This confirms the overall result of our cosmographic analyses.

In conclusion, we have demonstrated the power of the cosmographic approach applied to high-redshift probes and that there is a significant statistical tension between the observations and the theoretical prediction of the flat Λ CDM model, making the search for the most appropriate description of the Universe even more puzzling along with the quest of the reasons for these discrepancies.

10 Conclusions and future work

The aim of this thesis was to study the implementation and the impact in cosmology of QSOs and GRBs, as recent high-redshift probes. Given the immense power of these objects and their presence across all cosmological epochs, they provide a valuable tool for probing the expansion history of the Universe, which is inextricably linked to our understanding of the Universe on a large scale. This, in turn, allows us to shed light on the current cosmological tensions that the cosmological community is grappling with. At the same time, many fundamental questions about the emission physics of QSOs and GRBs remain unanswered, so addressing them will help to refine their role and implementation in the cosmological context.

In Chapter 1, we have introduced the cosmological framework of our work focusing on the current discrepancies and the search for a cosmological model that can solve these tensions and reproduce the observations of the Universe at different scales. In this context, we have described the role of SNe Ia, BAO, and CMB as well-established cosmological tools that probe different epochs in the evolution of the Universe.

In Chapters 2 and 3, we have delved into the QSO realm studying the X-UV relation and the cosmological QSO sample. Specifically, we have proved that the X-UV relation reliably turns QSOs into standard candles, thus extending the Hubble diagram of SNe Ia into a region previously unexplored, up to redshift $z \sim 7$. In this regard, we have reported new results on the choice of the indicators for the X and UV luminosities and the effective contribution of the intrinsic dispersion of the relation. Indeed, concerning the former, we presented an analysis of the relation carried out on a sample of ~ 1800 objects with a complete UV spectroscopic analysis and an X-ray spectroscopic analysis at redshifts higher than 1.9. We investigated which quantity works best as a proxy for the UV and X-ray emission. We found that the commonly used monochromatic fluxes at 2 keV and 2500 Å estimated from photometric data provide an almost as tight X-UV relation as other commonly available indicators and can be used to derive QSO distances. With regard to the intrinsic dispersion of the X-UV relation, we have investigated those factors that are known to contribute to the observed dispersion but which cannot be removed by the sample selection. Focusing on a subset of roughly 300 objects, we discerned that the variability contributes as much as 0.08 dex to the observed dispersion. We also employed a mock sample of QSOs to ascertain the effect of QSO inclination on the observed dispersion, determining its contribution to be approximately 0.06 dex. Yet, these results are of paramount importance and suggest that the intrinsic dispersion of

the relationship is minimal, potentially approaching zero. This implies a robust physical connection between the UV disc and the X-ray corona over a broad luminosity spectrum. Regarding the cosmological sample, we have outlined the criteria imposed on both the UV and X-ray observations to remove biased objects from the initial sample and build a sub-sample suitable for cosmological purposes. Still with respect to our QSO data set, we also focused on a sample of ~ 30 objects at $z \sim 3$ with high-quality measurements that was specifically chosen to be representative of the most luminous QSOs with a blue continuum emission. Our recent series of analyses showed that about two-thirds of the sources are clustered around the X-UV relation with a minimal scatter of 0.1 dex, while the remaining one-third appears to be X-ray under-luminous. We suggest that, at least in some of our X-ray weak QSOs, the corona may experience a radiatively inefficient phase due to the presence of a powerful accretion disc wind, which substantially reduces the accretion rate through the inner disc and therefore also the availability of seed photons for the Compton up-scattering, thus generating an X-ray weak QSO. In addition, the intrinsic dispersion of ~ 0.1 dex shown by the bright sample is significantly lower than the dispersion obtained in all our studies, confirming that, as stated before, most of the observed dispersion is due to observational issues rather than intrinsic effects.

Chapter 4 is instead devoted to the description of the physics of GRBs and their recent application to cosmology. Indeed, we revised the features of their light curves, their classification and possible progenitors, as well as the 3D “Dainotti” correlations in both X-ray and optical wavelengths, that turn GRBs into cosmological tools, together with their physical interpretations. In this context, we described the selection criteria that lead to the GRB sample applied for cosmological studies. In addition, we introduced the use of machine learning techniques to (i) reconstruct the light curves that present gaps to allow a more precise determination of the parameters and hence tighten the cosmological correlation and (ii) predict unknown redshifts to increase the sample size. Then, we have studied the impact that these methods can have on the use of GRBs as standalone cosmological probes. We obtained that they can significantly enhance the power of GRBs in constraining cosmological parameters, since we computed that in the year 2042 the optical GRB sample alone, once applied these machine learning techniques, will be able to constrain the current matter density parameter with the same precision as the Pantheon SNe Ia sample.

In Chapter 5, we dealt with the problems of selection biases and redshift evolution that can affect sources at high redshifts, as in the case of QSOs and GRBs, thus distorting or inducing a correlation among their physical quantities. In this context, we revised the Efron & Petrosian statistical method as a technique to correct for these effects and to test whether the investigated correlations are actually intrinsic to the physics of the source and not merely generated by the effects mentioned above. We hence demonstrated that both the X-UV and the 3D Dainotti relations are not affected by selection biases and redshift evolution and can therefore reliably be applied in cosmology. Furthermore, we detailed a novel method that we have developed to overcome the circularity problem of the Efron & Petrosian method. With this new technique, we do not need to assume a priori a cosmological model to apply the correction for selection biases and redshift evolution, but we express the evolutionary coefficient used to parameterize the correction

as a function of the free parameters of the cosmological model under consideration. In this way, the correction varies together with the cosmological parameters within the cosmological fit and the results are completely model-independent.

Chapter 6 presented our studies aimed at defining a QSO sub-sample that, as a standalone cosmological probe, can constrain the current matter density parameter with enhanced precision, ideally at the same level as SNe Ia. To this end, we presented two selection procedures based on the σ -clipping technique and the Huber regressor algorithm, respectively. Indeed, these methods allow us to retain from the initial sample only those sources that are closer to the ideal relation, thus reducing the intrinsic dispersion. As a consequence, the selected sample, if it still contains a sufficient number of sources, constrains the cosmological parameter under study with improved precision, compared to the original sample. More precisely, we showed that the precision level of Pantheon SNe Ia is reached with 983 QSOs selected using the σ -clipping technique.

In Chapter 7, we started the investigation of different cosmological models by using QSOs alone and combined with SNe Ia, and different combinations of QSOs with SNe Ia, BAO, and CMB. With these data sets, we studied the flat and non-flat standard cosmological model, the w CDM model, some extensions that require an equation of state of dark energy that evolves with the redshift, and a model that predicts an interaction in the dark sector. As a result, we obtained hints towards a value of H_0 around $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is intermediate between the values derived from SNe Ia and the CMB. In addition, our analyses showed a statistically significant discrepancy, greater than $2 - 3\sigma$, with the standard cosmological model favouring an equation of state of the dark energy $w < -1$ and a closed Universe with $\Omega_k < 0$. Furthermore, the presence of an interaction in the dark sector seems to be able to reconcile the data at different scales.

Still regarding the cosmological studies, Chapter 8 introduced our novel approach to the cosmological likelihoods. Indeed, we proved that, if the normalized residuals of the distance moduli (or equivalently the physical quantity used for the cosmological inference) are not distributed as a Gaussian, then it is not justified to employ a Gaussian likelihood in the cosmological fit. This is the case for QSOs, SNe Ia from both the Pantheon and Pantheon + samples, and BAO, while GRBs verify the Gaussian assumption. As a consequence, we derived the proper distributions for QSOs, SNe Ia, and BAO and we used them instead of the Gaussian likelihood. Remarkably, we obtained a significant reduction in the uncertainties of the inferred cosmological parameters when we employed the proper likelihood, compared to the use of the Gaussian likelihood. This marks the relevance of this statistical study to reach a higher precision of the cosmological parameters, with the aim of exploring the true nature of the current cosmological tensions.

Finally, in Chapter 9, the cosmographic approach was presented as a model-independent test of cosmological models. In this framework, we proposed a cosmographic logarithmic expansion, which overcomes the typical problems of the standard expansions. First, we validated this approach with mock samples of QSOs and SNe Ia, and then we applied it to test the standard cosmological model. In all our analyses with different combinations of QSOs and other probes, we found a strong discrepancy between the observational data and the prediction of the concordance model, in agreement with our cosmological results.

Based on these results, this thesis manifests several perspectives for future developments and projects, as we detail in the following:

- As we still lack a comprehensive theoretical interpretation of the X-UV relation that can actually explain the stability of the X-ray emission, future plans include the investigation of this mechanism. In this regard, we have presented in Section 2.1 a possible toy model, hence we can revise this model in the light of new findings. In particular, we can also consider the latest developments in the corona physics, including the polarization results from [341] and the recent simulation outcomes of the corona emission [342]. Any physical model of the relation should also take into account our results on the intrinsic dispersion and predict a tight relation over a wide range of QSO luminosities;
- Regarding the cosmological analyses, we plan to extend our study by testing cosmological models other than those reported in this work that have been proposed in the literature, with particular interest in cosmographic parameterisations, interacting dark energy models, and alternative theories of gravity;
- Future plans include also the application of machine learning techniques to QSOs, combined with other probes as well. Indeed, the Hubble diagram can be fitted non-parametrically with machine learning methods, such as Neural Networks [see e.g. 308]. Hence, our goal is to derive the cosmological model, or the class of cosmological models, that can properly reproduce the shape of the Hubble diagram obtained by machine learning.

List of Publications

Concerning this thesis:

High-redshift Cosmology by Gamma-Ray Bursts: an overview

Bargiacchi G., Dainotti, M.G. and Capozziello, S.

New Astronomy Reviews, 100 (2024)

[DOI](#)

Quasar cosmology II: joint analyses with Cosmic Microwave Background

Benetti, M., **Bargiacchi, G.**, Risaliti, G., Capozziello, S., Lusso, E. and Signorini, M.

Submitted to Physics of the Dark Universe

Quasars as Standard Candles V. Accounting for the dispersion in the L_X - L_{UV} relation down to ≤ 0.06 dex

Signorini, M., Risaliti, G., Lusso, E., Nardini, E., **Bargiacchi, G.**, Sacchi, A. and Trefoloni, B.

Astronomy & Astrophysics, 687, A32, 13 pp (2024)

[ArXiv:2312.08448](#)

Quasars as standard candles VI: spectroscopic validation of the cosmological sample

Trefoloni, B., Lusso, E., Nardini, E., Risaliti, G., Marconi, A., **Bargiacchi, G.**, Sacchi, A., and Signorini, M.

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